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Michel Mizony. What is our Universe now? For the century of the formula 15 written by de Sitter. 2017. hal-01629125v1

HAL Id: hal-01629125

<https://hal.science/hal-01629125v1>

Preprint submitted on 6 Nov 2017 (v1), last revised 2 Feb 2018 (v2)

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What is our Universe now ?

For the century of the formula 15 written by de Sitter

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Autumn 2017

Then the line-element becomes *

$$(15) \quad ds^2 = R^2 \{ d\omega'^2 - \sinh^2 \omega' (d\zeta'^2 + \sinh^2 \zeta' [d\psi^2 + \sin^2 \psi d\theta^2]) \}.$$



Résumé

Starting with the Friedmann-Lemaître (FL) metric g_{FL} of an isotropic universe, we give the radially inertial form of this metric which is a generalized Gullstrand-Painlevé form of metric g_{GP} . Then, for g_{GP} , the equivariant stress-energy tensor is convenient because it has a straightforward interpretation in terms of velocity and potential. The energy and the entropy are well-defined. For each model of the Universe, the osculating manifold is a de Sitter model. Moreover, if the Universe is the open cone of the future of a point (a big bang event), then this de Sitter model is an open, accelerated one. So, we could easily confront this model with local observations through the osculating de Sitter model, taking into account the observed SNIa and the Hubble parameter $H(z)$ for redshifts $z \leq 2$. The recent data about $H(z)$ provide a tool to estimate cosmological parameters for the de Sitter models and their Milne limits; we find : $H_0 = 65 \pm 2 \text{ km/s/Mpc}$, $\Omega_0 = 0.05 \pm 0.02$, and an age = $15.2 \pm 0.3 \text{ Gyr}$. In other words, our Universe contains only baryonic matter.

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1 Introduction

We describe the Universe by a Friedmann-Lemaître universe model (\mathcal{U}, g_{FL}) , i.e. spatially homogeneous and isotropic. Most often, g_{FL} is expressed in its Robertson-Walker form

$$g_{FL} = ds_{FL}^2 = d\tau^2 - R^2(\tau) (dx^2 + f_k^2(x) d\omega^2), \quad (1)$$

where $f_k(x) = x, \sin(x), \sinh(x)$ according to the sign $k = 0, 1$ or -1 of the spatial curvature, and where $d\omega^2$ is the element of spherical angle.

We will give different expressions of *the same* metric g_{FL} defined at the event $E_0 = (\text{here, today}) \equiv \{\tau = \tau_o, x = \theta = \phi = 0\}$.

It is convenient to start from the *locally inertial form* relative to E_0 , defined after defining $\rho := R(\tau_o) x = R_o x$, as

$$ds_{FL}^2 = d\tau^2 - \frac{R^2(\tau)}{R^2(\tau_o)} \left(d\rho^2 + R^2(\tau_o) f_k^2\left(\frac{\rho}{R(\tau_o)}\right) d\omega^2 \right). \quad (2)$$

The slight change with respect to (1) emphasizes that, rigorously, an inertial metric must have the Minkowskian form, and that, in (1), x is an *angular* coordinate.

The model is characterized by a scale factor $R(\tau)$, τ being the cosmic time. We have the Hubble parameter $H(\tau) \equiv \dot{R}(\tau)/R(\tau)$, the deceleration parameter $q(\tau) \equiv -\ddot{R}R/\dot{R}^2$ that we assume negative (corresponding to an accelerating universe model), and the Einstein equation

$$H(\tau)^2 + \frac{k}{R(\tau)^2} = \frac{8\pi G\rho(\tau)}{3} \equiv \Omega(\tau) H^2(\tau). \quad (3)$$

We include in the density parameter Ω the contribution of the cosmological constant. *Present* values of these quantities are written with a zero index : $\Omega(\tau_0) = \Omega_0$, etc.

In section 2, the generalized Gullstrand-Painlevé form is given and the case of the de Sitter models is developed. In section 2.1 we address the global problems, without considering an hypothetic equation of state, but only the entropy equation. In section 2.2 we address the de Sitter models. In section 2.3 we address the de Sitter model as an osculating manifold to a flat Λ CDM model. The local observations developed in section 3 underline the usefulness and the suitability of the accelerated de Sitter models, and also of the limit case of the Milne's models. After a discussion in section 4, the conclusion ends with revisiting some classical problems.

2 The radially inertial form of a FL metric

In order to obtain a radially inertial form, the basic equation (2) is important and avoids a mathematical mistake that is too often written in papers and books.

2.1 The general case

We start from the usual form (2) of the metric, which is expressed in the locally inertial frame for the event $E_0 = (\text{today, here})$. The change of variables $(\tau, \rho) \mapsto$

$(\tau, r \equiv R(\tau) f_k[\frac{\rho}{R(\tau_0)}])$; with the help of the Einstein's equation (3) this change of variables leads straightforwardly to the form

$$g_{GP} = ds_{FL}^2 = d\tau^2 - \frac{(dr - r H(\tau) d\tau)^2}{1 + (1 - \Omega(\tau)) H^2(\tau) r^2} - r^2 d\omega^2. \quad (4)$$

The metric g_{GP} looks like a generalised Gullstrand-Painlevé metric but with an hyperbolic space part ($k = -1$).

