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Modeling dependent competing failure processes with degradation-shock dependence

Mengfei Fan ^{1,2}, Zhiguo Zeng ³, Enrico Zio ^{3,4} and Rui Kang ^{1,2}

¹ School of Reliability and Systems Engineering, Beihang University, Beijing, China

² Center for Resilience and Safety of Critical Infrastructure, Beihang University, Beijing, China

³ EDF Foundation Chair on Systems Science and Energetic Challenge, CentraleSupélec, Université Paris-Saclay,
Grande Voie des Vignes, 92290 Chatenay-Malabry, France

⁴ Energy Department, Politecnico di Milano, Italy

fanmengfei@buaa.edu.cn, zhiguo.zeng@centralesupelec.fr, enrico.zio@ecp.fr, kangrui@buaa.edu.cn

Abstract

In this paper, we develop a new reliability model for dependent competing failure processes (DCFPs), which accounts for degradation-shock dependence. This is a type of dependence where random shock processes are influenced by degradation processes. The degradation-shock dependence is modeled by assuming that the intensity function of the nonhomogeneous Poisson process describing the random shock processes is dependent on the degradation processes. The dependence effect is modeled with reference to a classification of the random shocks in three “zones” according to their magnitudes, damage zone, fatal zone, and safety zone, with different effects on the system’s failure behavior. To the best of the authors’ knowledge, this type of dependence has not yet been considered in reliability models. Monte Carlo simulation is used to calculate the system reliability. A realistic application is presented with regards to the dependent failure behavior of a sliding spool, which is subject to two dependent competing failure processes, wear and clamping stagnation. It is shown that the developed model is capable of describing the dependent competing failure behaviors and their dependence.

Keywords

Dependent competing failure processes; degradation; shock; degradation-shock dependence; Monte Carlo simulation; sliding spool

Highlights

- A new reliability model is developed for degradation-shock dependence.
- The influence of degradations on random shocks is considered.
- Zone effect of random shocks magnitudes is considered.
- A case study of a sliding spool is conducted.

1 **Acronyms**

2	DCFP	Dependent competing failure processes
3	DTS	Degradation-Threshold-Shock
4	NHPP	Nonhomogeneous Poisson process

5 **Notation**

6	$x(t)$	Degradation at time t
7	θ	Model parameters for the soft failure process
8	$f_{\theta}(\mathbf{x})$	Probability density function of θ
9	$F_{\theta}(\mathbf{x})$	Cumulative distribution function of θ
10	D	Soft failure threshold
11	Y_i	Magnitude of the i th shock
12	W_i	Damage caused by the i th shock
13	α	Shock damage proportional coefficient
14	$f_Y(y)$	Probability density function of Y_i
15	$F_Y(y)$	Cumulative distribution function of Y_i
16	$\lambda(t)$	Intensity of random shocks at time t
17	λ_0	Initial intensity of random shocks
18	$N(t)$	Number of random shocks that have arrived by time t
19	$N_1(t)$	Number of random shocks that have arrived in the damage zone by time t
20	$N_2(t)$	Number of random shocks that have arrived in the fatal zone by time t
21	$N_3(t)$	Number of random shocks that have arrived in the safety zone by time t
22	H	Hard failure threshold due to damage shocks
23	$\gamma_a, \gamma_b, \gamma_c, \gamma_d$	Dependence factors

1. Introduction

Industrial components, systems and products are subject to multiple degradation processes and failure modes, i.e. wear, corrosion, fracture, fatigue, and etc. [1, 2]. Such processes are often “competing” in leading to failure [3] and may be dependent on each other in a number of ways, in which case they are referred to as dependent competing failure processes (DCFPs) [4]. Various dependencies have been discussed in literature, e.g., the statistical dependency described by joint probability distributions [5, 6], copulas [7, 8], and the functional dependency described by failure propagation and isolations [9-11], etc. In this paper, we consider the dependency between two types of failure processes: degradation failure processes (or soft failures) and catastrophic failure processes (or hard failures) [12]. In practice, soft failures are modeled by an evolving degradation process and hard failures are modeled by a random shock process. Models that consider both degradation and random shocks are called Degradation-Threshold-Shock (DTS) models [13, 14]. In literature, most DCFPs are modeled by DTS models.

An early DTS model was developed by Lemoine and Wenocur [15], where degradation is modeled by a diffusion process and fatal shocks are modeled by a Poisson process. Soft failures are defined with respect to the degradation reaching pre-established critical values and hard failures arrive in correspondence of the occurrence of fatal shocks. Klutke and Yang [16] have developed an availability model for systems subject to degradations and random shocks, where degradations occur at a constant rate and shocks follow a Poisson process. Marseguerra et al. [17] and Barata et al. [18] investigate condition-based maintenance optimizations for deteriorating systems subject to degradations and random shocks. Li and Pham [12, 19] develop a reliability model and an maintenance model for multistate degraded systems considering two degradation processes and a shock process. In a recent paper by Lin et al. [20], a DTS model has been developed, where the degradation process is modeled by a continuous-time semi Markov process and the shock process is modeled using a homogeneous Poisson process.

To consider the dependence between degradations and shocks, Peng et al. [4] assume that the arriving shock would bring a sudden increase to the normal degradation process and develop a probabilistic model to calculate the system’s reliability. Wang and Pham [21] use a similar approach to model DCFPs and determine the optimal imperfect preventive maintenance policy. Keedy and Feng [22] apply the model in [4] on a stent, where the degradation process is modeled by a PoF (Physics of failures) model. Song et al. [23] consider the reliability of a system whose components are subject to the DCFPs and distinct component shock sets [24].

Apart from the dependence considered in [4], Jiang et al. [25] consider a different kind of dependence, where the threshold of the degradation process is shifted by shocks. Rafiee et al. [26] develop a model in which the

1 degradation rates are shifted by different shock patterns. Wang et al. [27] explore another type of dependence by
2 assuming that a sudden change to the failure rate is caused by the arriving shocks. Jiang et al. [1] consider dependence
3 where shocks are divided into different zones, where each different zone has a different effect on the degradation
4 process.

5 As reviewed above, the majority of existing DCFP models focuses on modeling the influence of random shocks
6 on degradations (shock-degradation dependence). In engineering practice, however, there are components, systems
7 and products that are subject to DCFPs in which random shocks are influenced by degradations (degradation-shock
8 dependence). A typical engineering example is the sliding spool, which is a widely applied component in hydraulic
9 control systems [28]. A sliding spool is subject to two failure processes: wear, which is modeled as a degradation
10 process and clamping stagnation, which is modeled by random shocks [29]. As shown in Sasak and Yamamot [30],
11 the process of clamping stagnation is dependent on the wear process: as the wear process progresses, more wear
12 debris is generated. The debris will contaminate the hydraulic oil and further, increase the likelihood of clamping
13 stagnation [31, 32], which causes the degradation-shock dependence.

14 Compared to shock-degradation dependence, there are fewer models considering the degradation-shock
15 dependence. In [33], Fan et al. consider the degradation-shock dependence by assuming that the magnitude of random
16 shocks is increased by the degradation processes. Bagdonavicius et al. [34] have developed a model to consider the
17 degradation-shock dependence among a linear degradation process and several extreme shocks. In their model, the
18 intensities of the random shocks are assumed to depend only on the degradation level. Ye et al. [35] model the
19 degradation-shock dependence by assuming that the destructive probability of a shock is determined by the remaining
20 hazard of the degradation process. In [13, 36, 37], the intensity of the random shock is assumed to be a piecewise
21 function of the degradation level.

