A Model-Based Reliability Metric Considering Aleatory and Epistemic Uncertainty

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Zhiguo Zeng,[‡] Rui Kang,[†*], Meilin Wen[†], Enrico Zio,[‡,§]
[‡]Chair System Science and the Energy Challenge, Fondation Electricité de France (EDF), CentraleSupélec, Université Paris-Saclay, Grande Voie des Vignes, 92290 Chatenay-Malabry, France
[†]School of Reliability and Systems Engineering, Beihang University, Beijing, China
[§]Energy Department, Politecnico di Milano, Milano, Italy

***Abstract*—Model-based reliability analysis and assessment methods rely on models, which are assumed to be precise, to predict reliability. In practice, however, the precision of the model cannot be guaranteed due to the presence of epistemic uncertainty. In this paper, a new reliability metric, called belief reliability, is defined to explicitly account for epistemic uncertainty in model-based reliability analysis and assessment. A new method is developed to explicitly quantify epistemic uncertainty by measuring the effectiveness of the engineering analysis and assessment activities related to reliability. To evaluate belief reliability, an integrated framework is presented, where the contributions of design margin, aleatory uncertainty and epistemic uncertainty are integrated to yield a comprehensive and systematic description of reliability. The developed methods are demonstrated by two case studies.**

***Index Terms*—Reliability, physics-of-failure, epistemic uncertainty, model uncertainty, belief reliability**

I. Introduction

Reliability refers to the ability of a component or system to perform a required function for a given period of time when used under stated operating conditions [1]. Traditionally, reliability is measured by the probability that functional failure does not occur in the considered period of time and failure data are used for its estimation based on statistical methods [2]. In practice, however, failure data are often scarce (if available at all), which defies the use of classical statistical methods and challenges Bayesian methods with respect to the assumption of subjective prior distributions [3]. Due to the problem of limited failure data, model-based methods (*cf.* physics-of-failure (PoF) methods [4], structural reliability methods [5], *etc*.) are widely applied to predict reliability, by deterministically describing the degradation and failure processes using deterministic failure behavior models. More specifically, it is assumed that:

1) the failure behavior of a component or a system can be described by a deterministic model;
2) random variations in the variables of the deterministic model are the sole source of uncertainty.

The probabilistic quantification of reliability is, then, obtained by propagating uncertainties through the model analytically or numerically, *e.g.* by Monte Carlo simulation [6–8].

The random variations represent the uncertainty inherent in the physical behavior of the system and are referred to as aleatory uncertainty [9]. However, the model-based methods are also subject to epistemic uncertainty due to incomplete knowledge on the degradation and failure processes [10, 11]. According to Aven and Zio [12, 13], epistemic uncertainty may arise because:

1) the deterministic model cannot exactly describe the failure process, *e.g.*, due to incomplete understanding of the failure causes and mechanisms (model uncertainty, also known as structural uncertainty);
2) the precise values of the model parameters might not be accurately estimated due to lack of data in the actual operational and environmental conditions (parameter uncertainty).

In this paper, we introduce a new reliability metric, belief reliability, to explicitly consider the effect of epistemic uncertainty on the model-based methods. For illustrative purposes, we consider only model uncertainty in this paper. However, the framework can be easily extended to deal with parameter uncertainty.

In literature, various approaches have been developed to consider model uncertainty. Mosleh and Droguett reviewed a number of approaches for model uncertainty assessment and compared them in terms of theoretical foundations and domains of application [14, 15]. Among them, the alternate hypotheses approach and the adjustment factor approach are two most widely applied ones [16]. The alternate hypotheses approach identifies a family of possible alternate models and probabilistically combines the predictions of them based on Bayesian model averaging, where the probability of each model is evaluated from experimental data or expert judgements [17, 18]. Apostolakis [19] addressed the issue of model uncertainty in probabilistic risk assessment using the alternate hypotheses approach. Park and Grandhi [20] quantified the model probability in the alternate hypotheses approach by the measured deviations between experimental data and model predictions. In [21], two crack models were probabilistically combined using the alternate hypotheses approach to estimate the failure probability of a butt weld. Other applications of the alternate hypotheses approach include sediment transport models [22], identification of benchmark doses [23], precipitation modeling [24], *etc*.

In the adjustment factor approach, the model uncertainty is addressed by modifying a benchmark model (the one that

* Email of the corresponding author: kangrui@buaa.edu.cn

we have highest confidence in) with an adjustment factor, which is assumed to be uncertain, and is either added to or multiplied by the prediction results of the model [16, 25]. In [26], the adjustment factor approach was used to combine experts' estimates according to Bayes' theorem. Zio and Apostolakis [16] used the approach to assess the risk of radioactive waste repositories. Fischer and Grandhi [27] applied an adjustment factor to low-fidelities models so as to scale them to high-fidelity models. In a series of studies conducted by Park and Grandhi [25, 28–30], the adjustment factor approach was combined with the alternate hypotheses approach by introducing an adjustment factor to quantify the uncertainty in each alternate model; the model uncertainty was, then, evaluated by averaging all the models according to the alternate hypotheses approach.

The alternate hypotheses approach requires enumerating a set of mutually exclusive and collectively exhaustive models [15]. In the case of model-based reliability methods, however, it is impossible for us to enumerate all the possible models, which limits the application of the alternate hypotheses approach. Hence, we adopt the adjustment factor approach in this paper to develop a new reliability metric to describe the effect of epistemic uncertainty (model uncertainty) on the model-based reliability methods.

In the adjustment factor approaches, epistemic uncertainty is quantified by the adjustment factor, which is often determined based on validation test data (for example, see [18] or [30]). In practice, however, due to limited time and resources, it is hard, if not impossible, to gather sufficient validation test data. Resorting to expert judgements might offer an alternative solution (for example, see [16]), but they could be criticized for being too subjective. On the other hand, epistemic uncertainty relates to the knowledge on the component or system functions and failure behaviors: as this knowledge is accumulated, epistemic uncertainty is reduced. In the life cycle of a component or system, the knowledge is gained by implementing a number of reliability analysis-related engineering activities, whose purpose is to help designers better understand potential failure modes and mechanisms. For example, through Failure Mode, Effect and Criticality Analysis (FMECA), potential failure modes and their effects could be identified, so that the designer can better understand the product's failure behaviors [31]. Similar engineering activities include Failure Report, Analysis, and Corrective Action System (FRACAS) [32], Reliability Growth Test (RGT) [33], Reliability Enhancement Test (RET) [32], Reliability Simulation Test (RST) [34, 35], *etc*. In this paper, we develop a new quantification method for the epistemic uncertainty in the adjustment factor method, based on the effectiveness of these engineering activities.

