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# Shooting method for predicting the self-excited vibrations of nonlinear systems with planar friction

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## 1 Introduction

Squeal noise is still an issue for engineers and scientific communities. Many investigations over recent decades considering various specific features and developments have been proposed to study the problem of friction-induced vibration. Overviews can be found in [1, 2, 3]. Assuming that squeal noise is due to mode coupling instabilities, two types of numerical studies are classically done: the first one is based on the stability analysis. It allows one to determine the occurrence of the squeal events. The second one corresponds to the non-linear analysis leading to the calculation of the vibratory levels of the self-excited vibrations. In the present, both stability analysis and nonlinear time analysis are performed. Moreover, a shooting method for predicting the self-excited vibrations of nonlinear systems with planar friction is proposed. It is demonstrated that the shooting method with a specific initialization accelerates the time calculation of the non-linear analysis.

## 2 Problem formulation

The system under study is described in the Figure (1). It is composed of a mass  $m$  in frictional contact with a rigid plane moving with a constant rectilinear velocity denoted  $V$ . The mass is held against the moving plane by three springs and pressed by an external force  $F$ . Damping is also integrated as described in the Figure (1). The equations of motion can be written in matrix form denoted in the following :

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{R} + \mathbf{F} \quad (1)$$

Where  $\mathbf{U}$ ,  $\dot{\mathbf{U}}$  and  $\ddot{\mathbf{U}}$  are respectively the displacement, velocity and acceleration vectors regrouping the components in the three directions  $x$ ,  $y$  and  $z$ . The vector  $\mathbf{F}$  represented the external force. The vector  $\mathbf{R}$  includes the contact normal reaction and the friction force applied on the center of mass due to frictional contact with the plane.  $\mathbf{M}$ ,  $\mathbf{C}$  and  $\mathbf{K}$  are the mass, damping and stiffness matrices, respectively.

In this study, the contact between the mass and the plane is assumed to be unilateral without inter-penetration. It is modeled by Signorini's law. The friction is governed by Coulomb's law with a constant frictional coefficient  $\mu$ . These laws are non-linear and non-smooth.

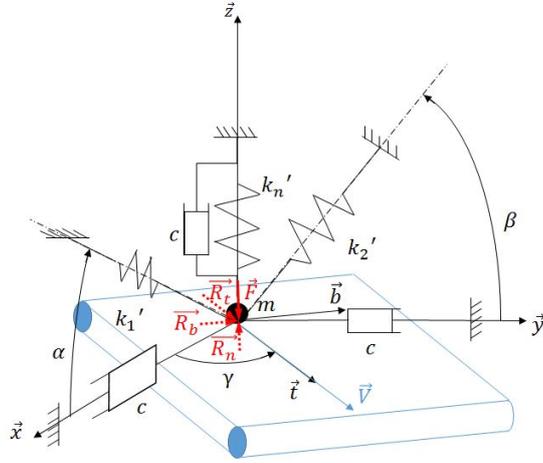


Figure 1: Description of the model

### 3 Stability analysis

In this section, the stability of the system is investigated. The behavior of small perturbations around the quasi-static sliding equilibrium is analyzed in order to determine the propensity of the squeal events. In this study the contact between the mass and the plane is supposed to be bilateral. The frictional reactions are then linearized around the static equilibrium and the complex eigenvalues of the system are calculated. If the real part of all the eigenvalues are negative the system is stable. In the other way, if at least one of the eigenvalues has a positive real part then the system is unstable, submitted to mode coupling and self-excited vibrations and squeal noise can occur. All the details of such analysis can be found in [4].

### 4 Non-linear Analysis

Based on the previous results, the self-excited vibrations are calculated. First of all, a classical time integration scheme is used. A modified  $\theta$ -method (allowing to verify the inelastic choc law) is used (see [5] and [6] for more details). The results are used as a reference to validate the shooting method

Then, an adaptation of the shooting method for autonomous system is proposed in order to reduce the computational costs. This method determines periodic solutions of the mechanical system by solving a two-point boundary-value problem (see [7] for more details). In this study, the system is submitted to self-excited vibrations, the period of these vibrations is one of the unknowns of the problem, as well as the displacements and velocities, and is integrated in the shooting method as explained in [7].

The problem of this method is its initialization. Indeed, the corrections on the initial conditions and the period are

calculated by a Newton-Raphson algorithm which is a local algorithm. If the shooting method is initialized too far of the stationary part looked for, the shooting method may not converge. In this paper, an energetic method is proposed. It is based on the observation of the behavior of a system submitted to autonomous vibrations. The amplitude of these vibrations increases until saturation of the non-linear forces. This saturation leads to the decrease of the contact injected power while the power dissipated by damping remains approximately constant. A periodic solution is looked for based on the unstable mode. The frictional contact forces are expressed in a way that they can be saturated. The averaged power balance on one cycle is then estimated. Assuming that the solution of the problem is a stationary state, the variation of mechanical energy is null, and if the solution is periodic and the external force is constant, the corresponding injected power is also null. The amplitude of the unstable mode  $q_0$  who balances the contact injected power and the damping dissipated power, is determined. The shooting method is finally initialized as a real projection on the unstable mode with the amplitude  $q_0$  and a period equals to unstable mode's one.

Figure (2) shows the result of the shooting method initialized with the energetic method. The stationary part is correctly approximated by the shooting method compared to the direct numerical integration. The blue star corresponds to the result of the energetic method and the initialization of the shooting method is satisfying.

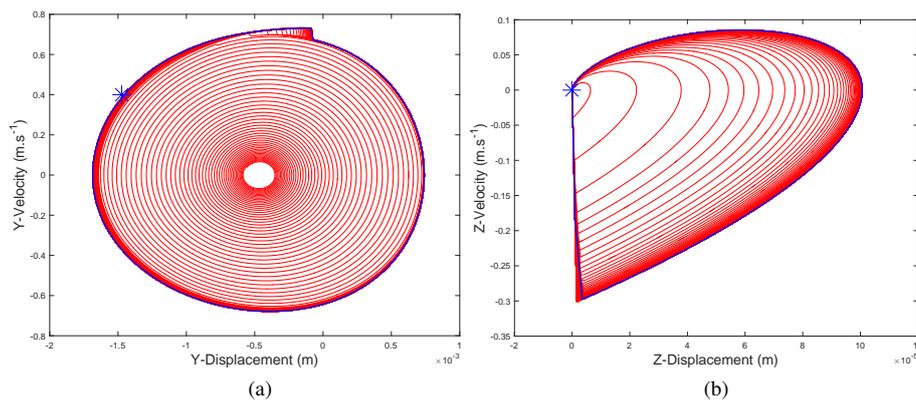


Figure 2: Comparaision of the phase spaces obtained by the direct numerical integration procedure (red) and the shooting method (blue) in the direction  $\vec{Y}$  (a) and  $\vec{Z}$  (b)

## 5 Conclusion

The present study investigated the problem of friction-induced self-excited vibrations. The stability analysis of the numerical model is performed in order to determine the mode coupling phenomena. Then, a nonlinear analysis is proposed to calculate the self-excited vibrations of the system. The potential of using a shooting method with a specific initialization has been demonstrated. Results are in perfect agreement with the reference model based on the classical time integration scheme. The proposed methodology based on the shooting method has the potential to drastically improve the computational costs.

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