22 The aforementioned models only consider a relatively simple type of random shock: the extreme shocks (also
23 referred to as traumatic shocks), where the arrival of a shock in the critical region causes the immediate failure of the
24 product [38]. In practice, however, more complex shock patterns, such as cumulative shocks [39], δ -shocks [38],
25 mixed shocks [40], run shocks [41], are frequently encountered. Since the 1990s, works on imperfect fault coverage
26 models [42-44] categorize the effects of a component fault into three types: single-point failure, which causes
27 immediate system failure, permanent coverage, which causes cumulative damages to the system and transient
28 restoration, which has no effect on the system. Based on a similar consideration, Jiang et al. discuss a shock pattern
29 called zone shocks, where shocks with different magnitudes might have different impacts on the failure behavior of

the product [1]. Zhou [45] reports an example of the zone shocks based on experimental results of a sliding spool: the sliding spools exhibit different failure behaviors when they are tested under different levels of shock magnitudes (measured by sizes of wear debris). To the best of our knowledge, the zone shocks process has only been considered in the shock-degradation dependence model [1]. However, after a careful literature review, we find that there are no existing models that consider both the degradation-shock dependence and the zone shocks.

In this paper, we develop a new DCFP model which considers degradation-shock dependence and zone shocks. The developed model contributes to the existing scientific literature and to engineering practice in two aspects:

- it allows for explicit modeling of the degradation-shock dependence, where random shocks are influenced by degradation processes and
- it allows for the consideration of zone shocks in the degradation-shock dependence model, where shocks with different magnitudes have different impacts on the components failure behaviors.

The remainder of the paper is organized as follows. Sect. 2 introduces the engineering motivation of the present research. In Sect. 3, the new reliability model is developed to describe failure behaviors of systems subject to competing soft failures and hard failures involving degradation-shock dependence. In Sect. 4, the developed model is applied to model the dependent failure behavior of a sliding spool subject to wear and clamping stagnation. Finally, the paper is concluded with a discussion on potential future works in Sect. 5.

2. Engineering problem setting

The model developed in this paper is motivated by the actual engineering problem of modeling the reliability of a sliding spool. Sliding spools are critical control components in hydraulic control systems [28]. As illustrated in Figure 1, a sliding spool is composed of a spool and a sleeve, where the spool slides in the sleeve to control hydraulic oil flows [46].

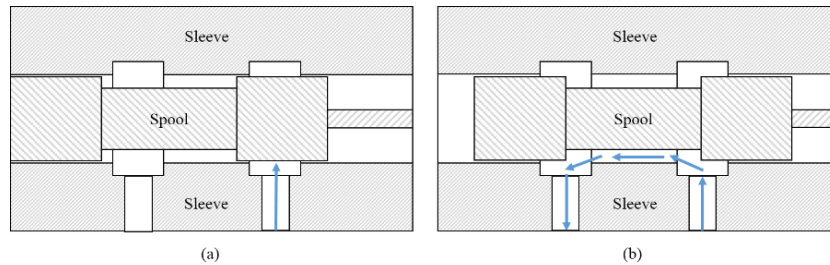


Figure 1 Illustration to a sliding spool: (a) Closed position; (b) Open position [46]

A sliding spool is subject to two failure mechanisms [29]. One is wear between the spool and the sleeve, and the other is clamping stagnation (also referred to as hydraulic locking or sticking), in which the spool is stuck in the

1 sleeve. In practice, wear can be modeled by a physical deterioration model, for example, the linear Archard model
2 suggested by Liao [29]. The modeling of clamping stagnation, however, is more complex because the factors
3 contributing to clamping stagnation are much more varied. According to the survey by Sasak and Yamamot [30], one
4 of the major causes of clamping stagnation is the sudden appearance of pollutant in the hydraulic oil, which can be
5 modeled by a random shock model. The pollutant may come inside the hydraulic system from the outside
6 environment or be generated by the hydraulic system itself. One significant source of internal pollutant is the wear
7 debris generated due to the wear of the sliding spool. Therefore, the random shock process is dependent on the wear
8 process: As the wear process progresses, more wear debris is generated. The debris will contaminate the hydraulic
9 oil and further, increase the likelihood of clamping stagnation [31, 32]. This kind of dependence, caused by the
10 influence of degradation on shocks, needs to be considered when developing the reliability model of the spool valve.

11 Furthermore, experimental results show that the most harmful effect of shocks is generated by wear debris,
12 whose sizes are either close to or much smaller than the clearance of the sliding spool [30, 45, 47]. This is because
13 the clamping stagnation is caused by two failure mechanisms, immediate stagnation and cumulative stagnation [45].
14 When a particle with a size close to the clearance is generated, it causes immediate stagnation of the sliding spool
15 [45], as shown in Figure 2 (a). If particle sizes are smaller than the clearance, the particles can enter the clearance
16 with the hydraulic oil and form filter cakes cumulatively [48]. When the filter cakes become large enough, cumulative
17 stagnation occurs, as shown in Figure 2 (b). According to Zhou [45], particles whose sizes are greater than the
18 clearance have little effect on clamping stagnation, because they are blocked outside the clearance. Hence, based on
19 their magnitudes, the random shocks affecting clamping stagnation can be classified into three zones with different
20 effects on the clamping stagnation. In other words, the sliding spool is subject to zone shocks.

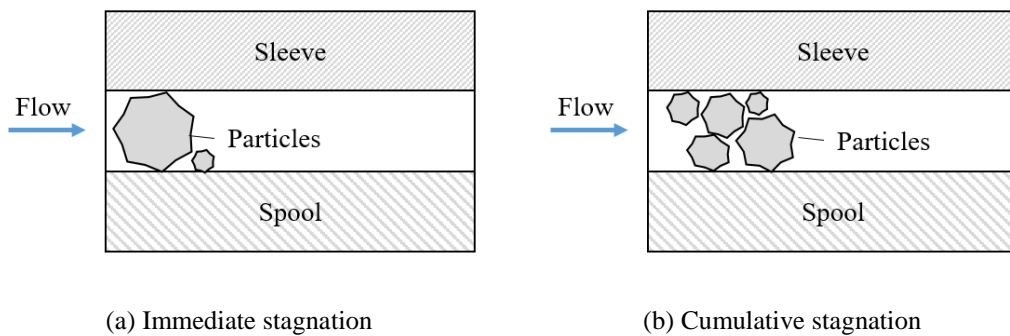


Figure 2 Two failure mechanisms leading to clamping stagnation [45]

21 Hence, to accurately describe the failure behavior of components like the sliding spool, a new model for DCFPs
22 is needed, which allows for: (1) the consideration of the degradation-shock dependence, where random shocks are

influenced by degradation processes and (2) the modeling of zone shocks, where random shocks with different magnitudes have different effects on the products' failure behaviors. This is the motivation of the new DCFP model, which will be developed in Sect. 3.