The contributions of this paper are summarized as follows:

1) a new reliability metric, the belief reliability, is developed to explicitly consider epistemic uncertainty in the model-based reliability methods;
2) a new method is developed to quantify epistemic uncertainty, based on the effectiveness of the engineering activities related to the reliability analysis and assessment of components and systems;
3) a method is developed to evaluate the belief reliability of components and systems, based on the integration of design margin, aleatory uncertainty and epistemic uncertainty.

The rest of the paper is organized as follows. In section II, belief reliability is defined to account for the effect of epistemic uncertainty in model-based reliability methods. In section III-B, epistemic uncertainty is quantified based on the effectiveness of the related engineering activities and a belief reliability evaluation method is developed. Section IV presents two case studies to demonstrate the developed methods. Finally, the paper is concluded in section V with a discussion on future works.

II. Definition of Belief Reliability

In this section, we introduce a new metric of reliability, belief reliability, to explicitly account for the influence of epistemic uncertainty on model-based reliability methods. We start with a brief introduction of the model-based reliability method in subsection II-A. Then, belief reliability is defined in subsection II-B.

A. Model-based reliability methods

For a general description of model-based reliability methods, we introduce the concepts of performance parameter and performance margin:

Definition 1 (Performance parameter)**.** Suppose failure occurs when a parameter p reaches a threshold value p_{th}. Then, the parameter p is referred to as a performance parameter, while the threshold value p_{th} is referred to as the functional failure threshold associated with p.

According to *Definition* 1, performance parameters and functional failure thresholds define the functional requirements on a system or a component, for which three categories exist in practice:

1) Smaller-the-better (STB) parameters: if failure occurs when $p \geq p_{th}$, then, the performance parameter p is a STB parameter.
2) Larger-the-better (LTB) parameters: if failure occurs when $p \leq p_{th}$, then, the performance parameter p is a LTB parameter.
3) Nominal-the-better (NTB) parameters: if failure occurs when $p \leq p_{th,L}$ or $p \geq p_{th,U}$, then, the performance parameter p is a NTB parameter.

Definition 2 (Performance margin)**.** Suppose p is a performance parameter and p_{th} is its associated functional failure threshold; then,

$$m = \begin{cases} \dfrac{p_{th} - p}{p_{th}}, & \text{if } p \text{ is STB,} \\ \dfrac{p - p_{th}}{p_{th}}, & \text{if } p \text{ is LTB,} \\ \min\left(\dfrac{p_{th,U} - p}{p_{th,U}}, \dfrac{p - p_{th,L}}{p_{th,L}}\right), & \text{if } p \text{ is NTB} \end{cases} \tag{1}$$

is defined as the (relative) performance margin associated with the performance parameter p.

Remark 1. From Definition 2, performance margin is a unitless quantity and failure occurs whenever $m \leq 0$.

In the model-based reliability methods, it is assumed that the performance margin can be described by a deterministic model, which is derived based on knowledge of the functional principles and failure mechanisms of the component [5, 36]. Conceptually, we assume that the performance margin model has the form

$$m = g_m(\mathbf{x}), \tag{2}$$

where $g_m(\cdot)$ denotes the deterministic model which predicts the performance margin and $\mathbf{x}$ is a vector of input variables.

In the design and manufacturing processes of a product, there are many uncertain factors influencing the input $\mathbf{x}$ of (2). Thus, the values of $\mathbf{x}$ may vary from product to product of the same type. Usually, this product-to-product variability is described by assuming that $\mathbf{x}$ is a vector of random variables with given probability density functions. Then, m is also a random variable and reliability R_p is defined as the probability that m is greater than zero. The subscript p is used to indicate that R_p is a probability measure. Given the probability density function of $\mathbf{x}$, denoted by $f_X(\cdot)$, R_p can be calculated by:

$$R_p = Pr\left(g_m(\mathbf{x}) > 0\right) = \int \cdots \int_{g_m(\mathbf{x})>0} f_X(\mathbf{x}) \mathrm{d}\mathbf{x}. \tag{3}$$

B. Definition of belief reliability

Belief reliability is defined in this subsection to explicitly account for the effect of epistemic uncertainty in model-based reliability methods. For this, we first define design margin and Aleatory Uncertainty Factor (AUF):

Definition 3 (Design margin)**.** Suppose the performance margin of a component or a system can be calculated by (2). Then, design margin m_d is defined as

$$m_d = g_m(\mathbf{x}_N), \tag{4}$$

where $\mathbf{x}_N$ is the nominal values of the parameters.

Definition 4 (Aleatory Uncertainty Factor (AUF))**.** Suppose R_p is the probabilistic reliability calculated from the performance margin model using (3). Then, AUF σ_m is defined as

$$\sigma_m = \frac{m_d}{Z_{R_p}}, \tag{5}$$

where Z_{R_p} is the value of the inverse cumulative distribution function of a standard normal distribution evaluated at R_p.

Further, let equivalent design margin M_E to be

$$M_E = m_d + \epsilon_m, \tag{6}$$

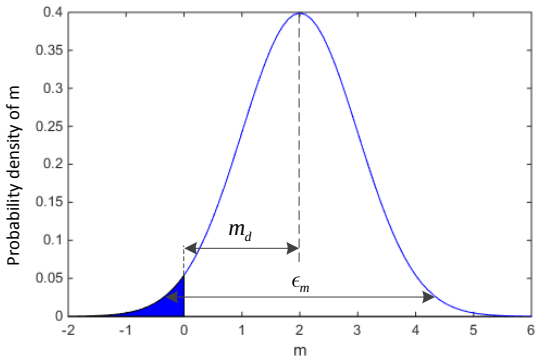
where $\epsilon_m \sim \text{Normal}(0, \sigma_m^2)$. It is easy to verify that $M_E \sim \text{Normal}(m_d, \sigma_m^2)$ and R_p can be calculated as the probability that $M_E > 0$, as shown in Figure 1 (a). Therefore, the probabilistic reliability can be quantified by the equivalent performance margin and further by m_d and σ_m, where

- m_d describes the inherent reliability of the product when all the input variables take their nominal values. Graphically, it measures the distance from the center of the equivalent performance margin distribution to the boundaries of the failure region, as shown in Figure 1 (a);
- σ_m accounts for the uncertainty resulting from the product-to-product random variations, *e.g.* the tolerance of manufacturing processes, the variability in material properties, *etc*. Usually, these random variations are controlled by engineering activities such as tolerance design, environmental stress screening, stochastic process control, *etc* [11].

