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# Simultaneous Information and Energy Transmission in the Interference Channel

Nizar Khalfet and Samir M. Perlaza

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# Simultaneous Information and Energy Transmission in the Interference Channel

Nizar Khalfet and Samir M. Perlaza

Project-Teams Socrate

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**Abstract:** In this report, the fundamental limits of simultaneous information and energy transmission in the two-user Gaussian interference channel (G-IC) with and without feedback are fully characterized. More specifically, an achievable and converse region in terms of information and energy transmission rates (in bits per channel use and energy-units per channel use, respectively) are identified. In both cases, with and without feedback, an achievability scheme based on power-splitting, common randomness, rate splitting, block-Markov superposition coding, and backward decoding is presented. Finally, converse regions for both cases are obtained using some of the existing outer bounds for information transmission rates, as well as a new outer bound for the energy transmission rate.

**Key-words:** Two-user Gaussian interference channel, simultaneous information and energy transmission, feedback, information-energy capacity region, achievability, converse.

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**Résumé :** Dans ce rapport, les limites fondamentales de la transmission simultanée d'information et d'énergie dans le canal Gaussien à interférence (G-IC) avec et sans voie de retour sont déterminées. Un ensemble des débits atteignables de transmission d'information et d'énergie (en bits par utilisation du canal et en unités d'énergie par utilisation du canal respectivement) est identifié. Pour les deux cas, un schéma d'atteignabilité basé sur power-splitting, common randomness, rate splitting, block-Markov superposition coding, et backward decoding est présenté. Finalement, une région converse pour les deux cas est obtenue en utilisant des techniques de majoration dans la littérature pour les débits d'information et aussi un majorant pour le débit d'énergie en utilisant la loi des grands nombres.

**Mots-clés :** Canal à interférence, voie de retour, transmission simultanée d'information et d'énergie

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## 1 Notation

Throughout this research report, sets are denoted with uppercase calligraphic letters, i.e.,  $\mathcal{X}$ . Random variables are denoted by uppercase letters, e.g.,  $X$ . The realization and the set of events from which the random variable  $X$  takes values are respectively denoted by  $x$  and  $\mathcal{X}$ . The probability distribution of  $X$  over the set  $\mathcal{X}$  is denoted  $P_X$ . Whenever a second random variable  $Y$  is involved,  $P_{XY}$  and  $P_{Y|X}$  denote respectively the joint probability distribution of  $(X, Y)$  and the conditional probability distribution of  $Y$  given  $X$ . Let  $N$  be a fixed natural number. An  $N$ -dimensional vector of random variables is denoted by  $\mathbf{X} = (X_1, X_2, \dots, X_N)^\top$  and a corresponding realization is denoted by  $\mathbf{x} = (x_1, x_2, \dots, x_N)^\top \in \mathcal{X}^N$ . Given  $\mathbf{X} = (X_1, X_2, \dots, X_N)^\top$  and  $(a, b) \in \mathbb{N}^2$ , with  $a < b \leq N$ , the  $(b-a+1)$ -dimensional vector of random variables formed by the components  $a$  to  $b$  of  $\mathbf{X}$  is denoted by  $\mathbf{X}_{(a,b)} = (X_a, X_{a+1}, \dots, X_b)^\top$ . The notation  $(\cdot)^+$  denotes the positive part operator, i.e.,  $(\cdot)^+ = \max(\cdot, 0)$  and  $\mathbb{E}_X[\cdot]$  denotes the expectation with respect to the distribution of the random variable  $X$ . The logarithm function is assumed to be base 2.

## 2 Gaussian Interference Channel with Energy Harvesting

Consider a Gaussian interference channel (G-IC) with a non-colocated energy harvester depicted in Figure 1 without feedback and in Figure 2 with perfect channel-output feedback. Transmitter  $i$ , with  $i \in \{1, 2\}$ , aims to execute two tasks: (a) an information transmission task and (b) an energy transmission task.

### 2.1 Information Transmission Task

From the information transmission standpoint, the goal of transmitter  $i$  is to convey an independent message index  $W_i \in \mathcal{W}_i = \{1, 2, \dots, 2^{NR_i}\}$  to receiver  $i$  using  $N$  channel input symbols  $X_{i,1}, X_{i,2}, \dots, X_{i,N}$ . The message index  $W_i$  is assumed to be uniformly distributed over  $\mathcal{W}$ . The channel coefficient from transmitter  $k$  to receiver  $i$ , with  $k \in \{1, 2\}$ , is denoted by  $h_{i,k} \in \mathbb{R}_+$ . At receiver  $i$ , during channel use  $n$ , input symbol  $X_{i,n}$  is observed at receiver  $i$  subject to the interference produced by the symbol  $X_{j,n}$  sent by transmitter  $j$ , with  $j \in \{1, 2\} \setminus \{i\}$ , and a real additive Gaussian noise  $Z_{i,n}$  with zero mean and variance  $\sigma_i^2$ . Hence, the channel output at receiver  $i$  during channel use  $n$ , denoted by  $Y_{i,n}$ , is:

$$Y_{i,n} = h_{i,i}X_{i,n} + h_{i,j}X_{j,n} + Z_{i,n}. \quad (1)$$

In the case without feedback, at each channel use  $n$ , the symbol  $X_{i,n}$  sent by transmitter  $i$  depends upon the message index  $W_i$  and a randomly generated index  $\Omega \in \mathbb{N}$  independent of  $W_1$  and  $W_2$ . Let  $f_{i,n}^{(N)} : \mathcal{W}_i \times \mathbb{N} \rightarrow \mathbb{R}$  be the encoding function at channel use  $n$ , such that for all  $n \in \{1, 2, \dots, N\}$ , the following holds:

$$X_{i,n} = f_{i,n}^{(N)}(W_i, \Omega). \quad (2)$$

In the case with feedback, the symbol  $X_{i,n}$  sent by transmitter  $i$  depends upon the indices  $W_i$  and  $\Omega$ , but also upon all previous channel-outputs  $Y_{i,1}, Y_{i,2}, \dots, Y_{i,n-d}$ , with  $d \in \mathbb{N}$  the feedback delay. In the following, without any loss of generality, it is assumed that  $d$  is equal to one channel use. More specifically,  $f_{i,1}^{(N)} : \mathcal{W}_i \times \mathbb{N} \rightarrow \mathbb{R}$  and for all  $n \in \{2, 3, \dots, N\}$ ,  $f_{i,n}^{(N)} : \mathcal{W}_i \times \mathbb{N} \times \mathbb{R}^{n-1} \rightarrow \mathbb{R}$  are encoding functions such that:

$$X_{i,1} = f_{i,1}^{(N)}(W_i, \Omega) \quad \text{and} \quad (3a)$$

$$X_{i,n} = f_{i,n}^{(N)}(W_i, \Omega, Y_{i,1}, Y_{i,2}, \dots, Y_{i,n-1}). \quad (3b)$$

In both cases, with and without feedback, the random index  $\Omega$  is assumed to be known by all transmitters and receivers. Moreover, channel input symbols  $X_{i,1}, X_{i,2}, \dots, X_{i,N}$  are subject to an average power constraint of the form

$$\frac{1}{N} \sum_{n=1}^N \mathbb{E}_{X_{i,n}} [X_{i,n}^2] \leq P_i, \quad (4)$$

where  $P_i$  denotes the average transmit power of transmitter  $i$  in energy units per channel use. The decoder of receiver  $i$  observes the channel outputs  $Y_{i,1}, Y_{i,2}, \dots, Y_{i,N}$  and uses a decoding function  $\phi_i^{(N)} : \mathbb{N} \times \mathbb{R}^N \rightarrow \mathcal{W}_i$ , to get an estimate of the message indices:

$$\widehat{W}_i = \phi_i^{(N)}(\Omega, Y_{i,1}, Y_{i,2}, \dots, Y_{i,N}), \quad (5)$$



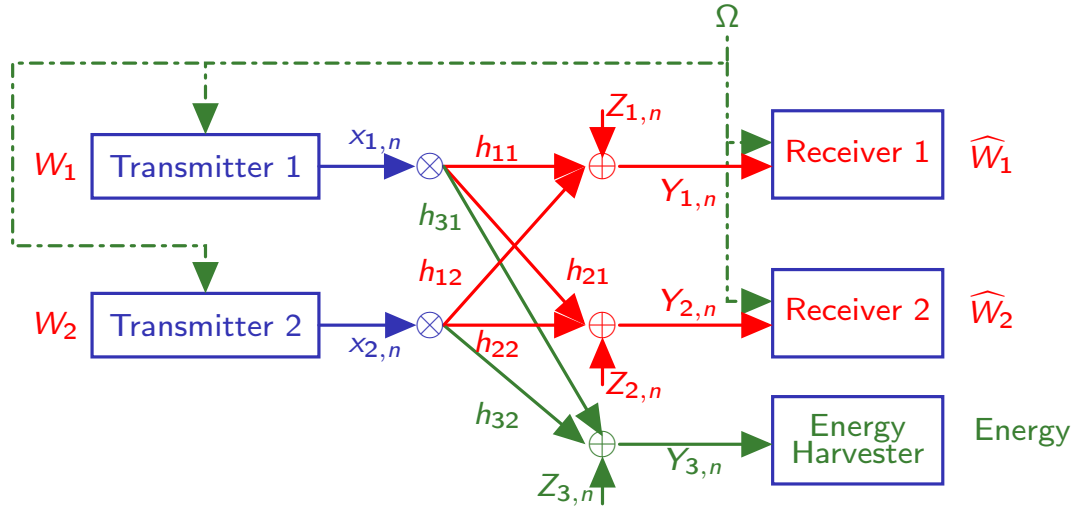


Figure 1: Two-user Gaussian interference channel with a non-cocated energy harvester at channel use  $n$ .

where  $\widehat{W}_i$  is an estimate of the message index  $W_i$ . The decoding error probability of a codebook of block-length  $N$ , denoted by  $P_{\text{DE}}^{(N)}$ , is given by

$$P_{\text{DE}}^{(N)} = \max \left[ \Pr \left[ \widehat{W}_1 \neq W_1 \right], \Pr \left[ \widehat{W}_2 \neq W_2 \right] \right]. \quad (6)$$

The signal to noise ratio (SNR) at receiver  $i$  is denoted by

$$\text{SNR}_i = \frac{|h_{i,i}|^2 P_i}{\sigma_i^2}. \quad (7a)$$

The interference to noise ratio (INR) at receiver  $i$  is denoted by

$$\text{INR}_i = \frac{|h_{i,j}|^2 P_j}{\sigma_i^2}, \text{ with } j \neq i. \quad (7b)$$

## 2.2 Energy Transmission Task

Let  $h_{3i} \in \mathbb{R}_+$  be the channel coefficient from transmitter  $i$  to the energy harvester (EH). The symbols sent by the transmitters during channel use  $n$  are observed by the EH subject to an additive Gaussian noise  $Z_{3,n}$  with zero mean and variance  $\sigma_3^2$ . More specifically, the channel output at the EH during channel use  $n$ , denoted by  $Y_{3,n}$ , is:

$$Y_{3,n} = h_{3,1}X_{1,n} + h_{3,2}X_{2,n} + Z_{3,n}. \quad (8)$$

From the energy transmission standpoint, the goal of both transmitters is to jointly guarantee an average energy rate at the EH. Let  $B^{(N)} : \mathbb{R}^N \rightarrow \mathbb{R}_+$  be a function that determines the energy

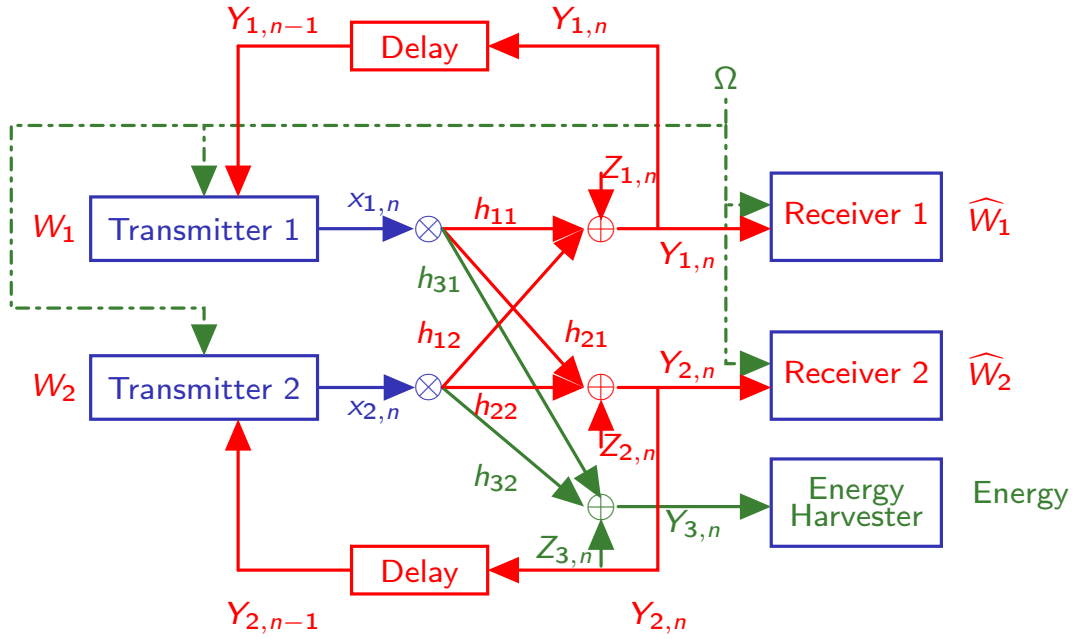


Figure 2: Two-user Gaussian interference channel with channel-output feedback and a non-collocated energy harvester during channel use  $n$ .

obtained from the channel outputs  $Y_{3,1}, Y_{3,2}, \dots, Y_{3,N}$ . In the following, this function is chosen to be the average energy rate (in energy-units per channel use) at the end of  $N$  channel uses. That is,

$$B^{(N)}(Y_{3,1}, Y_{3,2}, \dots, Y_{3,N}) \triangleq \frac{1}{N} \sum_{n=1}^N Y_{3,n}^2, \quad (9)$$

which implies that the energy carried by a given channel output  $Y_{3,t}$ , with  $t \in \{1, 2, \dots, N\}$ , is  $Y_{3,t}^2$ . This assumption is very optimistic given the dependency of the delivered DC power on higher order statistics of the channel input distribution [3, 4]. Nonetheless, from the fundamental limits point of view, the assumption in (9) has the justification of the assumption of perfect channel output feedback, i.e., any more realistic model would induce fundamental limits that would be more pessimistic than the results presented in the following sections.

The SNR of transmitter  $i$  at the EH is denoted by

$$\text{SNR}_{3i} = \frac{|h_{3,i}|^2 P_i}{\sigma_3^2}. \quad (10)$$

Note that the maximum average energy rate, denoted by  $B_{\max}$ , is:

$$B_{\max} = \sigma_3^2 \left( 1 + \text{SNR}_{31} + \text{SNR}_{32} + 2\sqrt{\text{SNR}_{31}\text{SNR}_{32}} \right). \quad (11)$$

which can be achieved in the asymptotic block-length regime when both channel inputs exhibit a correlation coefficient equal to one. Hence, given an energy rate  $B \in [0, B_{\max}]$ , the energy

shortage probability, denoted by  $P_{\text{ES}}^{(N)}$ , is:

$$P_{\text{ES}}^{(N)} \triangleq \Pr [B^{(N)}(\mathbf{Y}_3) < B]. \quad (12)$$

### 2.3 Simultaneous Information and Energy Transmission

The system is said to operate at the information-energy rate triplet  $(R_1, R_2, B) \in \mathbb{R}_+^3$  when both transmitter-receiver pairs use a transmit-receive configuration such that: (i) reliable communication at information rates  $R_1$  and  $R_2$  is ensured; and (ii) reliable energy transmission at energy rate  $B$  is ensured. A formal definition is given below.

**Definition 1 (Achievable Rates)** *The triplet  $(R_1, R_2, B) \in \mathbb{R}_+^3$  is achievable if for all  $i \in \{1, 2\}$ , there exists a sequence of encoding functions  $f_{i,1}^{(N)}, f_{i,2}^{(N)}, \dots, f_{i,N}^{(N)}$  and a decoding functions  $\phi_1^{(N)}$  and  $\phi_2^{(N)}$  such that both the average decoding error probability  $P_{\text{DE}}^{(N)}$  and the energy-shortage probability  $P_{\text{ES}}^{(N)}$  tend to zero as the block-length  $N$  tends to infinity. That is,*

$$\limsup_{N \rightarrow \infty} P_{\text{DE}}^{(N)} = 0 \text{ and} \quad (13a)$$

$$\limsup_{N \rightarrow \infty} P_{\text{ES}}^{(N)} = 0. \quad (13b)$$

Using Definition 1, the fundamental limits of simultaneous information and energy transmission in the G-IC can be described by the information-energy capacity region, defined as follows.

**Definition 2 (Information-Energy Capacity Region)** *The information-energy capacity region, denoted by  $\mathcal{E}^{\text{F}}$  in the case with feedback and  $\mathcal{E}$  in the case without feedback, corresponds to the closure of all achievable information-energy rate triplets  $(R_1, R_2, B)$ .*

## 3 Main Result

The main result consists of a description of the information-energy capacity regions with feedback  $\mathcal{E}^{\text{F}}$  and without feedback  $\mathcal{E}$ . Such a description is presented in the form of an *approximation* in the sense of the definition hereunder.

**Definition 3 (Approximation of a Set)** *Let  $n \in \mathbb{N}$  be fixed. A set  $\mathcal{X} \subset \mathbb{R}_+^n$  is approximated by the sets  $\underline{\mathcal{X}}$  and  $\overline{\mathcal{X}}$  if  $\underline{\mathcal{X}} \subseteq \mathcal{X} \subseteq \overline{\mathcal{X}}$  and  $\forall \mathbf{x} = (x_1, \dots, x_n) \in \overline{\mathcal{X}}$  then  $\left( (x_1 - \xi_1)^+, (x_2 - \xi_2)^+, \dots, (x_n - \xi_n)^+ \right) \in \underline{\mathcal{X}}$ , for some  $(\xi_1, \xi_2, \dots, \xi_n) \in \mathbb{R}_+^n$ .*

### 3.1 Case without Channel-Output Feedback

The information-energy capacity region  $\mathcal{E}$ , is approximated by the regions  $\underline{\mathcal{E}}$  (Theorem 1), which represents an information-energy achievable region, and  $\overline{\mathcal{E}}$  (Theorem 2), which represents an information-energy converse region.

#### 3.1.1 An Achievable Region

The following theorem introduces an achievable information-energy region.

**Theorem 1** *The information-energy capacity region  $\mathcal{E}$  contains the set  $\underline{\mathcal{E}} \subseteq \mathbb{R}_+^3$  of all rate tuples  $(R_1, R_2, B)$  that satisfy:*

$$R_1 \leq \frac{1}{2} \log \left( 1 + \frac{(1 - \lambda_{1e})\text{SNR}_1}{1 + \lambda_{2p}\text{INR}_1} \right), \quad (14a)$$

$$R_2 \leq \frac{1}{2} \log \left( 1 + \frac{(1 - \lambda_{2e})\text{SNR}_2}{1 + \lambda_{1p}\text{INR}_2} \right), \quad (14b)$$

$$R_1 + R_2 \leq \frac{1}{2} \log \left( \frac{1 + (1 - \lambda_{1e})\text{SNR}_1 + (1 - \lambda_{2e})\text{INR}_1}{1 + \lambda_{2p}\text{INR}_1} \right) + \frac{1}{2} \log \left( 1 + \frac{\lambda_{2p}\text{SNR}_2}{1 + \lambda_{1p}\text{INR}_2} \right), \quad (14c)$$

$$R_1 + R_2 \leq \frac{1}{2} \log \left( \frac{1 + (1 - \lambda_{2e})\text{SNR}_2 + (1 - \lambda_{1e})\text{INR}_2}{1 + \lambda_{1p}\text{INR}_2} \right) + \frac{1}{2} \log \left( 1 + \frac{\lambda_{1p}\text{SNR}_1}{1 + \lambda_{2p}\text{INR}_1} \right), \quad (14d)$$

$$R_1 + R_2 \leq \frac{1}{2} \log \left( \frac{1 + \lambda_{1p}\text{SNR}_1 + (1 - \lambda_{2e})\text{INR}_1}{1 + \lambda_{2p}\text{INR}_1} \right) + \frac{1}{2} \log \left( \frac{1 + \lambda_{2p}\text{SNR}_2 + (1 - \lambda_{1e})\text{INR}_2}{1 + \lambda_{1p}\text{INR}_2} \right) \quad (14e)$$

$$2R_1 + R_2 \leq \frac{1}{2} \log \left( \frac{1 + (1 - \lambda_{1e})\text{SNR}_1 + (1 - \lambda_{2e})\text{INR}_1}{1 + \lambda_{2p}\text{INR}_1} \right) + \frac{1}{2} \log \left( \frac{1 + \lambda_{2p}\text{SNR}_2 + (1 - \lambda_{1e})\text{INR}_2}{1 + \lambda_{1p}\text{INR}_2} \right) + \frac{1}{2} \log \left( 1 + \frac{\lambda_{1p}\text{SNR}_1}{1 + \lambda_{2p}\text{INR}_1} \right), \quad (14f)$$

$$R_1 + 2R_2 \leq \frac{1}{2} \log \left( \frac{1 + (1 - \lambda_{2e})\text{SNR}_2 + (1 - \lambda_{1e})\text{INR}_2}{1 + \lambda_{1p}\text{INR}_2} \right) + \frac{1}{2} \log \left( \frac{1 + \lambda_{1p}\text{SNR}_1 + (1 - \lambda_{2e})\text{INR}_1}{1 + \lambda_{2p}\text{INR}_1} \right) + \frac{1}{2} \log \left( 1 + \frac{\lambda_{2p}\text{SNR}_2}{1 + \lambda_{1p}\text{INR}_2} \right), \quad (14g)$$

$$B \leq \sigma_3^2 \left( 1 + \text{SNR}_{31} + \text{SNR}_{32} + 2\sqrt{\text{SNR}_{31}\text{SNR}_{32}}\sqrt{\lambda_{1e}\lambda_{2e}} \right), \quad (14h)$$

with  $(\lambda_{ip}, \lambda_{ie}) \in [0, 1]^2$  such that  $\lambda_{ip} + \lambda_{ie} \leq 1$ , for all  $i \in \{1, 2\}$ .

