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# A model-based reliability metric considering aleatory and epistemic uncertainty

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Abstract—Model-based reliability analysis and assessment methods rely on models, which are assumed to be precise, to predict reliability. In practice, however, the precision of the model cannot be guaranteed due to the presence of epistemic uncertainty. In this paper, a new reliability metric, called belief reliability, is defined to explicitly account for epistemic uncertainty in model-based reliability analysis and assessment. A new method is developed to explicitly quantify epistemic uncertainty by measuring the effectiveness of the engineering analysis and assessment activities related to reliability. To evaluate belief reliability, an integrated framework is presented, where the contributions of design margin, aleatory uncertainty and epistemic uncertainty are integrated to yield a comprehensive and systematic description of reliability. The developed methods are demonstrated by two case studies.

*Index Terms*—Reliability, physics-of-failure, epistemic uncertainty, model uncertainty, belief reliability

#### I. INTRODUCTION

Reliability refers to the ability of a component or system to perform a required function for a given period of time when used under stated operating conditions [1]. Traditionally, reliability is measured by the probability that functional failure does not occur in the considered period of time and failure data are used for its estimation based on statistical methods [2]. In practice, however, failure data are often scarce (if available at all), which defies the use of classical statistical methods and challenges Bayesian methods with respect to the assumption of subjective prior distributions [3]. Due to the problem of limited failure data, model-based methods (cf. physics-of-failure (PoF) methods [4], structural reliability methods [5], etc.) are widely applied to predict reliability, by deterministically describing the degradation and failure processes using deterministic failure behavior models. More specifically, it is assumed that:

- the failure behavior of a component or a system can be described by a deterministic model;
- 2) random variations in the variables of the deterministic model are the sole source of uncertainty.

The probabilistic quantification of reliability is, then, obtained by propagating uncertainties through the model analytically or numerically, *e.g.* by Monte Carlo simulation [6-8].

The random variations represent the uncertainty inherent in the physical behavior of the system and are referred to as aleatory uncertainty [9]. However, the model-based methods are also subject to epistemic uncertainty due to incomplete knowledge on the degradation and failure processes [10, 11]. According to Aven and Zio [12, 13], epistemic uncertainty may arise because:

- the deterministic model cannot exactly describe the failure process, *e.g.*, due to incomplete understanding of the failure causes and mechanisms (model uncertainty, also known as structural uncertainty);
- the precise values of the model parameters might not be accurately estimated due to lack of data in the actual operational and environmental conditions (parameter uncertainty).

In this paper, we introduce a new reliability metric, belief reliability, to explicitly consider the effect of epistemic uncertainty on the model-based methods. For illustrative purposes, we consider only model uncertainty in this paper. However, the framework can be easily extended to deal with parameter uncertainty.

In literature, various approaches have been developed to consider model uncertainty. Mosleh and Droguett reviewed a number of approaches for model uncertainty assessment and compared them in terms of theoretical foundations and domains of application [14, 15]. Among them, the alternate hypotheses approach and the adjustment factor approach are two most widely applied ones [16]. The alternate hypotheses approach identifies a family of possible alternate models and probabilistically combines the predictions of them based on Bayesian model averaging, where the probability of each model is evaluated from experimental data or expert judgements [17, 18]. Apostolakis [19] addressed the issue of model uncertainty in probabilistic risk assessment using the alternate hypotheses approach. Park and Grandhi [20] quantified the model probability in the alternate hypotheses approach by the measured deviations between experimental data and model predictions. In [21], two crack models were probabilistically combined using the alternate hypotheses approach to estimate the failure probability of a butt weld. Other applications of the alternate hypotheses approach include sediment transport models [22], identification of benchmark doses [23], precipitation modeling [24], etc.

In the adjustment factor approach, the model uncertainty is addressed by modifying a benchmark model (the one that

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we have highest confidence in) with an adjustment factor, which is assumed to be uncertain, and is either added to or multiplied by the prediction results of the model [16, 25]. In [26], the adjustment factor approach was used to combine experts' estimates according to Bayes' theorem. Zio and Apostolakis [16] used the approach to assess the risk of radioactive waste repositories. Fischer and Grandhi [27] applied an adjustment factor to low-fidelities models so as to scale them to high-fidelity models. In a series of studies conducted by Park and Grandhi [25, 28–30], the adjustment factor approach was combined with the alternate hypotheses approach by introducing an adjustment factor to quantify the uncertainty in each alternate model; the model uncertainty was, then, evaluated by averaging all the models according to the alternate hypotheses approach.

The alternate hypotheses approach requires enumerating a set of mutually exclusive and collectively exhaustive models [15]. In the case of model-based reliability methods, however, it is impossible for us to enumerate all the possible models, which limits the application of the alternate hypotheses approach. Hence, we adopt the adjustment factor approach in this paper to develop a new reliability metric to describe the effect of epistemic uncertainty (model uncertainty) on the model-based reliability methods.

In the adjustment factor approaches, epistemic uncertainty is quantified by the adjustment factor, which is often determined based on validation test data (for example, see [18] or [30]). In practice, however, due to limited time and resources, it is hard, if not impossible, to gather sufficient validation test data. Resorting to expert judgements might offer an alternative solution (for example, see [16]), but they could be criticized for being too subjective. On the other hand, epistemic uncertainty relates to the knowledge on the component or system functions and failure behaviors: as this knowledge is accumulated, epistemic uncertainty is reduced. In the life cycle of a component or system, the knowledge is gained by implementing a number of reliability analysis-related engineering activities, whose purpose is to help designers better understand potential failure modes and mechanisms. For example, through Failure Mode, Effect and Criticality Analysis (FMECA), potential failure modes and their effects could be identified, so that the designer can better understand the product's failure behaviors [31]. Similar engineering activities include Failure Report, Analysis, and Corrective Action System (FRACAS) [32], Reliability Growth Test (RGT) [33], Reliability Enhancement Test (RET) [32], Reliability Simulation Test (RST) [34, 35], etc. In this paper, we develop a new quantification method for the epistemic uncertainty in the adjustment factor method, based on the effectiveness of these engineering activities.