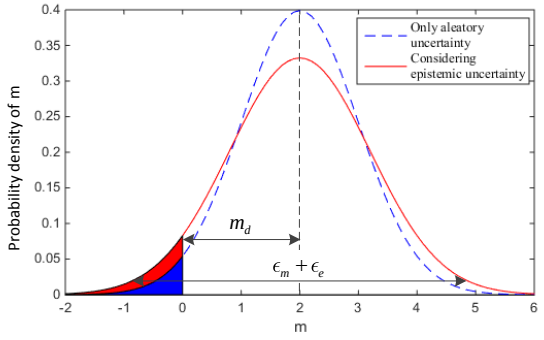
To further account for the effect of epistemic uncertainty, it is assumed that:

$$M_E = m_d + \epsilon_m + \epsilon_e, \tag{7}$$

where ϵ_e is an adjustment factor [16] and $\epsilon_e \sim \text{Normal}(0, \sigma_e^2)$. Parameter σ_e is defined as Epistemic Uncertainty Factor (EUF) and it quantifies the effect of epistemic uncertainty. The physical meaning of (7) is explained in Figure 1 (b): epistemic uncertainty introduces additional dispersion to the aleatory distribution of the equivalent performance margin. The degree of the dispersion is related to the knowledge we have on the failure process of the product, *i.e.*, the more knowledge we have, the less value σ_e takes.



(a) Aleatory distribution



(b) Effect of epistemic uncertainty

Fig. 1. Epistemic uncertainty effect on the distribution of the equivalent performance margin

Considering the assumption made in (7), we can, then, define the belief reliability as follows:

Definition 5 (Belief reliability)**.** The reliability metric

$$R_B = \Phi_N\left(\frac{m_d}{\sqrt{\sigma_m^2+\sigma_e^2}}\right) \tag{8}$$

is defined as belief reliability, where $\Phi_N(\cdot)$ is the cumulative distribution function of a standard normal random variable.

Belief reliability can be interpreted as our belief degree on the product reliability, based on the knowledge of design margin, aleatory uncertainty and epistemic uncertainty. In the following, we discuss respectively how design margin, aleatory uncertainty and epistemic uncertainty influence the value of belief reliability.

Discussion 1. It is obvious from (8) that $R_B \in [0,1]$, where

- $R_B = 0$ indicates that we believe for sure that a component or system is unreliable, *i.e.*, it cannot perform its desired function under stated time period and operated conditions.
- $R_B = 1$ indicates that we believe for sure that a component or system is reliable, *i.e.*, it can perform its desired function under stated time period and operated conditions.
- $R_B = 0.5$ indicates that we are most uncertain about the reliability of the component or system [37].
- $R_{B,A} > R_{B,B}$ indicates that we believe that product A is more reliable than product B.

Discussion 2 (Variation of R_B with the design margin). From (8), it is easy to see that R_B is an increasing function of m_d, as illustrated by Figure 2, which is in accordance with the intuitive fact that when the design margin is increased, the component or system becomes more reliable.

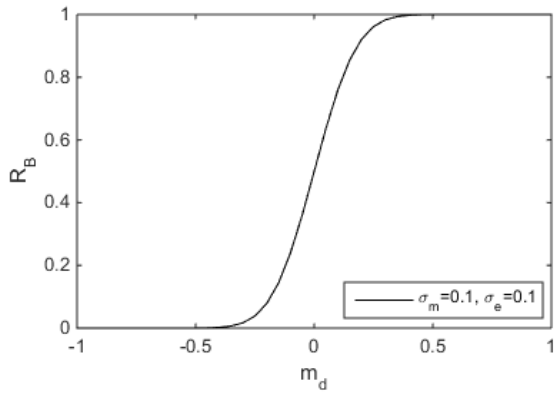


Fig. 2. Influence of m_d on R_B

Besides, it can be verified from (8) that if $m_d = 0$, $R_B = 0.5$. This is because when $m_d = 0$, the product is at borderline between working and failure. Therefore, we are most uncertain about its reliability (For details, please refer to the maximum uncertainty principle in [37]).

Discussion 3 (Variation of R_B with the aleatory uncertainty). In (8), the effect of aleatory uncertainty is measured by the AUF, σ_m. Figure 3 shows the variation of R_B with σ_m, when σ_e is fixed, for different values of m_d. It can be seen from Figure 3 that when m_d and σ_e are fixed, R_B approaches 0.5 as σ_m increases to infinity. The result is easy to understand, since $\sigma_m \to \infty$ indicates the fact that uncertainty has the greatest influence.

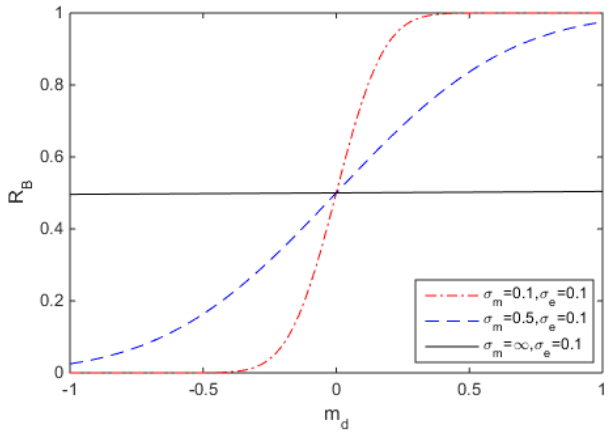


Fig. 3. Variation of R_B with σ_m

Discussion 4 (Variation of R_B with the epistemic uncertainty). In (8), the effect of epistemic uncertainty is measured by the EUF, σ_e. The variation of R_B with respect to σ_e is illustrated in Figure 4, with σ_m fixed to 0.2. From Figure 4, we can see that when $\sigma_e \to \infty$, R_B also approaches 0.5, for the same reason as the AUF.