*Proof:* The proof of Theorem 1 is presented in Appendix A. ■

### 3.1.2 A Converse Region

The following theorem introduces an information-energy converse region.

**Theorem 2** *The information-energy capacity region  $\mathcal{E}$  contains the set  $\bar{\mathcal{E}} \subseteq \mathbb{R}_+^3$  of all rate tuples*

$(R_1, R_2, B)$  that satisfy:

$$R_1 \leq \frac{1}{2} \log(1 + \beta_1 \text{SNR}_1), \quad (15a)$$

$$R_2 \leq \frac{1}{2} \log(1 + \beta_2 \text{SNR}_2), \quad (15b)$$

$$R_1 + R_2 \leq \frac{1}{2} \log(1 + \beta_1 \text{SNR}_1 + \beta_2 \text{INR}_1) + \frac{1}{2} \log\left(1 + \frac{\beta_2 \text{SNR}_2}{1 + \beta_2 \text{INR}_1}\right), \quad (15c)$$

$$R_1 + R_2 \leq \frac{1}{2} \log(1 + \beta_2 \text{SNR}_2 + \beta_1 \text{INR}_2) + \frac{1}{2} \log\left(1 + \frac{\beta_1 \text{SNR}_1}{1 + \beta_1 \text{INR}_2}\right), \quad (15d)$$

$$\begin{aligned} R_1 + R_2 &\leq \frac{1}{2} \log\left(1 + \frac{\beta_1 \text{SNR}_1 + \beta_2 \text{INR}_1 + \beta_1 \beta_2 \text{INR}_1 \text{INR}_2}{1 + \beta_1 \text{INR}_2}\right) \\ &\quad + \frac{1}{2} \log\left(1 + \frac{\beta_2 \text{SNR}_2 + \beta_1 \text{INR}_2 + \beta_1 \beta_2 \text{INR}_1 \text{INR}_2}{1 + \beta_2 \text{INR}_1}\right), \end{aligned} \quad (15e)$$

$$\begin{aligned} 2R_1 + R_2 &\leq \frac{1}{2} \log\left(1 + \frac{\beta_1 \text{SNR}_1}{1 + \beta_1 \text{INR}_2}\right) + \frac{1}{2} \log(1 + \beta_1 \text{SNR}_1 + \beta_2 \text{INR}_1) \\ &\quad + \frac{1}{2} \log\left(1 + \frac{\beta_2 \text{SNR}_2 + \beta_1 \text{INR}_2 + \beta_1 \beta_2 \text{INR}_1 \text{INR}_2}{1 + \beta_2 \text{INR}_1}\right), \end{aligned} \quad (15f)$$

$$\begin{aligned} R_1 + 2R_2 &\leq \frac{1}{2} \log\left(1 + \frac{\beta_2 \text{SNR}_2}{1 + \beta_2 \text{INR}_1}\right) + \frac{1}{2} \log(1 + \beta_2 \text{SNR}_2 + \beta_1 \text{INR}_2) \\ &\quad + \frac{1}{2} \log\left(1 + \frac{\beta_1 \text{SNR}_1 + \beta_2 \text{INR}_1 + \beta_1 \beta_2 \text{INR}_1 \text{INR}_2}{1 + \beta_1 \text{INR}_2}\right), \end{aligned} \quad (15g)$$

$$B \leq \sigma_3^2 \left(1 + \text{SNR}_{31} + \text{SNR}_{32} + 2\sqrt{\text{SNR}_{31} \text{SNR}_{32}} \sqrt{(1 - \beta_1)(1 - \beta_2)}\right), \quad (15h)$$

with  $(\beta_1, \beta_2) \in [0, 1]^2$ .

*Proof:* The proof of Theorem 2 is presented in Appendix B. ■

### 3.1.3 An Approximation to the Information-Energy Capacity Region

Using the inner region  $\underline{\mathcal{E}}$  and the outer region  $\bar{\mathcal{E}}$ , described respectively by Theorem 1 and Theorem 2, the information-energy capacity region  $\mathcal{E}$  can be approximated in the sense of Definition 3.

**Theorem 3 (Approximation of  $\mathcal{E}$ )** *Let  $\underline{\mathcal{E}} \subset \mathbb{R}_+^3$  and  $\bar{\mathcal{E}} \subset \mathbb{R}_+^3$  be the sets of tuples  $(R_1, R_2, B)$  described by Theorem 1 and Theorem 2, respectively. Then,*

$$\underline{\mathcal{E}} \subset \mathcal{E} \subset \bar{\mathcal{E}}, \quad (16)$$

and for all  $(R_1, R_2, B) \in \bar{\mathcal{E}}$  it follows that  $\left((R_1 - 1/2)^+, (R_2 - 1/2)^+, \left(B - \frac{B_{\max}}{2}\right)^+\right) \in \underline{\mathcal{E}}$ .

*Proof:* Note that for all  $(R_1, R_2, B) \in \bar{\mathcal{E}}$  and for all  $(R'_1, R'_2, B') \in \underline{\mathcal{E}}$ , there always exists a tuple  $(\beta_1, \beta_2, \lambda_{1e}, \lambda_{2e})$  such that:

$$\begin{aligned}
\frac{B - B'}{B_{\max}} &= \frac{2h_{3,1}h_{3,2}\sqrt{P_1P_2} \left( \sqrt{(1-\beta_1)(1-\beta_2)} - \sqrt{\lambda_{1e}\lambda_{2e}} \right)}{\sigma_3^2 + h_{3,1}^2P_1 + h_{3,2}^2P_2 + 2|h_{3,1}||h_{3,2}|\sqrt{P_1P_2}} \\
&\leq \frac{2\sqrt{\text{SNR}_{31}\text{SNR}_{32}}}{1 + \text{SNR}_{31} + \text{SNR}_{32} + 2\sqrt{\text{SNR}_{31}\text{SNR}_{32}}} \\
&\leq \frac{2\sqrt{\text{SNR}_{31}\text{SNR}_{32}}}{1 + 4\sqrt{\text{SNR}_{31}\text{SNR}_{32}}} \\
&\leq \frac{1}{2}.
\end{aligned}$$

The proof that  $R_1 - R'_1 \leq \frac{1}{2}$  and  $R_2 - R'_2 \leq \frac{1}{2}$  is presented in [5]. This completes the proof.  $\blacksquare$

### 3.2 Case with Perfect Channel-Output Feedback

The information-energy capacity region  $\mathcal{E}^{\text{F}}$ , is approximated by the regions  $\underline{\mathcal{E}}^{\text{F}}$  (Theorem 4) and  $\overline{\mathcal{E}}^{\text{F}}$  (Theorem 5).

#### 3.2.1 An Achievable Region

The following theorem introduces an achievable information-energy region.

**Theorem 4** *The information-energy capacity region  $\mathcal{E}^{\text{F}}$  contains the set  $\underline{\mathcal{E}}^{\text{F}} \subseteq \mathbb{R}_+^3$  of all rate tuples  $(R_1, R_2, B)$  that satisfy:*

$$R_1 \leq \frac{1}{2} \log \left( \frac{1 + (1 - \lambda_{1e})\text{SNR}_1 + (1 - \lambda_{2e})\text{INR}_1 + 2\rho\sqrt{\text{SNR}_1\text{INR}_1}}{1 + \lambda_{2p}\text{INR}_1} \right), \quad (17a)$$

$$R_1 \leq \frac{1}{2} \log \left( \frac{1 + (1 - (\rho + \lambda_{1e}))\text{INR}_2}{1 + \lambda_{1p}\text{INR}_2} \right) + \frac{1}{2} \log \left( \frac{1 + \lambda_{1p}\text{SNR}_1 + \lambda_{2p}\text{INR}_1}{1 + \lambda_{2p}\text{INR}_1} \right), \quad (17b)$$

$$R_2 \leq \frac{1}{2} \log \left( \frac{1 + (1 - \lambda_{2e})\text{SNR}_2 + (1 - \lambda_{1e})\text{INR}_2 + 2\rho\sqrt{\text{SNR}_2\text{INR}_2}}{1 + \lambda_{1p}\text{INR}_2} \right), \quad (17c)$$

$$R_2 \leq \frac{1}{2} \log \left( \frac{1 + (1 - (\rho + \lambda_{2e}))\text{INR}_1}{1 + \lambda_{2p}\text{INR}_1} \right) + \frac{1}{2} \log \left( \frac{1 + \lambda_{2p}\text{SNR}_1 + \lambda_{1p}\text{INR}_1}{1 + \lambda_{1p}\text{INR}_1} \right), \quad (17d)$$

$$\begin{aligned}
R_1 + R_2 &\leq \frac{1}{2} \log \left( \frac{1 + \lambda_{1p}\text{SNR}_1 + \lambda_{2p}\text{INR}_1}{1 + \lambda_{2p}\text{INR}_1} \right) \\
&\quad + \frac{1}{2} \log \left( \frac{1 + (1 - \lambda_{2e})\text{SNR}_2 + (1 - \lambda_{1e})\text{INR}_2 + 2\rho\sqrt{\text{SNR}_2\text{INR}_2}}{1 + \lambda_{1p}\text{INR}_2} \right), \quad (17e)
\end{aligned}$$

$$\begin{aligned}
R_1 + R_2 &\leq \frac{1}{2} \log \left( \frac{1 + \lambda_{2p}\text{SNR}_1 + \lambda_{1p}\text{INR}_1}{1 + \lambda_{1p}\text{INR}_1} \right) \\
&\quad + \frac{1}{2} \log \left( \frac{1 + (1 - \lambda_{1e})\text{SNR}_1 + (1 - \lambda_{2e})\text{INR}_1 + 2\rho\sqrt{\text{SNR}_1\text{INR}_1}}{1 + \lambda_{2p}\text{INR}_1} \right), \quad (17f)
\end{aligned}$$

$$B \leq \sigma_3^2 \left( 1 + \text{SNR}_{31} + \text{SNR}_{32} + 2\sqrt{\text{SNR}_{31}\text{SNR}_{32}}(\rho + \sqrt{\lambda_{1e}\lambda_{2e}}) \right), \quad (17g)$$

where  $(\rho, \lambda_{ip}, \lambda_{ie}) \in [0, 1]^3$  and  $\rho + \lambda_{ip} + \lambda_{ie} \leq 1$ , for all  $i \in \{1, 2\}$ .

*Proof:* The proof of Theorem 4 is presented in Appendix C.  $\blacksquare$

### 3.2.2 A Converse Region

The following theorem describes a converse region denoted by  $\bar{\mathcal{E}}^F$ .

**Theorem 5** *The information-energy capacity region  $\mathcal{E}^F$  is contained into the set  $\bar{\mathcal{E}}^F \in \mathbb{R}_+^3$ , which contains all rate tuples  $(R_1, R_2, B)$  that satisfy:*

$$R_1 \leq \frac{1}{2} \log \left( 1 + \beta_1 \text{SNR}_1 + \beta_2 \text{INR}_1 + 2\rho \sqrt{\beta_1 \text{SNR}_1 \beta_2 \text{INR}_1} \right), \quad (18a)$$

$$R_1 \leq \frac{1}{2} \log \left( 1 + \frac{\beta_1(1-\rho^2)\text{SNR}_1}{1+\beta_1(1-\rho^2)\text{INR}_2} \right) + \frac{1}{2} \log \left( 1 + \beta_1(1-\rho^2)\text{INR}_2 \right), \quad (18b)$$

$$R_2 \leq \frac{1}{2} \log \left( 1 + \beta_2 \text{SNR}_2 + \beta_1 \text{INR}_2 + 2\rho \sqrt{\beta_2 \text{SNR}_2 \beta_1 \text{INR}_2} \right), \quad (18c)$$

$$R_2 \leq \frac{1}{2} \log \left( 1 + \frac{\beta_2(1-\rho^2)\text{SNR}_2}{1+\beta_2(1-\rho^2)\text{INR}_1} \right) + \frac{1}{2} \log \left( 1 + \beta_2(1-\rho^2)\text{INR}_1 \right), \quad (18d)$$

$$R_1 + R_2 \leq \frac{1}{2} \log \left( 1 + \frac{\beta_1(1-\rho^2)\text{SNR}_1}{1+\beta_1(1-\rho^2)\text{INR}_2} \right) + \frac{1}{2} \log \left( 1 + \beta_2 \text{SNR}_2 + \beta_1 \text{INR}_2 + 2\rho \sqrt{\beta_2 \text{SNR}_2 \beta_1 \text{INR}_2} \right), \quad (18e)$$

$$R_1 + R_2 \leq \frac{1}{2} \log \left( 1 + \frac{\beta_2(1-\rho^2)\text{SNR}_2}{1+\beta_2(1-\rho^2)\text{INR}_1} \right) + \frac{1}{2} \log \left( 1 + \beta_1 \text{SNR}_1 + \beta_2 \text{INR}_1 + 2\rho \sqrt{\beta_1 \text{SNR}_1 \beta_2 \text{INR}_1} \right), \quad (18f)$$

$$B \leq \sigma_3^2 \left( 1 + \text{SNR}_{31} + \text{SNR}_{32} + 2\sqrt{\text{SNR}_{31}\text{SNR}_{32}(\rho\sqrt{\beta_1\beta_2} + \sqrt{(1-\beta_1)(1-\beta_2)})} \right), \quad (18g)$$

with  $(\beta_1, \beta_2, \rho) \in [0, 1]^3$ .

*Proof:* The proof of Theorem 5 is presented in Appendix D. ■

### 3.2.3 An Approximation to the Information-Energy Capacity Region

Using the inner region  $\underline{\mathcal{E}}^F$  and the outer region  $\bar{\mathcal{E}}^F$ , described respectively by Theorem 4 and Theorem 5, the information-energy capacity region  $\mathcal{E}^F$  can be approximated in the sense of Definition 3.

**Theorem 6 (Approximation of  $\mathcal{E}^F$ )** *Let  $\underline{\mathcal{E}}^F \subset \mathbb{R}_+^3$  and  $\bar{\mathcal{E}}^F \subset \mathbb{R}_+^3$  be the sets of tuples  $(R_1, R_2, B)$  described by Theorem 4 and Theorem 5, respectively. Then,*

$$\underline{\mathcal{E}}^F \subset \mathcal{E}^F \subset \bar{\mathcal{E}}^F, \quad (19)$$

and for all  $(R_1, R_2, B) \in \bar{\mathcal{E}}^F$  it follows that  $\left( (R_1 - 1)^+, (R_2 - 1)^+, \left( B - \frac{B_{\max}}{2} \right)^+ \right) \in \underline{\mathcal{E}}^F$ .

*Proof:* Note that for all  $(R_1, R_2, B) \in \bar{\mathcal{E}}^F$  and for all  $(R'_1, R'_2, B') \in \underline{\mathcal{E}}^F$ , there always exists

a tuple  $(\beta_1, \beta_2, \lambda_{1e}, \lambda_{2e}, \rho)$  such that:

$$\begin{aligned} \frac{B - B'}{B_{\max}} &\leq \frac{2h_{3,1}h_{3,2}\sqrt{P_1P_2} \left( \sqrt{(1-\beta_1)(1-\beta_2)} + \rho\sqrt{\beta_1\beta_2} - \rho' - \sqrt{\lambda_{1e}\lambda_{2e}} \right)}{\sigma_3^2 + h_{3,1}^2P_1 + h_{3,2}^2P_2 + 2h_{3,1}h_{3,2}\sqrt{P_1P_2}} \\ &\leq \frac{2\sqrt{\text{SNR}_{31}\text{SNR}_{32}}}{1 + \text{SNR}_{31} + \text{SNR}_{32} + 2\sqrt{\text{SNR}_{31}\text{SNR}_{32}}} \\ &\leq \frac{2\sqrt{\text{SNR}_{31}\text{SNR}_{32}}}{1 + 4\sqrt{\text{SNR}_{31}\text{SNR}_{32}}} \\ &\leq \frac{1}{2}. \end{aligned}$$

The proof of the inequalities  $R_1 - R'_1 \leq 1$  and  $R_2 - R'_2 \leq 1$  is presented in [6]. This completes the proof.  $\blacksquare$

### 3.3 Maximum Energy Rate Improvement with Feedback

Consider the following sets of energy rates:  $\underline{\mathcal{B}} = \{b \in \mathbb{R}_+ : (R_1, R_2, b) \in \underline{\mathcal{E}}\}$ ,  $\overline{\mathcal{B}} = \{b \in \mathbb{R}_+ : (R_1, R_2, b) \in \overline{\mathcal{E}}\}$ ,  $\underline{\mathcal{B}}_F = \{b \in \mathbb{R}_+ : (R_1, R_2, b) \in \underline{\mathcal{E}}^F\}$ , and  $\overline{\mathcal{B}}_F = \{b \in \mathbb{R}_+ : (R_1, R_2, b) \in \overline{\mathcal{E}}^F\}$ . The maximum improvement that can be achieved on the energy rate due to feedback can be shown to be at most a factor of two. The following proposition shows this by providing upper bounds on the ratios  $\frac{\max \underline{\mathcal{B}}_F}{\max \underline{\mathcal{B}}}$  and  $\frac{\max \overline{\mathcal{B}}_F}{\max \overline{\mathcal{B}}}$ .

**Proposition 1 (Rate improvement with Feedback)** The energy rate achievable in the two-user G-IC with perfect channel-output feedback can be twice the energy rate achievable in the two-user G-IC without feedback. That is,

$$1 < \frac{\max \underline{\mathcal{B}}_F}{\max \underline{\mathcal{B}}} \leq 2. \quad (20)$$

Any improvement beyond a factor of two is not feasible. That is,

$$1 < \frac{\max \overline{\mathcal{B}}_F}{\max \overline{\mathcal{B}}} \leq 2. \quad (21)$$

Before presenting the proof of Proposition 1, the following auxiliary result is presented

**Proposition 2** For all  $(x, y) \in [0, 1]^2$ , it holds that

$$\sqrt{xy} + \sqrt{(1-x)(1-y)} \leq 1. \quad (22)$$

*Proof:* Given that  $(\sqrt{x} - \sqrt{y})^2 = x + y - 2\sqrt{xy} \geq 0$ , it holds that

$$\sqrt{xy} \leq \frac{x+y}{2}. \quad (23)$$

Using (23), it holds that

$$\sqrt{xy} \leq \frac{x+y}{2}, \quad \text{and} \quad (24)$$

$$\sqrt{(1-x)(1-y)} \leq \frac{2-(x+y)}{2}. \quad (25)$$

Adding (24) and (25) yields the inequality in (22). This completes the proof.  $\blacksquare$

*Proof of Proposition 1:* The proof is divided into two parts. The first part presents the proof of (20). The second part presents the proof of (21)



1. Proof of (20): Let  $\max \underline{\mathcal{B}}_F$  be written in terms of  $(\rho, \lambda_{1p}, \lambda_{2p}, \lambda_{1e}, \lambda_{2e})$  and let also  $\max \underline{\mathcal{B}}$  be written in terms of  $(\rho', \lambda'_{1p}, \lambda'_{2p}, \lambda'_{1e}, \lambda'_{2e})$  such that:

$$\frac{\sup \underline{\mathcal{B}}_F}{\sup \underline{\mathcal{B}}} = \frac{1 + \text{SNR}_{31} + \text{SNR}_{32} + 2\sqrt{\text{SNR}_{31}\text{SNR}_{32}}(\rho + \sqrt{\lambda_{1e}\lambda_{2e}})}{1 + \text{SNR}_{31} + \text{SNR}_{32} + 2\sqrt{\text{SNR}_{31}\text{SNR}_{32}}\lambda'_{1e}\lambda'_{2e}} \quad (26)$$

$$\leq \frac{1 + \text{SNR}_{31} + \text{SNR}_{32} + 2\sqrt{\text{SNR}_{31}\text{SNR}_{32}}(\rho + \max(\lambda_{1e}, \lambda_{2e}))}{1 + \text{SNR}_{31} + \text{SNR}_{32}} \quad (27)$$

$$\leq 1 + \frac{2\sqrt{\text{SNR}_{31}\text{SNR}_{32}}}{1 + \text{SNR}_{31} + \text{SNR}_{32}} \quad (28)$$

$$\leq 1 + \frac{\text{SNR}_{31} + \text{SNR}_{32}}{1 + \text{SNR}_{31} + \text{SNR}_{32}} \quad (29)$$

$$\leq 2. \quad (30)$$

2. Proof of (21) : Let  $\max \bar{\mathcal{B}}_F$  be written in terms of  $(\rho, \beta_1, \beta_2) \in [0, 1]^3$  and let also  $\max \bar{\mathcal{B}}$  be written in terms of  $(\beta'_1, \beta'_2) \in [0, 1]^2$  such that:

$$\frac{\sup \bar{\mathcal{B}}_F}{\sup \bar{\mathcal{B}}} = \frac{1 + \text{SNR}_{31} + \text{SNR}_{32} + 2\sqrt{\text{SNR}_{31}\text{SNR}_{32}}(\rho\sqrt{\beta_1\beta_2} + \sqrt{(1-\beta_1)(1-\beta_2)})}{1 + \text{SNR}_{31} + \text{SNR}_{32} + 2\sqrt{\text{SNR}_{31}\text{SNR}_{32}}(1-\beta'_1)(1-\beta'_2)} \quad (31)$$

$$\leq 1 + \frac{2\sqrt{\text{SNR}_{31}\text{SNR}_{32}}}{1 + \text{SNR}_{31} + \text{SNR}_{32} + 2\sqrt{\text{SNR}_{31}\text{SNR}_{32}}(1-\beta_1)(1-\beta_2)} \quad (32)$$

$$\leq 1 + \frac{2\sqrt{\text{SNR}_{31}\text{SNR}_{32}}}{1 + \text{SNR}_{31} + \text{SNR}_{32}} \quad (33)$$

$$\leq 1 + \frac{\text{SNR}_{31} + \text{SNR}_{32}}{1 + \text{SNR}_{31} + \text{SNR}_{32}} \quad (34)$$

$$\leq 2, \quad (35)$$

where (32) follows from Proposition 2. This completes the proof. ■

## 4 Numerical Analysis

Consider the two-user G-ICs with and without channel-output feedback depicted in Figure 1 and Figure 2 with parameters  $\text{SNR}_1 = \text{SNR}_2 = 20$  dB,  $\text{INR}_1 = \text{INR}_2 = \text{SNR}_{31} = \text{SNR}_{32} = 10$  dB, and  $\sigma_3^2 = 1$ . The corresponding achievable region  $\underline{\mathcal{E}}$  and converse region  $\bar{\mathcal{E}}$  are shown in Figure 3. In the case with feedback, the corresponding achievable region  $\underline{\mathcal{E}}^F$  and converse region  $\bar{\mathcal{E}}^F$  are shown in Figure 4. Note the strict inclusions  $\underline{\mathcal{E}} \subset \bar{\mathcal{E}}$  and  $\underline{\mathcal{E}}^F \subset \bar{\mathcal{E}}^F$  (Definition 3). Note also that for all  $B \leq 21$  energy units, the set of triplets  $\underline{\mathcal{E}}^F$  and the set of triplets  $\bar{\mathcal{E}}^F$  are prisms whose bases correspond to the inner and outer regions approximating the information capacity region presented in [7] and [5] respectively. For all  $B > 21$ , the trade-off between information transmission rates and the energy transmission rate becomes evident as both regions  $\underline{\mathcal{E}}^F$  and  $\bar{\mathcal{E}}^F$  monotonically shrink when  $B$  increases (Remark 3). The same observation can be made for the case without feedback.

