



## A pragmatic and systematic statistical analysis for identification of industrial robots

Mathieu Brunot, Alexandre Janot, Francisco Carrillo, Hugues Garnier

### ► To cite this version:

Mathieu Brunot, Alexandre Janot, Francisco Carrillo, Hugues Garnier. A pragmatic and systematic statistical analysis for identification of industrial robots. International Conference on Advanced Intelligent Mechatronics, AIM 2017, Jul 2017, Munich, Germany. hal-01635639

**HAL Id: hal-01635639**

**<https://hal.science/hal-01635639>**

Submitted on 15 Nov 2017

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



## Open Archive Toulouse Archive Ouverte (OATAO)

OATAO is an open access repository that collects the work of Toulouse researchers and makes it freely available over the web where possible.

This is an author-deposited version published in: <http://oatao.univ-toulouse.fr/>  
Eprints ID: 17990

**To cite this version:**

Brunot, Mathieu and Janot, Alexandre and Carrillo, Francisco and Garnier, Huges *A pragmatic and systematic statistical analysis for identification of industrial robots*. In: AIM2017 - International Conference on Advanced Intelligent Mechatronics, 3 July 2017 - 7 July 2017 (Munich, Germany).

Any correspondence concerning this service should be sent to the repository administrator: [staff-oatao@listes-diff.inp-toulouse.fr](mailto:staff-oatao@listes-diff.inp-toulouse.fr)

# A Pragmatic and Systematic Statistical Analysis for Identification of Industrial Robots

M. Brunot, A. Janot, F. Carrillo, and H. Garnier

**Abstract**—Identification of industrial robots is a prolific topic that has been deeply investigated over the last three decades. The standard method is based on the use of the inverse dynamic model and the least-squares estimation (IDIM-LS method) while robots are operating in closed loop by tracking exciting trajectories. Recently, in order to secure the consistency of the parameters estimates, an instrumental variable (IV) approach, called IDIM-IV method, has been designed and experimentally validated. However, the statistical analysis of estimates was not treated. Surprisingly, this topic is rarely addressed in mechatronics whereas it has been deeply investigated in automatic control. This paper aims at bridging the gap between these two communities by presenting a pragmatic statistical analysis of the IDIM-IV estimates. This analysis consists of a two-step procedure: first, the consistency of the IDIM-IV estimates is validated by the Revised Durbin-Wu-Hausman test, and then the statistical analysis of the IDIM-IV residuals is treated. This two-step approach is experimentally validated on the TX40 robot.

## I. INTRODUCTION

Parametric identification of rigid industrial robots has been deeply investigated over the last three decades see [1]-[9] and the references therein. The usual method is based on the use of the inverse dynamic model (IDM) and the least-squares estimation (IDIM-LS method) while robots are operating in closed loop by tracking exciting trajectories.

More recently, an instrumental variable (IV) approach relevant for industrial robots, called IDIM-IV method, has been designed in order to secure the consistency of the estimates in [8]. Furthermore, in order to validate the construction of the set of instruments, a method called the Revised Durbin-Wu-Hausman test (Revised DWH-test) has been developed in [9]. This IV approach was experimentally validated on the TX40 robot but the statistical analysis of estimates was not treated in a systematic way. Surprisingly, statistical analysis of parameters estimates and/or residuals is rarely addressed in mechatronics (see e.g. [10]-[13]) whereas it has been deeply investigated in automatic control (see e.g.

[14]-[18] and the references given therein). This may be explained by the fact that these works are mostly theoretically oriented, validated on low-dimensional linear systems with few real world applications in the field of mechatronics. Finally, it should be stressed that researchers and engineers in mechatronics prefer to keep a physical interpretation of the results.

This paper aims at bridging the gap between the communities of automatic control and mechatronics by proposing and validating a pragmatic and systematic statistical analysis of the IDIM-IV estimates and residuals. This analysis consists of a two-step procedure: the consistency of the IDIM-IV estimates is first validated by the Revised DWH-test and the statistical analysis of the IDIM-IV residuals is then treated. This two-step procedure is experimentally validated on the TX40 robot. The rest of the paper is organized as follows: section II introduces the basics of robot modeling while section III recalls the IDIM-IV method; section IV presents the two-step statistical procedure while a deep experimental validation on the TX40 robot is presented in section V; finally, section VI concludes the paper.

## II. ROBOT MODELLING

### A. The Inverse and Direct Dynamic Models

For robot having  $n$  moving links, the IDM expresses the joint torques in terms of the joint positions, velocities and accelerations [19]. It is given by

$$\boldsymbol{\tau}_{idm} = \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}), \quad (1)$$

with  $\boldsymbol{\tau}_{idm}$  the  $(n \times 1)$  vector of joint torques;  $\mathbf{q}$ ,  $\dot{\mathbf{q}}$  and  $\ddot{\mathbf{q}}$  are the  $(n \times 1)$  vector of joint positions, velocities and accelerations, respectively;  $\mathbf{M}(\mathbf{q})$  the  $(n \times n)$  inertia matrix of the robot;  $\mathbf{N}(\mathbf{q}, \dot{\mathbf{q}})$  the  $(n \times 1)$  vector that gathers the Coriolis, centrifugal, gravity and friction torques.

The direct dynamic model (DDM) provides the joint accelerations as a function of the joint positions, velocities and torques. It is given by

$$\ddot{\mathbf{q}} = \mathbf{M}^{-1}(\mathbf{q}) \left( \boldsymbol{\tau}_{idm} - \mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}) \right). \quad (2)$$

### B. Standard and base parameters

The standard parameters of a link  $j$  are the 6 components of inertia tensor  $XX_j$ ,  $XY_j$ ,  $XZ_j$ ,  $YY_j$ ,  $YZ_j$  and  $ZZ_j$ ; the 3 components of the first moments  $MX_j$ ,  $MY_j$

M. Brunot is with ONERA the French Aerospacelab and Laboratoire Génie de Production, Ecole Nationale d'Ingénieurs de Tarbes, 2 avenue Edouard Belin, BP 74025, 31055 Toulouse Cedex 4, France, (e-mail: [Mathieu.Brunot@onera.fr](mailto:Mathieu.Brunot@onera.fr)).