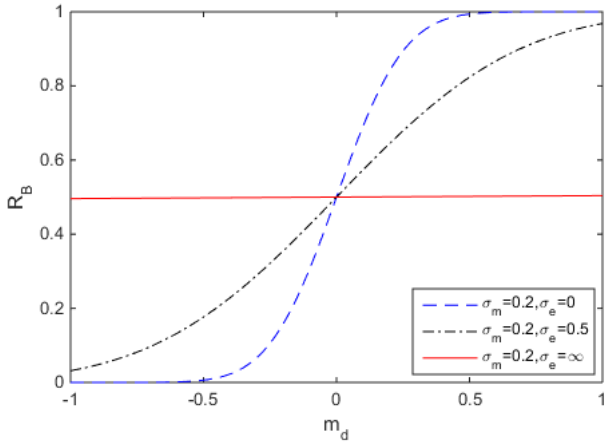


Fig. 4. Variation of R_B with σ_e

Besides, it can be shown from (8) and assumption (3) that as $\sigma_e \to 0$, R_B approaches the R_p calculated by the model-based reliability methods using equation (3). This is a natural result since $\sigma_e = 0$ is the ideal case for which there is no epistemic uncertainty, so that the product failure behavior is accurately predicted by the deterministic performance margin model and the aleatory uncertainty.

In practice, we always have $m_d \geq 0$ and $\sigma_e > 0$. Therefore,

$$R_B \leq R_p \tag{9}$$

where R_p is the probabilistic reliability predicted by (3) under the same conditions. Equation (9) shows that using belief reliability yields a more conservative evaluation result than using the probabilistic reliability, because belief reliability considers the effect of insufficient knowledge on the reliability evaluations.

III. Evaluation of Belief Reliability

In this section, we discuss how to evaluate the belief reliability for a given product. A general framework for belief reliability evaluation is first given in subsection III-A. Then, a method is presented for evaluating epistemic uncertainty and determining the value of the EUF.

A. Belief reliability evaluation

The R_B defined in (8) incorporates the contributions of design margin m_d, aleatory uncertainty (represented by σ_m) and epistemic uncertainty (represented by σ_e). The contributions from the three factors should be evaluated individually and then, combined to evaluate the belief reliability of a component. Detailed procedures are presented in Figure 5.

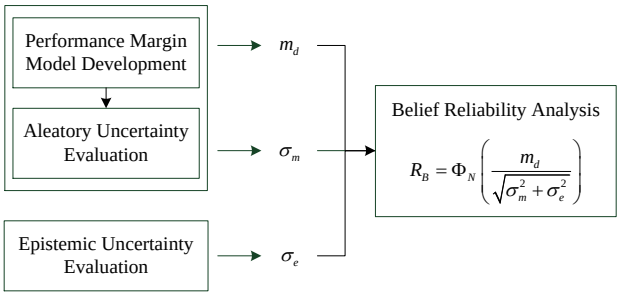


Fig. 5. Procedures for component belief reliability evaluation

Four steps comprise the evaluation procedure:

1) Performance Margin Model Development: First, a deterministic performance margin model is developed to predict the value of the performance margin m. The performance margin model can be developed based on knowledge of underlying functional principles and physics of failures. For a detailed discussion on how to develop performance margin models, readers might refer to [38] and [39].

2) Aleatory Uncertainty Evaluation: Next, the values of m_d and σ_m are determined. The value of m_d is calculated based on (4), where all the input parameters of the performance margin model take their nominal values. To calculated the value of σ_m, the probabilistic reliability R_p is calculated first by propagating aleatory uncertainty in the model parameters according to (3). Either structural reliability methods [5] or Monte Carlo simulations [7] might be used for the calculation. Then, σ_m can be calculated by combining m_d and R_p using (5).

3) Epistemic Uncertainty Evaluation: The value of σ_e is, then, determined by evaluating the effect and potential impact of epistemic uncertainty. In practice, epistemic uncertainty relates to the knowledge on the component or system functions and failure behaviors: as this knowledge is accumulated, epistemic uncertainty is reduced. Hence, in this paper, we relate epistemic uncertainty to our state of knowledge on the product and its failure process and assess the value of σ_e based on the effectiveness of engineering activities that generate our knowledge base. Details on how to evaluate the value of σ_e is given in Section III-B.

4) Belief Reliability Evaluation: Following steps 1) - 3), the values of m_d, σ_m and σ_e are determined. Then, the belief reliability can be evaluated according to (8).

B. Quantification of epistemic uncertainty

In this section, we develop a method to quantify epistemic uncertainty based on the state of knowledge. In subsection III-B1, we discuss how to evaluate the state of knowledge, and then, in subsection III-B2, we quantify the effect of epistemic uncertainty in terms of σ_e.

1) Evaluation of the state of knowledge: In the life cycle of a component or system, the knowledge on the products' failure behavior is gained by implementing a number of engineering activities of reliability analysis, whose purposes are to help designers better understand potential failure modes and mechanisms. In this paper, we refer to these engineering activities as epistemic uncertainty-related (EU-related) engineering activities. Table I lists some commonly encountered EU-related engineering activities and discusses their contributions to gaining knowledge and reducing epistemic uncertainty, where FMECA stands for Failure Mode, Effect and Criticality Analysis, FRACAS stands for Failure Reporting, Analysis, and Corrective Action System, RET stands for Reliability Enhancement Test, RGT stands for Reliability Growth Test and RST stands for Reliability Simulation Test.

In this paper, we make an assumption that the state of knowledge is directly related to the effectiveness of the EU-related engineering activities. Suppose there are n EU-related engineering activities in a product life cycle. Let $y_i, i = 1, 2, \cdots, n$ denote the effectiveness of the EU-related engineering activities, where $y_i \in [0, 1]$; the more effective the engineering activity is, the larger value the corresponding y_i takes. The values of y_i are determined by asking experts to evaluate the effectiveness of the EU-related engineering activities, based on a set of predefined evaluation criteria.