The contributions of this paper are summarized as follows:

- a new reliability metric, the belief reliability, is developed to explicitly consider epistemic uncertainty in the model-based reliability methods;
- a new method is developed to quantify epistemic uncertainty, based on the effectiveness of the engineering activities related to the reliability analysis and assessment of components and systems;
- 3) a method is developed to evaluate the belief reliability

of components and systems, based on the integration of design margin, aleatory uncertainty and epistemic uncertainty.

The rest of the paper is organized as follows. In section II, belief reliability is defined to account for the effect of epistemic uncertainty in model-based reliability methods. In section III-B, epistemic uncertainty is quantified based on the effectiveness of the related engineering activities and a belief reliability evaluation method is developed. Section IV presents two case studies to demonstrate the developed methods. Finally, the paper is concluded in section V with a discussion on future works.

#### II. DEFINITION OF BELIEF RELIABILITY

In this section, we introduce a new metric of reliability, belief reliability, to explicitly account for the influence of epistemic uncertainty on model-based reliability methods. We start with a brief introduction of the model-based reliability method in subsection II-A. Then, belief reliability is defined in subsection II-B.

#### A. Model-based reliability methods

For a general description of model-based reliability methods, we introduce the concepts of performance parameter and performance margin:

**Definition 1** (Performance parameter). Suppose failure occurs when a parameter p reaches a threshold value  $p_{th}$ . Then, the parameter p is referred to as a performance parameter, while the threshold value  $p_{th}$  is referred to as the functional failure threshold associated with p.

According to *Definition* 1, performance parameters and functional failure thresholds define the functional requirements on a system or a component, for which three categories exist in practice:

- 1) Smaller-the-better (STB) parameters: if failure occurs when  $p \ge p_{th}$ , then, the performance parameter p is a STB parameter.
- 2) Larger-the-better (LTB) parameters: if failure occurs when  $p \leq p_{th}$ , then, the performance parameter p is a LTB parameter.
- 3) Nominal-the-better (NTB) parameters: if failure occurs when  $p \leq p_{th,L}$  or  $p \geq p_{th,U}$ , then, the performance parameter p is a NTB parameter.

**Definition 2** (Performance margin). Suppose p is a performance parameter and  $p_{th}$  is its associated functional failure threshold; then,

$$m = \begin{cases} \frac{p_{th} - p}{p_{th}}, & \text{if } p \text{ is STB}, \\ \frac{p - p_{th}}{p_{th}}, & \text{if } p \text{ is LTB}, \\ \min\left(\frac{p_{th,U} - p}{p_{th,U}}, \frac{p - p_{th,L}}{p_{th,L}}\right), & \text{if } p \text{ is NTB} \end{cases}$$
(1)

is defined as the (relative) performance margin associated with the performance parameter p.

*Remark* 1. From Definition 2, performance margin is a unitless quantity and failure occurs whenever  $m \leq 0$ .

In the model-based reliability methods, it is assumed that the performance margin can be described by a deterministic model, which is derived based on knowledge of the functional principles and failure mechanisms of the component [5, 36]. Conceptually, we assume that the performance margin model has the form

$$m = g_m(\mathbf{x}),\tag{2}$$

where  $g_m(\cdot)$  denotes the deterministic model which predicts the performance margin and **x** is a vector of input variables.

In the design and manufacturing processes of a product, there are many uncertain factors influencing the input  $\mathbf{x}$  of (2). Thus, the values of  $\mathbf{x}$  may vary from product to product of the same type. Usually, this product-to-product variability is described by assuming that  $\mathbf{x}$  is a vector of random variables with given probability density functions. Then, m is also a random variable and reliability  $R_p$  is defined as the probability that m is greater than zero. The subscript p is used to indicate that  $R_p$  is a probability measure. Given the probability density function of  $\mathbf{x}$ , denoted by  $f_X(\cdot)$ ,  $R_p$  can be calculated by:

$$R_p = Pr\left(g_m(\mathbf{x}) > 0\right) = \int \cdots \int_{g_m(\mathbf{x}) > 0} f_X(\mathbf{x}) \mathrm{d}\mathbf{x}.$$
 (3)

#### B. Definition of belief reliability

Belief reliability is defined in this subsection to explicitly account for the effect of epistemic uncertainty in model-based reliability methods. For this, we first define design margin and Aleatory Uncertainty Factor (AUF):

**Definition 3** (Design margin). Suppose the performance margin of a component or a system can be calculated by (2). Then, design margin  $m_d$  is defined as

$$m_d = g_m(\mathbf{x}_N),\tag{4}$$

where  $\mathbf{x}_N$  is the nominal values of the parameters.

**Definition 4** (Aleatory Uncertainty Factor (AUF)). Suppose  $R_p$  is the probabilistic reliability calculated from the performance margin model using (3). Then, AUF  $\sigma_m$  is defined as

$$\sigma_m = \frac{m_d}{Z_{R_p}},\tag{5}$$

where  $Z_{R_p}$  is the value of the inverse cumulative distribution function of a standard normal distribution evaluated at  $R_p$ .

Further, let equivalent design margin  $M_E$  to be

$$M_E = m_d + \epsilon_m,\tag{6}$$

where  $\epsilon_m \sim \text{Normal}(0, \sigma_m^2)$ . It is easy to verify that  $M_E \sim \text{Normal}(m_d, \sigma_m^2)$  and  $R_p$  can be calculated as the probability that  $M_E > 0$ , as shown in Figure 1 (a). Therefore, the probabilistic reliability can be quantified by the equivalent performance margin and further by  $m_d$  and  $\sigma_m$ , where

•  $m_d$  describes the inherent reliability of the product when all the input variables take their nominal values. Graphically, it measures the distance from the center of the equivalent performance margin distribution to the boundaries of the failure region, as shown in Figure 1 (a);

•  $\sigma_m$  accounts for the uncertainty resulting from the product-to-product random variations, *e.g.* the tolerance of manufacturing processes, the variability in material properties, *etc.* Usually, these random variations are controlled by engineering activities such as tolerance design, environmental stress screening, stochastic process control, *etc* [11].