Figure 5 shows the pairs  $(R_2, B)$  that are in the sets  $\{(R_2, B) \in \mathbb{R}_+^2 : (r_1, R_2, B) \in \bar{\mathcal{E}}\}$  (solid line) and  $\{(R_2, B) \in \mathbb{R}_+^2 : (r_1, R_2, B) \in \bar{\mathcal{E}}^F\}$  (dashed line), with  $r_1 = 0$  and  $r_1 = 3$ . Note that thanks

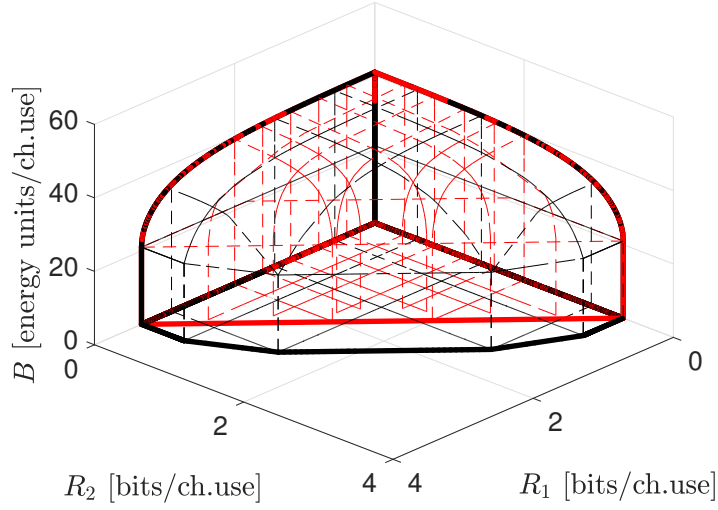


Figure 3: 3-D superposition of  $\mathcal{E}$  and  $\bar{\mathcal{E}}$ , with parameters  $\text{SNR}_1 = \text{SNR}_2 = 20$  dB,  $\text{INR}_1 = \text{INR}_2 = \text{SNR}_{31} = \text{SNR}_{32} = 10$  dB,  $\sigma_3^2 = 1$ .

to feedback, the information rate  $R_2$  can be increased one bit per channel use while keeping both the information rate  $R_1$  and the energy rate  $B$  invariant.

Figure 6 shows the set of pairs  $(R_1, R_2)$  that are in the sets  $\{(R_1, R_2) \in \mathbb{R}_+^2 : (R_1, R_2, b) \in \bar{\mathcal{E}}\}$  (solid line) and  $\{(R_1, R_2) \in \mathbb{R}_+^2 : (R_1, R_2, b) \in \bar{\mathcal{E}}^F\}$  (dashed line), with  $b = 21$  and  $b = 35$ . Note that thanks to feedback, both the information rates  $R_1$  and  $R_2$  can be increased more than half a bit per channel use while keeping the energy rate  $B$  constant.

Figure 7 shows the ratio  $\frac{\max \bar{B}_F}{\max \bar{B}}$  for different ratios of SNR at the EH, i.e.,  $\text{SNR}_{31} = \text{SNR}_{32} = \text{SNR}$ ;  $\frac{\text{SNR}_{31}}{2} = \text{SNR}_{32} = \text{SNR}$ ; and  $\frac{\text{SNR}_{31}}{10} = \text{SNR}_{32} = \text{SNR}$ , respectively. Note that the upper bound in Proposition 1 is tight in the case in the symmetric case.

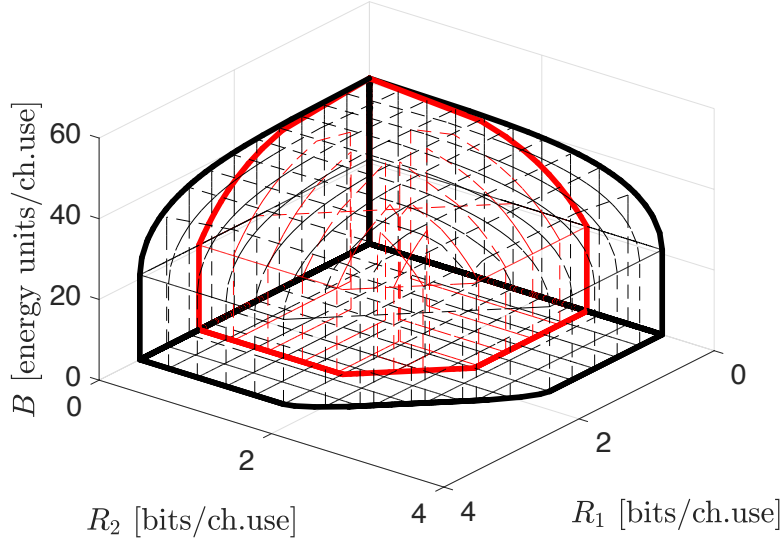


Figure 4: 3-D superposition of  $\mathcal{E}^F$  and  $\bar{\mathcal{E}}^F$ , with parameters  $\text{SNR}_1 = \text{SNR}_2 = 20$  dB,  $\text{INR}_1 = \text{INR}_2 = \text{SNR}_{31} = \text{SNR}_{32} = 10$  dB and  $\sigma_3^2 = 1$ .

## Appendices

### A Proof of Theorem 1

This proof is divided into two parts. The first part consists of the proof of (14a)-(14g). The second part consists of the proof of (14h).

#### A.1 Proof of (14a)-(14g)

**Codebook Generation:** Fix a strictly positive joint probability distribution:

$P_{VU_1U_2S_1S_2}(v, u_1, u_2, s_1, s_2) = P_V(v)P_{U_1|V}(u_1|v)P_{U_2|V}(u_2|v)P_{S_1|U_1V}(s_1|u_1, v)P_{S_2|U_2V}(s_2|u_2, v)$ , for all  $(v, u_1, u_2, s_1, s_2) \in \mathcal{X}_1 \cap \mathcal{X}_2 \times (\mathcal{X}_1 \times \mathcal{X}_2)^2$ . Let  $R_E, R_{1,C}, R_{2,C}, R_{1,P}$  and  $R_{2,P}$  be non negative real numbers. Generate  $2^{NR_E}$  i.i.d N-length codewords  $\mathbf{v}(\omega) = (v_1(\omega), \dots, v_N(\omega))$  according to

$$P_{\mathbf{V}}(\mathbf{v}(\omega)) = \prod_{m=1}^N P_V(v_m(\omega)), \quad (36)$$

with  $\omega \in \{1, 2, \dots, 2^{NR_E}\}$ . For encoder 1, for each codeword  $\mathbf{v}(\omega)$ , generate  $2^{NR_{1,C}}$  i.i.d. N-length codewords  $\mathbf{u}_1(\omega, i) = (u_{1,1}(\omega, i), \dots, u_{1,N}(\omega, i))$  according to

$$P_{U_1|V}(\mathbf{u}_1(\omega, i)|\mathbf{v}(\omega)) = \prod_{m=1}^N P_{U_1|V}(u_{1,m}(\omega, i)|v_m(\omega)), \quad (37)$$

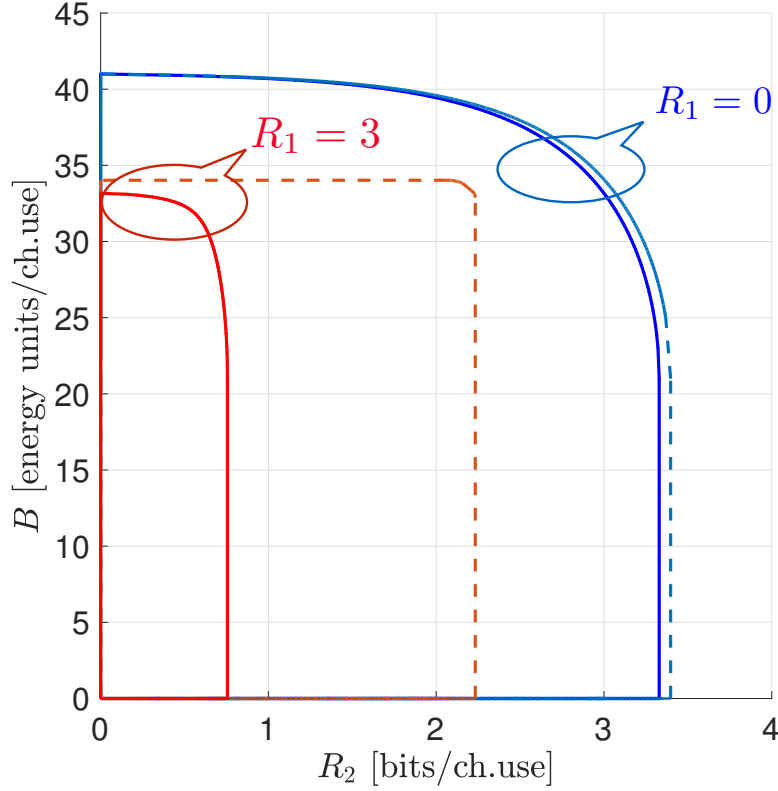


Figure 5: Convex hull of pairs  $(R_2, B)$  that are in the sets  $\{(R_2, B) \in \mathbb{R}_+^2 : (r_1, R_2, B) \in \bar{\mathcal{E}}\}$  (solid line) and  $\{(R_2, B) \in \mathbb{R}_+^2 : (r_1, R_2, B) \in \bar{\mathcal{E}}^F\}$  (dashed line), with  $r_1 \in \{0, 3\}$ . Parameters  $\text{SNR}_1 = \text{SNR}_2 = 20$  dB,  $\text{INR}_1 = \text{INR}_2 = \text{SNR}_{31} = \text{SNR}_{32} = 10$  dB, and  $\sigma_3^2 = 1$ .

with  $i \in \{1, \dots, 2^{NR_{1,C}}\}$ . For each pair of codewords  $\mathbf{u}_1(\omega, i)$  and  $\mathbf{v}(\omega)$ , generate  $2^{NR_{1,P}}$  i.i.d.  $N$ -length codewords  $\mathbf{s}_1(\omega, i, j) = (s_{1,1}(\omega, i, j), \dots, s_{1,N}(\omega, i, j))$  according to

$$P_{S_1|U_1V}(\mathbf{s}_1(\omega, i, j)|\mathbf{u}_1(\omega, i), \mathbf{v}(\omega)) = \prod_{m=1}^N P_{S_1|U_1V}(s_{1,m}(\omega, i, j)|u_{1,m}(\omega, i), v_m(\omega)), \quad (38)$$

with  $j \in \{1, \dots, 2^{NR_{1,P}}\}$ . For encoder 2, for each codeword  $\mathbf{v}(\omega)$ , with  $\omega \in \{1, 2, \dots, 2^{NR_E}\}$  generate  $2^{NR_{2,C}}$  i.i.d.  $N$ -length codewords  $\mathbf{u}_2(\omega, k) = (u_{2,1}(\omega, k), \dots, u_{2,N}(\omega, k))$  according to

$$P_{U_2|V}(\mathbf{u}_2(\omega, k)|\mathbf{v}(\omega)) = \prod_{m=1}^N P_{U_2|V}(u_{2,m}(\omega, k)|v_m(\omega)), \quad (39)$$

with  $k \in \{1, \dots, 2^{NR_{2,C}}\}$ . For each pair of codewords  $\mathbf{u}_2(\omega, k)$  and  $\mathbf{v}(\omega)$ , generate  $2^{NR_{2,P}}$  i.i.d.  $N$ -length codewords  $\mathbf{s}_2(\omega, k, l) = (s_{2,1}(\omega, k, l), \dots, s_{2,N}(\omega, k, l))$  according to

$$P_{S_2|U_2V}(\mathbf{s}_2(\omega, k, l)|\mathbf{u}_2(\omega, k), \mathbf{v}(\omega)) = \prod_{m=1}^N P_{S_2|U_2V}(s_{2,m}(\omega, k, l)|u_{2,m}(\omega, k), v_m(\omega)), \quad (40)$$

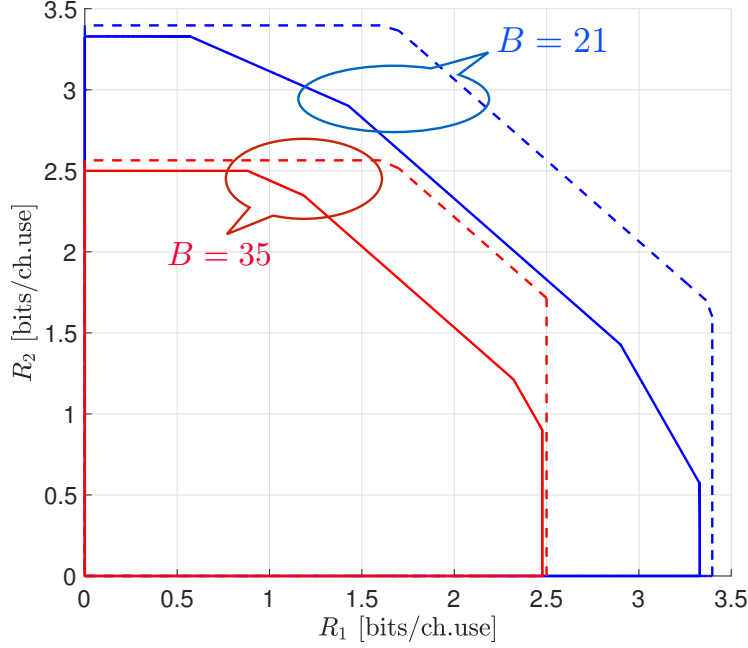


Figure 6: The ratio  $\frac{\sup \bar{E}_F}{\sup \bar{B}}$  for different ratios of SNR at the EH, i.e.,  $\text{SNR}_{31} = \text{SNR}_{32} = \text{SNR}$ ;  $\frac{\text{SNR}_{31}}{2} = \text{SNR}_{32} = \text{SNR}$ ; and  $\frac{\text{SNR}_{31}}{10} = \text{SNR}_{32} = \text{SNR}$ , respectively. Parameters  $\text{SNR}_1 = \text{SNR}_2 = 20$  dB,  $\text{INR}_1 = \text{INR}_2 = \text{SNR}_{31} = \text{SNR}_{32} = 10$  dB, and  $\sigma_3^2 = 1$ .

with  $l \in \{1, \dots, 2^{NR_{2,P}}\}$ .

**Encoding:** Denote by  $(W_i, \Omega) \in \{1, 2, \dots, 2^{N(R_{i,C} + R_{i,P})}\} \times \{1, 2, \dots, 2^{NR_E}\}$  the message index and the random message index of transmitter  $i$ . Let  $W_i$  be represented by the message index  $W_{i,C} \in \{1, 2, \dots, 2^{NR_{i,C}}\}$  and the message index  $W_{i,P} \in \{1, 2, \dots, 2^{NR_{i,P}}\}$ . Transmitter  $i$  sends the codeword  $\mathbf{x}_i = \theta_i \left( \mathbf{v}(\Omega), \mathbf{u}_i(\Omega, W_{i,C}), \mathbf{s}_i(\Omega, W_{i,C}, W_{i,P}) \right)$ , where  $\theta_i : (\mathcal{X}_1 \cap \mathcal{X}_2)^N \times \mathcal{X}_i^N \times \mathcal{X}_i^N \rightarrow \mathcal{X}_i^N$  is a function that transforms the codewords  $\mathbf{v}(\Omega)$ ,  $\mathbf{u}_i(\Omega, W_{i,C})$ , and  $\mathbf{s}_i(\Omega, W_{i,C}, W_{i,P})$  into a unique  $N$ -dimensional channel input vector  $\mathbf{x}_i$ .

**Decoding:** Given the channel output  $\mathbf{y}_1$ , receiver 1 estimates the unique tuple  $(\Omega, \widehat{W}_{1,C}, \widehat{W}_{1,P})$  that satisfies:

$$\left( \mathbf{v}(\Omega), \mathbf{u}_1(\Omega, \widehat{W}_{1,C}), \mathbf{s}_1(\Omega, \widehat{W}_{1,C}, \widehat{W}_{1,P}), \mathbf{u}_2(\Omega, \widehat{W}_{2,C}), \mathbf{y}_1 \right) \in \mathcal{T}_{VU_1S_1U_2Y_1}^{(N,\epsilon)}, \quad (41)$$

where  $\Omega$  are assumed to be perfectly known by both transmitters and receivers. The set  $\mathcal{T}_{VU_1S_1U_2Y_1}^{(N,\epsilon)}$  represents the set of jointly typical sequences of the random variables  $V, U_1, S_1, U_2$ , and  $Y_1$ , with  $\epsilon > 0$ . Finally, receiver 2 follows a similar decoding scheme.