For example, the effectiveness of FMECA can be evaluated based on eight elements, as shown in Table II. For each element, experts are invited to evaluate their performances according to the criteria listed in Table II. Based on the evaluated performance, a score can be assigned to each element, denoted by $S_1, S_2, \cdots, S_8$. Then, the effectiveness of FMECA, denoted by y_1, can be determined by

$$y_1 = \frac{1}{8}\sum_{i=1}^{8} S_i. \tag{10}$$

The effectiveness of other EU-related engineering activities can be evaluated in a similar way, so that the values

TABLE I
EXAMPLES OF EU-RELATED ENGINEERING ACTIVITIES

Activities	Contributions to gaining knowledge and reducing epistemic uncertainty
FMECA	FMECA helps designers to identify potential failure modes and understand their effects, so as to increase the designer's knowledge about potential failures [31].
FRACAS	By implementing FRACAS, knowledge on potential failure modes and mechanisms is accumulated based on previously ocurred failures and corrective actions [32].
RGT	In a RGT, cycles of Test Analysis and Fix (TAAF) are repeated until the product reaches its reliability requirements. In this way, designers' knowledge on the failure modes and mechanisms is accumulated [33].
RET	As the RGT, RET reduces epistemic uncertainty by stimulating potential failures, but using highly accelerated stresses, which can generate failures that are hard to be identified by analyses or conventional tests [32].
RST	In a RST, simulation tests are conducted based on physics-of-failure models to identify weak design points for the products. Knowledge of potential failure modes can be accumulated in this way [34, 35].

TABLE II
EVALUATION CRITERIA FOR FMECA

Notations	Elements	Criteria	Scores
S_1	Definition of failures	The failure definition is clear and unambiguous, so that a complete analysis of failure modes could be conducted.	$S_1 = 3$
		The failure definition is defined, but unclear or ambiguous.	$S_1 = 1$
		The failures are undefined.	$S_1 = 0$
S_2	Coverage of failure modes	The considered failure modes cover all that have occurred historically in the products with similar functions, use and environmental conditions.	$S_2 = 3$
		A few uncritical failure modes are not considered in the analysis.	$S_2 = 1$
		A lot of critical failure modes are not considered in the analysis.	$S_2 = 0$
S_3	Completeness of failure mode analysis	The considered failure modes include both catastrophic failures as well as degradation failures.	$S_3 = 3$
		Only one of the two types is considered.	$S_3 = 1$
		None of the two types is considered.	$S_3 = 0$
S_4	Credibility of information sources	The considered failure modes come from real historical data of the product or similar products.	$S_4 = 3$
		The considered failure modes come from literature.	$S_4 = 1$
		The considered failure modes come from expert judgements.	$S_4 = 0$
S_5	Completeness of failure cause analysis	The analysis takes into account all the possible failure causes.	$S_5 = 3$
		A few noncritical failure causes are not considered in the analysis.	$S_5 = 1$
		A lot of critical failure causes are not considered in the analysis.	$S_5 = 0$
S_6	Completeness of failure effect analysis	Failure effects in both component and system levels are analyzed.	$S_6 = 3$
		Only one of the two levels is considered.	$S_6 = 1$
		Failure effects are not considered in the analysis.	$S_6 = 0$
S_7	Credibility of data sources	Criticality analysis is based on field data.	$S_7 = 3$
		Criticality analysis is based on data from literature.	$S_7 = 1$
		Criticality analysis is based on data from expert judgements.	$S_7 = 0$
S_8	Effectiveness of design improvements	Over 90% of the exposed design weaknesses are modified.	$S_8 = 3$
		The modified design weaknesses are between 60%-90%.	$S_8 = 1$
		Less than 60% of the exposed design weaknesses are modified.	$S_8 = 0$

for $y_1, y_2, \cdots, y_n$ can be determined. Then, the state of knowledge about the potential failures of the component or system can be evaluated as the weighted average of $y_i, i = 1, 2, \cdots, n$:

$$y = \sum_{i=1}^{n} \omega_i y_i, \tag{11}$$

where ω_i is the relative importance of the ith engineering activity for the characterization of the potential failure behaviors, where $\sum_{i=1}^{n} \omega_i = 1$.

2) Determination of EUF: Having determined the value of y, we need to define a function $\sigma_e = h(y)$, through which σ_e is determined. Since σ_e is a measure of the severity of epistemic uncertainty and y measures the state of knowledge, σ_e is negatively dependent on y. Theoretically, any monotonic decreasing function of y could serve as $h(y)$. In practice, the form of $h(y)$ reflects the decision maker attitude towards epistemic uncertainty and is related to the complexity of the product. Therefore, we propose $h(y)$ to be

$$h(y) = \begin{cases} \dfrac{1}{3\sqrt{y}} \cdot m_d, & \text{for simple products;} \\ \dfrac{1}{3y^6} \cdot m_d, & \text{for complex products;} \\ \dfrac{1}{3y^2} \cdot m_d, & \text{for medium complex products.} \end{cases} \tag{12}$$

By letting $\sigma_m = 0$ and m_d fixed to a constant value, the attitudes of the decision maker for different products can be investigated (see Figure 6):

- for simple products, R_B is a convex function of y, indicating that even when y is small, we can gather enough knowledge on the product function and failure behaviors, so that we can assign a high value to the belief reliability;
- for complex products, R_B is a concave function of y, indicating that only when y is large we can gather sufficient knowledge on the product function and failure behaviors, so that we can assign a high value to the belief

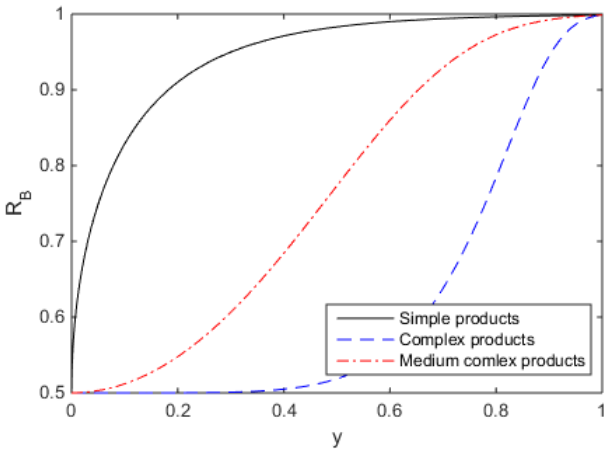


Fig. 6. Different attitudes of the decision maker towards epistemic uncertainty

reliability;

- the $h(y)$ for medium complex products lies between the two extremes.