To further account for the effect of epistemic uncertainty, it is assumed that:

$$M_E = m_d + \epsilon_m + \epsilon_e,\tag{7}$$

where  $\epsilon_e$  is an adjustment factor [16] and  $\epsilon_e \sim \text{Normal}(0, \sigma_e^2)$ . Parameter  $\sigma_e$  is defined as Epistemic Uncertainty Factor (EUF) and it quantifies the effect of epistemic uncertainty. The physical meaning of (7) is explained in Figure 1 (b): epistemic uncertainty introduces additional dispersion to the aleatory distribution of the equivalent performance margin. The degree of the dispersion is related to the knowledge we have on the failure process of the product, *i.e.*, the more knowledge we have, the less value  $\sigma_e$  takes.



(b) Effect of epistemic uncertainty

Fig. 1. Epistemic uncertainty effect on the distribution of the equivalent performance margin

Considering the assumption made in (7), we can, then, define the belief reliability as follows:

**Definition 5** (Belief reliability). The reliability metric

$$R_B = \Phi_N \left( \frac{m_d}{\sqrt{\sigma_m^2 + \sigma_e^2}} \right) \tag{8}$$

is defined as belief reliability, where  $\Phi_N(\cdot)$  is the cumulative distribution function of a standard normal random variable.

Belief reliability can be interpreted as our belief degree on the product reliability, based on the knowledge of design margin, aleatory uncertainty and epistemic uncertainty. In the following, we discuss respectively how design margin, aleatory uncertainty and epistemic uncertainty influence the value of belief reliability.

Discussion 1. It is obvious from (8) that  $R_B \in [0, 1]$ , where

- $R_B = 0$  indicates that we believe for sure that a component or system is unreliable, *i.e.*, it cannot perform its desired function under stated time period and operated conditions.
- $R_B = 1$  indicates that we believe for sure that a component or system is reliable, *i.e.*, it can perform its desired function under stated time period and operated conditions.
- $R_B = 0.5$  indicates that we are most uncertain about the reliability of the component or system [37].
- $R_{B,A} > R_{B,B}$  indicates that we believe that product A is more reliable than product B.

Discussion 2 (Variation of  $R_B$  with the design margin). From (8), it is easy to see that  $R_B$  is an increasing function of  $m_d$ , as illustrated by Figure 2, which is in accordance with the intuitive fact that when the design margin is increased, the component or system becomes more reliable.



Discussion 3 (Variation of  $R_B$  with the aleatory uncertainty). In (8), the effect of aleatory uncertainty is measured by the AUF,  $\sigma_m$ . Figure 3 shows the variation of  $R_B$  with  $\sigma_m$ , when  $\sigma_e$  is fixed, for different values of  $m_d$ . It can be seen from Figure 3 that when  $m_d$  and  $\sigma_e$  are fixed,  $R_B$  approaches 0.5 as  $\sigma_m$  increases to infinity. The result is easy to understand, since  $\sigma_m \to \infty$  indicates the fact that uncertainty has the greatest influence.



Fig. 3. Variation of  $R_B$  with  $\sigma_m$ 

Discussion 4 (Variation of  $R_B$  with the epistemic uncertainty). In (8), the effect of epistemic uncertainty is measured by the EUF,  $\sigma_e$ . The variation of  $R_B$  with respect to  $\sigma_e$  is illustrated in Figure 4, with  $\sigma_m$  fixed to 0.2. From Figure 4, we can see that when  $\sigma_e \rightarrow \infty$ ,  $R_B$  also approaches 0.5, for the same reason as the AUF.



Fig. 4. Variation of  $R_B$  with  $\sigma_e$ 

Fig. 2. Influence of  $m_d$  on  $R_B$ 

Besides, it can be verified from (8) that if  $m_d = 0$ ,  $R_B = 0.5$ . This is because when  $m_d = 0$ , the product is at borderline between working and failure. Therefore, we are most uncertain about its reliability (For details, please refer to the maximum uncertainty principle in [37]).

Besides, it can be shown from (8) and assumption (3) that as  $\sigma_e \rightarrow 0$ ,  $R_B$  approaches the  $R_p$  calculated by the modelbased reliability methods using equation (3). This is a natural result since  $\sigma_e = 0$  is the ideal case for which there is no epistemic uncertainty, so that the product failure behavior is accurately predicted by the deterministic performance margin model and the aleatory uncertainty. In practice, we always have  $m_d \ge 0$  and  $\sigma_e > 0$ . Therefore,

$$R_B \le R_p \tag{9}$$

where  $R_p$  is the probabilistic reliability predicted by (3) under the same conditions. Equation (9) shows that using belief reliability yields a more conservative evaluation result than using the probabilistic reliability, because belief reliability considers the effect of insufficient knowledge on the reliability evaluations.

#### **III. EVALUATION OF BELIEF RELIABILITY**

In this section, we discuss how to evaluate the belief reliability for a given product. A general framework for belief reliability evaluation is first given in subsection III-A. Then, a method is presented for evaluating epistemic uncertainty and determining the value of the EUF.

#### A. Belief reliability evaluation

The  $R_B$  defined in (8) incorporates the contributions of design margin  $m_d$ , aleatory uncertainty (represented by  $\sigma_m$ ) and epistemic uncertainty (represented by  $\sigma_e$ ). The contributions from the three factors should be evaluated individually and then, combined to evaluate the belief reliability of a component. Detailed procedures are presented in Figure 5.