A comoving galaxy is defined by $d\rho = 0$, which implies $dr = r H(\tau) d\tau$. This formulation of the Hubble law remains exact at any time. We denote with $v = r H(\tau)$ the radial velocity. In the denominator the total energy $(1 - \Omega(\tau)) H^2(\tau) r^2$ gives the potential Φ defined by $2 \Phi = \Omega(\tau) H^2(\tau) r^2$ so we have the well-defined acceleration equation :

$$\gamma = \frac{dv}{d\tau} = \frac{\partial v}{\partial \tau} + v \frac{\partial v}{\partial r} = \frac{\partial \Phi}{\partial r} + \frac{\partial \Phi}{v \partial \tau}. \quad (5)$$

The potential Φ being well-defined, we can compute the stress-energy tensor ; but the inertial metric g_{GP} has a cross term, so if we want a thermodynamic interpretation of this tensor (cf. Weinberg chap. 2.11) [2], we must compute the equivariant form of the stress-energy tensor which is not symmetric (it is an ordinary mathematical fact which comes from the cross term).

First, the components of the non null equivariant Einstein tensor G^μ_ν , with v and Φ , and using the identity (5), are given by

$$\begin{aligned} G^0_0 &= -2 \frac{\Phi + r \frac{\partial \Phi}{\partial r}}{r^2} \\ G^1_0 &= 2 \frac{\frac{\partial \Phi}{\partial \tau}}{r} \\ G^1_1 &= -2 \frac{vr \frac{\partial \Phi}{\partial r} + v\Phi + r \frac{\partial \Phi}{\partial \tau}}{r^2 v} \\ G^2_2 = G^3_3 &= -\frac{2v^2 \frac{\partial \Phi}{\partial r} + r \left(\frac{\partial^2 \Phi}{\partial r^2} \right) v^2 + v \frac{\partial \Phi}{\partial \tau} + rv \frac{\partial^2 \Phi}{\partial r \partial \tau} - r \left(\frac{\partial v}{\partial r} \right) \frac{\partial \Phi}{\partial \tau}}{rv^2}. \end{aligned} \quad (6)$$

We must underline that $G^0_1 = 0$ is equivalent to $\frac{\partial v}{\partial \tau} + v \frac{\partial v}{\partial r} = \frac{\partial \Phi}{\partial r} + \frac{\partial \Phi}{v \partial \tau}$, cf. (5).

We can notice also that the G^1_0 term is related to the entropy, and the conservation law $G^\mu_{\nu;\mu} = 0$ is verified.

Otherwise, the non null terms of the equivariant Einstein tensor G^μ_ν , with the 3 cosmological parameters $H(\tau)$, $\Omega(\tau)$ and $q(\tau)$, is even shorter, but it hides the acceleration equation (5) :

$$\begin{aligned} G^0_0 &= -3 H(\tau)^2 \Omega(\tau) \\ G^1_0 &= H(\tau)^3 r (\Omega(\tau) + q(\tau)) \\ G^1_1 = G^2_2 = G^3_3 &= -H(\tau)^2 (\Omega(\tau) - 2q(t)). \end{aligned} \quad (7)$$

After this study of the geometric tensor, the first member of Einstein's equations, let us pass to the second member of the equations $G^\mu_\nu = -\kappa T^\mu_\nu$, where $\kappa = 8\pi G_N/c^4$ where G_N is the Newton constant; the non-zero elements are :

$$\begin{aligned} T^0_0 &= \rho \\ T^1_0 &= v(\rho + p) \\ T^1_1 &= T^2_2 = T^3_3 = -p, \end{aligned} \tag{8}$$

where ρ denotes a matter-energy density and p a "pressure". Note that p and ρ are always defined, as the pressure and energy density measured by an observer in a locally inertial frame happens to be moving with the fluid at the instant of measurement. The cross term T^1_0 , whose writing follows from the identity $G^1_0 = v(G^0_0 - G^1_1)$, would translate to an "energy dissipation or exchange" of this thermodynamic fluid; the nullity of T^1_0 is equivalent to the constancy of the entropy for a universe model, (cf. [2] formula 15.6.13).

The fundamental consequence of these three forms of the same tensor is coming from the following query : What does the nullity of the G^1_0 , or T^1_0 terms imply ?

We have two answers : first, $\frac{\partial}{\partial \tau} \Phi = 0$, i.e. $H^2(\tau) \Omega(\tau)$ is constant, and second $\rho + p = 0$. Moreover, if, by analogy with thermodynamics, $G^1_0 = 0$ translates to a constant entropy, then there exists no dissipation because the universe model has no exterior part. So, it may exist only an exchange between two or more fluids in the universe model.

There are two cases :

- i) there exists only one fluid, moving with the inertial frame, and we obtain all the de Sitter universe models,
- ii) there exists another non comoving fluid which exchange energy with the comoving one ; this is possible, a radiative fluid is then a convenient choice.