IV. Case Studies

In this section, we apply the developed belief reliability to evaluate the reliability of two engineering components/systems, *i.e.,* a Hydraulic Servo Actuator (HSA) in Section IV-A and a Single Board Computer (SBC) in Section IV-B. A comparison is also made on both cases with respect to the traditional probabilistic reliability metrics.

A. Hydraulic Servo Actuator (HSA)

The HSA considered in this paper comprises the six components, as listed in Table III. The schematic of the HSA is given in Figure 7.

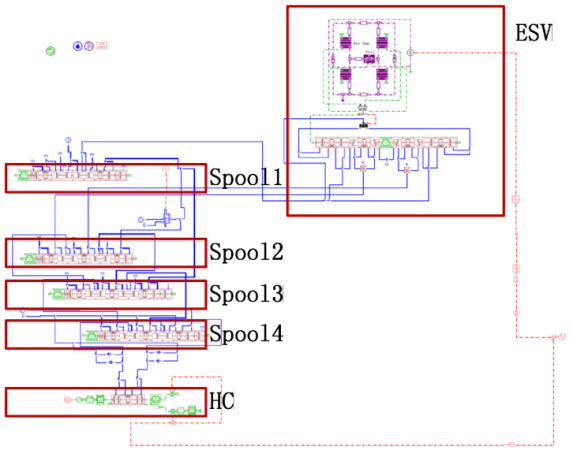


Fig. 7. Schematic of the AMESim model to predict p_{HSA}

The required function of the HSA is to transform input electrical signals, x_{input} , into the displacement of the hydraulic cylinder (HC). The performance parameter of the HSA is the attenuation ratio measured in dB:

$$p_{\text{HSA}} = -20 \lg \frac{A_{\text{HC}}}{A_{\text{obj}}}, \tag{13}$$

where, A_{HC} denotes the amplitude of the HC displacements when input signal x_{input} is a sinusoidal signal, and A_{obj} is the objective value of A_{HC}. Failure occurs when $p_{\text{HSA}} \geq p_{th} = 3$(dB). The belief reliability of the HSA is evaluated following the procedures in Figure 5.

1) Performance Margin Model Development: The performance margin model is developed in two steps. First, a model for the p_{HSA} is developed based on hydraulic principles, with the help of commercial software AMESim. The AMESim model is given in Figure 7. Coherently with (2), the model in Figure 7 is written as

$$p_{\text{HSA}} = g_{\text{HSA}}(\mathbf{x}_{\text{HSA}}). \tag{14}$$

Second, as p_{HSA} is a STB performance parameter, the performance margin of the HSA can be determined according to (1):

$$m_{\text{HSA}} = \frac{1}{p_{th}} \left(p_{th} - g_{\text{HSA}}(\mathbf{x}_{\text{HSA}}) \right). \tag{15}$$

2) Aleatory Uncertainty Evaluation: The $\mathbf{x}_{\text{HSA}}$ comprises six parameters, namely, the clearances on diameters (CoDs) of the six components of the HSA. The CoDs are subject to aleatory uncertainties from production and manufacturing processes, which are quantified by the tolerances in Table III. For simplicity of illustration, it is assumed that all the six parameters follow normal distributions. Following the '3σ' principle (for references, see [40]), the probability density function for each parameter is determined and given in Table III. The value of m_d is calculated by (4), where the nominal values are given in Table III. The resulting m_d is 0.6928 (dB). The values of σ_m is determined using Monte Carlo simulations with a sample size $N = 3000$. The resulting σ_m is 0.0353 (dB).

3) Epistemic Uncertainty Evaluation: Then, we need to determine the value of σ_e. In the development of the HSA, five EU-related engineering activities, i.e., FMECA, FRACAS, RGT, RET and RST have been conducted. Let $y_i, i = 1, 2, \cdots, 5$ denote the five engineering activities, respectively. The values of y_is can be determined by evaluating the effectiveness of these engineering activities, based on the procedures illustrated in Section III-B1. The result is $y_1 = 0.70$, $y_2 = 0.90$, $y_3 = 0.80$, $y_4 = 0.85$, $y_5 = 0.70$. In this case study, the engineering activities are assumed to have equal weights, $\omega_1 = \omega_2 = \cdots = \omega_5 = 1/5$, and then, according to (11), $y = 0.79$. Since the HSA has medium complexity, according to (12),

$$\sigma_e = \frac{1}{3y^2} \cdot m_d = 0.3700. \tag{16}$$

4) Belief Reliability Evaluation: Finally, the belief reliability can be predicted using (8) and the result is shown in Table IV. If we only consider the aleatory uncertainty, probabilistic reliability can be predicted using (3), whose value is also presented in Table IV for comparisons. The result shows that,

TABLE III
COMPONENTS AND TOLERANCES OF THE HSA

Component	ESV	Spool 1	Spool 2	Spool 3	Spool 4	HC
Parameters	CoD x_1	CoD x_2	CoD x_3	CoD x_4	CoD x_5	CoD x_6
Tolerances ($\times 10^{-3}$mm)	1 ± 0.015	7 ± 0.15	7 ± 0.15	7 ± 0.15	7 ± 0.15	10 ± 1.5
Distributions	N$(1, 0.005^2)$	N$(7, 0.05^2)$	N$(7, 0.05^2)$	N$(7, 0.05^2)$	N$(7, 0.05^2)$	N$(10, 0.5^2)$

ESV: Electrohydraulic servo valve
HC: Hydraulic cylinder

as expected, epistemic uncertainty reduces our confidence that the product will perform its function as designed, whereas probabilistic reliability would lead to overconfidence.