A. Janot is with ONERA the French Aerospacelab, 2 avenue Edouard Belin, BP 74025, 31055 Toulouse Cedex 4, France, (Tel: +33 (0)5 62 25 27 73; e-mail: [Alexandre.Janot@oncert.fr](mailto:Alexandre.Janot@oncert.fr)).

F. Carrillo is with the Laboratoire Génie de Production, Ecole Nationale d'Ingénieurs de Tarbes, 47 avenue d'Azereix, BP 1629, 65016 Tarbes cedex, France, (e-mail: [Francisco.Carrillo@enit.fr](mailto:Francisco.Carrillo@enit.fr)).

H. Garnier is with Université de Lorraine and Centre de Recherche en Automatique de Nancy, UMR 7039, 54506 Vandoeuvre-les-Nancy, France (e-mail: [hugues.garnier@univ-lorraine.fr](mailto:hugues.garnier@univ-lorraine.fr)).

and  $MZ_j$ ; the mass  $m_j$ ; the inertia of the drive chain  $I_{a_j}$ ; the viscous friction coefficient  $Fv_j$ ; and the Coulomb friction (or dry friction) coefficient  $Fc_j$ . The base parameters are the minimum number of dynamic parameters from which the IDM can be calculated. The set of base parameters can be determined numerically by the use of the QR decomposition, [20]. Relation (1) is thus rewritten as

$$\boldsymbol{\tau}_{idm} = \mathbf{IDM}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) \boldsymbol{\beta}, \quad (3)$$

where  $\mathbf{IDM}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})$  is the  $(n \times b)$  matrix of basis functions;  $\boldsymbol{\beta}$  is the  $(b \times 1)$  vector of base parameters; and  $b$  is the number of base parameters.

### C. The IDIM model

Because of uncertainties (measurement noise, model mismatch ...), the  $(n \times 1)$  vector of the actual joint torques  $\boldsymbol{\tau}$  differs from  $\boldsymbol{\tau}_{idm}$  by a  $(n \times 1)$  vector of error  $\mathbf{e}$  i.e.

$$\boldsymbol{\tau} = \boldsymbol{\tau}_{idm} + \mathbf{e} = \mathbf{IDM}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) \boldsymbol{\beta} + \mathbf{e}. \quad (4)$$

The filtered positions are obtained offline by processing  $\mathbf{q}$  through a lowpass Butterworth filter in both the forward and reverse directions while  $(\dot{\mathbf{q}}, \ddot{\mathbf{q}})$  are calculated offline using a central differentiation algorithm of the filtered positions. Finally, a parallel decimation of the vector of measurements and each column of the observation matrix is carried out. All the details are given in [4]. The following over-determined system is thus obtained

$$\mathbf{y}(\boldsymbol{\tau}) = \mathbf{X}(\hat{\mathbf{q}}, \hat{\dot{\mathbf{q}}}, \hat{\ddot{\mathbf{q}}}) \boldsymbol{\beta} + \boldsymbol{\varepsilon}, \quad (5)$$

with  $\mathbf{y}(\boldsymbol{\tau})$  is the  $(r \times 1)$  measurements vector built from the actual torques  $\boldsymbol{\tau}$ ;  $\mathbf{X}(\hat{\mathbf{q}}, \hat{\dot{\mathbf{q}}}, \hat{\ddot{\mathbf{q}}})$  is the  $(r \times b)$  observation matrix built from  $(\hat{\mathbf{q}}, \hat{\dot{\mathbf{q}}}, \hat{\ddot{\mathbf{q}}})$  obtained by bandpass filtering of the measurements of  $\mathbf{q}$ ;  $\boldsymbol{\varepsilon}$  is the  $(r \times 1)$  vector of error terms; and  $r$  is the number of rows in (5).

## III. THE INSTRUMENTAL VARIABLE APPROACH FOR ROBOT IDENTIFICATION

Robots being identified while they are operating in closed loop, an IV approach has to be used in order to secure the consistency of the estimates, [17]. Equation (5) is, in fact, the reduced form of the more general model defined by

$$\begin{cases} \mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \\ \mathbf{X} = \mathbf{Z}\boldsymbol{\Pi} + \mathbf{V} \end{cases}, \quad (6)$$

where  $\mathbf{Z}$  is the  $(r \times b)$  instrumental matrix;  $\boldsymbol{\Pi}$  is the  $(b \times b)$  matrix of constant coefficients to be identified; and  $\mathbf{V}$  is a  $(r \times b)$  matrix of error terms. The columns of  $\mathbf{Z}$  are called instruments. If the following assumptions hold  $\text{rank}(\mathbf{Z}) = b$ ,

$E(\mathbf{Z}^T \boldsymbol{\varepsilon}) = \mathbf{0}$ ,  $E(\mathbf{Z}^T \mathbf{V}) = \mathbf{0}$  and  $E(\mathbf{V}) = \mathbf{0}$ , then  $\mathbf{Z}$  is said valid. The LS estimate of  $\boldsymbol{\Pi}$ , denoted  $\hat{\boldsymbol{\Pi}}$ , is given by  $\hat{\boldsymbol{\Pi}} = (\mathbf{Z}^T \mathbf{Z})^{-1} \mathbf{Z}^T \mathbf{X}$ .