We shall consider the first case in the next subsection ; but before, we want to point out that the so-called dust fluid, with a null pressure, does not fulfill this entropy condition. This is the case for the Λ CDM models which contain such a fluid. Where is the problem ? Too often a dust fluid is taken, a priori, based on a strong analogy with thermodynamics. It is taken with the covariant tensor coming from the diagonal Robertson-Walker metric ; this form of metric is not inertial, leading to a misunderstanding ; but, a Gullstrand-Painlevé-like form of this metric is inertial so we can only then apply the thermodynamic analogy with the equivariant stress-energy tensor as a consequence of general relativity.

So, if we have a dust fluid which is comoving, then it has a pressure p and a density ρ such that $p + \rho = 0$; we think this fact to come from a basic principle of general relativity which asserts that the gravitational mass is equal to the inertial one. Indeed, the de Sitter expanding models fulfill the Mach principle which asserts that the inertial mass of a body comes from all the content of the Universe ; if so, then we have a proof for main results about fluids in general relativity without the analogy with thermodynamics, but with the use of what is inherited from an inertial frame. Starting from another theoretical point of view, the conservation of global energy for the Universe, H. Telkamp [4] found the same result : the open, flat or closed de Sitter models.

2.2 The de Sitter case

As it appears that the de Sitter models for isotropic universe models are basic, we develop this case but only for de Sitter models coming from a big bang. The study of the de Sitter manifold was made in 1917 ([1]) and for the model of the expanding Universe, de Sitter gives the metric which is very close to the Robertson-Walker form, long before Friedmann or Lemaitre, here his formula 15 with $R=1/\lambda$, $\omega' = \lambda \tau$, etc. :

$$ds^2 = d\tau^2 - \frac{\sinh^2 \lambda \tau}{\lambda^2} (d\alpha^2 + \sinh^2 \alpha d\omega^2), \quad (9)$$

where the curvature λ is a non-negative real number, so we have an infinity of de Sitter manifold. Each inertial form is given by :

$$ds^2 = d\tau^2 - \frac{(dr - r H(\tau) d\tau)^2}{1 + \frac{r^2 \lambda^2}{\sinh^2 \lambda \tau}} - r^2 d\omega^2. \quad (10)$$

These two forms of the same metric are defined on the same open part of a de Sitter manifold which is isomorphic to the $SO(1,4)/SO(1,3)$ manifold, where $SO(1,4)$ is the ten dimensional de Sitter group and $SO(1,3)$ is the Lorentz group, see [23] for details. The generalized Gullstrand-Painlevé form (10) is introduced in ([3]). For these de Sitter models, the function $H^2(\tau) \Omega(\tau)$ is constant, more precisely :

$$\lambda^2 = H^2(\tau) \Omega(\tau) = -H^2(\tau) q(\tau). \quad (11)$$

Let us take the initial conditions at the event (now, here) for the three cosmological parameters H_0, q_0, Ω_0 ; as $\Omega_0 = -q_0$, then the set of de Sitter models is two dimensional. We choose H_0 and q_0 as parameters in order to confront these models with observations. The time $\tau = 0$ is the time of the big bang event, so τ_0 denotes “now”.

The Milne models can be viewed as the limit of these de Sitter models when the curvature λ , tends to 0, H_0 being fixed; one Milne model for each value of H_0 , one de Sitter model for each pair (λ, H_0) .

As the redshift z is a useful cosmological observable, we will provide here some formulas.

Lemma : Let us take the two parameters $(H_0, q_0 = -\Omega_0)$ characterizing one of these models of de Sitter with negative curvature ($0 < \Omega_0 < 1$); we have :

i) The time of the emission of a photon received at τ_0 with a redshift z :

$$\tau(z) = \frac{1}{H_0 \sqrt{-q_0}} \operatorname{arsinh}\left(\frac{\sqrt{-q_0}}{\sqrt{1+q_0}(1+z)}\right), \quad (12)$$

and for $z = 0$, the age of the Universe,

$$\tau_0 = \frac{1}{H_0 \sqrt{-q_0}} \operatorname{arsinh}\left(\frac{\sqrt{-q_0}}{\sqrt{1+q_0}}\right). \quad (13)$$

ii) The Hubble function $z \rightarrow H(z)$ is merely :

$$H(z) = H_0 \sqrt{q_0 z^2 + 2 q_0 z + z^2 + 2 z + 1}. \quad (14)$$

iii) The angular distance is given by :

$$dA_{dS}(z) = \frac{1}{-H_0 q_0 (1+z)} \left((1+z) - \sqrt{1 + (1+q_0)z(2+z)} \right). \quad (15)$$

These are classical formulas [5] formula 237, [6].