3. DCFP model considering degradation-shock dependence

3.1 System descriptions

As shown in Figure 3, we consider a DCFP that comprises two failure processes: soft failures due to a degradation process (Figure 3 (a)) and hard failures due to random shocks. According to their magnitudes, random shocks are divided into three zones: damage zone, fatal zone and safety zone [1]. As shown in Figure 3 (b-1), a shock in the damage zone generates a cumulative damage to the system; as shown in Figure 3 (b-2), a shock in the fatal zone causes immediate failure of the system; a shock in the safety zone has no effect on the system's failure behavior. Note that the three shock zones in Figure 3 are shown for illustrative purposes only. In different applications, case-specific shock zones can be defined based on the magnitude of the shocks.

Degradation-shock dependence exists among the failure processes, that is, the arrival rate of the random shocks is dependent on the degradation levels, as illustrated in Figure 3. Failures occur whenever one of the three events happens:

- the degradation process reaches its threshold (denoted by T_1 in Figure 3);
- the cumulative damage resulting from the shocks in the damage zone exceeds its threshold (denoted by T_2 in Figure 3);
- a shock in the fatal zone occurs (denoted by T_3 in Figure 3).

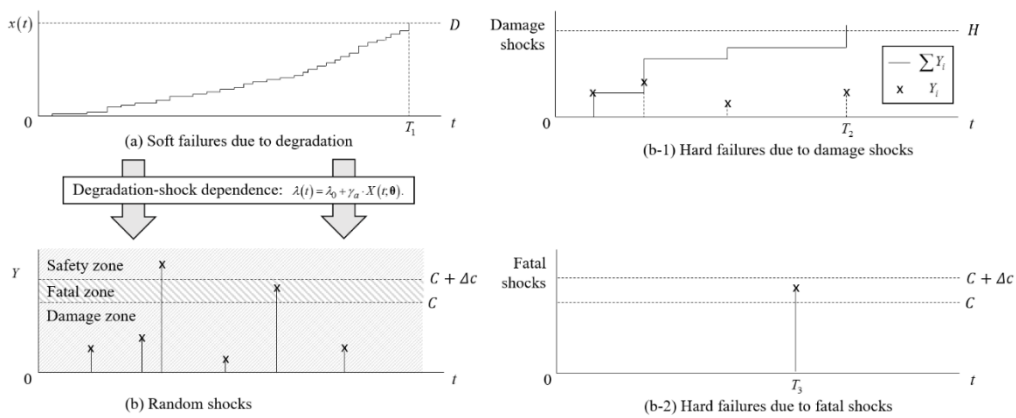


Figure 3 Soft failures due to degradation and hard failures due to random shocks

Additional assumptions of this model include:

- (1) The degradation process can be modeled by the general path model proposed by Lu and Meeker [49]:

$$x(t) = X(t; \boldsymbol{\theta}), \quad (1)$$

where $x(t)$ is the degradation measure and $X(\cdot)$ is the deterministic degradation path, which can be obtained from knowledge of physics of failure [50]. Vector $\boldsymbol{\theta}$ in (1) contains a set of random parameters that represent the influence of uncertainties. It should be noted that the general path model in (1) is a general degradation model, for each specific case, the form of $X(\cdot)$ in (1) could be specified to describe the case-specific degradation mechanism.

- (2) The random shocks can be modeled by a nonhomogeneous Poisson process (NHPP) with intensity function $\lambda(t)$, so that the arrival shocks prior to t , denoted by $N(t)$, follow a Poisson distribution with mean

$$a(t) = \int_0^t \lambda(u) du \quad [51]:$$

$$P(N(t) = n) = \frac{e^{-a(t)} a(t)^n}{n!}. \quad (2)$$

- (3) The magnitude of each shock, denoted by Y_i , is an independent and identically distributed (i.i.d.) random variable.

- (4) The damage caused by the i th shock in damage zone, denoted by W_i , is proportional to its magnitude Y_i with coefficient α :

$$W_i = \alpha Y_i. \quad (3)$$

- (5) To consider the degradation-shock dependence, we introduce a dependence factor γ_a and assume:

$$\lambda(t) = \lambda_0 + \gamma_a \cdot X(t; \boldsymbol{\theta}). \quad (4)$$

Remark 1: the description of the shock damage in the form of equation (3) belongs to a type of commonly used damage model, which assumes that the shock damage is linearly dependent on the shock magnitude. Similar damage models can be found in [1, 52].

Remark 2: the assumption that the intensity of the NHPP is a linear function of the degradation level is adapted from the assumption proposed by Mercer [53]. In Mercer's model, the traumatic failure intensity is of the form $\lambda(t) + c \cdot Z(t)$, where $Z(t)$ is a degradation function describing the piecewise constant degradation process. Also,

note that λ_0 in (4) denotes the initial intensity of the NHPP.

3.2 Modeling zone shocks

Once a random shock arrives, we should first classify it into one of the three zones according to its magnitude.

Let P_1, P_2, P_3 denote the probabilities that the i th shock belongs to the damage, fatal and safety zone, respectively.

It is clear that $P_1 + P_2 + P_3 = 1$. Because the magnitude of each shock is an i.i.d. random variable, the value of P_1, P_2 and P_3 can be easily determined from the distribution of Y_i .

Let us define $N(t)$ as the number of shocks which has arrived prior to time t and $N_1(t), N_2(t), N_3(t)$ be the number of shocks that have fallen into the damage, fatal and safety zone prior to time t , respectively. According to [54], if a NHPP with intensity $\lambda(t)$ is randomly split into two sub-processes with probabilities P_1 and P_2 , where $P_1 + P_2 = 1$, then, the resulting sub-processes are independent NHPPs with intensities $P_1\lambda(t)$ and $P_2\lambda(t)$.

The conclusion can be easily generalized to the three-sub-processes case, which leads to the following proposition:

Proposition 1: The number of shocks falling into damage and fatal zones, denoted by $N_1(t)$ and $N_2(t)$ respectively, follow NHPPs with intensity functions $P_1\lambda(t)$ and $P_2\lambda(t)$.

From Proposition 1, the probability mass function of $N_1(t)$ and $N_2(t)$ can be derived:

$$\begin{aligned} P(N_1(t) = n) &= \frac{e^{-P_1 a(t)} (P_1 a(t))^n}{n!}, \\ P(N_2(t) = n) &= \frac{e^{-P_2 a(t)} (P_2 a(t))^n}{n!}, \end{aligned} \quad (5)$$

where $a(t) = \int_0^t \lambda(u) du$. Proposition 1 and equation (5) will be used in the next section to develop the reliability model.

3.3 Reliability modeling

It can be seen from Figure 3 that a reliable system at time t should satisfy the following conditions: (1) the degradation process does not exceed its threshold D before t ; (2) no shocks in the fatal zone occur before t and (3) the cumulative damage resulting from shocks in the damage zone does not exceed its threshold H before t . Therefore, we can express the reliability at time t as:

$$R(t) = \sum_{i=0}^{\infty} P\left(X(t; \mathbf{0}) < D, \sum_{j=1}^{N_1(t)} W_j < H, N_2(t) = 0 \mid N_1(t) = i\right) \cdot P(N_1(t) = i). \quad (6)$$

Equation (6) comes from a direct application of the total probability formula.