For robot identification,  $\mathbf{Z}$  is constructed from simulated data that are the outputs of the simulation of the DDM (2). Its simulation is performed assuming the same reference trajectories and the same control law structure for both the actual and the simulated robots. In addition, the simulation makes use of the IV estimates obtained at the previous iteration,  $\hat{\boldsymbol{\beta}}_{IV}^{k-1}$ , and this defines an iterative process (also called bootstrapping algorithm, [14]). All the details are given in [8]. At step  $k$ , where  $k$  is the  $k$ th IV estimates, the  $(n \times 1)$  vectors of simulated joint accelerations,  $\ddot{\mathbf{q}}_s$ , is given by

$$\ddot{\mathbf{q}}_s = \mathbf{M}^{-1}(\mathbf{q}_s, \hat{\boldsymbol{\beta}}_{IV}^{k-1}) (\boldsymbol{\tau}_s - \mathbf{N}(\mathbf{q}_s, \dot{\mathbf{q}}_s, \hat{\boldsymbol{\beta}}_{IV}^{k-1})), \quad (7)$$

$\mathbf{q}_s$  and  $\dot{\mathbf{q}}_s$  are the  $(n \times 1)$  vectors of simulated joint positions and velocities, respectively;  $\boldsymbol{\tau}_s$  is the  $(n \times 1)$  vector of the simulated torques. Finally, after simulating the DDM and the parallel decimation that is still required because it induces a low-pass filtering, at step  $k$ , one has

$$\mathbf{Z} = \mathbf{X}(\mathbf{q}_s, \dot{\mathbf{q}}_s, \ddot{\mathbf{q}}_s, \hat{\boldsymbol{\beta}}_{IV}^{k-1}). \quad (8)$$

The IDIM-IV estimates are then given by

$$\hat{\boldsymbol{\beta}}_{IV}^k = (\mathbf{Z}^T \mathbf{X})^{-1} \mathbf{Z}^T \mathbf{y}. \quad (9)$$

This bootstrapping process is run until  $\max_{i=1, \dots, b} \frac{|\hat{\boldsymbol{\beta}}_{IV}^k(i) - \hat{\boldsymbol{\beta}}_{IV}^{k-1}(i)|}{|\hat{\boldsymbol{\beta}}_{IV}^{k-1}(i)|} \leq \text{tol}$ , where  $\hat{\boldsymbol{\beta}}_{IV}^k(i)$ , the  $i$ th compent of  $\hat{\boldsymbol{\beta}}_{IV}^k$ , is the IDIM-IV estimate of  $\boldsymbol{\beta}(i)$  at step  $k$  and  $\text{tol}$  is ideally chosen to get a good compromise between rapid convergence and good accuracy. The IDIM-IV estimates obtained at the last iteration are denoted  $\hat{\boldsymbol{\beta}}_{IV}$ . Once the algorithm has converged to  $\hat{\boldsymbol{\beta}}_{IV}$ , the instruments must be validated. To do so, it is suggested to run the Revised DWH-test introduced in [9]. The construction of  $\mathbf{Z}$  is valid if the Wald-test accepts the following hypothesis

$$H_0 : \hat{\boldsymbol{\Pi}} = \mathbf{I}_b. \quad (10)$$

Loosely speaking, the Revised DWH-test is a formal test that checks if  $\mathbf{X}$  differs from  $\mathbf{Z}$  by the error matrix  $\mathbf{V}$ . The full details of the Revised DWH-test are given in [9].

## IV. A PRAGMATIC STATISTICAL ANALYSIS FOR ROBOT IDENTIFICATION

### A. Modeling of error

In automatic control, the error is often modeled with

$$\boldsymbol{\varepsilon} = H(z^{-1})\boldsymbol{\varepsilon}_N = D(z^{-1})/C(z^{-1})\boldsymbol{\varepsilon}_N, \quad (11)$$

where  $H(z^{-1}) = D(z^{-1})/C(z^{-1})$  is a discrete-time filter that colors  $\boldsymbol{\varepsilon}_N$  that is assumed to be a zero-mean white noise with a covariance matrix given by  $\boldsymbol{\Omega}_{\boldsymbol{\varepsilon}_N} = \sigma_{\boldsymbol{\varepsilon}_N}^2 \mathbf{I}_r$  where  $\mathbf{I}_r$  is the  $(r \times r)$  identity matrix;  $C(z^{-1}) = 1 + c_1 z^{-1} + \dots + c_{n_c} z^{-n_c}$  and  $D(z^{-1}) = 1 + d_1 z^{-1} + \dots + d_{n_d} z^{-n_d}$  are the  $n_c$ -degree denominator and the  $n_d$ -degree numerator of  $H(z^{-1})$  respectively; and  $z^{-1}$  is the delay operator. Let  $\boldsymbol{\beta}_H = [c_1 \quad c_{n_c} \quad d_1 \quad d_{n_d}]^T$  be the  $((n_c + n_d) \times 1)$  vector of the parameters of  $H(z^{-1})$  to be identified.

When dealing with identification of real-world systems,  $H(z^{-1})$  has not a straight physical interpretation. In fact, the user has two choices: it may exhibit an error in the model as it may indicate that data are oversampled compared with the maximum bandwidth of the position closed-loop control denoted  $\omega_{dyn}$ . So, how to make a decision between these two choices?