TABLE IV
COMPARISON BETWEEN PROBABILISTIC RELIABILITY AND BELIEF RELIABILITY

Types of reliability measures	Results
Probabilistic reliability calculated by (3)	0.9999
Belief reliability calculated by (8)	0.9688

Another major difference between belief reliability and probabilistic reliability is that belief reliability allows for the consideration of EU-related engineering activities in the reliability assessment, which are neglected in the probability-based reliability evaluation. For example, if the effectiveness of the EU-related engineering activities is increased from $y_1 = 0.70$, $y_2 = 0.90$, $y_3 = 0.80$, $y_4 = 0.85$, $y_5 = 0.70$ to $y_1 = y_2 = \cdots = y_5 = 0.9$, then, the belief reliability will increase from $R_{B,0} = 0.9688$ to $R_{B,1} = 0.9921$. In other words, in order to enhance the belief reliability, one not only needs to increase the design margin and reduce aleatory uncertainty by design, but also needs to reduce epistemic uncertainty by improving the state of knowledge, whereas probabilistic reliability focuses only on the former two aspects.

B. Single Board Computer

A SBC, as shown in Figure 8 [41], is chosen to demonstrate the time-dependent belief reliability analysis for electrical systems.

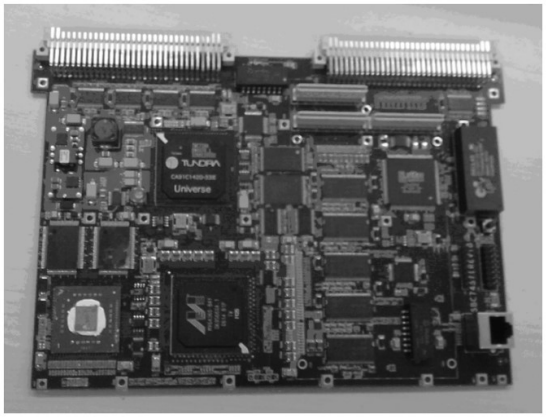


Fig. 8. A SBC [41]

A probabilistic reliability analysis was conducted in [41] based on the parts-counting reliability prediction method in [42]. The times to failure of both the components are assumed to be exponentially distributed and their failure rates are predicted based on the database in [42], as shown in Table V. The failure rate of the SBC can, then, be calculated by summing over all the components' failure rates. Hence, the predicted probabilistic reliability is

$$R_p(t) = \exp\{-1.186 \times 10^{-6} t\}, \tag{17}$$

where the unit of t is hour.

TABLE V
PREDICTED FAILURE RATES OF THE SBC [41]

Components	Number	Predicted failure rate ($\times 10^{-9}$ (h^{-1}))
IC	51	384.8
Crystal oscillator	4	56
Inductance	6	6.56
Connector	9	32.76
Capacitor	631	40.60
Resistance	545	648.91
Others	20	16.16
Total	1266	1186

The probabilistic reliability in (17) is a time-dependent function. To further evaluate the belief reliability, first note by substituting (5) into (8), we have

$$R_B = \frac{1}{\sqrt{\left(\frac{1}{Z_{R_p}}\right)^2 + \left(\frac{\sigma_e}{m_d}\right)^2}}. \tag{18}$$

Since R_p is time-dependent, the belief reliability is also a time-dependent function and can be calculated by using (18) recursively at each time t:

$$R_B(t) = \frac{1}{\sqrt{\left(\frac{1}{Z_{R_p(t)}}\right)^2 + \left(\frac{\sigma_e}{m_d}\right)^2}}, \tag{19}$$

where $R_p(t)$ is the time-dependent probabilistic reliability and σ_e is the EUF evaluated using the procedures in Section III-B.

The effectiveness of the five EU-related engineering activities, i.e., FMECA, FRACAS, RGT, RET and RST, can be assessed using the procedures illustrated in Section III-B1: $y_1 = 0.60$, $y_2 = 0.80$, $y_3 = 0.70$, $y_4 = 0.75$, $y_5 = 0.55$. As the previous case study, we also assume that the five activities have equal weights. From (11), $y = 0.68$. By assessing the configuration of the SBC, it is determined that it has medium complexity. Therefore, by substituting (12) and (17) into (19), the belief reliability of the SBC can be calculated, as shown in Figure 9.

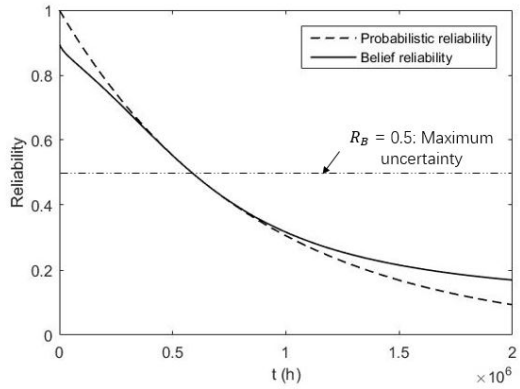


Fig. 9. Belief reliability of the SBC

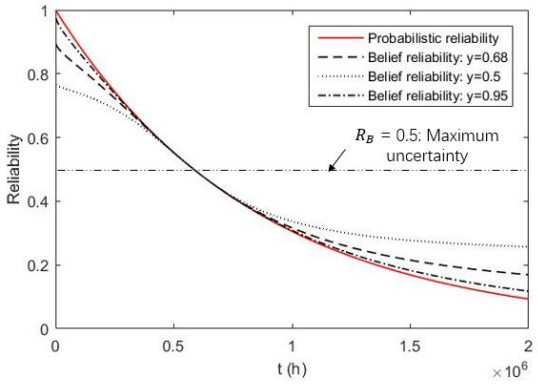


Fig. 10. Belief reliability of the SBC

It can be seen from Figure 9 that the belief reliability curve is more close to $R_B = 0.5$ than the probabilistic reliability. This is because $R_B = 0.5$ corresponds to the state of maximum uncertainty, since we cannot differentiate whether the system is more likely to be working or failure (for details, please refer to maximum uncertainty principle in [37]). Since belief reliability considers the influence of epistemic uncertainty, it yields a more uncertain result than the probabilistic reliability.

A sensitivity analysis is conducted with respect to y to further investigate the influence of epistemic uncertainty on belief reliability. The results are given in Figure 10. It can be seen from Figure 10 that the value of y significantly impacts R_B: a larger value of y, which indicates improvements on the effectiveness of the EU-related engineering activities, tends to make the belief reliability moving towards the probabilistic reliability; while a lower value of y tends to make the belief reliability moving towards 0.5, which is the state of maximum uncertainty. This demonstrates that, compared to the traditional probabilistic reliability, belief reliability allows for the explicit consideration of epistemic uncertainty and EU-related engineering activities in the reliability assessment. In other words, in order to enhance the belief reliability, one not only needs to increase the design margin and reduce aleatory uncertainty by design, but also needs to reduce epistemic uncertainty by improving the state of knowledge.