**Error Probability Analysis:** an error might occur at receiver 1 if the indices  $W_{1,C}$  and  $W_{1,P}$  are not decoded correctly given that  $\Omega$  is known by both transmitters and receivers. These errors might arise for two reasons: (i) there does not exist a tuple  $(\Omega, \widehat{W}_{1,C}, \widehat{W}_{1,P})$ , for at least one  $\widehat{W}_{2,C}$  that satisfy (41), or (ii) there exist several tuples  $(\Omega, \widehat{W}_{1,C}, \widehat{W}_{1,P})$ , for at least one  $\widehat{W}_{2,C}$  that simultaneously satisfy (41). From the asymptotic equipartition property (AEP) [8], the probability of an error due to (i) tends to zero when  $N$  grows to infinity. Consider the error due to (ii) and

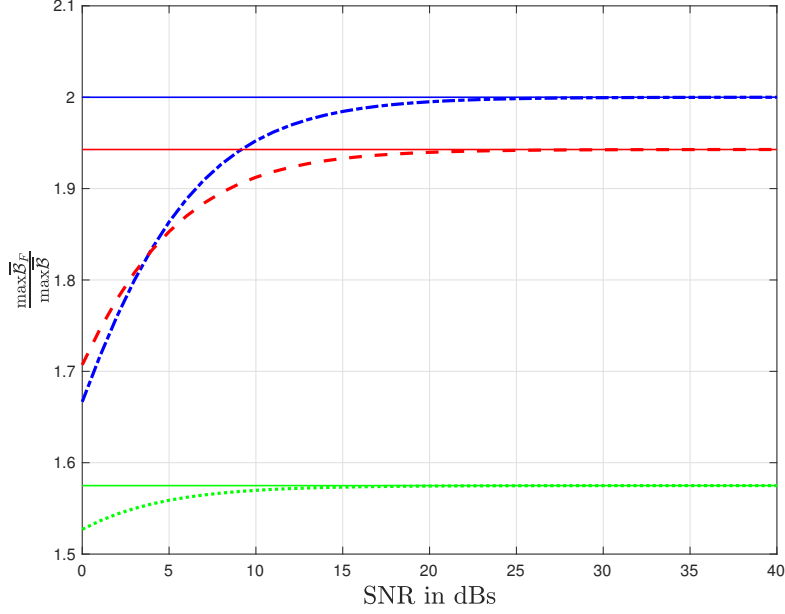


Figure 7: The ratio  $\frac{\max \bar{B}_E}{\max \bar{B}}$  for different ratios of SNR at the EH, i.e.,  $\text{SNR}_{31} = \text{SNR}_{32} = \text{SNR}$ ;  $\frac{\text{SNR}_{31}}{2} = \text{SNR}_{32} = \text{SNR}$ ; and  $\frac{\text{SNR}_{31}}{10} = \text{SNR}_{32} = \text{SNR}$ , respectively. Parameters  $\text{SNR}_1 = \text{SNR}_2 = 20$  dB,  $\text{INR}_1 = \text{INR}_2 = \text{SNR}_{31} = \text{SNR}_{32} = 10$  dB, and  $\sigma_3^2 = 1$ .

define the event  $E_{ijk}$  that describes the case in which the codewords  $\mathbf{v}(\Omega)$ ,  $\mathbf{u}_1(i, \Omega)$ ,  $\mathbf{s}_1(i, j, \Omega)$ , and  $\mathbf{u}_2(k, \Omega)$  are jointly typical with a given  $\mathbf{y}_1$ . Assume now that the codeword to be decoded corresponds to the indices  $(i, j, k) = (1, 1, 1)$ . This is without loss of generality due to the symmetry of the code. Note that no error is declared when codewords  $\left( \mathbf{v}(\Omega), \mathbf{u}_1(\Omega, 1), \mathbf{s}_1(\Omega, 1, 1), \mathbf{u}_2(\Omega, \hat{k}) \right)$ , where  $\hat{k} \neq 1$ , are the only jointly typical sequences with the received sequence  $\mathbf{y}_1$ . This is due to the fact that receiver 1 is interested only on the indices  $i$  and  $j$ . Then, the probability of error  $P_e$  due to the event  $(ii)$ , can be bounded as follows:

$$\begin{aligned}
 P_{\text{DE}}^{(N)} &= \Pr \left[ \bigcup_{(i,j) \neq (1,1)} \bigcup_{k=1}^{2^{NR_{2,C}}} E_{ijk} \right] \\
 &\leq \sum_{i \neq 1, j \neq 1, k \neq 1} \Pr[E_{ijk}] + \sum_{i=1, j \neq 1, k=1} \Pr[E_{ijk}] + \sum_{i \neq 1, j \neq 1, k=1} \Pr[E_{ijk}] \\
 &\quad + \sum_{i \neq 1, j=1, k \neq 1} \Pr[E_{ijk}] + \sum_{i \neq 1, j=1, k=1} \Pr[E_{ijk}] + \sum_{i=1, j \neq 1, k \neq 1} \Pr[E_{ijk}]. \quad (42)
 \end{aligned}$$

For all  $i \in \{2, 3, \dots, 2^{NR_{1,C}}\}$ , the following holds

$$\Pr[(E_{i11})] = \Pr\left[(\mathbf{V}, \mathbf{U}_1, \mathbf{S}_1, \mathbf{U}_2, \mathbf{Y}_1) \in \mathcal{T}_{\mathbf{V}\mathbf{U}_1\mathbf{S}_1\mathbf{U}_2\mathbf{Y}_1}^{(N,\epsilon)}\right] \quad (43a)$$

$$= \sum_{(\mathbf{v}, \mathbf{u}_1, \mathbf{s}_1, \mathbf{u}_2, \mathbf{y}_1) \in \mathcal{T}_{\mathbf{V}\mathbf{U}_1\mathbf{S}_1\mathbf{U}_2\mathbf{Y}_1}^{(N,\epsilon)}} P_{\mathbf{U}_1\mathbf{S}_1|\mathbf{V}}(\mathbf{u}_1, \mathbf{s}_1|\mathbf{v}) P_{\mathbf{U}_2\mathbf{Y}_1|\mathbf{V}}(\mathbf{u}_2, \mathbf{y}_1|\mathbf{v}) P_{\mathbf{V}}(\mathbf{v}), \quad (43b)$$

$$\leq \left| \mathcal{T}_{\mathbf{V}\mathbf{U}_1\mathbf{S}_1\mathbf{U}_2\mathbf{Y}_1}^{(N,\epsilon)} \right| 2^{-N(H(\mathbf{U}_1, \mathbf{S}_1|\mathbf{V}) + H(\mathbf{U}_2, \mathbf{Y}_1|\mathbf{V}) + H(\mathbf{V}) - 3\epsilon)} \quad (43c)$$

$$\leq 2^{-N(H(\mathbf{U}_1, \mathbf{S}_1|\mathbf{V}) + H(\mathbf{U}_2, \mathbf{Y}_1|\mathbf{V}) + H(\mathbf{V}) - H(\mathbf{V}, \mathbf{U}_1, \mathbf{S}_1, \mathbf{U}_2, \mathbf{Y}_1) - 4\epsilon)} \quad (43d)$$

$$= 2^{-N(H(\mathbf{U}_1, \mathbf{S}_1|\mathbf{V}) + H(\mathbf{U}_2, \mathbf{Y}_1|\mathbf{V}) - H(\mathbf{U}_1, \mathbf{S}_1, \mathbf{U}_2, \mathbf{Y}_1|\mathbf{V}) - 4\epsilon)} \quad (43e)$$

$$= 2^{-N(I(\mathbf{U}_1, \mathbf{S}_1; \mathbf{U}_2, \mathbf{Y}_1|\mathbf{V}) - 4\epsilon)} \quad (43f)$$

$$= 2^{-N(I(\mathbf{U}_1, \mathbf{S}_1; \mathbf{Y}_1|\mathbf{U}_2, \mathbf{V}) + I(\mathbf{U}_1, \mathbf{S}_1; \mathbf{U}_2|\mathbf{V}) - 4\epsilon)} \quad (43g)$$

$$= 2^{-N(I(\mathbf{U}_1, \mathbf{S}_1; \mathbf{Y}_1|\mathbf{U}_2, \mathbf{V}) - 4\epsilon)} \quad (43h)$$

$$= 2^{-N(I(\mathbf{X}_1; \mathbf{Y}_1|\mathbf{U}_2, \mathbf{V}) - 4\epsilon)}, \quad (43i)$$

where (43f) follows from the fact that, for any three random variables  $X$ ,  $Y$ , and  $Z$  it holds that

$$I(X; Y|Z) = H(X|Z) + H(Y|Z) - H(X, Y|Z); \quad (44)$$

and (43i) follows from the fact that  $X_i = \theta_i(S_i, U_i, V)$ . Note that the probability operator  $\Pr[\cdot]$  in (43a) applies with a probability distribution  $P_{\mathbf{V}\mathbf{U}_1\mathbf{S}_1\mathbf{U}_2\mathbf{Y}_1}$  that factorizes as  $P_{\mathbf{V}}P_{\mathbf{U}_1\mathbf{S}_1|\mathbf{V}}P_{\mathbf{U}_2\mathbf{Y}_1|\mathbf{V}}$  given that all the codewords  $\mathbf{s}_1$  and  $\mathbf{u}_1$  are independent from the given channel output  $\mathbf{y}_1$ .

For all  $j \in \{2, 3, \dots, 2^{NR_{1,P}}\}$ , the following holds:

$$\Pr(E_{1j1}) = \Pr\left[(\mathbf{V}, \mathbf{U}_1, \mathbf{S}_1, \mathbf{U}_2, \mathbf{Y}_1) \in \mathcal{T}_{\mathbf{V}\mathbf{U}_1\mathbf{S}_1\mathbf{U}_2\mathbf{Y}_1}^{(N,\epsilon)}\right] \quad (45a)$$

$$= \sum_{(\mathbf{v}, \mathbf{u}_1, \mathbf{s}_1, \mathbf{y}_1) \in \mathcal{T}_{\mathbf{V}\mathbf{U}_1\mathbf{S}_1\mathbf{U}_2\mathbf{Y}_1}^{(N,\epsilon)}} P_{\mathbf{S}_1|\mathbf{U}_1\mathbf{V}}(\mathbf{s}_1|\mathbf{u}_1, \mathbf{v}) P_{\mathbf{Y}_1|\mathbf{U}_1\mathbf{U}_2\mathbf{V}}(\mathbf{y}_1|\mathbf{u}_1, \mathbf{u}_2, \mathbf{v}) P_{\mathbf{U}_1\mathbf{U}_2\mathbf{V}}(\mathbf{u}_1, \mathbf{u}_2, \mathbf{v}), \quad (45b)$$

$$\leq \left| \mathcal{T}_{\mathbf{V}\mathbf{U}_1\mathbf{S}_1\mathbf{U}_2\mathbf{Y}_1}^{(N,\epsilon)} \right| 2^{-N(H(\mathbf{S}_1|\mathbf{V}, \mathbf{U}_1) + H(\mathbf{Y}_1|\mathbf{V}, \mathbf{U}_1, \mathbf{U}_2) + H(\mathbf{V}, \mathbf{U}_1, \mathbf{U}_2) - 3\epsilon)} \quad (45c)$$

$$\leq 2^{-N(H(\mathbf{S}_1|\mathbf{V}, \mathbf{U}_1) + H(\mathbf{Y}_1|\mathbf{V}, \mathbf{U}_1, \mathbf{U}_2) + H(\mathbf{V}, \mathbf{U}_1, \mathbf{U}_2) - H(\mathbf{V}, \mathbf{U}_1, \mathbf{S}_1, \mathbf{U}_2, \mathbf{Y}_1) - 4\epsilon)} \quad (45d)$$

$$= 2^{-N(H(\mathbf{S}_1|\mathbf{V}, \mathbf{U}_1, \mathbf{U}_2) + H(\mathbf{Y}_1|\mathbf{V}, \mathbf{U}_1, \mathbf{U}_2) - H(\mathbf{S}_1, \mathbf{Y}_1|\mathbf{U}_1, \mathbf{U}_2, \mathbf{V}) - 4\epsilon)} \quad (45e)$$

$$= 2^{-N(I(\mathbf{S}_1; \mathbf{Y}_1|\mathbf{U}_1, \mathbf{U}_2, \mathbf{V}) - 4\epsilon)} \quad (45f)$$

$$= 2^{-N(I(\mathbf{X}_1; \mathbf{Y}_1|\mathbf{U}_1, \mathbf{U}_2, \mathbf{V}) - 4\epsilon)}, \quad (45g)$$

where (45f) follows from (44); and (45g) follows from the fact that  $X_i = \theta_i(S_i, U_i, V)$ . Note that the probability operator  $\Pr[\cdot]$  in (45a) applies with a probability distribution  $P_{\mathbf{V}\mathbf{U}_1\mathbf{S}_1\mathbf{U}_2\mathbf{Y}_1}$  that factorizes as  $P_{\mathbf{S}_1|\mathbf{U}_1\mathbf{V}}P_{\mathbf{Y}_1|\mathbf{U}_1\mathbf{U}_2\mathbf{V}}P_{\mathbf{U}_1\mathbf{U}_2\mathbf{V}}$  given that the codeword  $\mathbf{s}_1$  is independent from the given channel output  $\mathbf{y}_1$ .

For all  $i \in \{2, 3, \dots, 2^{NR_{1,C}}\}$  and  $j \in \{2, 3, \dots, 2^{NR_{1,P}}\}$  the following holds:

$$\Pr(E_{ij1}) = \Pr\left[(\mathbf{V}, \mathbf{U}_1, \mathbf{S}_1, \mathbf{U}_2, \mathbf{Y}_1) \in \mathcal{T}_{VU_1S_1U_2Y_1}^{(N,\epsilon)}\right] \quad (46a)$$

$$= \sum_{(\mathbf{v}, \mathbf{u}_1, \mathbf{s}_1, \mathbf{u}_2, \mathbf{y}_1) \in \mathcal{T}_{VU_1S_1U_2Y_1}^{(N,\epsilon)}} P_{U_1S_1|V}(\mathbf{u}_1, \mathbf{s}_1|\mathbf{v}) P_{U_2Y_1|V}(\mathbf{u}_2, \mathbf{y}_1|\mathbf{v}) P_V(\mathbf{v}), \quad (46b)$$

$$\leq 2^{-N(H(U_1, S_1|V) + H(U_2, Y_1|V) + H(V) - H(V, U_1, S_1, U_2, Y_1) - 4\epsilon)} \quad (46c)$$

$$= 2^{-N(H(U_1, S_1|V) + H(U_2, Y_1|V) - H(U_1, S_1, U_2, Y_1|V) - 4\epsilon)} \quad (46d)$$

$$= 2^{-N(I(U_1, S_1; U_2, Y_1|V) - 4\epsilon)} \quad (46e)$$

$$= 2^{-N(I(U_1, S_1; Y_1|U_2, V) + I(U_1, S_1; U_2|V) - 4\epsilon)} \quad (46f)$$

$$= 2^{-N(I(U_1, S_1; Y_1|U_2, V) - 4\epsilon)} \quad (46g)$$

$$= 2^{-N(I(X_1; Y_1|U_2, V) - 4\epsilon)}, \quad (46h)$$

where (46e) follows from (44); and (46h) follows from the fact that  $X_i = \theta_i(S_i, U_i, V)$ . Note that the probability operator  $\Pr[\cdot]$  in (46a) applies with a probability distribution  $P_{VU_1S_1U_2Y_1}$  that factorizes as  $P_V P_{U_1S_1|V} P_{U_2Y_1|V}$  given that all the codewords  $\mathbf{s}_1$  and  $\mathbf{u}_1$  are independent from the output of the channel output  $\mathbf{y}_1$ .

For all  $i \in \{2, 3, \dots, 2^{NR_{1,C}}\}$  and  $k \in \{2, 3, \dots, 2^{NR_{2,C}}\}$  the following holds:

$$\Pr(E_{i1k}) = \Pr\left[(\mathbf{V}, \mathbf{U}_1, \mathbf{S}_1, \mathbf{U}_2, \mathbf{Y}_1) \in \mathcal{T}_{VU_1S_1U_2Y_1}^{(N,\epsilon)}\right] \quad (47a)$$

$$= \sum_{(\mathbf{v}, \mathbf{u}_1, \mathbf{s}_1, \mathbf{u}_2, \mathbf{y}_1) \in \mathcal{T}_{VU_1S_1U_2Y_1}^{(N,\epsilon)}} P_{U_1S_1U_2|V}(\mathbf{u}_1, \mathbf{s}_1, \mathbf{u}_2|\mathbf{v}) P_{Y_1|V}(\mathbf{y}_1|\mathbf{v}) P_V(\mathbf{v}), \quad (47b)$$

$$\leq \left| \mathcal{T}_{VU_1S_1U_2Y_1}^{(N,\epsilon)} \right| 2^{-N(H(U_1, S_1, U_2|V) + H(Y_1|V) + H(V) - 3\epsilon)} \quad (47c)$$

$$\leq 2^{-N(H(U_1, S_1, U_2|V) + H(Y_1|V) + H(V) - H(V, U_1, S_1, U_2, Y_1) - 4\epsilon)} \quad (47d)$$

$$= 2^{-N(H(U_1, S_1, U_2|V) + H(Y_1|V) - H(Y_1, U_1, S_1, U_2|V) - 4\epsilon)} \quad (47e)$$

$$= 2^{-N(I(S_1, U_1, U_2; Y_1|V) - 4\epsilon)} \quad (47f)$$

$$= 2^{-N(I(X_1, U_2; Y_1|V) - 4\epsilon)}, \quad (47g)$$

where (47f) follows from (44); and (47g) follows from the fact that  $X_i = \theta_i(S_i, U_i, V)$ . Note that the probability operator  $\Pr[\cdot]$  in (47a) applies with a probability distribution  $P_{VU_1S_1U_2Y_1}$  that factorizes as  $P_{U_1S_1U_2|V} P_{Y_1|V} P_V$  given that all the codewords  $\mathbf{s}_1$ ,  $\mathbf{u}_1$  and  $\mathbf{u}_2$  are independent from the given channel output  $\mathbf{y}_1$ .

For all  $i \in \{2, 3, \dots, 2^{NR_{1,P}}\}$  and  $j \in \{2, 3, \dots, 2^{NR_{2,C}}\}$  the following holds:

$$\Pr(E_{ij1}) = \Pr\left[(\mathbf{V}, \mathbf{U}_1, \mathbf{S}_1, \mathbf{U}_2, \mathbf{Y}_1) \in \mathcal{T}_{VU_1S_1U_2Y_1}^{(N,\epsilon)}\right] \quad (48a)$$

$$= \sum_{(\mathbf{v}, \mathbf{u}_1, \mathbf{s}_1, \mathbf{u}_2, \mathbf{y}_1) \in \mathcal{T}_{VU_1S_1U_2Y_1}^{(N,\epsilon)}} P_{U_1S_1|V}(\mathbf{u}_1, \mathbf{s}_1|\mathbf{v}) P_{U_2Y_1|V}(\mathbf{u}_2, \mathbf{y}_1|\mathbf{v}) P_V(\mathbf{v}), \quad (48b)$$

$$\leq 2^{-N(H(U_1, S_1|V) + H(U_2, Y_1|V) + H(V) - H(V, U_1, S_1, U_2, Y_1) - 4\epsilon)} \quad (48c)$$

$$= 2^{-N(H(U_1, S_1|V) + H(U_2, Y_1|V) - H(U_1, S_1, U_2, Y_1|V) - 4\epsilon)} \quad (48d)$$

$$= 2^{-N(I(U_1, S_1; U_2, Y_1|V) - 4\epsilon)} \quad (48e)$$

$$= 2^{-N(I(U_1, S_1; Y_1|U_2, V) + I(U_1, S_1; U_2|V) - 4\epsilon)} \quad (48f)$$

$$= 2^{-N(I(U_1, S_1; Y_1|U_2, V) - 4\epsilon)} \quad (48g)$$

$$= 2^{-N(I(X_1; Y_1|U_2, V) - 4\epsilon)}, \quad (48h)$$



where (48e) follows from (44); and (48h) follows from the fact that  $X_i = \theta_i(S_i, U_i, V)$ . Note that the probability operator  $\Pr[\cdot]$  in (48a) applies with a probability distribution  $P_{\mathbf{V}U_1\mathbf{s}_1U_2\mathbf{Y}_1}$  that factorizes as  $P_{\mathbf{V}}P_{U_1\mathbf{s}_1|\mathbf{V}}P_{U_2\mathbf{Y}_1|\mathbf{V}}$  given that all the codewords  $\mathbf{s}_1$  and  $\mathbf{u}_1$  are independent from the output of the channel output  $\mathbf{y}_1$ .

For all  $i \in \{2, 3, \dots, 2^{NR_{1,C}}\}$ ,  $j \in \{2, 3, \dots, 2^{NR_{1,P}}\}$  and  $k \in \{2, 3, \dots, 2^{NR_{2,C}}\}$  the following holds

$$\Pr(E_{ijk}) = \Pr\left[(\mathbf{V}, \mathbf{U}_1, \mathbf{s}_1, \mathbf{U}_2, \mathbf{Y}_1) \in \mathcal{T}_{\mathbf{V}U_1\mathbf{s}_1U_2\mathbf{Y}_1}^{(N,\epsilon)}\right] \quad (49a)$$

$$= \sum_{(\mathbf{v}, \mathbf{u}_1, \mathbf{s}_1, \mathbf{u}_2, \mathbf{y}_1) \in \mathcal{T}_{\mathbf{V}U_1\mathbf{s}_1U_2\mathbf{Y}_1}^{(N,\epsilon)}} P_{U_1\mathbf{s}_1U_2|\mathbf{V}}(\mathbf{u}_1, \mathbf{s}_1, \mathbf{u}_2|\mathbf{v}) P_{\mathbf{Y}_1|\mathbf{V}}(\mathbf{y}_1|\mathbf{v}) P_{\mathbf{V}}(\mathbf{v}), \quad (49b)$$

$$\stackrel{(b)}{\leq} \left| \mathcal{T}_{\mathbf{V}U_1\mathbf{s}_1U_2\mathbf{Y}_1}^{(N,\epsilon)} \right| 2^{-n(H(U_1, S_1, U_2|V) + H(Y_1|V) + H(V) - 3\epsilon)} \quad (49c)$$

$$\leq 2^{-N(H(U_1, S_1, U_2|V) + H(Y_1|V) + H(V) - H(V, U_1, S_1, U_2, Y_1) - 4\epsilon)} \quad (49d)$$

$$= 2^{-N(H(U_1, S_1, U_2|V) + H(Y_1|V) - H(Y_1, U_1, S_1, U_2|V) - 4\epsilon)} \quad (49e)$$

$$= 2^{-N(I(S_1, U_1, U_2; Y_1|V) - 4\epsilon)} \quad (49f)$$

$$= 2^{-N(I(X_1, U_2; Y_1|V) - 4\epsilon)}, \quad (49g)$$

where (49f) follows from (44); and (49g) follows from the fact that  $X_i = \theta_i(S_i, U_i, V)$ . Note that the probability operator  $\Pr[\cdot]$  in (49a) applies with a probability distribution  $P_{\mathbf{V}U_1\mathbf{s}_1U_2\mathbf{Y}_1}$  that factorizes as  $P_{U_1\mathbf{s}_1U_2|\mathbf{V}}P_{\mathbf{Y}_1|\mathbf{V}}P_{\mathbf{V}}(\mathbf{v})$  given that all the codewords  $\mathbf{s}_1$ ,  $\mathbf{u}_1$  and  $\mathbf{u}_2$  are independent from the given channel output  $\mathbf{y}_1$ .