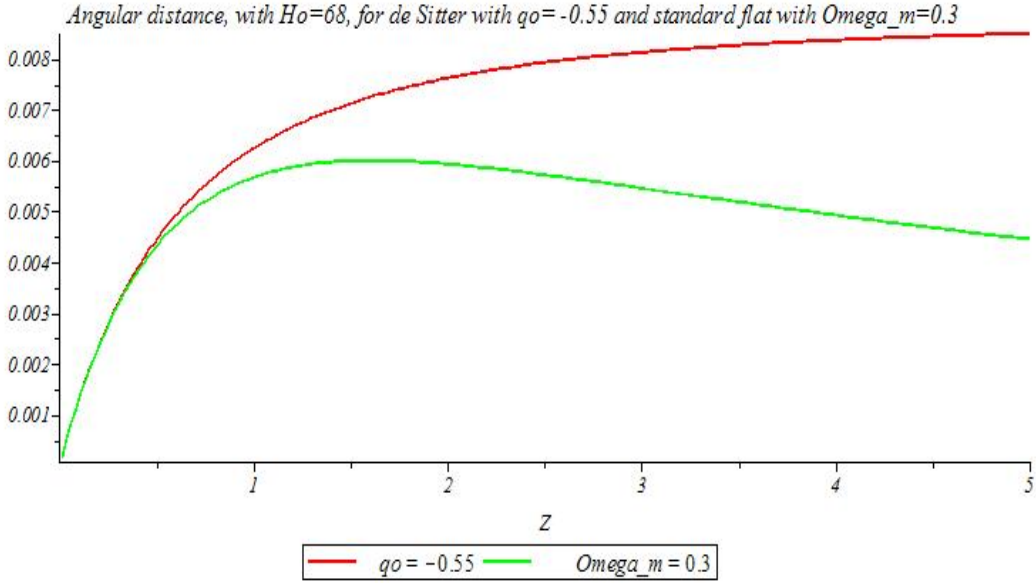
2.3 The osculating de Sitter model to a flat Λ CDM one

A flat Λ CDM has two initial parameters : the Hubble parameter H_0 and the density parameter Ω_m of the comoving fluid (dust without pressure and dark matter).

For the flat Λ CDM the angular distance is given by an integral formula :

$$dA_{\Lambda CDM}(z) = \frac{1}{(1+z) H_0} \int_0^z \frac{1}{\sqrt{\Omega_m (1+x)^3 + 1 - \Omega_m}} dx. \quad (16)$$

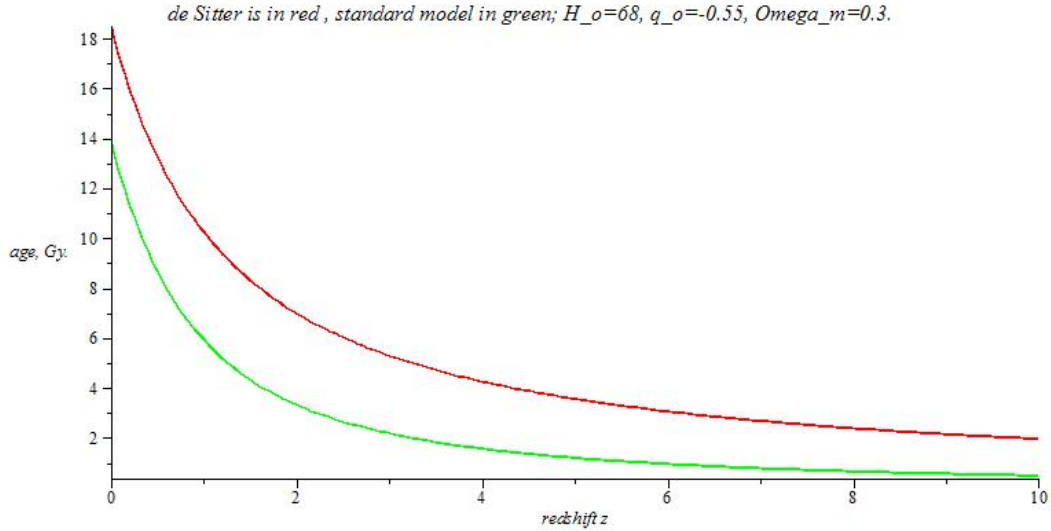
If we want that $dA_{\Lambda CDM}(z) \equiv dA_{dS}(z)$ near $z = 0$, then $q_0 = \frac{3}{2} \Omega_m - 1$. For example, for the recent Planck Λ CDM model with $\Omega_m = 0.3$ and $H_0 = 68$, cf. ([7]), we have the de Sitter osculating model today defined by $q_0 = -0.55$ and the same Hubble value, see figure 1.



We have also an integral formula for the flat Λ CDM model :

$$\tau(z) = 1/H_0 \int_0^{1/(1+z)} \frac{1}{\sqrt{\Omega_m/x + (1 - \Omega_m)x^2}} dx. \quad (17)$$

For the figure 2 the same initial values are taken.



The theoretical error is very big for the age because we made an integration for all z and not only for z less than 0.6. Thus, the misunderstanding about frames (comoving and inertial) became a big mathematical mistake if we consider the universe model as a whole. The problems are the same for the values coming from the WMAP collaboration ([8]). Two facts seem weird or puzzling : first, the fact that the angular distance is decreasing after $z > 1.5$ and, second, the very accurate values given by the each team with, for example, ages with more three exact digits.