For ease of illustration, we define three events E_1, E_2, E_3 and let them denote the events $X(t; \boldsymbol{\theta}) < D, \sum_{j=1}^{N_1(t)} W_j < H, N_2(t) = 0$, respectively. Because the degradation-shock dependence is considered (see (4)), E_1, E_2, E_3 are dependent. However, once the value of $\boldsymbol{\theta}$ is fixed, the three events become conditional-independent [50]. Therefore, we can apply the total probability formula on $\boldsymbol{\theta}$ in (6), which leads to:

$$R(t) = \sum_{i=0}^{\infty} \int_{\boldsymbol{\theta}} P(E_1, E_2, E_3 | N_1(t) = i, \boldsymbol{\theta} = \mathbf{x}) \cdot P(N_1(t) = i | \boldsymbol{\theta} = \mathbf{x}) \cdot f_{\boldsymbol{\theta}}(\mathbf{x}) d\mathbf{x}, \quad (7)$$

where $f_{\boldsymbol{\theta}}(\mathbf{x})$ is the probability density function of $\boldsymbol{\theta}$. Considering the fact that E_1, E_2, E_3 are conditional-independent given $\boldsymbol{\theta}$, (7) becomes:

$$R(t) = \sum_{i=0}^{\infty} \int_{\boldsymbol{\theta}} P(E_1 | A, B) \cdot P(E_2 | A, B) \cdot P(E_3 | A, B) \cdot P(A | B) \cdot f_{\boldsymbol{\theta}}(\mathbf{x}) d\mathbf{x}, \quad (8)$$

where event A denotes $N_1(t) = i$ and event B denotes $\boldsymbol{\theta} = \mathbf{x}$. To use (8) to calculate the reliability, expressions for $P(E_1 | A, B)$, $P(E_2 | A, B)$, $P(E_3 | A, B)$ as well as $P(A | B)$ should be determined.

To determine $P(E_1 | A, B)$, first note that it denotes the probability that $X(t; \boldsymbol{\theta}) < D$ given that $\boldsymbol{\theta} = \mathbf{x}$. Because $X(\cdot)$ is a deterministic degradation path model and $\boldsymbol{\theta}$ is the only random variable [48], we have

$$P(E_1 | A, B) = \begin{cases} 1 & \text{if } X(t; \mathbf{x}) < D, \\ 0 & \text{if } X(t; \mathbf{x}) \geq D. \end{cases} \quad (9)$$

The probability $P(E_2 | A, B)$ is expressed as

$$P(E_2 | A, B) = P\left(\sum_{j=1}^{N_1(t)} W_j < H \mid N_1(t) = i, \boldsymbol{\theta} = \mathbf{x}\right), \quad (10)$$

which can be further expanded once the distribution of Y_i is determined.

The probability $P(E_3 | A, B)$ is the probability that no fatal shocks occur given $\boldsymbol{\theta} = \mathbf{x}$ and $N_1(t) = i$. From equation (4) and Proposition 1, we have

$$P(E_3 | A, B) = \exp\left\{-P_2 \int_0^t (\lambda_0 + \gamma_a \cdot X(u; \mathbf{x})) du\right\}, \quad (11)$$

where P_2 is the probability that the arrival shock belongs to the fatal zone.

Finally, from (5), $P(A | B)$ is given by:

$$P(A|B) = \frac{e^{-P_1 \Lambda(t; \mathbf{x})} (P_1 \Lambda(t; \mathbf{x}))^i}{i!}, \quad (12)$$

where P_1 is the probability that the arrival shock belongs to the damage zone and $\Lambda(t; \mathbf{x})$ is given by:

$$\Lambda(t; \mathbf{x}) = \int_0^t (\lambda_0 + \gamma_a \cdot X(u; \mathbf{x})) du. \quad (13)$$

The reliability model developed in (8) is an extension to the model in [1], obtained by considering the degradation-shock dependence described by (4). If we let $\gamma_a = 0$, which indicates that there is no dependency between the degradation process and the shock process, the reliability model in (8) degenerates to

$$R(t) = \sum_{i=0}^{\infty} P\left(X(t) < D, \sum_{j=1}^{N_1(t)} W_j < H\right) \frac{e^{-\lambda(P_1 + P_2)t} (\lambda P_1 t)^i}{i!}. \quad (14)$$

Equation (14) is in accordance with the model developed in [1], where $P\left(X(t) < D, \sum_{j=1}^{N_1(t)} W_j < H\right)$ corresponds to the $\int_0^H f_{X_s}(x_s | t, n) dx_s$ in [1].

3.4 Reliability model quantification by Monte Carlo simulation

Due to the complexity of (8), the analytical solution of the reliability is hard to obtain. A numerical approach based on Monte Carlo simulation is, then, developed to compute the system reliability, by the following pseudo-code:

Algorithm 1 Monte Carlo simulation for DCFP model considering degradation-shock dependence

Set t, M, n .

For $j = 1 : n$,

Sample a θ_j from F_{θ} .

Calculate $x(t)$ by (1).

Calculate $P^j(E_1|A, B)$ and $P^j(E_3|A, B)$ by (9) and (11), respectively.

Sample a set of Y , $\{Y_1, Y_2, \dots, Y_M\}$ from F_Y . Pick those belonging to the damage zone and reassemble them as a new set $\{Y_1, Y_2, \dots, Y_m\}$.

For $i = 1 : m$,

Calculate $P_i^j(A|B)$ by (12) and (13).

Calculate $\sum_{Y_k \in \{Y_1, \dots, Y_i\}} \alpha Y_k$ by adding the first i elements in $\{Y_1, Y_2, \dots, Y_m\}$.

Calculate $P_i^j(E_2|A, B)$ by (15):

$$P(E_2|A,B) = \begin{cases} 1 & \text{if } \sum_{Y_k \in \{Y_1, \dots, Y_i\}} \alpha Y_k < H, \\ 0 & \text{if } \sum_{Y_k \in \{Y_1, \dots, Y_i\}} \alpha Y_k \geq H. \end{cases} \quad (15)$$

End For

Calculate $R_j(t)$ by (16):

$$R_j(t) = \sum_{i=0}^m P^j(E_1|A,B) P_i^j(E_2|A,B) P^j(E_3|A,B) P_i^j(A|B). \quad (16)$$

End For

Calculate $R(t)$ by (17):

$$R(t) = \frac{1}{n} \sum_{j=1}^n R_j(t). \quad (17)$$

1

2 The computational complexity of the simulation method is $O(M \times n \times n_t) \times O_{obj}$, where n_t is the evaluated
 3 number of t , O_{obj} is the computational complexity of each evaluation of the objective function, i.e., the
 4 $P^j(E_1|A,B) P_i^j(E_2|A,B) P^j(E_3|A,B) P_i^j(A|B)$ in (16).

5 To implement the algorithm in practice, the values for M, n and n_t need to be set. The values of M and
 6 n determine the accuracy of the reliability estimation at each time point t . Therefore, they should be determined
 7 based on the requirements on the confidence bands. It is well known that the error of Monte Carlo simulation is
 8 proportional to $1/\sqrt{M \times n}$. Therefore, the appropriate sample sizes could be determined by trial-and-error: determine
 9 an initial sample size and check if the confidence band satisfies the requirement; then, adjust the sample sizes based
 10 on the proportional relation between sample sizes and simulation errors. The value of n_t , reflects the closeness of
 11 the point wise approximation values of the reliability function, output by the algorithm, to the true values: the larger
 12 n_t , the better the approximation. Proper balance is needed between the accuracy of the approximation and the
 13 computational complexity of the algorithm.