Fig. 5. Procedures for component belief reliability evaluation

Four steps comprise the evaluation procedure:

1) Performance Margin Model Development: First, a deterministic performance margin model is developed to predict the value of the performance margin m. The performance margin model can be developed based on knowledge of underlying functional principles and physics of failures. For a detailed discussion on how to develop performance margin models, readers might refer to [38] and [39].

2) Aleatory Uncertainty Evaluation: Next, the values of  $m_d$ and  $\sigma_m$  are determined. The value of  $m_d$  is calculated based on (4), where all the input parameters of the performance margin model take their nominal values. To calculated the value of  $\sigma_m$ , the probabilistic reliability  $R_p$  is calculated first by propagating aleatory uncertainty in the model parameters according to (3). Either structural reliability methods [5] or Monte Carlo simulations [7] might be used for the calculation. Then,  $\sigma_m$  can be calculated by combining  $m_d$  and  $R_p$  using (5). 3) Epistemic Uncertainty Evaluation: The value of  $\sigma_e$  is, then, determined by evaluating the effect and potential impact of epistemic uncertainty. In practice, epistemic uncertainty relates to the knowledge on the component or system functions and failure behaviors: as this knowledge is accumulated, epistemic uncertainty is reduced. Hence, in this paper, we relate epistemic uncertainty to our state of knowledge on the product and its failure process and assess the value of  $\sigma_e$  based on the effectiveness of engineering activities that generate our knowledge base. Details on how to evaluate the value of  $\sigma_e$ is given in Section III-B.

4) Belief Reliability Evaluation: Following steps 1) - 3), the values of  $m_d$ ,  $\sigma_m$  and  $\sigma_e$  are determined. Then, the belief reliability can be evaluated according to (8).

#### B. Quantification of epistemic uncertainty

In this section, we develop a method to quantify epistemic uncertainty based on the state of knowledge. In subsection III-B1, we discuss how to evaluate the state of knowledge, and then, in subsection III-B2, we quantify the effect of epistemic uncertainty in terms of  $\sigma_e$ .

1) Evaluation of the state of knowledge: In the life cycle of a component or system, the knowledge on the products' failure behavior is gained by implementing a number of engineering activities of reliability analysis, whose purposes are to help designers better understand potential failure modes and mechanisms. In this paper, we refer to these engineering activities as epistemic uncertainty-related (EU-related) engineering activities. Table I lists some commonly encountered EU-related engineering activities and discusses their contributions to gaining knowledge and reducing epistemic uncertainty, where FMECA stands for Failure Mode, Effect and Criticality Analysis, FRACAS stands for Failure Reporting, Analysis, and Corrective Action System, RET stands for Reliability Enhancement Test, RGT stands for Reliability Growth Test and RST stands for Reliability Simulation Test.

In this paper, we make an assumption that the state of knowledge is directly related to the effectiveness of the EU-related engineering activities. Suppose there are n EU-related engineering activities in a product life cycle. Let  $y_i, i = 1, 2, \dots, n$  denote the effectiveness of the EU-related engineering activities, where  $y_i \in [0, 1]$ ; the more effective the engineering activity is, the larger value the corresponding  $y_i$  takes. The values of  $y_i$  are determined by asking experts to evaluate the effectiveness of the EU-related engineering activities, based on a set of predefined evaluation criteria.

For example, the effectiveness of FMECA can be evaluated based on eight elements, as shown in Table II. For each element, experts are invited to evaluate their performances according to the criteria listed in Table II. Based on the evaluated performance, a score can be assigned to each element, denoted by  $S_1, S_2, \dots, S_8$ . Then, the effectiveness of FMECA, denoted by  $y_1$ , can be determined by

$$y_1 = \frac{1}{8} \sum_{i=1}^8 S_i. \tag{10}$$

The effectiveness of other EU-related engineering activities can be evaluated in a similar way, so that the values

 TABLE I

 Examples of EU-related engineering activities

Activities	Contributions to gaining knowledge and reducing epistemic uncertainty
FMECA	FMECA helps designers to identify potential failure modes and understand their effects, so as to increase the designer's
	knowledge about potential failures [31].
FRACAS	By implementing FRACAS, knowledge on potential failure modes and mechanisms is accumulated based on previously
	ocurred failures and corrective actions [32].
RGT	In a RGT, cycles of Test Analysis and Fix (TAAF) are repeated until the product reaches its reliability requirements. In
	this way, designers' knowledge on the failure modes and mechanisms is accumulated [33].
RET	As the RGT, RET reduces epistemic uncertainty by stimulating potential failures, but using highly accelerated stresses,
	which can generate failures that are hard to be identified by analyses or conventional tests [32].
RST	In a RST, simulation tests are conducted based on physics-of-failure models to identify weak design points for the products.
	Knowledge of potential failure modes can be accumulated in this way [34, 35].

TABLE II	
EVALUATION CRITERIA FOR FMECA	

Notations	Elements	Criteria	Scores
$S_1$	Definition of failures	The failure definition is clear and unambiguous, so that a complete analysis of failure	$S_1 = 3$
		modes could be conducted.	
		The failure definition is defined, but unclear or ambiguous.	$S_1 = 1$
		The failures are undefined.	$S_1 = 0$
$S_2$	Coverage of failure modes	The considered failure modes cover all that have occurred historically in the products	$S_2 = 3$
		with similar functions, use and environmental conditions.	
		A few uncritical failure modes are not considered in the analysis.	$S_2 = 1$
		A lot of critical failure modes are not considered in the analysis.	$S_2 = 0$
$S_3$	Completeness of failure mode analysis	The considered failure modes include both catastrophic failures as well as degradation	$S_3 = 3$
		failures.	
		Only one of the two types is considered.	$S_3 = 1$
		None of the two types is considered.	$S_3 = 0$
$S_4$	Credibility of information sources	The considered failure modes come from real historical data of the product or similar	$S_4 = 3$
		products.	
		The considered failure modes come from literature.	$S_4 = 1$
		The considered failure modes come from expert judgements.	$S_4 = 0$
$S_5$	Completeness of failure cause analysis	The analysis takes into account all the possible failure causes.	$S_5 = 3$
		A few noncritical failure causes are not considered in the analysis.	$S_5 = 1$
		A lot of critical failure causes are not considered in the analysis.	$S_{5} = 0$
$S_6$	Completeness of failure effect analysis	Failure effects in both component and system levels are analyzed.	$S_6 = 3$
		Only one of the two levels is considered.	$S_6 = 1$
		Failure effects are not considered in the analysis.	$S_{6} = 0$
$S_7$	Credibility of data sources	Criticality analysis is based on field data.	$S_7 = 3$
		Criticality analysis is based on data from literature.	$S_7 = 1$
		Criticality analysis is based on data from expert judgements.	$S_7 = 0$
$S_8$	Effectiveness of design improvements	Over 90% of the exposed design weaknesses are modified.	$S_8 = 3$
		The modified design weaknesses are between 60%-90%.	$S_8 = 1$
		Less than $60\%$ of the exposed design weaknesses are modified.	$S_8 = 0$