Using (43), (45), (46), (47), (48), and (49) in (42), the following holds:

$$\begin{aligned} P_{\text{DE}}^{(N)} &\leq 2^{N(R_{1c} - I(X_1; Y_1|U_2, V) + 4\epsilon)} 2^{N(R_{1p} - I(X_1; Y_1|U_1, U_2, V) + 4\epsilon)} \\ &\quad + 2^{N(R_{1c} + R_{1p} - I(X_1; Y_1|U_2, V) + 4\epsilon)} + 2^{N(R_{1c} + R_{2c} - I(U_2, X_1; Y_1|V) + 4\epsilon)} \\ &\quad + 2^{N(R_{1p} + R_{2c} - I(U_2, X_1; Y_1|U_1, V) + 4\epsilon)} + 2^{N(R_{1p} + R_{1c} + R_{2c} - I(U_2, X_1; Y_1|V) + 4\epsilon)}. \end{aligned} \quad (50)$$

The same analysis of the probability of error holds for transmitter-receiver pair 2. Hence in general, from (50), reliable decoding holds under the following conditions:

$$R_{1p} \leq a_1, \quad (51a)$$

$$R_{1p} + R_{1c} \leq d_1, \quad (51b)$$

$$R_{1p} + R_{2c} \leq e_1, \quad (51c)$$

$$R_{1p} + R_{1c} + R_{2c} \leq g_1, \quad (51d)$$

$$-R_{1p} \leq 0, \quad (51e)$$

$$-R_{1c} \leq 0, \quad (51f)$$

$$R_{2p} \leq a_2, \quad (51g)$$

$$R_{2p} + R_{2c} \leq d_2, \quad (51h)$$

$$R_{2p} + R_{1c} \leq e_2, \quad (51i)$$

$$R_{1p} + R_{1c} + R_{2c} \leq g_2, \quad (51j)$$

$$-R_{2p} \leq 0, \quad (51k)$$

$$-R_{2c} \leq 0, \quad (51l)$$

where

$$a_1 = I(Y_1; X_1 | U_1, U_2, V), \quad (52a)$$

$$d_1 = I(Y_1; X_1 | U_2, V), \quad (52b)$$

$$e_1 = I(Y_1; X_1, U_2 | U_1, V), \quad (52c)$$

$$g_1 = I(Y_1; X_1, U_2 | V), \quad (52d)$$

$$a_2 = I(Y_2; X_2 | U_1, U_2, V), \quad (52e)$$

$$d_2 = I(Y_2; X_2 | U_1, V), \quad (52f)$$

$$e_2 = I(Y_2; X_2, U_1 | U_2, V), \text{ and} \quad (52g)$$

$$g_2 = I(Y_2; X_2, U_1 | V). \quad (52h)$$

The proof continues by applying a Fourier-Motzkin elimination process on (51). Set  $R_{1p} = R_1 - R_{1c}$ ,  $R_{2p} = R_2 - R_{2c}$  and eliminate  $R_{1p}$ ,  $R_{2p}$  from the set of inequalities (51) to obtain

$$R_1 - R_{1c} \leq a_1, \quad (53a)$$

$$R_1 \leq d_1, \quad (53b)$$

$$R_1 - R_{1c} + R_{2c} \leq e_1, \quad (53c)$$

$$R_1 + R_{2c} \leq g_1, \quad (53d)$$

$$-R_1 + R_{1c} \leq 0, \quad (53e)$$

$$-R_{1c} \leq 0, \quad (53f)$$

$$R_2 - R_{2c} \leq a_2, \quad (53g)$$

$$R_2 \leq d_2, \quad (53h)$$

$$R_2 - R_{2c} + R_{1c} \leq e_2, \quad (53i)$$

$$R_2 + R_{1c} \leq g_2, \quad (53j)$$

$$-R_2 + R_{2c} \leq 0, \quad (53k)$$

$$-R_{2c} \leq 0. \quad (53l)$$

Collect the inequalities in (53) that do not include  $R_{1c}$  to obtain:

$$R_1 \leq d_1, \quad (54a)$$

$$R_1 + R_{2c} \leq g_1, \quad (54b)$$

$$R_2 - R_{2c} \leq a_2, \quad (54c)$$

$$R_2 \leq d_2, \quad (54d)$$

$$-R_2 + R_{2c} \leq 0, \quad (54e)$$

$$-R_{2c} \leq 0. \quad (54f)$$

Also, collect the inequalities in (53) that include  $R_{1c}$  with positive coefficients to obtain:

$$R_2 - R_{2c} + R_{1c} \leq e_2, \quad (55a)$$

$$R_2 + R_{1c} \leq g_2, \quad (55b)$$

$$-R_1 + R_{1c} \leq 0. \quad (55c)$$

Furthermore, collect the inequalities in (53) that include  $R_{1c}$  with negative coefficients to obtain:

$$-R_{1c} \leq 0, \quad (56a)$$

$$R_1 - R_{1c} \leq a_1, \quad (56b)$$

$$R_1 - R_{1c} + R_{2c} \leq e_1. \quad (56c)$$

Eliminate  $R_{1c}$  by summing each inequality in (55) with (56) to obtain:

$$R_2 - R_{2c} \leq e_2, \quad (57a)$$

$$R_2 \leq g_2, \quad (57b)$$

$$-R_1 \leq 0, \quad (57c)$$

$$R_1 + R_2 - R_{2c} \leq a_1 + e_2, \quad (57d)$$

$$R_1 + R_2 \leq a_1 + g_2, \quad (57e)$$

$$0 \leq a_1, \quad (57f)$$

$$R_1 + R_2 \leq e_1 + e_2, \quad (57g)$$

$$R_1 + R_2 + R_{2c} \leq e_1 + g_2, \quad (57h)$$

$$R_{2c} \leq e_1. \quad (57i)$$

Collect the inequalities in (54) and (57) that do not include  $R_{2c}$  to obtain:

$$0 \leq a_1, \quad (58a)$$

$$R_1 \leq d_1, \quad (58b)$$

$$R_2 \leq d_2, \quad (58c)$$

$$R_2 \leq g_2, \quad (58d)$$

$$-R_1 \leq 0, \quad (58e)$$

$$R_1 + R_2 \leq a_1 + g_2, \quad (58f)$$

$$R_1 + R_2 \leq e_1 + e_2. \quad (58g)$$

Note that (58a) is redundant due to the positivity of mutual information. The inequality (58d) is redundant with respect to (58c), given that:

$$\begin{aligned} g_2 &= I(Y_2; X_2, U_1 | V) \\ &= I(Y_2; U_1 | V) + I(Y_2; X_2 | U_1, V) \\ &= d_2 + I(Y_2; U_1 | V) \\ &\geq d_2. \end{aligned} \quad (59)$$

Collect the inequalities in (54) and (57) that include  $R_{2c}$  with positive coefficients to obtain:

$$R_{2c} \leq e_1, \quad (60a)$$

$$R_1 + R_{2c} \leq g_1, \quad (60b)$$

$$R_1 + R_2 + R_{2c} \leq e_1 + g_2, \quad (60c)$$

$$-R_2 + R_{2c} \leq 0. \quad (60d)$$

Collect the inequalities in (54) and (57) that include  $R_{2c}$  with negative coefficients to obtain:

$$-R_{2c} \leq 0, \quad (61a)$$

$$R_2 - R_{2c} \leq a_2, \quad (61b)$$

$$R_2 - R_{2c} \leq e_2, \quad (61c)$$

$$R_1 + R_2 - R_{2c} \leq a_1 + e_2. \quad (61d)$$

The inequality (61c) is redundant with respect to (61b), as shown hereunder:

$$\begin{aligned}
 e_2 &= I(Y_2; X_2, U_1 | U_2, V) \\
 &= I(Y_2; U_1 | U_2, V) + I(Y_2; X_2 | U_2, U_1, V) \\
 &= a_2 + I(Y_2; U_1 | U_2, V) \\
 &\geq a_2.
 \end{aligned} \tag{62}$$

Eliminate  $R_{2c}$  by adding each inequality from (60) to each inequality (61) to obtain inequalities not including  $R_{2c}$ :

$$0 \leq e_1, \tag{63a}$$

$$R_1 \leq g_1, \tag{63b}$$

$$R_1 + R_2 \leq e_1 + g_2, \tag{63c}$$

$$-R_2 \leq 0, \tag{63d}$$

$$R_2 \leq a_2 + e_1, \tag{63e}$$

$$R_1 + R_2 \leq a_2 + g_1, \tag{63f}$$

$$R_1 + 2R_2 \leq a_2 + e_1 + g_2, \tag{63g}$$

$$0 \leq a_2, \tag{63h}$$

$$R_1 + R_2 \leq a_1 + e_2 + e_1, \tag{63i}$$

$$2R_1 + R_2 \leq a_1 + e_2 + g_1, \tag{63j}$$

$$2R_1 + 2R_2 \leq a_1 + e_2 + e_1 + g_2, \tag{63k}$$

$$R_1 \leq a_1 + e_2. \tag{63l}$$

Note that (63a) and (63h) are redundant due to the positivity of mutual information. The inequality (63b) is redundant with respect to (58b), as shown hereunder:

$$\begin{aligned}
 g_1 &= I(Y_1; X_1, U_2 | V) \\
 &= I(Y_1; U_2 | V) + I(Y_1; X_1 | U_2, V) \\
 &= d_1 + I(Y_1; U_2 | V) \\
 &\geq d_1.
 \end{aligned} \tag{64}$$

The inequality (63c) is redundant with respect to (58f), since

$$\begin{aligned}
 e_1 &= I(Y_1; X_1, U_2 | U_1, V) \\
 &= I(Y_1; U_2 | U_1, V) + I(Y_1; X_1 | U_1, U_2, V) \\
 &= a_1 + I(Y_1; U_2 | U_1, V) \\
 &\geq a_1.
 \end{aligned} \tag{65}$$

Note also that the inequality (63i) is redundant with respect to (58g). Furthermore, the inequality (63k) is redundant with respect to (58f) and (58g). Hence from (58) and (63), the system of

inequalities in (51) can be simplified as follows:

$$R_1 \leq d_1, \quad (66a)$$

$$R_1 \leq a_1 + e_2, \quad (66b)$$

$$R_2 \leq d_2, \quad (66c)$$

$$R_2 \leq a_2 + e_1, \quad (66d)$$

$$R_1 + R_2 \leq a_1 + g_2, \quad (66e)$$

$$R_1 + R_2 \leq a_2 + g_1, \quad (66f)$$

$$R_1 + R_2 \leq e_1 + e_2, \quad (66g)$$

$$2R_1 + R_2 \leq a_1 + g_1 + e_2, \quad (66h)$$

$$R_1 + 2R_2 \leq a_2 + g_2 + e_1. \quad (66i)$$

Finally, following the result of [9], the inequalities (66b) and (66d) can be dropped. Let  $k \in \{1, 2\}$  be fixed. Consider the following Gaussian input distribution for transmitter  $k$ :

$$V \sim \mathcal{N}(0, 1); U_k \sim \mathcal{N}(0, \lambda_{kc}); S_k \sim \mathcal{N}(0, \lambda_{kp}), \quad (67)$$

where  $X_k = \sqrt{P_k}S_k + \sqrt{P_k}U_k + \sqrt{\lambda_{ke}P_k}V$ ; and  $(\lambda_{kp}, \lambda_{kc}, \lambda_{ke}) \in [0, 1]^3$  and  $\lambda_{kp} + \lambda_{kc} + \lambda_{ke} \leq 1$ . By symmetry, it suffices to prove (14a), (14c), (14e) and (14f). The choice of the Gaussian input distribution in (67) yields:

$$\begin{aligned} I(X_1; Y_1 | U_2, V) &= h(Y_1 | U_2, V) - h(Y_1 | X_1, U_2, V) \\ &= \frac{1}{2} \log \left( 2\pi e \text{Var}[Y_1 | U_2, V] \right) - \frac{1}{2} \log \left( 2\pi e \text{Var}[Y_1 | X_1, U_2, V] \right) \\ &= \frac{1}{2} \log \left( 2\pi e (\sigma_1^2 + h_{1,1}^2 (1 - \lambda_{1e}) P_1 - h_{2,1}^2 \lambda_{2p} P_2) \right) \\ &\quad - \frac{1}{2} \log \left( 2\pi e (\sigma_1^2 + h_{2,1}^2 \lambda_{2p} P_2) \right) \\ &= \frac{1}{2} \log \left( 1 + \frac{(1 - \lambda_{1e}) \text{SNR}_1}{1 + \lambda_{2p} \text{INR}_1} \right), \end{aligned} \quad (68a)$$

$$\begin{aligned} I(X_1, U_2; Y_1 | V) &= h(Y_1 | V) - h(Y_1 | X_1, U_2, V) \\ &= \frac{1}{2} \log \left( 2\pi e \text{Var}[Y_1 | V] \right) - \frac{1}{2} \log \left( 2\pi e \text{Var}[Y_1 | X_1, U_2, V] \right) \\ &= \frac{1}{2} \log \left( 2\pi e (\sigma_1^2 + h_{1,1}^2 (1 - \lambda_{1e}) P_1 + h_{2,2}^2 (1 - \lambda_{2e}) P_2) \right) \\ &\quad - \frac{1}{2} \log \left( 2\pi e (\sigma_1^2 + h_{2,1}^2 \lambda_{2p} P_2) \right) \\ &= \frac{1}{2} \log \left( \frac{1 + (1 - \lambda_{1e}) \text{SNR}_1 + (1 - \lambda_{2e}) \text{INR}_1}{1 + \lambda_{2p} \text{INR}_1} \right), \end{aligned} \quad (68b)$$

$$\begin{aligned}
I(X_2; Y_2 | U_1, U_2, V) &= h(Y_2 | U_1, U_2, V) - h(Y_2 | U_1, U_2, X_2, V) \\
&= \frac{1}{2} \log \left( 2\pi e \text{Var}[Y_2 | U_1, U_2, V] \right) - \frac{1}{2} \log \left( 2\pi e \text{Var}[Y_2 | U_1, U_2, X_2, V] \right) \\
&= \frac{1}{2} \log \left( 2\pi e (\sigma_2^2 + h_{2,2}^2 \lambda_{2p} P_2 + h_{2,1}^2 \lambda_{1p} P_1) \right) \\
&\quad - \frac{1}{2} \log \left( 2\pi e (\sigma_2^2 + h_{2,1}^2 \lambda_{1p} P_1) \right) \\
&= \frac{1}{2} \log \left( 1 + \frac{\lambda_{2p} \text{SNR}_2}{1 + \lambda_{1p} \text{INR}_2} \right), \tag{68c}
\end{aligned}$$

$$\begin{aligned}
I(X_1, U_2; Y_1 | U_1, V) &= h(Y_1 | U_1, V) - h(Y_1 | U_1, U_2, X_1, V) \\
&= \frac{1}{2} \log \left( 2\pi e \text{Var}[Y_1 | U_1, V] \right) - \frac{1}{2} \log \left( 2\pi e \text{Var}[Y_1 | U_1, U_2, X_1, V] \right) \\
&= \frac{1}{2} \log \left( 2\pi e (\sigma_1^2 + h_{1,1}^2 \lambda_{1p} P_1 + h_{1,2}^2 (1 - \lambda_{2e}) P_2) \right) \\
&\quad - \frac{1}{2} \log \left( 2\pi e (\sigma_1^2 + h_{1,2}^2 \lambda_{2p} P_2) \right) \\
&= \frac{1}{2} \log \left( \frac{1 + \lambda_{1p} \text{SNR}_1 + (1 - \lambda_{2e}) \text{INR}_1}{1 + \lambda_{2p} \text{INR}_1} \right), \tag{68d}
\end{aligned}$$

$$\begin{aligned}
I(X_2, U_1; Y_2 | U_2, V) &= h(Y_2 | U_2, V) - h(Y_2 | U_2, U_1, X_2, V) \\
&= \frac{1}{2} \log \left( 2\pi e \text{Var}[Y_2 | U_2, V] \right) - \frac{1}{2} \log \left( 2\pi e \text{Var}[Y_2 | U_2, U_1, X_2, V] \right) \\
&= \frac{1}{2} \log \left( 2\pi e (\sigma_2^2 + h_{2,2}^2 \lambda_{2p} P_2 + h_{2,1}^2 (1 - \lambda_{1e}) P_1) \right) \\
&\quad - \frac{1}{2} \log \left( 2\pi e (\sigma_2^2 + h_{2,1}^2 \lambda_{1p} P_1) \right) \\
&= \frac{1}{2} \log \left( \frac{1 + \lambda_{2p} \text{SNR}_2 + (1 - \lambda_{1e}) \text{INR}_2}{1 + \lambda_{1p} \text{INR}_1} \right), \tag{68e}
\end{aligned}$$

which proves (14a), (14c) and (14e). Finally, using (68c), (68d) and (68e), the proof of (14f) follows immediately .

## A.2 Proof of (14h)

The choice of the channel input in (67) guarantees that the random variables  $Y_{3,1}, \dots, Y_{3,n}$  are independently and identically distributed. For all  $n \in \{1, 2, \dots, N\}$ ,  $Y_{3,n}$  follows a zero-mean Gaussian distribution with variance  $\bar{B}$  given by

$$\begin{aligned}
\bar{B} &= \mathbb{E} [Y_{3,n}^2] \\
&= \mathbb{E} [(h_{3,1} X_{1,n} + h_{3,2} X_{2,n} + Z_{3,n})^2] \\
&= h_{3,1}^2 \mathbb{E} [X_{1,n}^2] + h_{3,2}^2 \mathbb{E} [X_{2,n}^2] + 2h_{3,1} h_{3,2} \mathbb{E} [X_{1,n} X_{2,n}] + \sigma_3^2 \\
&= h_{3,1}^2 P_1 + h_{3,2}^2 P_2 + 2h_{3,1} h_{3,2} \mathbb{E} [X_{1,n} X_{2,n}] + \sigma_3^2 \\
&\leq h_{3,1}^2 P_1 + h_{3,2}^2 P_2 + 2h_{3,1} h_{3,2} \sqrt{\lambda_{1e} P_1 \lambda_{2e} P_2} + \sigma_3^2.
\end{aligned}$$

By the weak law of large numbers, it holds that

$$\lim_{N \rightarrow \infty} \Pr \left( |B^{(N)} - \bar{B}| > 0 \right) = 0. \tag{69}$$

Consequently,

$$\lim_{N \rightarrow \infty} \Pr(B^{(N)} > \bar{B}) = 0 \quad \text{and} \quad (70a)$$

$$\lim_{N \rightarrow \infty} \Pr(B^{(N)} < \bar{B}) = 0. \quad (70b)$$

From (70b), it holds that for any energy rate  $B$  which satisfies  $0 < B \leq \bar{B}$

$$\lim_{N \rightarrow \infty} \Pr(B^{(N)} < B) = 0. \quad (71)$$

This proves (14h) and completes the proof of Theorem 1.