What confidence can then be given to studies which, based on the a priori of a flat Λ CDM model, give extremely accurate results both with respect to BAO (Baryon Acoustic Oscillations) and to cosmic background radiation because they involve large redshifts ? If we compare the functions $z \rightarrow H(z)$ for de Sitter models on the one hand, and flat Λ CDM models on the other hand, the latter is of the order \sqrt{z} times the former. Another example near the redshift $z = 1100$ of the CMB : $\tau(1100) = 0.5 \cdot 10^6$ light years for the standard model and $\tau(1100) = 16 \cdot 10^6$ light years (32 times more), for the osculating de Sitter universe model (but without radiation, so it's just an approximate calculus).

It is also worth recalling that one of the consequences of the Machian nature of de Sitter models is, ipso facto, no resort to a hypothetical period of inflation is necessary to account for the isotropy of the cosmic background radiation and also for the primordial baryogenesis ([5]).

3 The de Sitter models and astronomical observations

For the observations which concern the de Sitter and Milne models, three very interesting papers exist, based on studies of supernovae (SNIa) : in 1998, A. Riess and his team [9] ; in 2010, F. Farley and in 2016, V. Lukovic and all [12] ; see also in 1999 [10]. They pointed out that the better values for the Hubble parameter are about from 63 to 65 km/s/Mpc for these models :

- i) in his historical paper of 1998, A. Riess said : *The Hubble constants as derived from the MLCs method, $65.2 \pm 1.3 \text{ km/s/Mpc}$, and from the template fitting approach, $63.8 \pm 1.3 \text{ km/s/Mpc}$, are extremely robust and attest to the consistency* [9], see also their emblematic figure that highlights many elements ;
- ii) in 2010, F. Farley gives us a nice proof that the Milne models explain the kinematic face of the SNIa observations and that H_0 is so around 63 km/s/Mpc ; see also [13] and [14]
- iii) let us present only the wonderful figure given by V. Lukovic and all in 2016 :

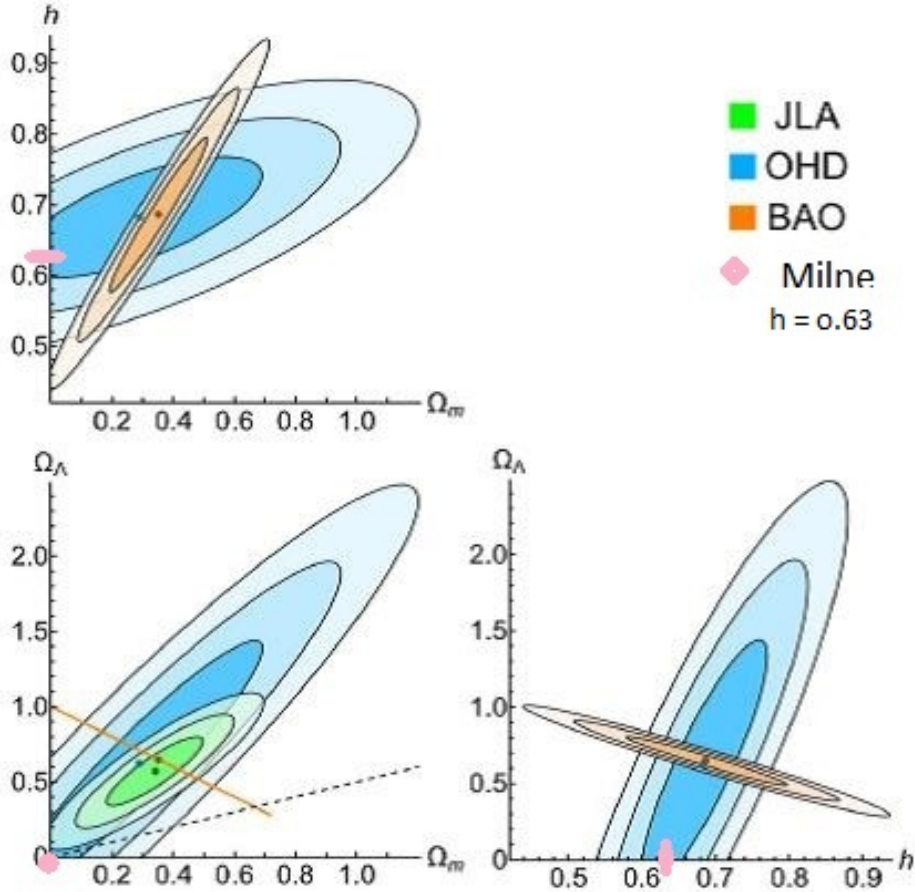


Fig. 2. 1σ , 2σ and 3σ confidence regions resulting from the fit of Λ CDM model to the single datasets as indicated in the top right panel. The dashed line in the $\Omega_m - \Omega_\Lambda$ plane represents the transition from the decelerating (below) to the accelerating (above) models.

We have added, on the three panels of this figure, where are the Milne models, using pink color ; the de Sitter models are defined, on these plots, by $\Omega_m = 0$ and $0 < \Omega_\Lambda < 1$. Let us remark that the BAO results are definitely not relevant for our study, cf. the red line on the $(\Omega_m, \Omega_\Lambda)$ plot, bottom left panel. Thus, our Ω_0 must be less than 0.3 and $60 \leq H_0 \leq 68$ (top panel), cf. also the paper of Buchert, Coley and others [15].