for  $y_1, y_2, \dots, y_n$  can be determined. Then, the state of knowledge about the potential failures of the component or system can be evaluated as the weighted average of  $y_i, i = 1, 2, \dots, n$ :

$$y = \sum_{i=1}^{n} \omega_i y_i,\tag{11}$$

where  $\omega_i$  is the relative importance of the *i*th engineering activity for the characterization of the potential failure behaviors, where  $\sum_{i=1}^{n} \omega_i = 1$ .

2) Determination of EUF: Having determined the value of y, we need to define a function  $\sigma_e = h(y)$ , through which  $\sigma_e$  is determined. Since  $\sigma_e$  is a measure of the severity of epistemic uncertainty and y measures the state of knowledge,  $\sigma_e$  is negatively dependent on y. Theoretically, any monotonic decreasing function of y could serve as h(y). In practice, the form of h(y) reflects the decision maker attitude towards epistemic uncertainty and is related to the complexity of the

product. Therefore, we propose h(y) to be

$$h(y) = \begin{cases} \frac{1}{3\sqrt{y}} \cdot m_d, & \text{for simple products;} \\ \frac{1}{3y^6} \cdot m_d, & \text{for complex products;} \\ \frac{1}{3y^2} \cdot m_d, & \text{for medium complex products.} \end{cases}$$
(12)

By letting  $\sigma_m = 0$  and  $m_d$  fixed to a constant value, the attitudes of the decision maker for different products can be investigated (see Figure 6):

- for simple products,  $R_B$  is a convex function of y, indicating that even when y is small, we can gather enough knowledge on the product function and failure behaviors, so that we can assign a high value to the belief reliability;
- for complex products,  $R_B$  is a concave function of y, indicating that only when y is large we can gather sufficient knowledge on the product function and failure behaviors, so that we can assign a high value to the belief



Fig. 6. Different attitudes of the decision maker towards epistemic uncertainty

reliability;

• the h(y) for medium complex products lies between the two extremes.

#### **IV. CASE STUDIES**

In this section, we apply the developed belief reliability to evaluate the reliability of two engineering components/systems, *i.e.*, a Hydraulic Servo Actuator (HSA) in Section IV-A and a Single Board Computer (SBC) in Section IV-B. A comparison is also made on both cases with respect to the traditional probabilistic reliability metrics.

#### A. Hydraulic Servo Actuator (HSA)

The HSA considered in this paper comprises the six components, as listed in Table III. The schematic of the HSA is given in Figure 7.



Fig. 7. Schematic of the AMESim model to predict  $p_{\text{HSA}}$ 

The required function of the HSA is to transform input electrical signals,  $x_{input}$ , into the displacement of the hydraulic

cylinder (HC). The performance parameter of the HSA is the attenuation ratio measured in dB:

$$p_{\rm HSA} = -20 \lg \frac{A_{\rm HC}}{A_{\rm obj}},\tag{13}$$

where,  $A_{\rm HC}$  denotes the amplitude of the HC displacements when input signal  $x_{\rm input}$  is a sinusoidal signal, and  $A_{\rm obj}$  is the objective value of  $A_{\rm HC}$ . Failure occurs when  $p_{\rm HSA} \ge p_{th} =$ 3(dB). The belief reliability of the HSA is evaluated following the procedures in Figure 5.

1) Performance Margin Model Development: The performance margin model is developed in two steps. First, a model for the  $p_{\text{HSA}}$  is developed based on hydraulic principles, with the help of commercial software AMESim. The AMESim model is given in Figure 7. Coherently with (2), the model in Figure 7 is written as

$$p_{\rm HSA} = g_{\rm HSA}(\mathbf{x}_{\rm HSA}). \tag{14}$$

Second, as  $p_{\text{HSA}}$  is a STB performance parameter, the performance margin of the HSA can be determined according to (1):

$$m_{\rm HSA} = \frac{1}{p_{th}} \left( p_{th} - g_{\rm HSA}(\mathbf{x}_{\rm HSA}) \right). \tag{15}$$

2) Aleatory Uncertainty Evaluation: The  $\mathbf{x}_{\text{HSA}}$  comprises six parameters, namely, the clearances on diameters (CoDs) of the six components of the HSA. The CoDs are subject to aleatory uncertainties from production and manufacturing processes, which are quantified by the tolerances in Table III. For simplicity of illustration, it is assumed that all the six parameters follow normal distributions. Following the ' $3\sigma$ ' principle (for references, see [40]), the probability density function for each parameter is determined and given in Table III. The value of  $m_d$  is calculated by (4), where the nominal values are given in Table III. The resulting  $m_d$  is 0.6928 (dB). The values of  $\sigma_m$  is determined using Monte Carlo simulations with a sample size N = 3000. The resulting  $\sigma_m$  is 0.0353 (dB).