## B Proof of Theorem 2

Fix an information-energy rate triplet  $(R_1, R_2, B)$  achievable with a given coding scheme (Definition 1). Denote by  $\mathbf{X}_1$  and  $\mathbf{X}_2$  the channel inputs resulting from transmitting the independent message  $W_1$  and  $W_2$  using such a coding scheme. Denote by  $\mathbf{Y}_1$  and  $\mathbf{Y}_2$  the corresponding channel outputs. The bounds (15a) and (15b) on  $R_1$  and  $R_2$  are trivial and can be obtained by removing the interference from the other user and calculating the point-to-point capacity:

$$NR_1 \leq \sum_{n=1}^N h(Y_{1,n}|X_{2,n}) - Nh(Z_1) + o(N) \quad \text{and} \quad (72)$$

$$NR_2 \leq \sum_{n=1}^N h(Y_{2,n}|X_{1,n}) - Nh(Z_2) + o(N), \quad (73)$$

where  $\frac{o(N)}{N}$  tends to zero as  $N$  tends to infinity. Define the following random variables:

$$T_1 = h_{2,1}X_{1,n} + Z_{2,n}, \quad (74)$$

$$T_2 = h_{1,2}X_{2,n} + Z_{1,n}, \quad (75)$$

$$U_1 = h_{2,1}X_{1,n} + Z'_{2,n}, \quad \text{and} \quad (76)$$

$$U_2 = h_{1,2}X_{2,n} + Z'_{1,n}, \quad (77)$$

where,  $Z'_1$  and  $Z'_2$  are real Gaussian random variables with zero mean and variances  $\sigma_1^2$  and  $\sigma_2^2$ , respectively, independent of each other and of  $(X_1, X_2, Z_1, Z_2)$ . Let  $\mathbf{T}_1$  and  $\mathbf{T}_2$  be the vectors:

$$\mathbf{T}_1 = (T_{1,1}, T_{1,2}, \dots, T_{1,N}) \quad \text{and} \quad (78)$$

$$\mathbf{T}_2 = (T_{2,1}, T_{2,2}, \dots, T_{2,N}). \quad (79)$$

The outer bound is established by using a genie aided argument. For all  $j \in \{1, 2\}$ , by Fano's inequality, it follows that

$$\begin{aligned} NR_j &= H(M_j) \\ &\leq I(M_j; \mathbf{Y}_j) + o(N) \\ &\leq I(\mathbf{X}_j; \mathbf{Y}_j) + o(N). \end{aligned} \quad (80)$$

Using the definition of mutual information, yields:

$$I(\mathbf{X}_1; \mathbf{Y}_1) = h(\mathbf{Y}_1) - h(\mathbf{Y}_1 | \mathbf{X}_1) \quad (81a)$$

$$= h(\mathbf{Y}_1) - h(\mathbf{T}_2 | \mathbf{X}_1) \quad (81b)$$

$$= h(\mathbf{Y}_1) - h(\mathbf{T}_2) \quad (81c)$$

$$\leq \sum_{n=1}^N h(Y_{1,n}) - h(\mathbf{T}_2). \quad (81d)$$

Consider the genie-aided channel in which a genie provides  $U_1$  and  $\mathbf{X}_2$  to receiver 1, then  $I(\mathbf{X}_1; \mathbf{Y}_1)$  can be upper bounded as follows:

$$I(\mathbf{X}_1; \mathbf{Y}_1) \leq I(\mathbf{X}_1; \mathbf{Y}_1, U_1, \mathbf{X}_2) \quad (82a)$$

$$= I(\mathbf{X}_1; U_1) + I(\mathbf{X}_1; \mathbf{X}_2 | U_1) + I(\mathbf{X}_1; \mathbf{Y}_1 | U_1, \mathbf{X}_2) \quad (82b)$$

$$= h(U_1) - h(U_1 | \mathbf{X}_1) + h(\mathbf{Y}_1 | U_1, \mathbf{X}_2) - h(\mathbf{Y}_1 | \mathbf{X}_1, U_1, \mathbf{X}_2) \quad (82c)$$

$$= h(\mathbf{T}_1) - h(U_1 | \mathbf{X}_1) + h(\mathbf{Y}_1 | U_1, \mathbf{X}_2) - h(\mathbf{Y}_1 | \mathbf{X}_1, U_1, \mathbf{X}_2) \quad (82d)$$

$$= h(\mathbf{T}_1) - h(U_1 | \mathbf{X}_1) + h(\mathbf{Y}_1 | U_1, \mathbf{X}_2) - h(\mathbf{T}_2 | \mathbf{X}_2) \quad (82e)$$

$$\leq h(\mathbf{T}_1) - h(\mathbf{Z}'_2) + \sum_{n=1}^N h(Y_{1,n} | U_{1,n}, X_{2,n}) - h(\mathbf{Z}_1) \quad (82f)$$

$$= h(\mathbf{T}_1) - Nh(\mathbf{Z}'_2) + \sum_{n=1}^N h(Y_{1,n} | U_{1,n}, X_{2,n}) - Nh(Z_{1,n}), \quad (82g)$$

where (82f) follows from the fact that conditioning does not increase the entropy.

Consider the genie-aided channel in which a genie provides  $U_1$  to receiver 1, then  $I(\mathbf{X}_1; \mathbf{Y}_1)$  can be upper bounded as follows:

$$I(\mathbf{X}_1; \mathbf{Y}_1) \leq I(\mathbf{X}_1; \mathbf{Y}_1, U_1) \quad (83a)$$

$$= I(\mathbf{X}_1; U_1) + I(\mathbf{X}_1; \mathbf{Y}_1 | U_1) \quad (83b)$$

$$= h(U_1) - h(U_1 | \mathbf{X}_1) + h(\mathbf{Y}_1 | U_1) - h(\mathbf{Y}_1 | \mathbf{X}_1, U_1) \quad (83c)$$

$$= h(\mathbf{T}_1) - h(U_1 | \mathbf{X}_1) + h(\mathbf{Y}_1 | U_1) - h(\mathbf{T}_2) \quad (83d)$$

$$\leq h(\mathbf{T}_1) - h(\mathbf{T}_2) - h(\mathbf{Z}'_2) + \sum_{i=1}^N h(Y_{1,n} | U_{1,n}) \quad (83e)$$

$$= h(\mathbf{T}_1) - h(\mathbf{T}_2) - Nh(\mathbf{Z}'_2) + \sum_{n=1}^N h(Y_{1,n} | U_{1,n}). \quad (83f)$$

where (83e) follows from the fact that conditioning does not increase the entropy.

Consider the genie-aided channel in which a genie provides  $\mathbf{X}_2$  to receiver 1, then  $I(\mathbf{X}_1; \mathbf{Y}_1)$  can be upper bounded as follows:

$$I(\mathbf{X}_1; \mathbf{Y}_1) \leq I(\mathbf{X}_1; \mathbf{Y}_1, \mathbf{X}_2) \quad (84a)$$

$$= I(\mathbf{X}_1; \mathbf{X}_2) + I(\mathbf{X}_1; \mathbf{Y}_1 | \mathbf{X}_2) \quad (84b)$$

$$= h(\mathbf{Y}_1 | \mathbf{X}_2) - h(\mathbf{Y}_1 | \mathbf{X}_1, \mathbf{X}_2) \quad (84c)$$

$$= h(\mathbf{Y}_1 | \mathbf{X}_2) - h(\mathbf{Z}_1) \quad (84d)$$

$$\leq \sum_{n=1}^N h(Y_{1,n} | X_{2,n}) - Nh(\mathbf{Z}_1). \quad (84e)$$



By symmetry, similar bounds can be established for  $I(\mathbf{X}_2; \mathbf{Y}_2)$ , namely,

$$I(\mathbf{X}_2; \mathbf{Y}_2) \leq \sum_{n=1}^N h(Y_{2,n}) - h(\mathbf{T}_1), \quad (85a)$$

$$I(\mathbf{X}_2; \mathbf{Y}_2) \leq h(\mathbf{T}_2) - Nh(Z'_1) + \sum_{n=1}^N h(Y_{2,n}|U_{2,n}, X_{1,n}) - Nh(Z_1), \quad (85b)$$

$$I(\mathbf{X}_2; \mathbf{Y}_2) \leq h(\mathbf{T}_2) - h(\mathbf{T}_1) - Nh(Z'_1) + \sum_{n=1}^N h(Y_{2,n}|U_{2,n}), \quad (85c)$$

$$I(\mathbf{X}_2; \mathbf{Y}_2) \leq \sum_{n=1}^N h(Y_{2,n}|X_{2,n}) - Nh(Z_1). \quad (85d)$$

The key idea of the proof is to consider a linear combination of the inequalities in (81d), (82g), (83f), (84e), (85a), (85b), (85c), and (85d), where all the terms on  $h(\mathbf{T}_1)$  and  $h(\mathbf{T}_2)$  are removed. Adding (81d) and (85b) and plugging into (80) yields the first bound on sum rate:

$$N(R_1 + R_2) \leq \sum_{n=1}^N h(Y_{1,n}) + \sum_{n=1}^N h(Y_{2,n}|U_{2,n}, X_{1,n}) - Nh(Z_2) - Nh(Z'_1) + o(N). \quad (86a)$$

Adding (82g) and (85a) and plugging into (80) yields the second bound on sum rate:

$$N(R_1 + R_2) \leq \sum_{n=1}^N h(Y_{2,n}) + \sum_{n=1}^N h(Y_{1,n}|U_{1,n}, X_{2,n}) - Nh(Z_1) - Nh(Z'_2) + o(N). \quad (86b)$$

Adding (83f) and (85c) and plugging into (80) yields the third bound on sum rate:

$$N(R_1 + R_2) \leq \sum_{n=1}^N h(Y_{1,n}|U_{1,n}) + \sum_{n=1}^N h(Y_{2,n}|U_{2,n}) - Nh(Z'_1) - Nh(Z'_2) + o(N). \quad (86c)$$

Adding (81d), (82g) and (85c) and plugging into (80) yields the first bound for the weighted sum rate:

$$\begin{aligned} N(2R_1 + R_2) &\leq \sum_{n=1}^N h(Y_{1,n}) + \sum_{n=1}^N h(Y_{1,n}|U_{1,n}, X_{2,n}) + \sum_{n=1}^N h(Y_{2,n}|U_{2,n}) \\ &\quad - N(h(Z_1) + h(Z_2) + h(Z'_1) + h(Z'_2)) + o(N). \end{aligned} \quad (86d)$$

Adding (85a), (83f) and (85b) and plugging into (80) yields the second bound for the weighted sum rate :

$$\begin{aligned} N(R_1 + 2R_2) &\leq \sum_{n=1}^N h(Y_{2,n}) + \sum_{n=1}^N h(Y_{2,n}|U_{2,n}, X_{1,n}) + \sum_{n=1}^N h(Y_{1,n}|U_{1,n}) \\ &\quad - N(h(Z_1) + h(Z_2) + h(Z'_1) + h(Z'_2)) + o(N). \end{aligned} \quad (86e)$$

Hence, the information rates can be upper bounded as follows:

$$NR_1 \leq \sum_{n=1}^N h(Y_{1,n}|X_{2,n}) - Nh(Z_1) + o(N), \quad (87a)$$

$$NR_2 \leq \sum_{n=1}^N h(Y_{2,n}|X_{1,n}) - Nh(Z_2) + o(N), \quad (87b)$$

$$N(R_1 + R_2) \leq \sum_{n=1}^N h(Y_{1,n}) + \sum_{n=1}^N h(Y_{2,n}|U_{2,n}, X_{1,n}) - Nh(Z_2) - Nh(Z'_1) + o(N), \quad (87c)$$

$$N(R_1 + R_2) \leq \sum_{n=1}^N h(Y_{2,n}) + \sum_{n=1}^N h(Y_{1,n}|U_{1,n}, X_{2,n}) - Nh(Z_1) - Nh(Z'_2) + o(N), \quad (87d)$$

$$N(R_1 + R_2) \leq \sum_{n=1}^N h(Y_{1,n}|U_{1,n}) + \sum_{n=1}^N h(Y_{2,n}|U_{2,n}) - Nh(Z'_1) - Nh(Z'_2) + o(N), \quad (87e)$$

$$\begin{aligned} N(2R_1 + R_2) &\leq \sum_{n=1}^N h(Y_{1,n}) + \sum_{n=1}^N h(Y_{1,n}|U_{1,n}, X_{2,n}) + \sum_{n=1}^N h(Y_{2,n}|U_{2,n}) \\ &\quad - N(h(Z_1) + h(Z_2) + h(Z'_1) + h(Z'_2)) + o(N), \end{aligned} \quad (87f)$$

$$\begin{aligned} N(R_1 + 2R_2) &\leq \sum_{n=1}^N h(Y_{2,n}) + \sum_{n=1}^N h(Y_{2,n}|U_{2,n}, X_{1,n}) + \sum_{n=1}^N h(Y_{1,n}|U_{1,n}) \\ &\quad - N(h(Z_1) + h(Z_2) + h(Z'_1) + h(Z'_2)) + o(N). \end{aligned} \quad (87g)$$

Using assumption (13b), for any  $\eta > 0$  there exist  $N_0(\eta)$  such that for any  $N \geq N_0(\eta)$  it holds that

$$\Pr [B^{(N)} < B] < \eta. \quad (88)$$

Equivalently,

$$\Pr [B^{(N)} \geq B] \geq 1 - \eta. \quad (89)$$

From Markov's inequality [10], the following holds:

$$B\Pr [B^{(N)} \geq B] \leq \mathbb{E}[B^{(N)}]. \quad (90)$$

Combining (89) and (90) yields

$$B(1 - \eta) \leq \mathbb{E}[B^{(N)}], \quad (91)$$

which can be written as

$$(B - \delta_N) \leq \mathbb{E}[B^{(N)}], \quad (92)$$

for some  $\delta_N > 0$  (for sufficiently large  $N$ ).

In the following, for all  $n \in \mathbb{N}$ , the bounds in (87) and (92) are evaluated assuming that the channel inputs  $X_{1,n}$  and  $X_{2,n}$  are arbitrary independent. Define the following parameters:

$$\mu_{i,n} \triangleq \mathbb{E}[X_{i,n}], \quad (93)$$

$$\gamma_{i,n}^2 \triangleq \text{Var}[X_{i,n}], \quad (94)$$

for  $n \in \{1, \dots, N\}$  and for  $i \in \{1, 2\}$ . The input sequence must satisfy the input power constraint in (4), which can be written, for  $i \in \{1, 2\}$ , as

$$\frac{1}{n} \sum_{n=1}^N \mathbb{E} [X_{i,n}^2] = \left( \frac{1}{N} \sum_{n=1}^N \gamma_{i,n}^2 \right) + \left( \frac{1}{N} \sum_{n=1}^N \mu_{i,n}^2 \right) \leq P_i. \quad (95)$$

Using these assumptions the following holds:

$$\begin{aligned} h(Y_{i,n}) &\leq \frac{1}{2} \log \left( 2\pi e \text{Var}[Y_{i,n}] \right) \\ &= \frac{1}{2} \log \left( 2\pi e (\sigma^2 + h_{i,i}^2 \gamma_{i,n}^2 + h_{i,j}^2 \gamma_{j,n}^2) \right), \end{aligned} \quad (96a)$$

$$\begin{aligned} h(Y_{i,n}|U_{i,n}, X_{j,n}) &\leq \frac{1}{2} \log \left( 2\pi e \text{Var}[Y_{i,n}|U_{i,n}, X_{j,n}] \right) \\ &\leq \frac{1}{2} \log \left( 2\pi e |\text{Cov}[h_{i,i} X_{i,n} + Z_{i,n}, U_{i,n}]| \right) - \frac{1}{2} \log \left( 2\pi e \text{Var}[U_{i,n}] \right) \\ &= \frac{1}{2} \log \left( \frac{\sigma_j^2 \gamma_{i,n}^2 h_{i,i}^2 + \sigma_1^2 \gamma_{i,n}^2 h_{j,i}^2 + \sigma_i^2 \sigma_j^2}{\gamma_{i,n}^2 h_{j,i}^2 + \sigma_j^2} \right) + \frac{1}{2} \log(2\pi e \sigma_i^2 \sigma_j^2) \\ &= \frac{1}{2} \log \left( 1 + \frac{\frac{h_{i,i}^2 \gamma_{i,n}^2}{\sigma_i^2}}{1 + \frac{h_{j,i}^2 \gamma_{i,n}^2}{\sigma_j^2}} \right) + \frac{1}{2} \log(2\pi e \sigma_i^2 \sigma_j^2), \end{aligned} \quad (96b)$$

$$\begin{aligned} h(Y_{i,n}|U_{i,n}) &\leq \frac{1}{2} \log \left( 2\pi e \text{Var}[Y_{i,n}|U_{i,n}] \right) \\ &\leq \frac{1}{2} \log \left( 2\pi e |\text{Cov}[Y_{i,n}, U_{i,n}]| \right) - \frac{1}{2} \log \left( 2\pi e \text{Var}[U_{i,n}] \right) \\ &= \frac{1}{2} \log \left( \frac{\sigma_j^2 \gamma_i^2 h_{i,i}^2 + \sigma_i^2 \gamma_{i,n}^2 h_{j,i}^2 + h_{i,j}^2 h_{j,i}^2 \gamma_{i,n}^2 \gamma_{j,n}^2 + \sigma_i^2 \sigma_j^2}{\gamma_i^2 h_{j,i}^2 + \sigma_j^2} \right) + \frac{1}{2} \log(2\pi e \sigma_i^2 \sigma_j^2) \\ &\leq \frac{1}{2} \log \left( 1 + \frac{\frac{h_{i,i}^2 \gamma_{i,n}^2}{\sigma_i^2} + \frac{h_{i,j}^2 \gamma_{j,n}^2}{\sigma_i^2} + \frac{\gamma_{i,n}^2 \gamma_{j,n}^2 h_{i,j}^2 h_{j,i}^2}{\sigma_i^2 \sigma_j^2}}{1 + \frac{\gamma_{i,n}^2 h_{j,i}^2}{\sigma_j^2}} \right) + \frac{1}{2} \log(2\pi e \sigma_i^2 \sigma_j^2), \end{aligned} \quad (96c)$$

Finally, plugging (96) in (87), it yields:

$$NR_1 \leq \sum_{n=1}^N \frac{1}{2} \log \left( 1 + \frac{\gamma_{1,n}^2 h_{1,1}^2}{\sigma_1^2} \right) + o(N), \quad (97a)$$

$$NR_2 \leq \sum_{n=1}^N \frac{1}{2} \log \left( 1 + \frac{\gamma_{2,n}^2 h_{2,2}^2}{\sigma_2^2} \right) + o(N), \quad (97b)$$

$$N(R_1 + R_2) \leq \sum_{n=1}^N \log \left( 1 + \frac{h_{1,1}^2 \gamma_{1,n}^2}{\sigma_1^2} + \frac{h_{1,2}^2 \gamma_{2,n}^2}{\sigma_1^2} \right) + \sum_{n=1}^N \frac{1}{2} \log \left( 1 + \frac{\frac{h_{2,2}^2 \gamma_{2,n}^2}{\sigma_2^2}}{1 + \frac{h_{1,2}^2 \gamma_{2,n}^2}{\sigma_1^2}} \right) + o(N), \quad (97c)$$

$$N(R_1 + R_2) \leq \sum_{n=1}^N \log \left( 1 + \frac{h_{2,2}^2 \gamma_{2,n}^2}{\sigma_2^2} + \frac{h_{2,1}^2 \gamma_{1,n}^2}{\sigma_2^2} \right) + \sum_{n=1}^N \frac{1}{2} \log \left( 1 + \frac{\frac{h_{1,1}^2 \gamma_{1,n}^2}{\sigma_1^2}}{1 + \frac{h_{2,1}^2 \gamma_{1,n}^2}{\sigma_2^2}} \right) + o(N), \quad (97d)$$

$$\begin{aligned}
 N(R_1 + R_2) &\leq \sum_{n=1}^N \frac{1}{2} \log \left( 1 + \frac{\frac{h_{1,1}^2 \gamma_{1,n}^2}{\sigma_1^2} + \frac{h_{1,2}^2 \gamma_{2,n}^2}{\sigma_1^2} + \frac{\gamma_{1,n}^2 \gamma_{2,n}^2 h_{1,2}^2 h_{2,1}^2}{\sigma_1^2 \sigma_2^2}}{1 + \frac{\gamma_{1,n}^2 h_{2,1}^2}{\sigma_2^2}} \right) \\
 &+ \sum_{n=1}^N \frac{1}{2} \log \left( 1 + \frac{\frac{h_{2,2}^2 \gamma_{2,n}^2}{\sigma_2^2} + \frac{h_{2,1}^2 \gamma_{1,n}^2}{\sigma_2^2} + \frac{\gamma_{1,n}^2 \gamma_{2,n}^2 h_{1,2}^2 h_{2,1}^2}{\sigma_1^2 \sigma_2^2}}{1 + \frac{\gamma_{2,n}^2 h_{1,2}^2}{\sigma_1^2}} \right) + o(N),
 \end{aligned} \tag{97e}$$

$$\begin{aligned}
 N(2R_1 + R_2) &\leq \sum_{n=1}^N \frac{1}{2} \log \left( 1 + \frac{\frac{h_{1,1}^2 \gamma_{1,n}^2}{\sigma_1^2}}{1 + \frac{h_{2,1}^2 \gamma_{1,n}^2}{\sigma_2^2}} \right) + \sum_{n=1}^N \log \left( 1 + \frac{h_{1,1}^2 \gamma_{1,n}^2}{\sigma_1^2} + \frac{h_{1,2}^2 \gamma_{2,n}^2}{\sigma_1^2} \right) \\
 &+ \sum_{n=1}^N \frac{1}{2} \log \left( 1 + \frac{\frac{h_{2,2}^2 \gamma_{2,n}^2}{\sigma_2^2} + \frac{h_{2,1}^2 \gamma_{1,n}^2}{\sigma_2^2} + \frac{\gamma_{1,n}^2 \gamma_{2,n}^2 h_{1,2}^2 h_{2,1}^2}{\sigma_1^2 \sigma_2^2}}{1 + \frac{\gamma_{2,n}^2 h_{1,2}^2}{\sigma_1^2}} \right) + o(N),
 \end{aligned} \tag{97f}$$

$$\begin{aligned}
 N(R_1 + 2R_2) &\leq \sum_{n=1}^N \frac{1}{2} \log \left( 1 + \frac{\frac{h_{2,2}^2 \gamma_{2,n}^2}{\sigma_2^2}}{1 + \frac{h_{1,2}^2 \gamma_{2,n}^2}{\sigma_1^2}} \right) + \sum_{n=1}^N \log \left( 1 + \frac{h_{2,2}^2 \gamma_{2,n}^2}{\sigma_2^2} + \frac{h_{2,1}^2 \gamma_{1,n}^2}{\sigma_2^2} \right) \\
 &+ \sum_{n=1}^N \frac{1}{2} \log \left( 1 + \frac{\frac{h_{1,1}^2 \gamma_{1,n}^2}{\sigma_1^2} + \frac{h_{1,2}^2 \gamma_{2,n}^2}{\sigma_1^2} + \frac{\gamma_{1,n}^2 \gamma_{2,n}^2 h_{1,2}^2 h_{2,1}^2}{\sigma_1^2 \sigma_2^2}}{1 + \frac{\gamma_{1,n}^2 h_{2,1}^2}{\sigma_2^2}} \right) + o(N).
 \end{aligned} \tag{97g}$$