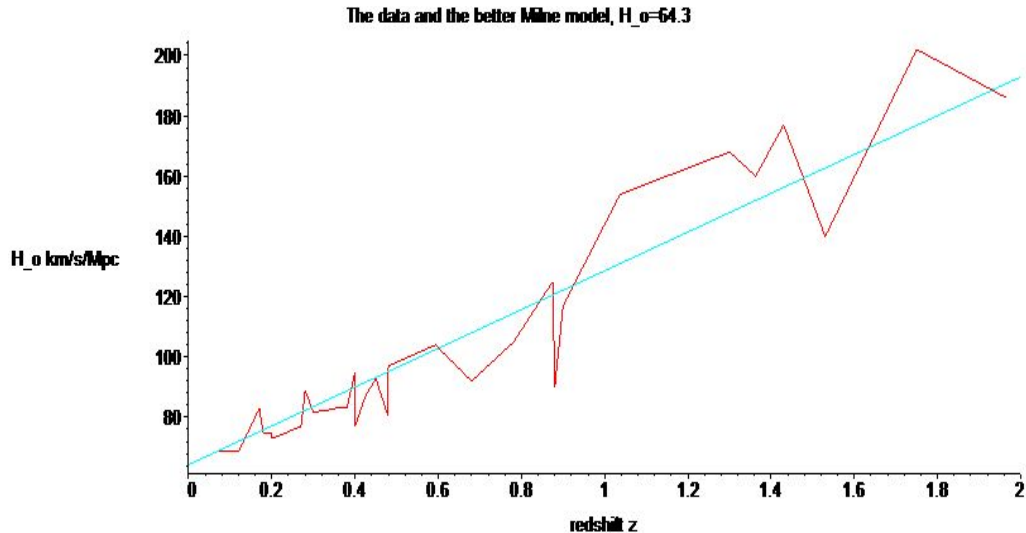
What is now the observational problem? The comoving density Ω_0 must be less than 0.3 and, more likely, even near 0 as Farley suggests. For a theoretical reason we have no need for dark energy as explained above via the inertial frame; we have also no need for dark matter to explain the flat curves of spiral galaxies, this is developed by many astrophysicists, see ([5]) and a bibliography in ([16]). A remark : the conformal gravity and the Einstein gravity are equivalent for the de Sitter universe models. Therefore, the 4% of baryonic matter is allowable.

For the respect of general relativity, we want to test the de Sitter models with other data than these coming from SNIa data. One year ago Duan and all give us several recent data about the function $z \rightarrow H(z)$ ([17]); thirty eight data, thanks to astronomers. But eight are coming from the erroneous mathematical use of the BAO. So we have thirty data coming from local measurements, in the table of the values $(z, H(z))$:

table := [[.7e-1, 69.], [.9e-1, 69.], [.12, 68.6], [.17, 83.], [.179, 75.], [.199, 75.], [.20, 72.9], [.27, 77.], [.28, 88.8], [.352, 83.], [.38, 83.], [.4, 95.], [.40, 77.], [.425, 87.1], [.45, 92.8], [.478, 80.9], [.48, 97.], [.593, 104.], [.68, 92.], [.781, 105.], [.875, 125.0], [.88, 90.0], [.9, 117.], [1.037, 154.], [1.3, 168.], [1.363, 160.], [1.43, 177.], [1.53, 140.], [1.75, 202.], [1.965, 186.]].

The accuracy of these measures, although not shown here, is small (uncertainties from 5 to 25% for most points), but the number of measures has been well-developed in recent years. A first small problem about these data, the smaller value of $H(z)$ is $H(0.12) = 68.6$, but the functions $\tau(z)$ given by 12 and also 17 are increasing, statistics are valid if H_0 is less than 68.6, stricto sensu.

As the function $H(z)$ is nearly linear for $0.07 \leq z \leq 2$, and for the small densities Ω_0 , we just use the least-square method for some values of H_0 to compute the values of Ω_0 . Before for the Mine models we have :



The linear regression is given by $H(z) = 59.8 (\pm 1.2) + 70.0 (\pm 1.6) z$, so a mathematical minimum error around 2% on $H(0)$; and for the Milne models by $H(z) = 64.32 (\pm 0.75) (1+z)$, so with a 1.2% error. These error bars are independent

of the uncertainties on the data. For the de Sitter models we must have $\Omega_0 = -q_0$ non-negative so $H(z) \leq H_0(1+z)$, the H_0 Milne model. Moreover, $\Omega_0 \geq 0.04$ the baryonic density, that gives a lower bound on Ω_0 and with the data, a lower limit on H_0 .

These first results from the data about the function $z \rightarrow H(z)$ confirm those of F. Farley ([11]) about the Milne universe and agree with the analysis of V. Lukovic (3).