14 4. Applications

15 4.1 Model development

16 In this section, the developed DCFP model is applied to describe the failure behavior of a sliding spool. The
 17 case study is artificially constructed, based on a realistic physical background, to illustrate the proposed modeling
 18 framework. As discussed in Sect. 2, the sliding spool is subject to two dependent failure processes, wear and clamping

stagnation. According to a study of physics-of-failures [29], the wear of the sliding spool increases smoothly for most of its lifetime, and therefore the wear process can be modeled by a linear model:

$$x(t) = X(t; \varphi, \beta) = \varphi + \beta t, \quad (18)$$

where the initial value φ is a constant and the degradation rate β is assumed to be a normal random variable.

Based on the analysis in Sect. 2, clamping stagnation is modeled by the zone shocks discussed in Sect. 3.2. The arrival of the particles is assumed to follow a nonhomogeneous Poisson process with intensity function $\lambda(t)$. The size of the i th particle getting into the clearance of the sliding spool, denoted by Y_i , is assumed to be an i.i.d. random variable with a probability distribution $f_Y(y)$. Based on the failure mechanisms of clamping stagnation discussed in Figure 2, Y_i is classified into three zones according to its magnitude:

- if $Y_i < C$, the shock belongs to the damage zone;
- if $C \leq Y_i < C + \Delta C$, the shock belongs to the fatal zone;
- if $Y_i \geq C + \Delta C$, the shock belongs to the safety zone,

where C is the clearance of the sliding spool and ΔC is the critical particle size. The probabilities that the i th shock belongs to the damage, fatal and safety zones, denoted by P_1, P_2, P_3 , respectively, are calculated by:

$$\begin{aligned} P_1 &= \int_0^C f_Y(y) dy, \\ P_2 &= \int_C^{C+\Delta C} f_Y(y) dy, \\ P_3 &= \int_{C+\Delta C}^{\infty} f_Y(y) dy. \end{aligned} \quad (19)$$

Moreover, degradation-shock dependence exists between the wear and the clamping stagnation, because wear processes generate wear debris that accelerates the arrival rate of the particles which cause the clamping stagnation [30, 31]. In our developed model, the degradation-shock dependence is modeled by (4). Because the wear process is modeled by (18), (4) becomes:

$$\lambda(t) = \lambda_0 + \gamma_a \cdot (\varphi + \beta t). \quad (20)$$

Substituting (18)-(20) into (8), the reliability of the sliding spool under the two dependent failure processes can be calculated.

4.2 Reliability evaluation

The numerical algorithm presented in Sect. 3.4 is used to evaluate the reliability of the sliding spool based on the developed model. In this application, the values for the model parameters are estimated by domain experts. For

illustrative purposes, we assume the expert estimations are accurate and given in Table 1. In practice, parameters related to degradation processes and random shocks, for example, the φ and β in Table 1, could be estimated from test data.

Table 1 Model parameters of the reliability model for the sliding spool

Parameter/distribution	Description	Values
φ	The initial wear magnitude	0 mm
β	The wear rate	$N(1 \times 10^{-4}, 1 \times 10^{-5})$ mm/s
D	The wear threshold	5 mm
$F_Y(y)$	The CDF for particle diameters	$N(1.2, 0.2)$
α	The shock damage proportional coefficient	1
H	The cumulative clamping stagnation threshold	7.5 mm
$[C, C + \Delta c]$	The range for fatal shocks	[1.5, 1.6]mm
λ_0	The initial intensity of random shocks	2.5×10^{-5} /s
Parameters for simulation	$n = 1000, M = 200, n_t = 1000, t = [0, 6 \times 10^4]s$	
Dependence factors	$\gamma_a = 0, 10^{-5}, 10^{-4}, 10^{-3}$	

4.3 Results and discussions

We compute the reliability of the sliding spool under different levels of dependence ($\gamma_a = 0, 1 \times 10^{-5}, 1 \times 10^{-4}, 1 \times 10^{-3}$), to investigate the failure behavior of the sliding spool. Results are given in Figure 4. In general, the reliability of the sliding spool increases with the decrease of γ_a . As expected, when $\gamma_a = 0$, indicating that there is no dependence between the two failure processes, $R(t)$ takes the largest values. This is mainly due to the dependence mechanism between wear and clamping stagnation. As the wear process progresses, more wear debris is generated and, the arrival rate of particles causing clamping stagnation increases. In this way, clamping stagnation is aggravated by wear. Therefore, positive dependence exists between the two failure mechanisms, as described by (20). Proactively, this suggests that if we can mitigate the dependence between the two failure processes, the reliability of the sliding spool could be enhanced. In practice, filters can be installed to stop the debris generated by wear processes from entering the hydraulic oil. In this way, the dependence between wear and clamping stagnation can be controlled, so that the reliability of the sliding spool can be increased.

Furthermore, it can be seen from Figure 4 that when γ_a is relatively small, the reliability changes slowly at

first, then declines sharply at about $4 \times 10^4 s$. As γ_a increases, the reliability curve decreases more smoothly. When γ_a becomes relatively large ($\gamma_a = 10^{-3}$), the declination of the reliability curve becomes more rapid. The main physical and mathematical reasons of the phenomenon are:

(1) A small γ_a indicates that the two failure mechanisms are almost independent. The sliding spool would experience two distinct failure behaviors. When t is small, because the wear process is far from its failure threshold, failures due to wear seldom occur. Hence, the failure behavior is primarily dominated by clamping stagnation. Because the rate of the clamping stagnation is relatively small, the sliding spool will have high reliability, which is indicated by the first part of the reliability curve.

As t increases, the wear level approaches its threshold. Therefore, failures are more likely to be caused by the wear processes. On the other hand, because γ_a is small, from (4), the rate of the clamping stagnation remains almost unchanged. Hence, when t is large, the failure behavior of the spool is primarily determined by wear, which leads to the second part of the reliability curve starting at around $t = 4 \times 10^4 s$.

(2) When γ_a is relatively large, the wear process has considerable influence on the shock process. Even when t is small, the failure probability of the sliding spool is relatively high due to the positive dependence between the two failure processes. Hence, the difference between the two failure behaviors in the independent case becomes small. Therefore, the reliability curve declines more smoothly comparing to the independent case.

(3) When γ_a becomes even larger, the reliability of the sliding spool decreases rapidly with time. This is because strong dependence exists between wear and clamping stagnation. Therefore, even at the early stages of the spool's life cycle, a lot of spools might fail due to the high risks of clamping stagnation, which results from the strong positive dependence on the wear process.