The expectation of the average received energy rate is given by

$$\begin{aligned}
 \mathbb{E} [B^{(N)}] &= \mathbb{E} \left[ \frac{1}{N} \sum_{n=1}^N Y_{3,n}^2 \right] \\
 &= \sigma_3^2 + h_{3,1}^2 \left( \frac{1}{N} \sum_{n=1}^N (\gamma_{1,n}^2 + \mu_{1,n}^2) \right) + h_{3,2}^2 \left( \frac{1}{N} \sum_{n=1}^N (\gamma_{2,n}^2 + \mu_{2,n}^2) \right) \\
 &+ 2h_{3,1}h_{3,2} \frac{1}{N} \sum_{n=1}^N \mu_{1,n}\mu_{2,n}.
 \end{aligned} \tag{98}$$

Using Cauchy-Schwarz inequality, the energy rate in (98) can be upper-bounded as follows:

$$\begin{aligned}
 \mathbb{E} [B^{(N)}] &\leq \sigma_3^2 + h_{3,1}^2 \left( \frac{1}{N} \sum_{n=1}^N (\gamma_{1,n}^2 + \mu_{1,n}^2) \right) + h_{3,2}^2 \left( \frac{1}{N} \sum_{n=1}^N (\gamma_{2,n}^2 + \mu_{2,n}^2) \right) \\
 &+ 2h_{3,1}h_{3,2} \left( \frac{1}{N} \sum_{n=1}^N \mu_{1,n}^2 \right)^{1/2} \left( \frac{1}{N} \sum_{n=1}^N \mu_{2,n}^2 \right)^{1/2}.
 \end{aligned} \tag{99}$$

Combining (92) and (99) yields the following upper-bound on the energy rate  $B$ :

$$\begin{aligned}
 B &\leq \sigma_3^2 + h_{3,1}^2 \left( \frac{1}{N} \sum_{n=1}^N (\gamma_{1,n}^2 + \mu_{1,n}^2) \right) + h_{3,2}^2 \left( \frac{1}{N} \sum_{n=1}^N (\gamma_{2,n}^2 + \mu_{2,n}^2) \right) \\
 &+ 2h_{3,1}h_{3,2} \left( \frac{1}{N} \sum_{n=1}^N \mu_{1,n}^2 \right)^{1/2} \left( \frac{1}{N} \sum_{n=1}^N \mu_{2,n}^2 \right)^{1/2} + \delta_N.
 \end{aligned} \tag{100}$$

In order to obtain a single-letterization of the upper-bound given by constraints (97) and (100), define also

$$\mu_i^2 \triangleq \frac{1}{N} \sum_{n=1}^N \mu_{i,t}^2, \quad i \in \{1, 2\}, \quad (101)$$

$$\gamma_i^2 \triangleq \frac{1}{N} \sum_{n=1}^N \gamma_{i,n}^2, \quad i \in \{1, 2\}, \quad (102)$$

Using these notations, the input power constraint in (153) can be rewritten as

$$\gamma_i^2 + \mu_i^2 \leq P_i, \quad \text{with } i \in \{1, 2\}. \quad (103)$$

By the concavity of the mutual information, applying Jensen's inequality to the bounds in (97) yields in the asymptotic blocklength regime:

$$R_1 \leq \frac{1}{2} \log \left( 1 + \frac{\gamma_1^2 h_{1,1}^2}{\sigma_1^2} \right), \quad (104a)$$

$$R_2 \leq \frac{1}{2} \log \left( 1 + \frac{\gamma_2^2 h_{2,2}^2}{\sigma_2^2} \right), \quad (104b)$$

$$R_1 + R_2 \leq \log \left( 1 + \frac{h_{1,1}^2 \gamma_1^2}{\sigma_1^2} + \frac{h_{1,2}^2 \gamma_2^2}{\sigma_1^2} \right) + \frac{1}{2} \log \left( 1 + \frac{\frac{h_{2,2}^2 \gamma_2^2}{\sigma_2^2}}{1 + \frac{h_{1,2}^2 \gamma_2^2}{\sigma_1^2}} \right), \quad (104c)$$

$$R_1 + R_2 \leq \log \left( 1 + \frac{h_{2,2}^2 \gamma_2^2}{\sigma_2^2} + \frac{h_{2,1}^2 \gamma_1^2}{\sigma_2^2} \right) + \frac{1}{2} \log \left( 1 + \frac{\frac{h_{1,1}^2 \gamma_1^2}{\sigma_1^2}}{1 + \frac{h_{2,1}^2 \gamma_1^2}{\sigma_2^2}} \right), \quad (104d)$$

$$R_1 + R_2 \leq \frac{1}{2} \log \left( 1 + \frac{\frac{h_{1,1}^2 \gamma_1^2}{\sigma_1^2} + \frac{h_{1,2}^2 \gamma_2^2}{\sigma_1^2} + \frac{\gamma_1^2 \gamma_2^2 h_{1,2}^2 h_{2,1}^2}{\sigma_1^2 \sigma_2^2}}{1 + \frac{\gamma_1^2 h_{2,1}^2}{\sigma_2^2}} \right) + \frac{1}{2} \log \left( 1 + \frac{\frac{h_{2,2}^2 \gamma_2^2}{\sigma_2^2} + \frac{h_{2,1}^2 \gamma_1^2}{\sigma_2^2} + \frac{\gamma_1^2 \gamma_2^2 h_{1,2}^2 h_{2,1}^2}{\sigma_1^2 \sigma_2^2}}{1 + \frac{\gamma_2^2 h_{1,2}^2}{\sigma_1^2}} \right), \quad (104e)$$

$$2R_1 + R_2 \leq \frac{1}{2} \log \left( 1 + \frac{\frac{h_{1,1}^2 \gamma_1^2}{\sigma_1^2}}{1 + \frac{h_{2,1}^2 \gamma_1^2}{\sigma_2^2}} \right) + \log \left( 1 + \frac{h_{1,1}^2 \gamma_1^2}{\sigma_1^2} + \frac{h_{1,2}^2 \gamma_2^2}{\sigma_1^2} \right) + \frac{1}{2} \log \left( 1 + \frac{\frac{h_{2,2}^2 \gamma_2^2}{\sigma_2^2} + \frac{h_{2,1}^2 \gamma_1^2}{\sigma_2^2} + \frac{\gamma_1^2 \gamma_2^2 h_{1,2}^2 h_{2,1}^2}{\sigma_1^2 \sigma_2^2}}{1 + \frac{\gamma_2^2 h_{1,2}^2}{\sigma_1^2}} \right), \quad (104f)$$

$$R_1 + 2R_2 \leq \frac{1}{2} \log \left( 1 + \frac{\frac{h_{2,2}^2 \gamma_2^2}{\sigma_2^2}}{1 + \frac{h_{1,2}^2 \gamma_2^2}{\sigma_1^2}} \right) + \log \left( 1 + \frac{h_{2,2}^2 \gamma_2^2}{\sigma_2^2} + \frac{h_{2,1}^2 \gamma_1^2}{\sigma_2^2} \right) + \frac{1}{2} \log \left( 1 + \frac{\frac{h_{1,1}^2 \gamma_1^2}{\sigma_1^2} + \frac{h_{1,2}^2 \gamma_2^2}{\sigma_1^2} + \frac{\gamma_1^2 \gamma_2^2 h_{1,2}^2 h_{2,1}^2}{\sigma_1^2 \sigma_2^2}}{1 + \frac{\gamma_1^2 h_{2,1}^2}{\sigma_2^2}} \right), \quad (104g)$$

and the upper-bound on the energy rate (100) yields

$$B \leq \sigma_3^2 + h_{3,1}^2 (\gamma_1^2 + \mu_1^2) + h_{3,2}^2 (\gamma_2^2 + \mu_2^2) + 2h_{3,1} h_{3,2} |\mu_1| |\mu_2|. \quad (104h)$$

To sum up, it has been shown so far that, in the limit when  $N$  tends to infinity, any information-energy rate triplet  $(R_1, R_2, B) \in \mathcal{E}$  can be bounded by the constraints in (104) for some  $\gamma_1, \gamma_2, \mu_1, \mu_2$  satisfying (103). Let  $\mathcal{R}(\gamma_1, \gamma_2, \mu_1, \mu_2)$  denote the set of information-energy rate triplets satisfying (104) for some  $\gamma_1, \gamma_2, \mu_1, \mu_2$  such that (103) is true. Thus, it holds that

$$\bar{\mathcal{E}} \subseteq \bigcup_{\substack{0 \leq \gamma_1^2 + \mu_1^2 \leq P_1 \\ 0 \leq \gamma_2^2 + \mu_2^2 \leq P_2}} \mathcal{R}(\gamma_1, \gamma_2, \mu_1, \mu_2). \quad (105)$$

In this union, it suffices to consider  $\mu_1 \geq 0, \mu_2 \geq 0$ , and  $\gamma_1, \gamma_2, \mu_1$ , and  $\mu_2$  that saturate the input power constraint (i.e., (103) holds with equality). Thus,

$$\bar{\mathcal{E}} \subseteq \bigcup_{\substack{0 \leq \gamma_1^2 + \mu_1^2 \leq P_1 \\ 0 \leq \gamma_2^2 + \mu_2^2 \leq P_2}} \mathcal{R}(\gamma_1^2, \gamma_2^2, \mu_1, \mu_2) \subseteq \bigcup_{\substack{\sigma_1^2 + \mu_1^2 = P_1 \\ \gamma_2^2 + \mu_2^2 = P_2}} \mathcal{R}(\gamma_1^2, \gamma_2^2, \mu_1, \mu_2). \quad (106)$$

Let  $\beta_i \in [0, 1]$  be defined as follows:

$$\beta_i \triangleq \frac{\gamma_i^2}{P_i} = \frac{P_i - \mu_i^2}{P_i}, \quad i \in \{1, 2\}. \quad (107)$$

With these notations, any region  $\mathcal{R}(\gamma_1^2, \gamma_2^2, \mu_1, \mu_2)$  in the union over all  $(\mu_1, \mu_2, \gamma_1, \gamma_2)$  that satisfy  $\gamma_1^2 + \mu_1^2 = P_1$  and  $\gamma_2^2 + \mu_2^2 = P_2$ , can be rewritten as follows:

$$R_1 \leq \frac{1}{2} \log \left( 1 + \frac{\beta_1 P_1 h_{1,1}^2}{\sigma_1^2} \right), \quad (108a)$$

$$R_2 \leq \frac{1}{2} \log \left( 1 + \frac{\beta_2 P_2 h_{2,2}^2}{\sigma_2^2} \right), \quad (108b)$$

$$R_1 + R_2 \leq \log \left( 1 + \frac{h_{1,1}^2 \beta_1 P_1}{\sigma_1^2} + \frac{h_{1,2}^2 \beta_2 P_2}{\sigma_1^2} \right) + \frac{1}{2} \log \left( 1 + \frac{\frac{h_{2,2}^2 \beta_2 P_2}{\sigma_2^2}}{1 + \frac{h_{1,2}^2 \beta_2 P_2}{\sigma_1^2}} \right), \quad (108c)$$

$$R_1 + R_2 \leq \log \left( 1 + \frac{h_{2,2}^2 \beta_2 P_2}{\sigma_2^2} + \frac{h_{2,1}^2 \beta_1 P_1}{\sigma_2^2} \right) + \frac{1}{2} \log \left( 1 + \frac{\frac{h_{1,1}^2 \beta_1 P_1}{\sigma_1^2}}{1 + \frac{h_{2,1}^2 \beta_1 P_1}{\sigma_2^2}} \right), \quad (108d)$$

$$R_1 + R_2 \leq \frac{1}{2} \log \left( 1 + \frac{\frac{h_{1,1}^2 \beta_1 P_1}{\sigma_1^2} + \frac{h_{1,2}^2 \beta_2 P_2}{\sigma_1^2} + \frac{\beta_1 \beta_2 P_1 P_2 h_{1,2}^2 h_{2,1}^2}{\sigma_1^2 \sigma_2^2}}{1 + \frac{\beta_1 P_1 h_{2,1}^2}{\sigma_2^2}} \right) + \frac{1}{2} \log \left( 1 + \frac{\frac{h_{2,2}^2 \beta_2 P_2}{\sigma_2^2} + \frac{h_{2,1}^2 \beta_1 P_1}{\sigma_2^2} + \frac{\beta_1 \beta_2 P_1 P_2 h_{1,2}^2 h_{2,1}^2}{\sigma_1^2 \sigma_2^2}}{1 + \frac{\beta_2 P_2 h_{1,2}^2}{\sigma_1^2}} \right), \quad (108e)$$

$$2R_1 + R_2 \leq \frac{1}{2} \log \left( 1 + \frac{\frac{h_{1,1}^2 \beta_1 P_1}{\sigma_1^2}}{1 + \frac{h_{2,1}^2 \beta_1 P_1}{\sigma_2^2}} \right) + \log \left( 1 + \frac{h_{1,1}^2 \beta_1 P_1}{\sigma_1^2} + \frac{h_{1,2}^2 \beta_2 P_2}{\sigma_1^2} \right) + \frac{1}{2} \log \left( 1 + \frac{\frac{h_{2,2}^2 \beta_2 P_2}{\sigma_2^2} + \frac{h_{2,1}^2 \beta_1 P_1}{\sigma_2^2} + \frac{\beta_1 \beta_2 P_1 P_2 h_{1,2}^2 h_{2,1}^2}{\sigma_1^2 \sigma_2^2}}{1 + \frac{\beta_2 P_2 h_{1,2}^2}{\sigma_1^2}} \right), \quad (108f)$$

$$R_1 + 2R_2 \leq \frac{1}{2} \log \left( 1 + \frac{\frac{h_{2,2}^2 \beta_2 P_2}{\sigma_2^2}}{1 + \frac{h_{1,2}^2 \beta_2 P_2}{\sigma_1^2}} \right) + \log \left( 1 + \frac{h_{2,2}^2 \beta_2 P_2}{\sigma_2^2} + \frac{h_{2,1}^2 \beta_1 P_1}{\sigma_2^2} \right) \quad (108g)$$

$$+ \frac{1}{2} \log \left( 1 + \frac{\frac{h_{1,1}^2 \beta_1 P_1}{\sigma_1^2} + \frac{h_{1,2}^2 \beta_2 P_2}{\sigma_1^2} + \frac{\beta_1 \beta_2 P_1 P_2 h_{1,2}^2 h_{2,1}^2}{\sigma_1^2 \sigma_2^2}}{1 + \frac{\beta_1 P_1 h_{2,1}^2}{\sigma_2^2}} \right),$$

$$B \leq \sigma_3^2 + h_{3,1}^2 P_1 + h_{3,2}^2 P_2 + 2h_{3,1} h_{3,2} \sqrt{(1 - \beta_1) P_1 (1 - \beta_2) P_2}, \quad (108h)$$

for some  $(\beta_1, \beta_2) \in [0, 1]^2$ . Hence, such a region contains all information-energy rate triplets  $(R_1, R_2, B)$  satisfying the constraints of Theorem 2 and this completes the proof of Theorem 2.

## C Proof of Theorem 4

This proof is divided into two parts. The first part consists of the proof of (17a)-(17f). The second part consists of the proof of (17g).

### C.1 Proof of (17a)-(17f)

**Codebook Generation:** Fix a strictly positive joint probability distribution:

$P_{V U_1 U_2 S_1 S_2}(v, u, u_1, u_2, s_1, s_2) = P_V(v) P_{U|V}(u|v) P_{U_1|UV}(u_1|u, v) P_{U_2|U,V}(u_2|u, v) P_{S_1|U U_1 V}(s_1|u, u_1, v) P_{S_2|U U_2 V}(s_2|u, u_2, v)$ , for all  $(v, u, u_1, u_2, x_1, x_2) \in \mathcal{X}_1 \cap \mathcal{X}_2 \times (\mathcal{X}_1 \cap \mathcal{X}_2) \times (\mathcal{X}_1 \times \mathcal{X}_2)$ . Let  $R_E, R_{1,C}, R_{2,C}, R_{1,P}$  and  $R_{2,P}$  be non-negative real numbers. For transmitter 1, generate  $2^{NR_E}$  i.i.d N-length codewords  $\mathbf{v}(\omega) = (v_1(\omega), \dots, v_N(\omega))$  according to

$$P_{\mathbf{V}}(\mathbf{v}(\omega)) = \prod_{m=1}^N P_V(v_m(\omega)), \quad (109)$$

with  $\omega \in \{1, 2, \dots, 2^{NR_E}\}$ . For each codeword  $\mathbf{v}(\omega)$ , generate  $2^{N(R_{1,C} + R_{2,C})}$  i.i.d. N-length codewords  $\mathbf{u}(\omega, s, r) = (u_1(\omega, s, r), \dots, u_N(\omega, s, r))$  according to

$$P_{U|V}(\mathbf{u}(\omega, s, r)|\mathbf{v}(\omega)) = \prod_{m=1}^N P_{U|V}(u_m(\omega, s, r)|v_m(\omega)), \quad (110)$$

with  $s \in \{1, \dots, 2^{NR_{1,C}}\}$  and  $r \in \{1, \dots, 2^{NR_{2,C}}\}$ . For transmitter 1, for each codeword  $\mathbf{u}(\omega, s, r)$ , generate  $2^{NR_{1,C}}$  i.i.d. N-length codewords  $\mathbf{u}_1(\omega, s, r, k) = (u_{1,1}(\omega, s, r, k), \dots, u_{1,N}(\omega, s, r, k))$  according to

$$P_{U_1|UV}(\mathbf{u}_1(\omega, s, r, k)|\mathbf{u}(s, r), \mathbf{v}(\omega)) = \prod_{m=1}^N P_{U_1|UV}(u_{1,m}(\omega, s, r, k)|u_m(\omega, s, r), v_m(\omega)), \quad (111)$$

with  $k \in \{1, \dots, 2^{NR_{1,C}}\}$ . For each tuple of codewords  $(\mathbf{v}(\omega), \mathbf{u}(\omega, s, r), \mathbf{u}_1(\omega, s, r, k))$ , generate  $2^{NR_{1,P}}$  i.i.d. N-length codewords  $\mathbf{s}_1(\omega, s, r, k, l) = (s_{1,1}(\omega, s, r, k, l), \dots, s_{1,N}(\omega, s, r, k, l))$  according to

$$P_{S_1|U_1 UV}(\mathbf{s}_1(\omega, s, r, k, l)|\mathbf{u}_1(\omega, s, r, k), \mathbf{u}(s, r), \mathbf{v}(\omega)) = \prod_{m=1}^N P_{S_1|U_1 UV}(s_{1,m}(\omega, s, r, k, l)|u_{1,m}(\omega, s, r, k), u_m(\omega, s, r), v_m(\omega)), \quad (112)$$

with  $l \in \{1, \dots, 2^{NR_{1,P}}\}$ .

For encoder 2, for each codeword  $\mathbf{u}(\omega, s, r)$ , generate  $2^{NR_{2,C}}$  i.i.d.  $N$ -length codewords  $\mathbf{u}_2(\omega, s, r, q) = (u_{2,1}(\omega, s, r, q), \dots, u_{2,N}(\omega, s, r, q))$  according to

$$P_{U_2|UV}(\mathbf{u}_2(\Omega, s, r, q)|\mathbf{u}(s, r), \mathbf{v}(\omega)) = \prod_{m=1}^N P_{U_{2,m}|UV} \left( u_{2,m}(\omega, s, r, q) | u_m(\omega, s, r), v_m(\omega) \right), \quad (113)$$

with  $q \in \{1, \dots, 2^{NR_{2,C}}\}$ . For each tuple of codewords  $(\mathbf{v}(\omega), \mathbf{u}(\omega, s, r), \mathbf{u}_2(\omega, s, r, q))$ , generate  $2^{NR_{2,P}}$  i.i.d.  $N$ -length codewords  $\mathbf{s}_2(\omega, s, r, q, z) = (s_{2,1}(\omega, s, r, q, z), \dots, s_{2,N}(\omega, s, r, q, z))$  according to

$$\begin{aligned} & P_{S_2|U_2UV}(\mathbf{s}_2(\omega, s, r, q, z)|\mathbf{u}_2(\omega, s, r, q), \mathbf{u}(\omega, s, r), \mathbf{v}(\omega)) \\ &= \prod_{m=1}^N P_{S_{2,m}|U_{2,m}UV} \left( s_{2,m}(\omega, s, r, q, z) | u_{2,m}(\omega, s, r, q), u_m(\omega, s, r), v_m(\omega) \right), \end{aligned} \quad (114)$$

with  $z \in \{1, \dots, 2^{NR_{2,P}}\}$ .