The results

$H_0 = 64.3$ km/s/Mpc :	Milne, $\Omega_0 = 0$	Age(Milne, 64.3)= 15.2 Gyr
$H_0 = 65$ km/s/Mpc :	$0.03 \leq \Omega_0 \leq 0.07$	$15.2 \text{ Gyr} \leq \text{Age}(65) \leq 15.4 \text{ Gyr}$
$H_0 = 67$ km/s/Mpc :	$0.06 \leq \Omega_0 \leq 0.17$	$14.9 \text{ Gyr} \leq \text{Age}(67) \leq 15.5 \text{ Gyr}$
$H_0 = 69$ km/s/Mpc :	$0.12 \leq \Omega_0 \leq 0.26$	$14.8 \text{ Gyr} \leq \text{Age}(69) \leq 15.6 \text{ Gyr}$

For a fixed H_0 among the set $\{63, 65, 67, 69\}$, by the least-square fit method, the H_0 -Milne curve is computed (in cyan color on the two following figures), after in black on figures, the development at order two of $H(z)$ at $z = 0$ is computed also by the least-square fit method, the de Sitter $H(z)$ tangent curve at zero is in green and the $H(z)$ -de Sitter curve such that $H(0) = H_0$ and $q_0 = -0.04$, the minimum baryonic density, is in pink. As for $H_0 = 63$ km/s/Mpc, the better fit is obtained for $\Omega_0 = 0.02$, this value is at the borderline and as for $H_0 = 69$ km/s/Mpc, the best fit is obtained for $\Omega_0 = 0.256$, this value is to be rejected because the de Sitter models contain neither dark energy nor dark matter. Even if the case $H_0 = 67$ km/s/Mpc is also borderline, and the figure looks interesting.

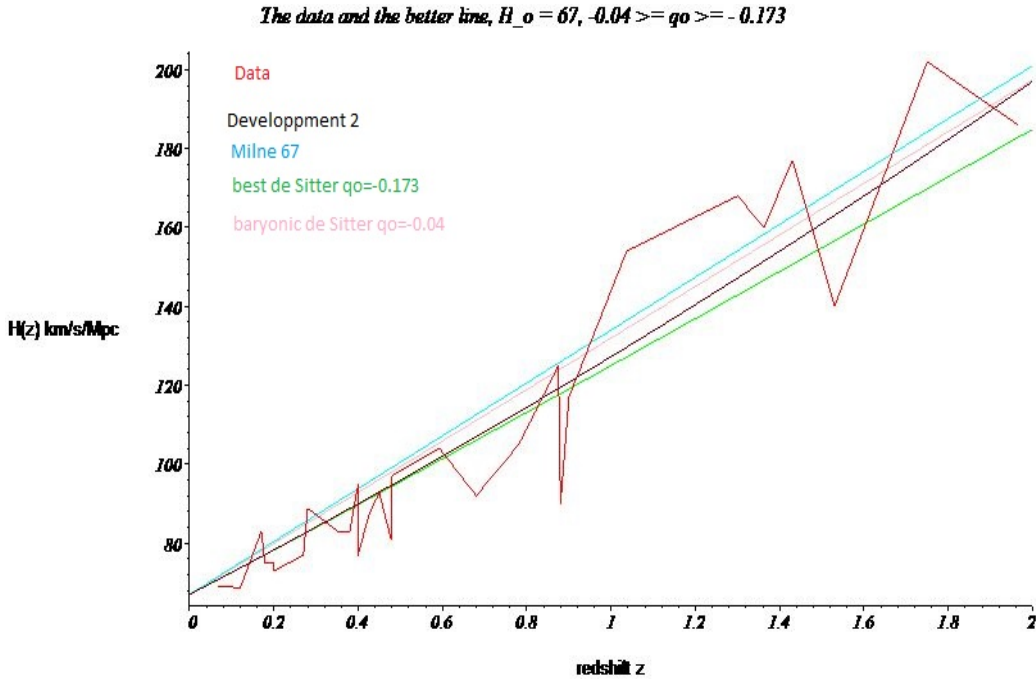


Figure 2 : for $H_0 = 67$ km/s/Mpc.