#### B. Extension for industrial robot identification

In automatic control, the literature addressing the identification of  $\boldsymbol{\beta}_H$  is vast. The most relevant approach consists in adopting a method inspired from the Refined Instrumental Variable approach that consists of a two-step algorithm, [17]: first, the physical parameters,  $\boldsymbol{\beta}$ , are identified; second, the parameters of the filters,  $\boldsymbol{\beta}_H$ , are estimated. This two-step procedure has proved its effectiveness through different types of systems, [17]. Before calculating the estimate of  $\boldsymbol{\beta}_H$ , the important result published by White in [21] has to be invoked. This result states that  $\hat{\boldsymbol{\varepsilon}}$ , the residuals obtained with the identification method, is a consistent estimate of  $\boldsymbol{\varepsilon}$  if the estimate of  $\boldsymbol{\beta}$  is consistent regardless the approach chosen by the user (e.g. LS or IV methods). For robot identification,  $\hat{\boldsymbol{\varepsilon}} = \mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}}_{IV}$  is a consistent estimate of  $\boldsymbol{\varepsilon}$  if the IDIM-IV estimates given by (9) are consistent. Since IV estimates are consistent if and only if  $\mathbf{Z}$  is valid,  $\hat{\boldsymbol{\varepsilon}}$  is a consistent estimate of  $\boldsymbol{\varepsilon}$  if (10) holds. In mechatronics, the estimation of  $\boldsymbol{\beta}_H$  makes sense if and only (10) holds. Otherwise, there is an error in the model that has to be solved.

$\boldsymbol{\beta}_H$  can be identified with the IVARMA function of the CAPTAIN toolbox. However, for practical purposes, an AR model is generally enough, see [17], and the physical interpretation of an AR model is usually easier than the interpretation of an ARMA model. It is therefore suggested to choose an AR model by imposing  $D(z^{-1}) = 1$  and identify the parameters of  $C(z^{-1})$  by using the AIC function of the CAPTAIN toolbox. Let  $\hat{C}(z^{-1})$  and  $\hat{H}(z^{-1})$  be the estimate of  $C(z^{-1})$  and  $H(z^{-1})$ , respectively. To obtain refined

IDIM-IV estimates, the columns of  $\mathbf{Z}$  and  $\mathbf{X}$  as well as  $\mathbf{y}$  are filtered by  $\hat{H}^{-1}(z^{-1})$ . Let  $\mathbf{Z}_f$ ,  $\mathbf{X}_f$  and  $\mathbf{y}_f$  the filtered instrumental matrix, the observation matrix and the vector of measurements, respectively. The refined IDIM-IV estimates and their associated covariance matrix are given by

$$\hat{\boldsymbol{\beta}}_{IV}^f = (\mathbf{Z}_f^T \mathbf{X}_f)^{-1} \mathbf{Z}_f^T \mathbf{y}_f, \quad \boldsymbol{\Sigma}_{IV} = \hat{\sigma}_{\boldsymbol{\varepsilon}_f}^2 (\mathbf{Z}_f^T \mathbf{Z}_f)^{-1}. \quad (12)$$

with  $\hat{\sigma}_{\boldsymbol{\varepsilon}_f}^2 = \|\hat{\boldsymbol{\varepsilon}}_f\|^2 / (r - b)$  with  $\hat{\boldsymbol{\varepsilon}}_f = \mathbf{y}_f - \mathbf{X}_f \hat{\boldsymbol{\beta}}_{IV}^f$ .  $\hat{\sigma}_{\hat{\boldsymbol{\beta}}_{IV}(i)}^2 = \boldsymbol{\Sigma}_{IV}(i, i)$  being the  $i^{\text{th}}$  diagonal coefficient of  $\boldsymbol{\Sigma}_{IV}$ , the relative standard deviation  $\% \hat{\sigma}_{\hat{\boldsymbol{\beta}}_{IV}(i)}$  is given by  $\% \hat{\sigma}_{\hat{\boldsymbol{\beta}}_{IV}(i)} = 100 \cdot \hat{\sigma}_{\hat{\boldsymbol{\beta}}_{IV}(i)} / |\hat{\boldsymbol{\beta}}_{IV}^f(i)|$  for  $|\hat{\boldsymbol{\beta}}_{IV}^f(i)| \neq 0$ .

#### C. Proposed statistical residual analysis

The pragmatic and systematic statistical residual analysis relevant for industrial robot identification is summarized below:

- Calculate the IDIM-IV estimates with the IDIM-IV method described in [8];
- Once the IDIM-IV has converged to  $\hat{\boldsymbol{\beta}}_{IV}$ , evaluate the quality of the instruments with the revised DWH-test described in [9];
- If the construction of  $\mathbf{Z}$  has been validated, then estimate  $\boldsymbol{\beta}_H$  in order to compute the refined IDIM-IV estimates given by (12).

### V. EXPERIMENTAL VALIDATION: APPLICATION TO THE TX40 ROBOT

#### A. Description of the TX40 robot

The Stäubli TX-40 robot has a serial structure with six rotational joints. The robot kinematics is defined using the modified Denavit and Hartenberg notation, [19]. The TX40 robot is characterized by a coupling between the joints 5 and 6. This coupling adds two new viscous and Coulomb friction parameters  $Fv_{m6}$  and  $Fc_{m6}$ . The TX40 has 86 standard dynamic parameters and 60 base parameters.

The identification of the dynamic parameters is carried out with one trajectory using the controller CS8C of Stäubli robots. The joint positions and torques are stored with a sampling measurement frequency  $f_m = 5\text{kHz}$ . The IDIM-IV method is initialized with the computer-aided-design values provided by the manufacturer except for the friction parameters that are initialized at zero.

#### B. The IDIM-IV method with an appropriate data filtering

The IDIM-IV method is carried out with a filtered position,  $\hat{\mathbf{q}}$ , calculated with a 50 Hz fourth-order Butterworth filter and with velocities,  $\hat{\dot{\mathbf{q}}}$ , and accelerations,  $\hat{\ddot{\mathbf{q}}}$ , calculated with a central difference algorithm of  $\hat{\mathbf{q}}$ . The maximum bandwidth for the sixth joint is 10 Hz leading to choose a 50 Hz cutoff frequency, [4]. The parallel decimation

is carried out with a lowpass Tchebyshef filter with a cutoff frequency of 20 Hz.