3) Epistemic Uncertainty Evaluation: Then, we need to determine the value of  $\sigma_e$ . In the development of the HSA, five EU-related engineering activities, i.e., FMECA, FRA-CAS, RGT, RET and RST have been conducted. Let  $y_i$ ,  $i = 1, 2, \dots, 5$  denote the five engineering activities, respectively. The values of  $y_i$ s can be determined by evaluating the effectiveness of these engineering activities, based on the procedures illustrated in Section III-B1. The result is  $y_1 = 0.70$ ,  $y_2 = 0.90$ ,  $y_3 = 0.80$ ,  $y_4 = 0.85$ ,  $y_5 = 0.70$ . In this case study, the engineering activities are assumed to have equal weights,  $\omega_1 = \omega_2 = \cdots = \omega_5 = 1/5$ , and then, according to (11), y = 0.79. Since the HSA has medium complexity, according to (12),

$$\sigma_e = \frac{1}{3y^2} \cdot m_d = 0.3700. \tag{16}$$

4) Belief Reliability Evaluation: Finally, the belief reliability can be predicted using (8) and the result is shown in Table IV. If we only consider the aleatory uncertainty, probabilistic reliability can be predicted using (3), whose value is also presented in Table IV for comparisons. The result shows that,

TABLE III COMPONENTS AND TOLERANCES OF THE HSA

Component	ESV	Spool 1	Spool 2	Spool 3	Spool 4	HC
Parameters	CoD $x_1$	CoD $x_2$	CoD $x_3$	CoD $x_4$	CoD $x_5$	CoD $x_6$
Tolerances $(\times 10^{-3} \text{mm})$	$1\pm 0.015$	$7\pm0.15$	$7\pm0.15$	$7\pm0.15$	$7\pm0.15$	$10\pm1.5$
Distributions	$N(1, 0.005^2)$	$N(7, 0.05^2)$	$N(7, 0.05^2)$	$N(7, 0.05^2)$	$N(7, 0.05^2)$	$N(10, 0.5^2)$

ESV: Electrohydraulic servo valve

HC: Hydraulic cylinder

as expected, epistemic uncertainty reduces our confidence that the product will perform its function as designed, whereas probabilistic reliability would lead to overconfidence.

TABLE IV Comparison between probabilistic reliability and belief reliability

Types of reliability measures	Results
Probabilistic reliability calculated by (3)	0.9999
Belief reliability calculated by (8)	0.9688

Another major difference between belief reliability and probabilistic reliability is that belief reliability allows for the consideration of EU-related engineering activities in the reliability assessment, which are neglected in the probabilitybased reliability evaluation. For example, if the effectiveness of the EU-related engineering activities is increased from  $y_1 = 0.70, y_2 = 0.90, y_3 = 0.80, y_4 = 0.85, y_5 = 0.70$ to  $y_1 = y_2 = \cdots = y_5 = 0.9$ , then, the belief reliability will increase from  $R_{B,0} = 0.9688$  to  $R_{B,1} = 0.9921$ . In other words, in order to enhance the belief reliability, one not only needs to increase the design margin and reduce aleatory uncertainty by design, but also needs to reduce epistemic uncertainty by improving the state of knowledge, whereas probabilistic reliability focuses only on the former two aspects.

#### B. Single Board Computer

A SBC, as shown in Figure 8 [41], is chosen to demonstrate the time-dependent belief reliability analysis for electrical systems.



Fig. 8. A SBC [41]

A probabilistic reliability analysis was conducted in [41] based on the parts-counting reliability prediction method in

[42]. The times to failure of both the components are assumed to be exponentially distributed and their failure rates are predicted based on the database in [42], as shown in Table V. The failure rate of the SBC can, then, be calculated by summing over all the components' failure rates. Hence, the predicted probabilistic reliability is

$$R_p(t) = \exp\{-1.186 \times 10^{-6}t\},\tag{17}$$

where the unit of t is hour.

 TABLE V

 PREDICTED FAILURE RATES OF THE SBC [41]

Components	Number	Predicted failure rate ( $\times 10^{-9}$ (h <sup>-1</sup> ))
IC	51	384.8
Crystal oscillator	4	56
Inductance	6	6.56
Connector	9	32.76
Capacitor	631	40.60
Resistance	545	648.91
Others	20	16.16
Total	1266	1186

The probabilistic reliability in (17) is a time-dependent function. To further evaluate the belief reliability, first note by substituting (5) into (8), we have

$$R_B = \frac{1}{\sqrt{\left(\frac{1}{Z_{R_p}}\right)^2 + \left(\frac{\sigma_e}{m_d}\right)^2}}.$$
(18)

Since  $R_p$  is time-dependent, the belief reliability is also a time-dependent function and can be calculated by using (18) recursively at each time t:

$$R_B(t) = \frac{1}{\sqrt{\left(\frac{1}{Z_{R_p(t)}}\right)^2 + \left(\frac{\sigma_e}{m_d}\right)^2}},$$
(19)

where  $R_p(t)$  is the time-dependent probabilistic reliability and  $\sigma_e$  is the EUF evaluated using the procedures in Section III-B.

The effectiveness of the five EU-related engineering activities, i.e., FMECA, FRACAS, RGT, RET and RST, can be assessed using the procedures illustrated in Section III-B1:  $y_1 = 0.60, y_2 = 0.80, y_3 = 0.70, y_4 = 0.75, y_5 = 0.55$ . As the previous case study, we also assume that the five activities have equal weights. From (11), y = 0.68. By assessing the configuration of the SBC, it is determined that it has medium complexity. Therefore, by substituting (12) and (17) into (19), the belief reliability of the SBC can be calculated, as shown in Figure 9.