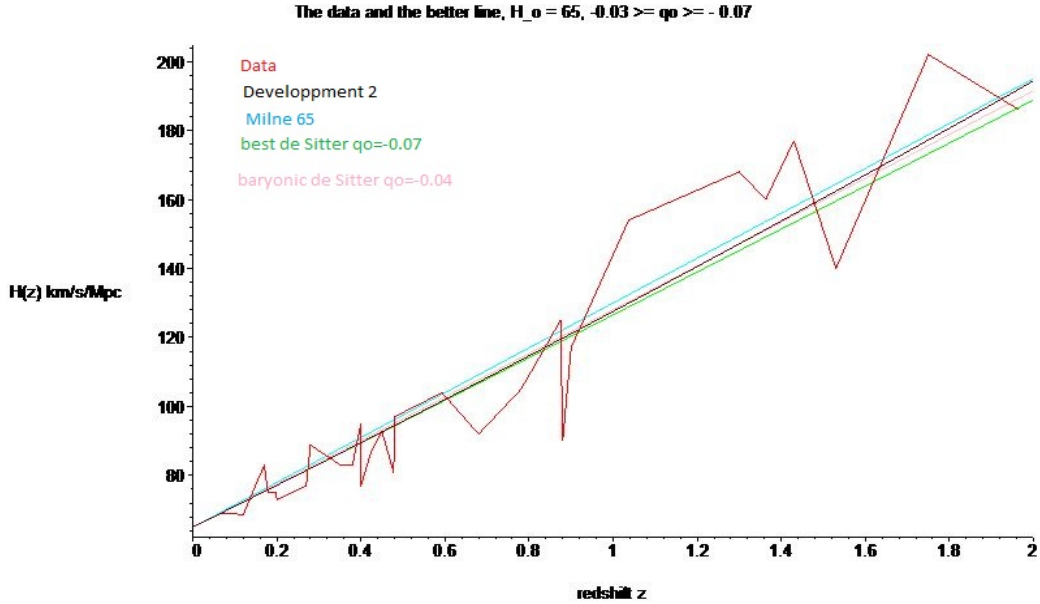


Figure 3 : for $H_0 = 65$ km/s/Mpc.

With these only thirty data the final result about the de Sitter universe is the following :

$$H_0 = 65 \pm 2 \text{ km/s/Mpc}, \Omega_0 = 0.05 \pm 0.02, \text{ Age} = 15.2 \pm 0.3 \text{ Gyr}. \quad (18)$$

4 Discussion

But, we do not have to take care, for this study, of the uncertainties on the data. In a few years, the data will be more numerous and more accurate and so it will be possible, with more theoretical statistics, to improve this first result.

Also we don't have to pay attention to the contribution of radiation ; even if this latter is tiny for a small redshift, for big redshifts, as the redshift 1100 for example, it would be necessary. If we added radiation, the scale factor $R(\tau)$ of the FL metric is equal to $b + c_1 \exp(\lambda \tau) + c_2 \exp(-\lambda \tau)$, with relations between the four constants ; $R(\tau)$ appears as a light modification of the de Sitter metric for $z \leq 2$.

The results (18) rest upon the a priori that, for theoretical reasons, it exists only baryonic matter around 4%, but it could be supposed that it exists a little unseen matter or even a little dark matter, but not in halos around galaxies [18], perhaps in clusters of galaxies or elsewhere ; so the density Ω_0 would be bigger and, ipso facto, the Hubble value now H_0 . This is compatible with the data $H_0 \leq 67$ i.e. $\Omega_0 \leq 0.17$, see the table of results and figure 2 above.

But a value of H_0 as high as 67 is not compatible with results coming from SNIa based on local methods, cf. for example the work of G. Tammann and B Reindl [19] who found $H_0 = 63.7 \pm 2.3$ km/s/Mpc or of V. Busti [20] who found $H_0 = 64.9 \pm 4.2$ km/s/Mpc and others.

“We emphasize here that the CMB estimates are highly model dependent” as Planck team said ([21] page 30), it is the same for the BAO.

5 Conclusion

In a first step, we have underlined a theoretical confusion : within a chart radially inertial, there is no need for dark energy. We also recalled why a huge mathematical mistake implies the need for dark matter to explain the flatness of the curves of rotation for spiral galaxies as many papers said since a long time (25 years). Thus, working in an inertial frame instead of a comoving one, the de Sitter models for the Universe appear to be the good theoretical models. It is well-known that these models are very good for the interpretation of the supernovae (a kinematic effect). The recent data about the Hubble parameter $H(z)$ was the occasion to confront the de Sitter models with these data. The results are beyond all that could be expected ; no conflict with the SNIa approach. No inflation, no problem of stability, no mystery about all which seems dark, but in conflict with the Λ CDM models. General relativity goes on, even if there remain many others problems, but the icing on the cake, the star HD140283 can extend his very long life quietly.

Among the problems to address :

- i) The BAO and the small fluctuations of the CMB, by using the inertial form of metric with radiation, a difficult problem even for the Λ CDM model [15].
- ii) The “Pioneer anomaly” which does not come from the dynamics of the universe model [3] but likely from the kinematics of the Milne model as is also the case for the SNIa.
- iii) The baryogenesis in this inertial frame for the Universe, and particularly the study of baryogenesis of the “lithium problem”.
- iv) If we want to glue together quantum mechanics and general relativity, the invariant de Sitter group is unavoidable ; for this goal, the Lie semigroup of causality of the de Sitter group is very interesting [22],[23], and maybe a good step for a well-posed problem to address this question.

Addendum : Let us return to the work of de Sitter : *On the curvature of space* [1] where the main metric (2B) he study is his metric (15), i.e. our metric 9. In this wonderful paper W. de Sitter said among others :

§7- "If in the future it should be proved that very distant objects have systematically positive apparent radial velocities, this would be an indication that the system B, and not A, would correspond to the truth."

§8- "We must then for " ρ_o " take the density not within the galactic system, but the average density over a unit of volume which is large compared with the mutual distances of the galactic systems. With the numerical data adopted above, this leads to $R = 5 \cdot 10^{13}$, and there would then be more than a billion galactic systems."

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