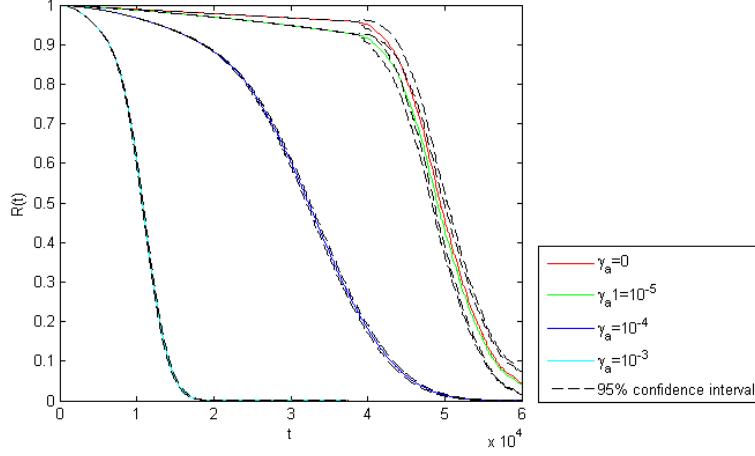


Figure 4 Reliability under different values of the dependence factor, γ_a

4.4 Some extensions

In real cases, the dependency behaviors might be more complicated. For example, as the debris accumulates, more wear and debris are expected. As a consequence, the intensity of the shock arrival process might not only depend on the degradation level, but also on the accumulated shock damage $\sum W$. Furthermore, the size of the wear debris might also be affected by the degradation levels. In addition, the degradation rate also depends on the accumulated shock damage, i.e., the shock-degradation dependence. In this section, we show how to extend the developed model to capture more complex dependency behaviors.

Except for the assumptions we made in Sect. 4.1, three additional assumptions are made:

- the intensity function of the arrival process of the wear debris, i.e., the $\lambda(t)$ in (20), depends on the degradation level $x(t)$ and the current cumulative shock damage $\sum W$:

$$\lambda(t) = \lambda_0 + \gamma_a x(t) + \gamma_b \sum W, \quad (21)$$

where γ_a, γ_b are dependence factors and λ_0 is the initial intensity of the arrival process.

- the mean of the debris size distribution $F_Y(y)$, denoted by $\mu_Y(t)$, is affected by the degradation level $x(t)$:

$$\mu_Y(t) = \mu_{Y_0} + \gamma_c \cdot x(t), \quad (22)$$

where γ_c is the dependence factor, and μ_{Y_0} is the initial value of $\mu_Y(t)$.

- the degradation rate, i.e., the β in (18), depends on the current cumulative shock damage $\sum W$:

$$\beta = \beta_0 + \gamma_d \cdot \sum W \quad (23)$$

where γ_d is the dependence factor.

The first two assumptions consider degradation-shock dependence, where the intensity of the shock arrival process and the distribution of shock magnitudes are both influenced by the degradation process. The last assumption describes shock-degradation dependence, where the degradation rate varies with the accumulated damages of the random shocks.

Monte Carlo method (with a sample size of 10^4) is used to investigate the effect of γ_b, γ_c and γ_d on the spool valve's reliability. The detailed procedures are given in the Appendix. As shown in Figure 5, when γ_b varies from 0 to 10^{-3} , the predicted reliability decreases significantly. This can be explained by the fact that γ_b measures the effect of cumulative shock damage $\sum W$ on the intensity function of the arrival process of the wear debris, as shown in (21), and on the system's degradation rate, as shown in (23). As $\sum W$ increases, on one hand, more shocks occur, which increases the probability of a hard failure; on the other hand, the system degrades more rapidly, which accelerates the arrival of a soft failure. Therefore, γ_b might has a significant effect on the system reliability.

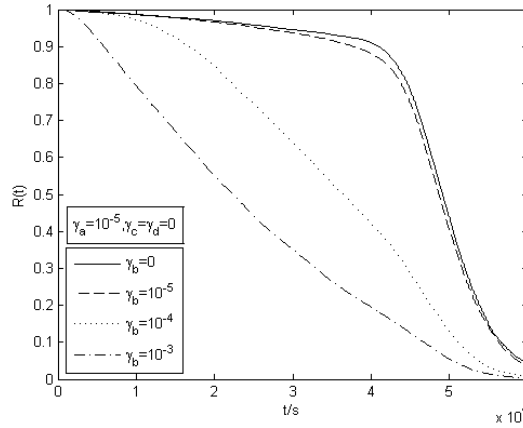


Figure 5 Effect and sensitivity of γ_b on the predicted reliability

As shown in Figure 6, when γ_c varies from 0 to 10^{-3} , the predicted reliability remains almost unchanged. This can be explained by the fact that γ_c measures the effect of the degradation magnitude $x(t)$ on the mean of the debris size distribution $\mu_Y(t)$, as shown in (22). In this case, the initial value of $\mu_Y(t)$ was in the damage shock zone; as $x(t)$ increases, $\mu_Y(t)$ moves first into the fatal shock zone, and then into the safety shock zone.

When $x(t)$ is relatively small, its effect on $\mu_Y(t)$ is negligible. However, when $x(t)$ is large enough, since $\mu_Y(t)$ has already entered the safety shock zone, most random shocks would also be in the safety shock zone. The failure process in this period is dominated by the degradation process. As a result, the system's reliability is not sensitive to γ_c and, therefore, the effect of γ_c can be ignored in modeling the sliding spool.

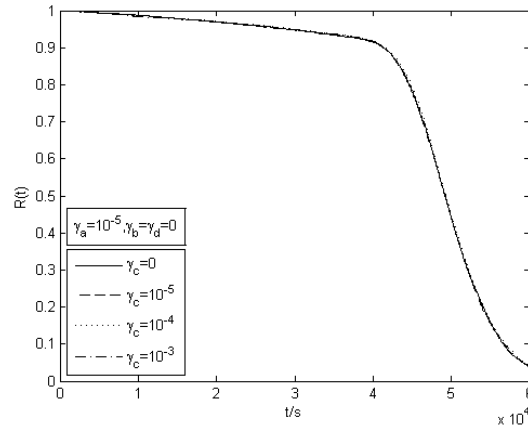


Figure 6 Effect and sensitivity of γ_c on the predicted reliability

As shown in Figure 7, when γ_d varies from 0 to 10^{-3} , the predicted reliability decreases significantly. This can be explained by the fact that γ_d measures the effect of cumulative shock damage $\sum W$ on the degradation rate β , as shown in (23). As γ_d increases, the degradation process becomes more rapid, according to (23), which, then, creates more random shocks due to the degradation-shock dependence (Eq. (20)). The system's reliability is significantly reduced by the mutually enhanced dependences.

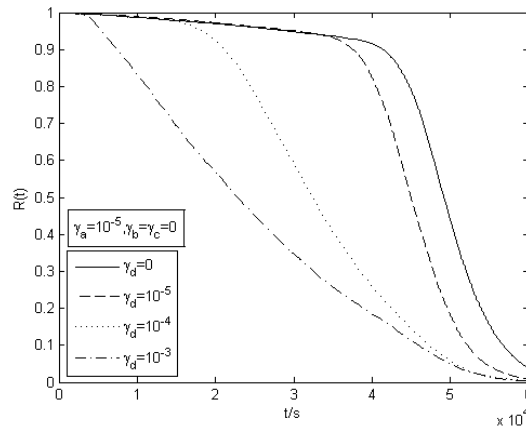


Figure 7 Effect and sensitivity of γ_d on the predicted reliability

5. Conclusions

In this paper, a new reliability model is developed for dependent competing failure processes. In the developed

model, failure of a component could be caused by either “soft” failure (described by a degradation model) or “hard” failure (described by a shock model), and the degradation-shock dependence exists among the failure processes (where the intensity of the shock process is dependent on the degradation process and the effect of dependence is classified into different zones according to the magnitude of the random shocks). Since degradation-shock dependence is considered, the shock process is influenced by the degradation process but the degradation process is not influenced by the shock process. The developed model contributes to existing ones in two aspects. First, it considers a new type of dependence, the degradation-shock dependence, where random shocks are influenced by degradation processes. Second, it combines zone shocks into the degradation-shock dependence, where random shocks are classified into different zones according to their magnitudes and different zones have different effects on the failure behavior. The developed model is applied to describe the dependent failure behavior of an actual product, a sliding spool, which is subject to wear and clamping stagnation.