The IDIM-IV method has converges to the parameters estimates given in Table 1 after 3 iterations. Their relative standard deviations are given in the “*App*” column of Table 1. Only the set of essential parameters that has been calculated with the F-test (see [8]) is given. From 60 base parameters, only 28 are enough to completely describe the dynamics of the TX40 robot. The construction of  $\mathbf{Z}$  is evaluated with the procedure described in [9]. The results obtained validate the instruments since the Wald-test accepts the hypothesis  $H_0 : \hat{\mathbf{\Pi}} = \mathbf{I}_b$ . The instruments being valid, the IDIM-IV estimates can be therefore considered as consistent.

The plot provided by the ACF function of the CAPTAIN toolbox (see [17]) shown in Fig. 1 suggests that  $\hat{\mathbf{e}}$  can be considered as white since there are not significant correlations between the samples. Furthermore, the plot of the histogram of  $\hat{\mathbf{e}}$  illustrated in Fig. 2 matches a Gaussian distribution and all these results validate the hypothesis  $\hat{\mathbf{e}} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_r)$ . By applying an appropriate data filtering such as the one described in [4], one has  $H(z^{-1}) = 1$  and the IDIM-IV estimates are refined.

#### C. The IDIM-IV method with an inappropriate data filtering

The IDIM-IV method is carried out with the positions  $\hat{\mathbf{q}}$  filtered with a 500 Hz fourth-order Butterworth filter and with velocities,  $\hat{\dot{\mathbf{q}}}$ , and accelerations,  $\hat{\ddot{\mathbf{q}}}$ , calculated with a central difference algorithm of  $\hat{\mathbf{q}}$ . The parallel decimation is carried out with a lowpass Tchebyshef filter with a cut-off frequency of 500 Hz. Those cut-off frequencies are chosen in a total arbitrary way as a user not familiar with robot identification or a beginner might do.

After 3 iterations, the IDIM-IV method converges to the values given in Table 1. Their relative standard deviations are given in the “*Inap*” column of Table 1. Since the Wald-test still accepts the hypothesis  $H_0 : \hat{\mathbf{\Pi}} = \mathbf{I}_b$ , the construction of  $\mathbf{Z}$  is still valid and the IDIM-IV estimates are considered as consistent. However, their relative deviations are smaller than those given in the “*App*” column of Table 1 and this raises the following question: which estimates are the most efficient? To answer this question, the statistical analysis of  $\hat{\mathbf{e}}$  has to be performed.

The plot provided by the ACF function shown in Fig. 3 suggests that there are significant correlations between the samples of  $\hat{\mathbf{e}}$ . Despite the fact that the plot of the histogram of  $\hat{\mathbf{e}}$  illustrated in Fig. 4 looks like a Gaussian distribution, these results reject the hypothesis  $\hat{\mathbf{e}} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_r)$ . Such plots are good reason for concerns and the engineer/researcher has to find the origin of such correlations. In this case, because the construction of  $\mathbf{Z}$  has been validated by the revised DWH-test, the significant correlations observed are due to the fact that data are oversampled in comparison with the maximum bandwidth of the six closed loops in position. In other word, these correlations are not due to an error in the

model and it comes out that the standard deviations given in Table 2 are underestimated. The ACF plot suggesting a AR(50) model denoted  $H_{AR(50)}(z^{-1})$ , the parameters of  $H_{AR(50)}(z^{-1})$  are estimated by running the AIC function of the CAPTAIN toolbox and  $\mathbf{y}$ , the columns of  $\mathbf{X}$  and  $\mathbf{Z}$  are then filtered by  $H_{AR(50)}^{-1}(z^{-1})$  in order to calculate the refined IDIM-IV estimates. The results given in Table 2 are very close to those provided in section V.B: the standard deviations stick to those given in Table 1 and there are no significant correlations between the samples of  $\hat{\mathbf{e}}$  since the plot obtained with the ACF function (not shown here) is very close to the one provided in Fig. 1. It should be mentioned that the use of an ARMA model that can be estimated by utilizing the IVARMA function of the CAPTAIN toolbox does not improve the results.

Finally, it is not surprising that the AIC function has estimated an AR(50) model. The maximum bandwidth being 10 Hz, there is no useful information beyond this frequency. Data have been decimated at 500 Hz which is 50 times greater than the maximum bandwidth. Then, to remove all those useless samples that contain no information and to whiten the residual, the AIC function returns an AR(50) model. If data are now decimated at 100 Hz, then the AIC function returns an AR(10) model. This experimental result shows that the parallel decimation based on the use of a lowpass Tchebyshef filter used in [4] is equivalent with the parallel filtering based on the use of prefilters commonly utilized in the automatic control community, [14]-[17].

#### D. The IDIM-IV method with an appropriate data filtering and an error in the model

The gear ratios being greater than 25, the user can assume that the parameters of gravity and the off-diagonal elements of inertia matrices do not significantly contribute to the dynamics. These parameters and their associated columns are removed from the IDM and the data are filtered as explained in section V.B. The IDIM-IV method has converged to the solution given in Table 3 after 5 iterations. These values are not compatible with those given in Table 1. For the inertia parameters of joints 1, 2, 3 and 4, the Wald-test rejects the hypothesis that  $\mathbf{Z}$  is a valid instrumental matrix whereas the set of instruments of joint 5 and 6 is valid. This is explained by the fact that the gravity parameters and the off-diagonal components of inertia matrices are practically null which means that removing them from the dynamic model has no consequences for those joints. The construction of  $\mathbf{Z}$  being rejected, the IDIM-IV estimates are expected biased.