Fig. 9. Belief reliability of the SBC



Fig. 10. Belief reliability of the SBC

It can be seen from Figure 9 that the belief reliability curve is more close to  $R_B = 0.5$  than the probabilistic reliability. This is because  $R_B = 0.5$  corresponds to the state of maximum uncertainty, since we cannot differentiate whether the system is more likely to be working or failure (for details, please refer to maximum uncertainty principle in [37]). Since belief reliability considers the influence of epistemic uncertainty, it yields a more uncertain result than the probabilistic reliability.

A sensitivity analysis is conducted with respect to y to further investigate the influence of epistemic uncertainty on belief reliability. The results are given in Figure 10. It can be seen from Figure 10 that the value of y significantly impacts  $R_B$ : a larger value of y, which indicates improvements on the effectiveness of the EU-related engineering activities, tends to make the belief reliability moving towards the probabilistic reliability; while a lower value of y tends to make the belief reliability moving towards 0.5, which is the state of maximum uncertainty. This demonstrates that, compared to the traditional probabilistic reliability, belief reliability allows for the explicit consideration of epistemic uncertainty and EUrelated engineering activities in the reliability assessment. In other words, in order to enhance the belief reliability, one not only needs to increase the design margin and reduce aleatory uncertainty by design, but also needs to reduce epistemic uncertainty by improving the state of knowledge.

#### V. CONCLUSION

In this paper, a new metric of belief reliability has been introduced to explicitly incorporate the influence of epistemic uncertainty into model-based methods of reliability assessments. To quantify the effect of epistemic uncertainty, an evaluation method is proposed, based on the effectiveness of engineering activities related to reliability analysis and assessment. The proposed belief reliability evaluation method integrates design margin, aleatory uncertainty and epistemic uncertainty for a comprehensive and systematic characterization of reliability. Two numerical case studies demonstrate the benefits of belief reliability compared to the traditional probability-based reliability metrics, with the explicit consideration of epistemic uncertainty.

Compared to the traditional probabilistic reliability metrics, belief reliability explicitly considers the effect of epistemic uncertainty and allows considering EU-related engineering activities in reliability assessment. We believe that as a new reliability metric, belief reliability is beneficial in reliability engineering practices, since epistemic uncertainty is a severe problem for real-world products, especially for those in design and development phases. An interesting future work is to define a mathematical theory to model belief reliability and its time-dependence. Various mathematical theories dealing with epistemic uncertainty can be considered, *e.g.*, Bayesian theory, evidence theory, possibility theory, uncertainty theory, *etc.* Besides, methods of scoring the effectiveness of engineering activities should be further investigated.

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#### REFERENCES

- C. Ebeling, An introduction to reliability and maintainability engineering. Long Grove, Ill.: Waveland Press,, 2nd ed. ed., 2010.
- [2] W. Meeker and L. Escobar, *Statistical methods for reliability data*. New York :: Wiley, 1998. "A Wiley-Interscience publication.".
- [3] T. Aven, P. Baraldi, R. Flage, and E. Zio, Uncertainty in Risk Assessment: The Representation and Treatment of Uncertainties by Probabilistic and Non-probabilistic Methods. John Wiley and Sons, 2013.
- [4] M. Pecht, D. Dasgupta, A., and C. Leonard, "The reliability physics approach to failure prediction modelling," *Quality and Reliability Engineering International*, vol. 6, no. 4, pp. 267–273, 1990.
- [5] J. Collins, H. Busby, and G. Staab, *Mechanical design* of machine elements and machines. Wiley, 2009.

- [6] Z. Mohaghegh and M. Modarres, "A probabilistic physics-of-failure approach to common cause failures in reliability assessment of structures and components," in *Transactions of the American Nuclear Society*, vol. 105 of *Transactions of the American Nuclear Society*, pp. 635–637, 2011.
- [7] E. Zio, *The Monte Carlo simulation method for system reliability and risk analysis.* Springer, 2013.
- [8] D. Zhang, X. Han, C. Jiang, J. Liu, and Q. Li, "Timedependent reliability analysis through response surface method," *Journal of Mechanical Design*, vol. 139, no. 4, p. 041404, 2017.
- [9] T. Aven and B. Krohn, "A new perspective on how to understand, assess and manage risk and the unforeseen," *Reliability Engineering and System Safety*, vol. 121, pp. 1–10, 2014.
- [10] A. Kiureghian and O. Ditlevsen, "Aleatory or epistemic? does it matter?," *Structural Safety*, vol. 31, no. 2, pp. 105–112, 2009.
- [11] Z. Zeng, R. Kang, M. Wen, and Y. Chen, "Measuring reliability during product development considering aleatory and epistemic uncertainty," 2015.
- [12] T. Aven and E. Zio, "Model output uncertainty in risk assessment," *International Journal of Performability En*gineering, vol. 9, no. 5, 2013.
- [13] T. Bjerga, T. Aven, and E. Zio, "An illustration of the use of an approach for treating model uncertainties in risk assessment," *Reliability Engineering and System Safety*, vol. 125, pp. 46–53, 2014.
- [14] e. a. Mosleh, A., Model Uncertainty: Its Characterization and Quantification. Center for Reliability Engineering, University of Maryland, 1995.
- [15] E. Droguett and A. Mosleh, "Bayesian treatment of model uncertainty for partially applicable models," *Risk Analysis*, vol. 34, no. 2, pp. 252–270, 2014.
- [16] E. Zio and G. Apostolakis, "Two methods for the structured assessment of model uncertainty by experts in performance assessments of radioactive waste repositories," *Reliability Engineering and System Safety*, vol. 54, no. 2, pp. 225–241, 1996.
- [17] D. Draper, "Assessment and propagation of model uncertainty," *Journal of the Royal Statistical Society. Series B* (*Methodological*), pp. 45–97, 1995.
- [18] E. Droguett and A. Mosleh, "Bayesian methodology for model uncertainty using model performance data," *Risk Analysis*, vol. 28, no. 5, pp. 1457–1476, 2008.
- [19] G. Apostolakis, "The concept of probability in safety assessments of technological systems," *Science*, vol. 250, no. 4986, p. 1359, 1990.
- [20] I. Park and R. V. Grandhi, "Quantifying multiple types of uncertainty in physics-based simulation using bayesian model averaging," *AIAA Journal*, vol. 49, no. 5, pp. 1038–1045, 2011.
- [21] R. Zhang and S. Mahadevan, "Model uncertainty and bayesian updating in reliability-based inspection," *Structural Safety*, vol. 22, no. 2, pp. 145–160, 2000.
- [22] S. M. Sabatine, J. D. Niemann, and B. P. Greimann, "Evaluation of parameter and model uncertainty in simple

applications of a 1d sediment transport model," *Journal of Hydraulic Engineering*, vol. 141, no. 5, p. 04015002, 2015.