In the future, additional work will be done to further improve the developed model. First of all, in this paper we only consider a simple dependence scenario, where linear dependence exists between the intensity function and the degradation processes. In the future, the influence of other dependence relations, possibly nonlinear, will also be considered. Second, in this paper we only consider the degradation-shock dependence. In the future, the combination of two types of dependencies will be considered. Third, in this paper we assume that the parameters of the model are perfectly known. In practice, however, many times precise values of the parameters are not available due to limited data and knowledge. Hence, the model is subject to epistemic uncertainty, whose effect will also be studied.

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References

- [1] L. Jiang, Q. Feng and D. W. Coit, "Modeling zoned shock effects on stochastic degradation in dependent failure processes," *IIE Transactions*, vol. 47, pp. 460-470, 2015.
- [2] Z. Zeng, R. Kang and Y. Chen. "Using PoF models to predict system reliability considering failure collaboration." *Chinese Journal of Aeronautics*, vol. 29, pp. 1294-1301, 2016.
- [3] W. Huang and R. G. Askin, "Reliability analysis of electronic devices with multiple competing failure modes involving performance aging degradation," *Quality and Reliability Engineering International*, vol. 19, pp. 241-254, 2003.
- [4] H. Peng, Q. Feng and D. W. Coit, "Reliability and maintenance modeling for systems subject to multiple

dependent competing failure processes," *IIE Transactions*, vol. 43, pp. 12 - 22, 2011.

[5] C. D. Lai and M. Xie, *Stochastic Ageing and Dependence for Reliability*. USA: Springer, 2006.

[6] J. Navarro and T. Rychlik, "Comparisons and bounds for expected lifetimes of reliability systems," *European Journal of Operational Research*, vol. 207, pp. 309-317, 2010.

[7] Y. Wang and H. Pham, "Modeling the dependent competing risks with multiple degradation processes and random shock using time-varying copulas," *IEEE Transactions on Reliability*, vol. 61, pp. 13-22, 2012.

[8] C. Bunea and T. Bedford, "The effect of model uncertainty on maintenance optimization," *IEEE Transactions on Reliability*, vol. 51, pp. 486-493, 2002.

[9] C. Wang, L. Xing and G. Levitin, "Competing failure analysis in phased-mission systems with functional dependence in one of phases," *Reliability Engineering & System Safety*, vol. 108, pp. 90-99, 2012.

[10] C. Wang, L. Xing and G. Levitin, "Reliability analysis of multi-trigger binary systems subject to competing failures," *Reliability Engineering & System Safety*, vol. 111, pp. 9-17, 2013.

[11] Y. Wang, L. Xing, H. Wang, and G. Levitin, "Combinatorial analysis of body sensor networks subject to probabilistic competing failures," *Reliability Engineering & System Safety*, vol. 142, pp. 388-398, 2015.

[12] W. Li and H. Pham, "Reliability modeling of multi-state degraded systems with multi-competing failures and random shocks," *IEEE Transactions on Reliability*, vol. 54, pp. 297-303, 2005.

[13] N. C. Caballé I. T. Castro, C. J. Pérez, and J. M. Lanza-Gutiérrez, "A condition-based maintenance of a dependent degradation-threshold-shock model in a system with multiple degradation processes," *Reliability Engineering & System Safety*, vol. 134, pp. 98-109, 2015.

[14] A. Lehmann, "Joint modeling of degradation and failure time data," *Journal of Statistical Planning and Inference*, vol. 139, pp. 1693-1706, 2009.

[15] A. J. Lemoine and M. L. Wenocur, "On failure modeling," *Naval research logistics quarterly*, vol. 32, pp. 497-508, 1985.

[16] G. Klutke and Y. Yang, "The availability of inspected systems subject to shocks and graceful degradation," *IEEE Transactions on Reliability*, vol. 51, pp. 371-374, 2002.

[17] M. Marseguerra, E. Zio and L. Podofillini, "Condition-based maintenance optimization by means of genetic algorithms and Monte Carlo simulation," *Reliability Engineering and System Safety*, vol. 77, pp. 151 - 165, 2002.

[18] J. Barata, C. G. Soares, M. Marseguerra, and E. Zio, "Simulation modelling of repairable multi-component deteriorating systems for 'on condition' maintenance optimisation," *Reliability Engineering and System Safety*, vol. 76, pp. 255 - 264, 2002.

[19] W. Li and H. Pham, "An inspection-maintenance model for systems with multiple competing processes," *IEEE Transactions on Reliability*, vol. 54, pp. 318-327, 2005.

[20] Y. Lin, Y. Li and E. Zio, "Integrating random shocks into multi-state physics models of degradation processes for component reliability assessment," *IEEE Transactions on Reliability*, vol. 64, pp. 154-166, 2015.

[21] Y. Wang and H. Pham, "A multi-objective optimization of imperfect preventive maintenance policy for dependent competing risk systems with hidden failure," *IEEE Transactions on Reliability*, vol. 60, pp. 770-781, 2011.

[22] E. Keedy and Q. Feng, "Reliability analysis and customized preventive maintenance policies for stents with stochastic dependent competing risk processes," *IEEE Transactions on Reliability*, vol. 62, pp. 887-897, 2013.

[23] S. Song, D. W. Coit, Q. Feng, and H. Peng, "Reliability analysis for multi-component systems subject to multiple dependent competing failure processes," *IEEE Transactions on Reliability*, vol. 63, pp. 331-345, 2014.

[24] S. Song, D. W. Coit and Q. Feng, "Reliability for systems of degrading components with distinct component shock sets," *Reliability Engineering & System Safety*, vol. 132, pp. 115-124, 2014.

[25] L. Jiang, Q. Feng and D. W. Coit, "Reliability and maintenance modeling for dependent competing failure processes with shifting failure thresholds," *IEEE Transactions on Reliability*, vol. 61, pp. 932-948, 2012.

[26] K. Rafiee, Q. Feng and D. W. Coit, "Reliability modeling for dependent competing failure processes with changing degradation rate," *IIE Transactions (Institute of Industrial Engineers)*, vol. 46, pp. 483-496, 2014.

[27] Z. Wang, H. Z. Huang, Y. Li, and N. C. Xiao, "An approach to reliability assessment under degradation and shock process," *IEEE Transactions on Reliability*, vol. 60, pp. 852-863, 2011.

[28] N. D. Vaughan, P. E. Pomeroy and D. G. Tilley, "The contribution of erosive wear to the performance degradation of sliding spool servovalves," *Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology*, vol. 212, pp. 437-451, 1998.