The plot provided by the ACF function shown in Fig. 5 suggests that there are some correlations between the samples of  $\hat{\mathbf{e}}$ . Despite the fact that the plot of the histogram of  $\hat{\mathbf{e}}$  illustrated in Fig. 6 is not far from a Gaussian distribution, these results reject the hypothesis  $\hat{\mathbf{e}} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_r)$ . The plot provided by the ACF function suggests an AR(10) model that is, compared with the AR(50) obtained with an inappropriate data filtering, not a real reason for concerns from a practical point of view. This experimental result clearly shows that it is quite difficult to address the physical interpretation of

$H_{AR(10)}(z^{-1})$ : it may simply indicate that data are oversampled compared with the bandwidth of interest (as in the case of an inappropriate data filtering) or highlight an error in the model (as in this case). Without evaluating the validity of  $\mathbf{Z}$  by running the Revised DWH-test, it would have been impossible to make a physical interpretation of  $H_{AR(10)}(z^{-1})$ . This result may explain why the methodologies developed in the automatic control have not yet well penetrated the field of mechatronics; they cannot be applied in a straightforward way because the points of view adopted in mechatronics are not the same as those adopted in automatic control. Nonetheless, in this paper, a pragmatic statistical analysis inspired from the methods developed in the automatic control community has been designed and successfully applied to a 6-DOF industrial robot.

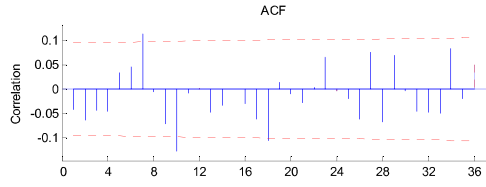


Fig. 1. Autocorrelation of the IDIM-IV error obtained with an appropriate filtering.

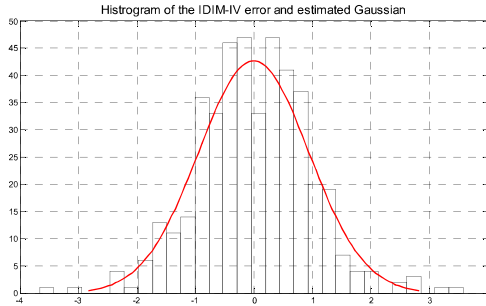


Fig. 2. Histogram of the IDIM-IV error and its estimated Gaussian obtained with an appropriate data filtering.

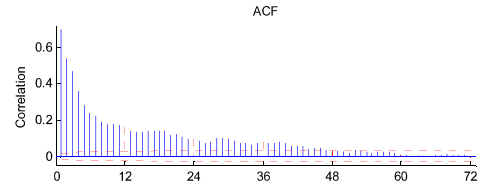


Fig. 3. Autocorrelation of the IDIM-IV error obtained with an inappropriate filtering.

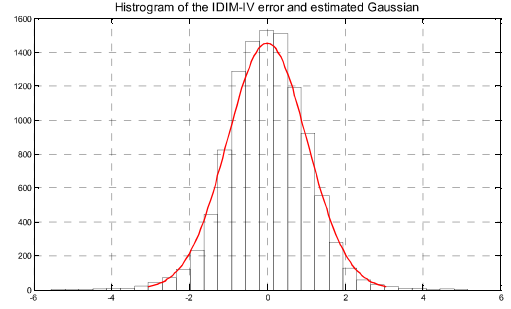


Fig. 4. Histogram of the IDIM-IV error and its estimated Gaussian obtained with an inappropriate data filtering.

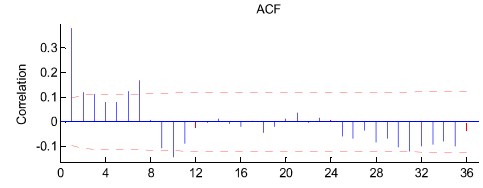


Fig. 5. Autocorrelation of the IDIM-IV error obtained with an appropriate filtering and an error in the model.

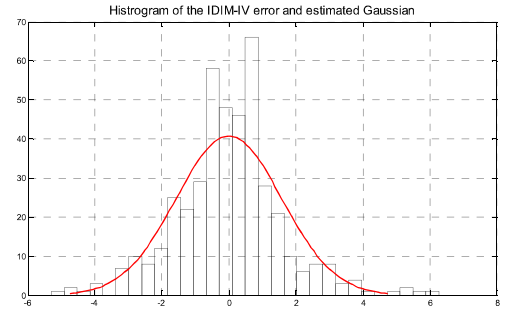


Fig. 6. Histogram of the IDIM-IV error and its estimated Gaussian obtained with an appropriate data filtering and an error in the model.

Table 1: IDIM-IV ESTIMATES OBTAINED AFTER 3 ITERATIONS, RELATIVE DEVIATIONS OBTAINED WITH AN APPROPRIATE (RESP. INAPPROPRIATE) DATA FILTERING, COLUMN APP (%) (RESP. INAP (%))