- [23] S. B. Kim, S. M. Bartell, and D. L. Gillen, "Estimation of a benchmark dose in the presence or absence of hormesis using posterior averaging," *Risk Analysis*, vol. 35, no. 3, pp. 396–408, 2015.
- [24] A. V. Dyrrdal, A. Lenkoski, T. L. Thorarinsdottir, and F. Stordal, "Bayesian hierarchical modeling of extreme hourly precipitation in norway," *Environmetrics*, vol. 26, no. 2, pp. 89–106, 2015.
- [25] M. E. Riley and R. V. Grandhi, "Quantification of modelform and predictive uncertainty for multi-physics simulation," *Computers and Structures*, vol. 89, no. 11-12, pp. 1206–1213, 2011.
- [26] A. Mosleh and G. Apostolakis, "The assessment of probability distributions from expert opinions with an application to seismic fragility curves," *Risk Analysis*, vol. 6, no. 4, pp. 447–461, 1986.
- [27] C. Corey Fischer and R. V. Grandhi, "Utilizing an adjustment factor to scale between multiple fidelities within a design process: A stepping stone to dialable fidelity design," in 16th AIAA Non-Deterministic Approaches Conference, 16th AIAA Non-Deterministic Approaches Conference, 2014.
- [28] I. Park and R. V. Grandhi, "A bayesian statistical method for quantifying model form uncertainty and two model combination methods," *Reliability Engineering and System Safety*, vol. 129, pp. 46–56, 2014.
- [29] M. E. Riley, R. V. Grandhi, and R. Kolonay, "Quantification of modeling uncertainty in aeroelastic analyses," *Journal of Aircraft*, vol. 48, no. 3, pp. 866–873, 2011.
- [30] I. Park, H. K. Amarchinta, and R. V. Grandhi, "A bayesian approach for quantification of model uncertainty," *Reliability Engineering and System Safety*, vol. 95, no. 7, pp. 777–785, 2010.
- [31] C. Carlson, Effective FMEAs: Achieving safe, reliable, and economical products and processes using failure mode and effects analysis, vol. 1. John Wiley and Sons, 2012.
- [32] D. W. Benbow and H. W. Broome, *The Certified Reliability Engineer Handbook*. ASQ Quality Press, 2012.
- [33] G. Yang, *Life cycle reliability engineering*. Hoboken, N.J. :: John Wiley and Sons, 2007.
- [34] M. Pecht and A. Dasgupta, "Physics-of-failure: an approach to reliable product development," in *Integrated Reliability Workshop*, Integrated Reliability Workshop, (Lake Tahoe, US), pp. pp. 1–4, IEEE, 1995. Integrated Reliability Workshop, 1995. Final Report., International.
- [35] Y. Chen, L. Gao, and R. Kang, "Research on reliability simulation prediction of electronic product based on physisc of failure model," *Journal of CAEIT*, vol. 8, no. 5, pp. 444–448, 2013.
- [36] Z. Zeng, R. Kang, and Y. Chen, "A physics-of-failurebased approach for failure behavior modeling: With a focus on failure collaborations," 2014.
- [37] B. Liu, Uncertainty Theory: A Branch of Mathematics for Modeling Human Uncertainty. Berlin: Springer-Verlag,

2010.

- [38] Z. Zeng, R. Kang, and Y. Chen, "Using pof models to predict system reliability considering failure collaboration," *Chinese Journal of Aeronautics*, vol. 29, no. 5, pp. 1294–1301, 2016.
- [39] Z. Zeng, Y. Chen, E. Zio, and R. Kang, "A compositional method to model dependent failure behavior based on pof models," *Chinese Journal of Aeronautics*, 2017.
- [40] G. Zhai, Y. Zhou, and X. Ye, "A tolerance design method for electronic circuits based on performance degradation," *Quality and Reliability Engineering International*, no. Online first, p. DOI: 10.1002/qre.1621, 2013.
- [41] C. Ran, C. Ying, and K. Rui, "Electronic product reliability prediction methods and case studies based on bellcore standards [j]," *Electronics Quality*, vol. 6, p. 026, 2010.
- [42] T. SR332, "Reliability prediction procedure for electronic equipment," 2001.



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#### ACRONYMS

AUF	Aleatory Uncertainty Factor
ESV	Electrohydraulic Servo Valve
EU	Epistemic Uncertainty
EUF	Epistemic Uncertainty Factor
FMECA	Failure Mode, Effect and Criticality Analysis
FRACAS	Failure Report, Analysis, and Corrective Action System
HC	Hydraulic Cylinder
HSA	Hydraulic Servo Actuator
LTB	Larger-the-better
NTB	Nominal-the-better
RGT	Reliability Growth Test
RET	Reliability Enhancement Test
RST	Reliability Simulation Test
SBC	Single Board Computer
STB	Smaller-the-better

#### NOTATIONS

- *m* Performance margin
- *p* Performance parameter
- $p_{th}$  Functional threshold
- $R_B$  Belief reliability
- *R<sub>p</sub>* Probabilistic reliability
- $m_d$  Design margin
- $\sigma_m$  Aleatory uncertainty factor
- $\sigma_e$  Epistemic uncertainty factor
- y Effectiveness of the EU-related engineering activ
  - ities