V. Conclusion

In this paper, a new metric of belief reliability has been introduced to explicitly incorporate the influence of epistemic uncertainty into model-based methods of reliability assessments. To quantify the effect of epistemic uncertainty, an evaluation method is proposed, based on the effectiveness of engineering activities related to reliability analysis and assessment. The proposed belief reliability evaluation method integrates design margin, aleatory uncertainty and epistemic uncertainty for a comprehensive and systematic characterization of reliability. Two numerical case studies demonstrate the benefits of belief reliability compared to the traditional probability-based reliability metrics, with the explicit consideration of epistemic uncertainty.

Compared to the traditional probabilistic reliability metrics, belief reliability explicitly considers the effect of epistemic uncertainty and allows considering EU-related engineering activities in reliability assessment. We believe that as a new reliability metric, belief reliability is beneficial in reliability engineering practices, since epistemic uncertainty is a severe problem for real-world products, especially for those in design and development phases. An interesting future work is to define a mathematical theory to model belief reliability and its time-dependence. Various mathematical theories dealing with epistemic uncertainty can be considered, *e.g.*, Bayesian theory, evidence theory, possibility theory, uncertainty theory, *etc*. Besides, methods of scoring the effectiveness of engineering activities should be further investigated.

Acknowledgment

This work has been performed within the initiative of the Center for Resilience and Safety of Critical Infrastructures (CRESCI, http://cresci.cn). The research of Dr. Zhiguo Zeng is partially supported by Natural Science Foundation of China (NSFC) under grant No. 71601010. The research of Dr. Meilin Wen is supported by Natural Science Foundation of China (NSFC) under grant No. 71671009. The research of Prof. Rui Kang is supported by Natural Science Foundation of China (NSFC) under grant No. 61573043.

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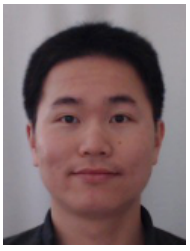

Zhiguo Zeng Zhiguo Zeng was born in 1989. He received his Bachelor's degree on Quality and Reliability Engineering on 2011 and his Ph.D. degree on Systems Engineering on 2015, both from Beihang university, China. Currently, he is a postdoc researcher at Chair on System Science and the Energy Challenge, Fondation Electricite de France (EDF), CentraleSupélec, Université Paris-Saclay. His researches focus on uncertainty quantification and analysis, belief reliability theory, dynamic risk assessment and business continuity modeling and analysis.

Rui Kang Rui Kang received the B.S. and M.S. degrees in the Department of Automatic Control from Beihang University (BUAA), Beijing, China, 1987 and 1990, respectively. Now he is the Yangtze River scholar and the Chair Professor with the School of Reliability and Systems Engineering, BUAA. He has developed six courses and has published nine books and more than 100 research papers. Prof. Kang serves as associated editor for several world-renowned journals in reliability, including IEEE Transactions on Reliability and Journal of Risk and Reliability. His main research interests include reliability of complex systems, prognostics and health management system.

Meilin Wen Meilin Wen received the B.S. degree in mathematics from Tianjin University in 2003, and Ph.D. degrees in mathematics from Tsinghua University in 2008. Currently, she is an assistant professor in School of Reliability and Systems Engineering, Beihang University. Her research interests include uncertainty theory, uncertain programming, uncertain reliability optimization, data envelope analysis under uncertain environments.

Enrico Zio Enrico Zio (BSc in nuclear engng., Politecnico di Milano, 1991; MSc in mechanical engng., UCLA, 1995; PhD, in nuclear engng., Politecnico di Milano, 1995; PhD, in nuclear engng., MIT, 1998) is Director of the Chair on System Science and the Energy Challenge, Fondation Electricite de France (EDF), CentraleSupélec, Université Paris-Saclay, full professor, President and Rector's delegate of the Alumni Association and past-Director of the Graduate School at Politecnico di Milano. He was the Chairman of the European Safety and Reliability Association ESRA (2010-2014), member of the scientific committee of the Accidental Risks Department of the French National Institute for Industrial Environment and Risks, member of the Korean Nuclear society and China Prognostics and Health Management society, and past-Chairman of the Italian Chapter of the IEEE Reliability Society. He was an Associate Editor of IEEE Transactions on Reliability and editorial board member in various international scientific journals, among which Reliability Engineering and System Safety, Journal of Risk and Reliability, International Journal of Performability Engineering, Environment, Systems and Engineering, International Journal of Computational Intelligence Systems. He has functioned as Scientific Chairman of three International Conferences and as Associate General Chairman of two others. His research focuses on the characterization and modeling of the failure/repair/maintenance behavior of components, complex systems and critical infrastructures for the study of their reliability, availability, maintainability, prognostics, safety, vulnerability and security, mostly using a computational approach based on advanced Monte Carlo simulation methods, soft computing techniques and optimization heuristics. He is author or co-author of five international books and more than 170 papers on international journals.

ACRONYMS

AUF	Aleatory Uncertainty Factor
ESV	Electrohydraulic Servo Valve
EU	Epistemic Uncertainty
EUF	Epistemic Uncertainty Factor
FMECA	Failure Mode, Effect and Criticality Analysis
FRACAS	Failure Report, Analysis, and Corrective Action System
HC	Hydraulic Cylinder
HSA	Hydraulic Servo Actuator
LTB	Larger-the-better
NTB	Nominal-the-better
RGT	Reliability Growth Test
RET	Reliability Enhancement Test
RST	Reliability Simulation Test
SBC	Single Board Computer
STB	Smaller-the-better

NOTATIONS

m	Performance margin
p	Performance parameter
p_{th}	Functional threshold
R_B	Belief reliability
R_p	Probabilistic reliability
m_d	Design margin
σ_m	Aleatory uncertainty factor
σ_e	Epistemic uncertainty factor
y	Effectiveness of the EU-related engineering activities