|                 | $\hat{\beta}_{IV}$ | App (%) | Inap (%) |                  | $\hat{\beta}_{IV}$ | App (%) | Inap (%) |
|-----------------|--------------------|---------|----------|------------------|--------------------|---------|----------|
| ZZ <sub>1</sub> | 1.25               | 1.3     | 0.71     | Fc <sub>3</sub>  | 6.0                | 1.9     | 0.93     |
| Fv <sub>1</sub> | 8.20               | 0.7     | 0.49     | MX <sub>4</sub>  | -0.02              | 20.0    | 19.6     |
| Fc <sub>1</sub> | 6.55               | 2.6     | 1.81     | Ia <sub>4</sub>  | 0.03               | 9.4     | 3.98     |
| XX <sub>2</sub> | -0.48              | 2.9     | 1.74     | Fv <sub>4</sub>  | 1.15               | 1.5     | 0.55     |
| XZ <sub>2</sub> | -0.16              | 4.8     | 2.85     | Fc <sub>4</sub>  | 2.27               | 2.6     | 1.02     |
| ZZ <sub>2</sub> | 1.09               | 1.2     | 0.65     | MY <sub>5</sub>  | -0.03              | 14.0    | 6.17     |
| MX <sub>2</sub> | 2.21               | 2.9     | 1.57     | Ia <sub>5</sub>  | 0.04               | 11.0    | 4.36     |
| Fv <sub>2</sub> | 5.68               | 1.2     | 0.65     | Fv <sub>5</sub>  | 1.90               | 2.0     | 0.75     |
| Fc <sub>2</sub> | 7.77               | 2.1     | 1.78     | Fc <sub>5</sub>  | 2.80               | 3.5     | 1.35     |
| XX <sub>3</sub> | 0.13               | 10.0    | 5.79     | Ia <sub>6</sub>  | 0.01               | 10.9    | 4.28     |
| ZZ <sub>3</sub> | 0.12               | 8.8     | 4.79     | Fv <sub>6</sub>  | 0.69               | 1.6     | 0.61     |
| MY <sub>3</sub> | -0.60              | 2.3     | 1.17     | Fc <sub>6</sub>  | 2.00               | 2.8     | 1.09     |
| Ia <sub>3</sub> | 0.10               | 9.2     | 4.43     | Fv <sub>m6</sub> | 0.63               | 1.8     | 0.59     |
| Fv <sub>3</sub> | 2.03               | 1.8     | 0.80     | Fc <sub>m6</sub> | 1.81               | 4.2     | 1.60     |

Table 2: IDIM-IV ESTIMATES OBTAINED AFTER 3 ITERATIONS AND THE ESTIMATED AR(50) MODEL IDENTIFIED

|                 | $\hat{\beta}_{IV} (\% \hat{\sigma}_{\hat{\beta}_{IV}})$ |                  | $\hat{\beta}_{IV} (\% \hat{\sigma}_{\hat{\beta}_{IV}})$ |
|-----------------|---------------------------------------------------------|------------------|---------------------------------------------------------|
| ZZ <sub>1</sub> | 1.25 (1.29%)                                            | Fc <sub>3</sub>  | 5.97 (1.62%)                                            |
| Fv <sub>1</sub> | 8.21 (0.68%)                                            | MX <sub>4</sub>  | -0.02 (19.50%)                                          |
| Fc <sub>1</sub> | 6.56 (2.55%)                                            | Ia <sub>4</sub>  | 0.03 (9.13%)                                            |
| XX <sub>2</sub> | -0.48 (1.99%)                                           | Fv <sub>4</sub>  | 1.16 (1.35%)                                            |
| XZ <sub>2</sub> | -0.16 (4.24%)                                           | Fc <sub>4</sub>  | 2.25 (2.53%)                                            |
| ZZ <sub>2</sub> | 1.08 (1.15%)                                            | MY <sub>5</sub>  | -0.03 (13.74%)                                          |
| MX <sub>2</sub> | 2.22 (2.80%)                                            | Ia <sub>5</sub>  | 0.04 (10.12%)                                           |
| Fv <sub>2</sub> | 5.68 (1.13%)                                            | Fv <sub>5</sub>  | 1.95 (1.65%)                                            |
| Fc <sub>2</sub> | 7.78 (1.98%)                                            | Fc <sub>5</sub>  | 2.80 (3.11%)                                            |
| XX <sub>3</sub> | 0.13 (9.91%)                                            | Ia <sub>6</sub>  | 0.01 (10.1%)                                            |
| ZZ <sub>3</sub> | 0.11 (8.72%)                                            | Fv <sub>6</sub>  | 0.69 (1.47%)                                            |
| MY <sub>3</sub> | -0.60 (2.27%)                                           | Fc <sub>6</sub>  | 2.00 (2.34%)                                            |
| Ia <sub>3</sub> | 0.10 (8.87%)                                            | Fv <sub>m6</sub> | 0.64 (1.71%)                                            |
| Fv <sub>3</sub> | 2.05 (1.71%)                                            | Fc <sub>m6</sub> | 1.79 (4.05%)                                            |

Table 3: IDIM-IV ESTIMATES OBTAINED AFTER 5 ITERATIONS AND AN APPROPRIATE DATA FILTERING AND AN ERROR IN THE MODEL

|                 | $\hat{\beta}_{IV} (\% \hat{\sigma}_{\hat{\beta}_{IV}})$ |                  | $\hat{\beta}_{IV} (\% \hat{\sigma}_{\hat{\beta}_{IV}})$ |
|-----------------|---------------------------------------------------------|------------------|---------------------------------------------------------|
| ZZ <sub>1</sub> | 1.08 (3.5%)                                             | Fv <sub>4</sub>  | 1.20 (5.8%)                                             |
| Fv <sub>1</sub> | 8.17 (3.6%)                                             | Fc <sub>4</sub>  | 2.17 (10.0%)                                            |
| Fc <sub>1</sub> | 6.48 (11.0%)                                            | Ia <sub>5</sub>  | 0.05 (29.3%)                                            |
| ZZ <sub>2</sub> | 1.20 (2.0%)                                             | Fv <sub>5</sub>  | 1.89 (7.3%)                                             |
| Fv <sub>2</sub> | 5.83 (5.8%)                                             | Fc <sub>5</sub>  | 2.75 (12.6%)                                            |
| Fc <sub>2</sub> | 6.80 (11.0%)                                            | Ia <sub>6</sub>  | 0.01 (33.0%)                                            |
| ZZ <sub>3</sub> | 0.27 (6.7%)                                             | Fv <sub>6</sub>  | 0.69 (5.4%)                                             |
| Ia <sub>3</sub> | 0.07 (40.0%)                                            | Fc <sub>6</sub>  | 2.0 (9.3%)                                              |
| Fv <sub>3</sub> | 2.22 (7.6%)                                             | Fv <sub>m6</sub> | 0.64 (5.9%)                                             |
| Fc <sub>3</sub> | 5.53 (9.5%)                                             | Fc <sub>m6</sub> | 1.70 (16.0%)                                            |
| Ia <sub>4</sub> | 0.05 (31.1%)                                            |                  |                                                         |

## VI. CONCLUSION

In this paper, a pragmatic and systematic statistical analysis relevant for identification of rigid industrial robots has been presented and experimentally validated on the 6 degrees-of-freedom TX40 robot manufactured and sold by Staubli. It is worth to note that statistical analysis of estimates is rarely addressed in Mechatronics whereas it has been deeply investigated in Automatic Control.