[29] X. Liao, "Research on the wear mechanism and life modeling method of aero-hydraulic spool valve," Degree of Master, Beijing, China: Beihang University, 2014.

[30] A. Sasak and T. Yamamot, "A review of studies of hydraulic lock," *Lubrication Engineering*, vol. 49, pp. 585-593, 1993.

[31] S. Raadnui and S. Kleesuwan, "Low-cost condition monitoring sensor for used oil analysis," *Wear*, vol. 259, pp. 1502-1506, 2005.

- [32] Y. S. Wang, M. J. Zhang and D. F. Liu, "A compact on-line particle counter sensor for hydraulic oil contamination detection," *Applied Mechanics and Materials*, vol. 130-134, pp. 4198-4201, 2011.
- [33] J. Fan, S. G. Ghurye and R. A. Levine, "Multicomponent lifetime distributions in the presence of ageing," *Journal of applied probability*, vol. 37, pp. 521-533, 2000.
- [34] V. Bagdonavicius, A. Bikelis and V. Kazakevicius, "Statistical analysis of linear degradation and failure time data with multiple failure modes," *Life Data Analysis*, pp. 65-81, 2004.
- [35] Z. S. Ye, L. C. Tang and H. Y. Xu, "A distribution-based systems reliability model under extreme shocks and natural degradation," *IEEE Transactions on Reliability*, vol. 60, pp. 246-256, 2011.
- [36] K. T. Huynh, I. T. Castro, A. Barros, and C. B érenguer, "Modeling age-based maintenance strategies with minimal repairs for systems subject to competing failure modes due to degradation and shocks," *European Journal of Operational Research*, vol. 218, pp. 140-151, 2012.
- [37] K. T. Huynh, A. Barros, C. B érenguer, and I. T. Castro, "A periodic inspection and replacement policy for systems subject to competing failure modes due to degradation and traumatic events," *Reliability Engineering & System Safety*, vol. 96, pp. 497-508, 2011.
- [38] H. M. Bai, Z. H. Li and X. B. Kong, "Generalized Shock Models Based on a Cluster Point Process," *IEEE Transactions on Reliability*, vol. 55, pp. 542-550, 2006.
- [39] X. X. Huang and J. Q. Chen, "Time-dependent reliability model of deteriorating structures based on stochastic processes and Bayesian inference methods," *Journal of Engineering Mechanics*, vol. 141, 2015.
- [40] A. Gut, "Mixed shock models," *Bernoulli*, vol. 7, pp. 541-555, 2001.
- [41] F. Mallor and E. Omey, "Shocks, runs and random sums," *Journal of Applied Probability*, vol. 38, pp. 438-448, 2001.
- [42] S. V. Amari, J. B. Dugan, R. B. Misra, "A Separable Method for Incorporating Imperfect Fault-Coverage into Combinatorial Models", *IEEE Transactions on Reliability*, vol. 48, pp. 267-274, 1999
- [43] L. Xing, "Reliability Evaluation of Phased-Mission Systems with Imperfect Fault Coverage and Common-Cause Failures," *IEEE Transactions on Reliability*, vol. 56, pp. 58-68, 2007.
- [44] J. Xiang, F. Machida, K. Tadano, and Y. Maeno, "An Imperfect Fault Coverage Model with Coverage of Irrelevant Components," *IEEE Transactions on Reliability*, vol. 64, pp. 320-332, 2015.
- [45] Z. Zhou, "Research on mechanism of contaminant lock for hydraulic valve," *Chinese Hydraulics & Pneumatics*, pp. 15-17, 1994. (In Chinese)
- [46] Y. J. Yang, W. W. Peng, D. B. Meng, S. P. Zhu, and H. Z. Huang, "Reliability analysis of direct drive electrohydraulic servo valves based on a wear degradation process and individual differences," *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, vol. 228, pp. 621-630, 2014.
- [47] F. J. Li, and S. Y. Zhou, "Determination of critical size of contamination of spool valve," *Machine Tool & Hydraulics*, pp. 51-54, 1996. (In Chinese)
- [48] C. S. Zheng et al., "Contaminant lock force and filter cake forming mechanism of hydraulic spool valves," *Lubrication Engineering*, pp.14-19, 2014. (In Chinese)
- [49] C. J. Lu and W. O. Meeker, "Using degradation measures to estimate a time-to-failure distribution," *Technometrics*, vol. 35, pp. 161-174, 1993.
- [50] M. Pecht and A. Dasgupta, "Physics-of-failure: an approach to reliable product development," Lake Tahoe, CA, USA, 1995, pp. 1-4.
- [51] S. M. Ross, *Introduction to Probability Models*, 10 ed. Boston: Academic Press, 2010.
- [52] S. L. Song, D. W. Coit and Q. M. Feng, "Reliability analysis of multiple-component series systems subject to hard and soft failures with dependent shock effects," *IIE Transactions*, vol. 48, pp. 720-735, 2016.
- [53] A. Mercer, "On wear depending renewal processes," *J. Roy. Stat. Soc.*, vol. 23, pp. 368-376, 1961.
- [54] G. R. Grimmett and D. R. Stirzaker, *Probability and Random Processes*, 3 ed. USA: Oxford University Press, 2001.

1 Appendix

Algorithm 2 Monte Carlo simulation for DCFP model considering multiple dependences in Sect. 4.4

Step 1: Set samplesize, $i = 0, x = 0, \sum W = 0, \tau = 0$;

Step 2: If $i > \text{samplesize}$, stop;

else generate a β_s from F_β ;

Step 3: Update the intensity function $\lambda(t) = \lambda_0 + \gamma_a \left(x + (\beta_s + \gamma_d \cdot \sum W)(t - \tau) \right) + \gamma_b \sum W$;

Generate an arrival time $\Delta\tau$ from a NHPP with intensity function $\lambda(t)$ using thinning method [51]:

Set T_{\max} ; Calculate $\lambda_M = \lambda(T_{\max})$; $tt = \tau$;

While 1

$\Delta\tau \leftarrow$ Generate a random number from $\exp(\lambda_M)$; $tt = tt + \Delta\tau$;

If $tt < T_{\max}$, $p_{acc} = \lambda(tt) / \lambda_M$; Accept $\Delta\tau$ with probability p_{acc} ; Return $\Delta\tau = tt - \tau$;

Else $tt = T_{\max}$; Increase the value of T_{\max} ; Calculate $\lambda_M = \lambda(T_{\max})$; Continue;

Endwhile

$\tau = \tau + \Delta\tau$;

Step 4: $x = x + (\beta_s + \gamma_d \cdot \sum W) \Delta\tau$;

If $x > D$, $i = i + 1, t_f(i) = (D - x) / (\beta_s + \gamma_d \cdot \sum W) + \tau$, go to Step 2;

Else go to Step 5;

Step 5: $\mu_Y = \mu_{Y_0} + \gamma_c \cdot x$;

Generate $Y \sim \text{Normal}(\mu_Y, \sigma_Y^2)$;

If $Y < C$, go to Step 6;

Else if $Y < C + \Delta C$, $i = i + 1, t_f(i) = \tau$, go to Step 2;

Step 6: $\sum W = \sum W + \alpha \cdot Y$;

If $\sum W \geq H$, $i = i + 1, t_f(i) = \tau$, go to Step 2;

Else go to Step 3;