This analysis consists of a two-step procedure: first, the dynamic parameters of robot are estimated with the Inverse-Dynamic-Model and Instrumental-Variables method, their consistency is then validated by the Revised Durbin-Wu-Hausman test; second, the statistical analysis of the IDIM-IV residuals is treated. This two-step procedure is inspired from the IDIM-LS approach often used in Robotics and the Refined Instrumental Variables method that is an iterative identification procedure popular in the field of Automatic Control. In a sense, this paper aimed at bridging the gap between the mechatronics and automatic control communities.

Future works concern the use of this two-step statistical analysis for other real-world systems such as electrical motors, unmanned aerial vehicles and flexible aircraft.

## REFERENCES

- [1] Atkeson, C. G., An, C. H., and Hollerbach, J. M., "Estimation of Inertial Parameters of Manipulator Loads and Links," *International Journal of Robotics Research*, 1986, Vol. 5(3), pp. 101-119.
- [2] Kawasaki, H., and Nishimura, K., "Terminal-Link Parameter Estimation and Trajectory Control," *IEEE Trans. on Robotics and Automation*, 1985, Vol. 4(5), pp. 485-490.
- [3] Swevers, J., Ganseman, C., Tukul, D.-B., DeSchutter, J., and VanBrussel, H., "Optimal Robot Excitation and Identification," *IEEE Trans. on Robotics and Automation*, 1997, Vol. 13(5), pp. 730-740.
- [4] Gautier, M., "Dynamic Identification of Robots with Power Model," in: *Proc. of IEEE International Conference on Robotics and Automation*, Albuquerque, New Mexico, USA, 20-25 April 1997, pp. 1922-1927.
- [5] Olsen, M. M., Swevers, J., and Verdonck, W., "Maximum Likelihood Identification of a Dynamic Robot Model: Implementation Issues," *International Journal of Robotics Research*, 2002, Vol. 21(2), pp. 89 – 96.
- [6] Swevers, J., Verdonck, W., and De Schutter, J., "Dynamic model identification for industrial robots - Integrated experiment design and parameter estimation," *IEEE control systems magazine*, 2007, Vol. 27(5), pp. 58-71.
- [7] Gautier, M., Janot, A., and Vandanjon, P.O., "A New Closed-Loop Output Error Method for Parameter Identification of Robot Dynamics," *IEEE Transactions on Control System Technology*, 2013, Vol. 21(2), pp. 428-444.
- [8] Janot, A., Vandanjon, P.O., and Gautier, M., "A Generic Instrumental Variable Approach for Industrial Robots Identification," *IEEE Transactions on Control Systems Technology*, 2014, Vol. 22(1), pp.132-145.
- [9] Janot, A., Vandanjon, P.-O., Gautier, M., "A revised DWH-test for rigid industrial robots identification," *Control Engineering Practice*, 2016, Vol. 48, pp. 52–62.
- [10] Puthenpura, S.C., and Sinha N.K., "Identification of Continuous-Time Systems Using Instrumental Variables with Application to an Industrial Robot," *IEEE Trans. on Industrial Electronics*, 1986, Vol. IE-33(3), pp. 224 – 229.
- [11] Östring, M., Gunnarsson, S., and Norrlöf, M., "Closed-loop identification of an industrial robot containing flexibilities," *Control Engineering Practice*, 2003, Vol. 11, pp. 291-300.
- [12] Calanca, A., Capisani, L. M., Ferrara, A., and Magnani, L., "MIMO Closed Loop Identification of an Industrial Robot," *IEEE Trans. on Control System Technology*, 2011, Vol. 19(5), pp. 1214-1224.
- [13] Boeren, F., Bruijnen, D., and Oomen, T., "Enhancing feedforward controller tuning via instrumental variables: with application to nanopositioning," *International Journal of Control*, 2017, Vol. 90(4), pp. 746-764.
- [14] Söderström, T., and Stoica, P., "System Identification," Prentice Hall International, New York, 1989.
- [15] Ljung, L., "System identification: Theory of user," Information and system sciences series, Prentice Hall, 1999.
- [16] Gamier, H., and Wang, L., "Identification of Continuous-time Models from Sampled Data," Springer, 2008.
- [17] Young, P.C., "Recursive Estimation and Time-Series Analysis: An Introduction for the Student and Practitioner", Berlin: Springer-Verlag, 2011.
- [18] Gamier, H., "Direct continuous-time approaches to system identification. Overview and benefits for practical applications," *European Journal of Control*, 2015, Vol. 54, pp. 50-62.
- [19] Khalil, W., and Dombre, E., "Modeling, identification and control of robots", Hermes Penton, London, 2002.
- [20] Gautier M., "Numerical calculation of the base inertial parameters of robots," *Journal of Robotics Systems*, 1991, Vol. 8(4), pp. 485-506.
- [21] White, H., "A heteroskedasticity-consistent covariance matrix estimator and a direct test for heteroskedasticity," *Econometrica*, 1980, Vol. 48(4), pp. 817-838.