**Encoding:** Consider Markov encoding over  $T$  blocks. Let  $W_i^{(t)}$  be the message index transmitted during block  $t$ . Let  $i$  also be represented by the message index  $W_{i,C}^{(t)} \in \{1, 2, \dots, 2^{NR_{i,C}}\}$  and the message index  $W_{i,P}^{(t)} \in \{1, 2, \dots, 2^{NR_{i,P}}\}$ . The message index  $\Omega^{(t)}$  is known by both transmitters and receivers. At encoding step  $t$ , with  $t \in \{1, 2, \dots, T\}$ , transmitter 1 sends the codeword  $\mathbf{x}_1^{(t)} = \theta_1 \left( \mathbf{v}^{(t)}(\Omega^{(t)}), \mathbf{u} \left( \Omega^{(t)}, W_{1,C}^{(t-1)}, W_{2,C}^{(t-1)} \right), \mathbf{u}_1 \left( \Omega^{(t)}, W_{1,C}^{(t-1)}, W_{2,C}^{(t-1)}, W_{1,C}^{(t)} \right), \right.$

$\left. \mathbf{s}_1 \left( \Omega^{(t)}, W_{1,C}^{(t-1)}, W_{2,C}^{(t-1)}, W_{1,C}^{(t)}, W_{1,P}^{(t)} \right) \right)$ , where  $\theta_1 : \mathcal{X}^N \times (\mathcal{X}_1 \cup \mathcal{X}_2)^N \times \mathcal{X}_1 \times \mathcal{X}_1^N \rightarrow \mathcal{X}_1^N$  is a function that transforms the codewords  $\mathbf{v}^{(t)}(\Omega)$ ,  $\mathbf{u} \left( \Omega^{(t)}, W_{1,C}^{(t-1)}, W_{2,C}^{(t-1)} \right)$ ,  $\mathbf{u}_1 \left( \Omega^{(t)}, W_{1,C}^{(t-1)}, W_{2,C}^{(t-1)}, W_{1,C}^{(t)} \right)$ , and  $\mathbf{s}_1 \left( \Omega^{(t)}, W_{1,C}^{(t-1)}, W_{2,C}^{(t-1)}, W_{1,C}^{(t)}, W_{1,P}^{(t)} \right)$  into the  $N$ -dimensional vector  $\mathbf{x}_1^{(t)}$ . The indices  $W_{1,C}^{(0)} = W_{1,C}^{(T)} = s^*$  and  $W_{2,C}^{(0)} = W_{2,C}^{(T)} = r^*$ , and the pair  $(s^*, r^*) \in \{1, 2, \dots, 2^{NR_{1,C}}\} \times \{1, 2, \dots, 2^{NR_{2,C}}\}$  are pre-defined and known by both receivers and transmitters. Transmitter 2 follows a similar encoding scheme.

**Decoding:** Both receivers decode their message indices at the end of block  $T$  in a backward decoding fashion. At each decoding step  $t$ , with  $t \in \{1, 2, \dots, T\}$ , receiver 1 obtains the indices  $\left( \widehat{W}_{1,C}^{(T-t)}, \widehat{W}_{2,C}^{(T-t)}, \widehat{W}_{1,P}^{(T-(t-1))} \right)$  from the given channel output  $\mathbf{y}_1$ .

The tuple  $\left( \widehat{W}_{1,C}^{(T-t)}, \widehat{W}_{2,C}^{(T-t)}, \widehat{W}_{1,P}^{(T-(t-1))} \right)$  is the unique tuple that satisfy:

$$\begin{aligned} & \left( \mathbf{v}(\Omega^{(t)}), \mathbf{u} \left( \Omega^{(t)}, \widehat{W}_{1,C}^{(T-t)}, \widehat{W}_{2,C}^{(T-t)} \right), \mathbf{u}_1 \left( \Omega^{(t)}, \widehat{W}_{1,C}^{(T-t)}, \widehat{W}_{2,C}^{(T-t)}, W_{1,C}^{(T-(t-1))} \right), \right. \\ & \left. \mathbf{s}_1 \left( \Omega^{(t)}, \widehat{W}_{1,C}^{(T-t)}, \widehat{W}_{2,C}^{(T-t)}, W_{1,C}^{(T-(t-1))}, W_{1,P}^{(T-(t-1))} \right), \mathbf{u}_2 \left( \Omega^{(t)}, \widehat{W}_{1,C}^{(T-t)}, \widehat{W}_{2,C}^{(T-t)}, W_{2,C}^{(T-(t-1))} \right), \right. \\ & \left. \mathbf{y}_1^{(T-(t-1))} \right) \in \mathcal{T}_{VUU_1S_1U_2Y_1}^{(N,\epsilon)}, \end{aligned} \quad (115)$$

where  $W_{1,C}^{(T-(t-1))}$  and  $W_{2,C}^{(T-(t-1))}$  are assumed to be perfectly decoded in the previous decoding step  $t-1$ . The set  $\mathcal{T}_{VUU_1S_1U_2Y_1}^{(N,\epsilon)}$  represent the set of jointly typical sequences of the random variables  $V, U, U_1, S_1, U_2$ , and  $Y_1$ , with  $\epsilon > 0$ . Finally, receiver 2 follows a similar decoding scheme.

**Probability of Error Analysis:** An error might occur during encoding step  $t > 2$  at transmitter



1 if the index  $\widehat{W}_{2,C}^{(t-1)}$  is not correctly decoded. Without any loss of generality, let  $W_{2,C}^{(t-1)} = 1$  and  $\widehat{W}_{2,C}^{(t-1)} = 1$ . Define the event  $E_k$  that describes the case in which there exists another message index  $k \neq 1$  that satisfies:  $\left( \mathbf{v}(\Omega^{(t)}), \mathbf{u}(\Omega^{(t)}, W_{1,C}^{(t-2)}, W_{2,C}^{(t-2)}), \mathbf{u}_1(\Omega^{(t)}, W_{1,C}^{(t-2)}, W_{2,C}^{(t-2)}, W_{1,C}^{(t-1)}), \mathbf{s}_1(\Omega^{(t)}, W_{1,C}^{(t-2)}, W_{2,C}^{(t-2)}, W_{1,C}^{(t-1)}, W_{1,P}^{(t-1)}), \mathbf{u}_2(\Omega^{(t)}, W_{1,C}^{(t-2)}, W_{2,C}^{(t-2)}, k) \right) \in \mathcal{T}_{VUU_1S_1U_2Y_1}^{(N,\epsilon)}$ , with  $t \in \{2, 3, \dots, T\}$  and  $W_{2,C}^{(t-2)}$  is assumed to be perfectly decoded in the previous block  $t-1$ . Then, the probability of event  $E_k$  can be bounded as follows:

$$\Pr(E_k) = \Pr \left[ (\mathbf{V}, \mathbf{U}, \mathbf{U}_1, \mathbf{S}_1, \mathbf{U}_2, \mathbf{Y}_1) \in \mathcal{T}_{VUU_1S_1U_2Y_1}^{(N,\epsilon)} \right] \quad (116a)$$

$$= \sum_{(\mathbf{v}, \mathbf{u}, \mathbf{u}_1, \mathbf{s}_1, \mathbf{y}_1) \in \mathcal{T}_{VUU_1S_1U_2Y_1}^{(N,\epsilon)}} P_{\mathbf{V}}(\mathbf{v}) P_{\mathbf{U}, \mathbf{U}_1, \mathbf{S}_1, \mathbf{Y}_1 | \mathbf{V}}(\mathbf{u}, \mathbf{u}_1, \mathbf{s}_1, \mathbf{y}_2 | \mathbf{v}) P_{\mathbf{U}_2 | \mathbf{V}}(\mathbf{u}_2 | \mathbf{v}), \quad (116b)$$

$$\leq \left| \mathcal{T}_{VUU_1S_1U_2Y_1}^{(N,\epsilon)} \right| 2^{-N(H(U, U_1, S_1, Y_1 | V) + H(U_2 | V) + H(V) - 4\epsilon)} \quad (116c)$$

$$\leq 2^{-N(H(U, U_1, S_1, Y_1 | V) + H(U_2 | V) + H(V) - H(U, U_1, U_2, S_1, Y_1) - 4\epsilon)} \quad (116d)$$

$$= 2^{-N(H(U, U_1, S_1, Y_1 | V) + H(U_2 | V) - H(U, U_1, U_2, S_1, Y_1 | V) - 4\epsilon)} \quad (116e)$$

$$= 2^{-N(I(U, U_1, S_1, Y_1; U_2 | V) - 4\epsilon)} \quad (116f)$$

$$= 2^{-N(I(U, X_1, Y_1; U_2 | V) - 4\epsilon)} \quad (116g)$$

$$= 2^{-N(I(Y_1; U_2 | X_1, V, U) - 4\epsilon)}, \quad (116h)$$

where (116f) follows from (44); and (116h) follows from the fact that  $X_i = \theta_i(U, U_i, S_i, V)$ . Note that the probability operator  $\Pr[\cdot]$  in (116a) applies with a probability distribution  $P_{\mathbf{V}UU_1\mathbf{X}_1U_2\mathbf{Y}_1}$  that factorizes as  $P_{\mathbf{V}}P_{\mathbf{U}, \mathbf{U}_1, \mathbf{X}_1, \mathbf{Y}_1 | \mathbf{V}}P_{\mathbf{U}_2 | \mathbf{V}}$  given that all the codewords  $\mathbf{u}_2$  are independent from the given channel output  $\mathbf{y}_1$ . The error probability becomes arbitrarily small (as  $N$  goes to infinity) if

$$R_{2c} \leq I(U_2; Y_1 | X_1, U, V). \quad (117)$$

An error might occur during the (backward) decoding step  $t$  if the indices  $\widehat{W}_{1,C}^{(T-t)}, \widehat{W}_{2,C}^{(T-t)}$  or  $\widehat{W}_{1,P}^{(T-(t-1))}$  are not decoded correctly given that the indices  $W_{1,C}^{(T-(t-1))}$  and  $W_{2,C}^{(T-(t-1))}$  were correctly decoded in the previous decoding step  $t-1$ . These errors might arise for two reasons: (i) there does not exist a tuple  $\left( \widehat{W}_{1,C}^{(T-t)}, \widehat{W}_{2,C}^{(T-t)}, W_{1,P}^{(T-(t-1))} \right)$  that satisfies (115), or (ii) there exist several tuples  $\left( \widehat{W}_{1,C}^{(T-t)}, \widehat{W}_{2,C}^{(T-t)}, W_{1,P}^{(T-(t-1))} \right)$  that simultaneously satisfy (115). From the asymptotic equipartition property [8], the probability of error due to (i) tends to zero when  $N$  grows to infinity. Consider the error due to (ii) and define the event  $E_{srl}$  that describes the case in which the codewords  $\mathbf{v}(\Omega^{(t)}), \mathbf{u}(\Omega^{(t)}, s, r), \mathbf{u}_1(\Omega^{(t)}, s, r, W_{1,C}^{(T-(t-1))}), \mathbf{s}_1(\Omega^{(t)}, s, r, W_{1,C}^{(T-(t-1))}, l)$ , and  $\mathbf{u}_2(\Omega^{(t)}, s, r, W_{2,C}^{(T-(t-1))})$  are jointly typical with  $\mathbf{y}_1^{(T-(t-1))}$  during decoding step  $t$ . Assume now that the codeword to be decoded at decoding step  $t$  corresponds to the indices  $(s, r, l) = (1, 1, 1)$ . This is without loss of generality due to the symmetry of the code. Then, the probability of error

due to (ii) during decoding step  $t$ , can be bounded as follows:

$$\begin{aligned}
 & \Pr \left[ \bigcup_{(s,r,l) \neq (1,1,1)} E_{srl} \right] \\
 & \leq \sum_{s \neq 1, r \neq 1, l \neq 1} \Pr[E_{srl}] + \sum_{s \neq 1, r \neq 1, l=1} \Pr[E_{srl}] + \sum_{s \neq 1, r=1, l \neq 1} \Pr[E_{srl}] \\
 & \quad + \sum_{s \neq 1, r=1, l=1} \Pr[E_{srl}] + \sum_{s=1, r \neq 1, l \neq 1} \Pr[E_{srl}] + \sum_{s=1, r \neq 1, l=1} \Pr[E_{srl}] \\
 & \quad + \sum_{s=1, r=1, l \neq 1} \Pr[E_{srl}], \tag{118}
 \end{aligned}$$

For all  $s \in \{2, 3, \dots, 2^{NR_{1,C}}\}$ , the following holds

$$\Pr[E_{s11}] = \Pr \left[ (\mathbf{V}, \mathbf{U}, \mathbf{U}_1, \mathbf{S}_1, \mathbf{U}_2, \mathbf{Y}_1) \in \mathcal{T}_{VUU_1S_1U_2Y_1}^{(N,\epsilon)} \right] \tag{119a}$$

$$= \sum_{(\mathbf{v}, \mathbf{u}, \mathbf{u}_1, \mathbf{s}_1, \mathbf{u}_2, \mathbf{y}_1) \in \mathcal{T}_{VUU_1S_1U_2Y_1}^{(N,\epsilon)}} P_{\mathbf{V}}(\mathbf{v}) P_{\mathbf{U}, \mathbf{U}_1, \mathbf{S}_1, \mathbf{U}_2 | \mathbf{V}}(\mathbf{u}, \mathbf{u}_1, \mathbf{s}_1, \mathbf{u}_2 | \mathbf{v}) P_{\mathbf{Y}_1 | \mathbf{V}}(\mathbf{y}_1 | \mathbf{v}), \tag{119b}$$

$$\leq \left| \mathcal{T}_{VUU_1S_1U_2Y_1}^{(N,\epsilon)} \right| 2^{-N(H(U, U_1, U_2, S_1 | V) + H(Y_1 | V) + H(V) - 4\epsilon)} \tag{119c}$$

$$\leq 2^{-N(H(U, U_1, U_2, S_1 | V) + H(Y_1 | V) + H(V) - H(U, U_1, U_2, S_1, Y_1, V) - 4\epsilon)} \tag{119d}$$

$$= 2^{-N(H(U, U_1, U_2, S_1 | V) + H(Y_1 | V) - H(U, U_1, U_2, S_1, Y_1 | V) - 4\epsilon)} \tag{119e}$$

$$= 2^{-N(I(U, U_1, U_2, S_1; Y_1 | V) - 4\epsilon)} \tag{119f}$$

$$= 2^{-N(I(U, U_2, X_1; Y_1 | V) - 4\epsilon)}, \tag{119g}$$

where (119f) follows from (44); and (119g) follows from the fact that  $X_i = \theta_i(U, U_i, S_i, V)$ . Note that the probability operator  $\Pr[\cdot]$  in (119a) applies with a probability distribution  $P_{VUU_1S_1U_2Y_1}$  that factorizes as  $P_{\mathbf{V}}P_{\mathbf{U}, \mathbf{U}_1, \mathbf{S}_1, \mathbf{U}_2 | \mathbf{V}}P_{\mathbf{Y}_1 | \mathbf{V}}$  given that all the codewords  $\mathbf{u}, \mathbf{u}_1, \mathbf{s}_1$ , and  $\mathbf{u}_2$  are independent from the output of the channel output  $\mathbf{y}_1$ . For all  $r \in \{2, 3, \dots, 2^{NR_{2,C}}\}$ , the following holds

$$\Pr[E_{1r1}] = \Pr \left[ (\mathbf{V}, \mathbf{U}, \mathbf{U}_1, \mathbf{S}_1, \mathbf{U}_2, \mathbf{Y}_1) \in \mathcal{T}_{VUU_1S_1U_2Y_1}^{(N,\epsilon)} \right] \tag{120a}$$

$$= \sum_{(\mathbf{v}, \mathbf{u}, \mathbf{u}_1, \mathbf{s}_1, \mathbf{u}_2, \mathbf{y}_1) \in \mathcal{T}_{VUU_1S_1U_2Y_1}^{(N,\epsilon)}} P_{\mathbf{V}}(\mathbf{v}) P_{\mathbf{U}, \mathbf{U}_1, \mathbf{S}_1, \mathbf{U}_2 | \mathbf{V}}(\mathbf{u}, \mathbf{u}_1, \mathbf{s}_1, \mathbf{u}_2 | \mathbf{v}) P_{\mathbf{Y}_1 | \mathbf{V}}(\mathbf{y}_1 | \mathbf{v}), \tag{120b}$$

$$\leq \left| \mathcal{T}_{VUU_1S_1U_2Y_1}^{(N,\epsilon)} \right| 2^{-N(H(U, U_1, U_2, S_1 | V) + H(Y_1 | V) + H(V) - 4\epsilon)} \tag{120c}$$

$$\leq 2^{-N(H(U, U_1, U_2, S_1 | V) + H(Y_1 | V) + H(V) - H(U, U_1, U_2, S_1, Y_1, V) - 4\epsilon)} \tag{120d}$$

$$= 2^{-N(H(U, U_1, U_2, S_1 | V) + H(Y_1 | V) - H(U, U_1, U_2, S_1, Y_1 | V) - 4\epsilon)} \tag{120e}$$

$$= 2^{-N(I(U, U_1, U_2, S_1; Y_1 | V) - 4\epsilon)} \tag{120f}$$

$$= 2^{-N(I(U, U_2, X_1; Y_1 | V) - 4\epsilon)}. \tag{120g}$$

where (120f) follows from (44); and (120g) follows from the fact that  $X_i = \theta_i(U, U_i, S_i, V)$ . Note that the probability operator  $\Pr[\cdot]$  in (120a) applies with a probability distribution  $P_{VUU_1S_1U_2Y_1}$  that factorizes as  $P_{\mathbf{V}}P_{\mathbf{U}, \mathbf{U}_1, \mathbf{S}_1, \mathbf{U}_2 | \mathbf{V}}P_{\mathbf{Y}_1 | \mathbf{V}}$  given that all the codewords  $\mathbf{u}, \mathbf{u}_1, \mathbf{s}_1$ , and  $\mathbf{u}_2$  are independent from the output of the channel output  $\mathbf{y}_1$ . For all  $s \in \{2, 3, \dots, 2^{NR_{1,C}}\}$  and

$r \in \{2, 3, \dots, 2^{NR_2, C}\}$ , the following holds

$$\Pr[E_{sr1}] = \Pr\left[(\mathbf{V}, \mathbf{U}, \mathbf{U}_1, \mathbf{S}_1, \mathbf{U}_2, \mathbf{Y}_1) \in \mathcal{T}_{VUU_1S_1U_2Y_1}^{(N, \epsilon)}\right] \quad (121a)$$

$$= \sum_{(\mathbf{v}, \mathbf{u}, \mathbf{u}_1, \mathbf{s}_1, \mathbf{u}_2, \mathbf{y}_1) \in \mathcal{T}_{VUU_1S_1U_2Y_1}^{(N, \epsilon)}} P_{\mathbf{V}}(\mathbf{v}) P_{\mathbf{U}, \mathbf{U}_1, \mathbf{S}_1, \mathbf{U}_2 | \mathbf{V}}(\mathbf{u}, \mathbf{u}_1, \mathbf{s}_1, \mathbf{u}_2 | \mathbf{v}) P_{\mathbf{Y}_1 | \mathbf{V}}(\mathbf{y}_1 | \mathbf{v}), \quad (121b)$$

$$\leq \left| \mathcal{T}_{VUU_1S_1U_2Y_1}^{(N, \epsilon)} \right| 2^{-N(H(U, U_1, U_2, S_1 | V) + H(Y_1 | V) + H(V) - 4\epsilon)} \quad (121c)$$

$$\leq 2^{-N(H(U, U_1, U_2, S_1 | V) + H(Y_1 | V) + H(V) - H(U, U_1, U_2, S_1, Y_1, V) - 4\epsilon)} \quad (121d)$$

$$= 2^{-N(H(U, U_1, U_2, S_1 | V) + H(Y_1 | V) - H(U, U_1, U_2, S_1, Y_1 | V) - 4\epsilon)} \quad (121e)$$

$$= 2^{-N(I(U, U_1, U_2, S_1; Y_1 | V) - 4\epsilon)} \quad (121f)$$

$$= 2^{-N(I(U, U_2, X_1; Y_1 | V) - 4\epsilon)}, \quad (121g)$$

where (121f) follows from (44); and (121g) follows from the fact that  $X_i = \theta_i(U, U_i, S_i, V)$ . Note that the probability operator  $\Pr[\cdot]$  applies in (121a) with a probability distribution  $P_{VUU_1S_1U_2Y_1}$  that factorizes as  $P_{\mathbf{V}}P_{\mathbf{U}, \mathbf{U}_1, \mathbf{S}_1, \mathbf{U}_2 | \mathbf{V}}P_{\mathbf{Y}_1 | \mathbf{V}}$  given that all the codewords  $\mathbf{u}$ ,  $\mathbf{u}_1$ ,  $\mathbf{s}_1$ , and  $\mathbf{u}_2$  are independent from the output of the channel output  $\mathbf{y}_1$ . For all  $s \in \{2, 3, \dots, 2^{NR_1, C}\}$  and  $l \in \{2, 3, \dots, 2^{NR_1, P}\}$ , the following holds

$$\Pr[E_{s1l}] = \Pr\left[(\mathbf{V}, \mathbf{U}, \mathbf{U}_1, \mathbf{S}_1, \mathbf{U}_2, \mathbf{Y}_1) \in \mathcal{T}_{VUU_1S_1U_2Y_1}^{(N, \epsilon)}\right] \quad (122a)$$

$$= \sum_{(\mathbf{v}, \mathbf{u}, \mathbf{u}_1, \mathbf{s}_1, \mathbf{u}_2, \mathbf{y}_1) \in \mathcal{T}_{VUU_1S_1U_2Y_1}^{(N, \epsilon)}} P_{\mathbf{V}}(\mathbf{v}) P_{\mathbf{U}, \mathbf{U}_1, \mathbf{S}_1, \mathbf{U}_2 | \mathbf{V}}(\mathbf{u}, \mathbf{u}_1, \mathbf{s}_1, \mathbf{u}_2 | \mathbf{v}) P_{\mathbf{Y}_1 | \mathbf{V}}(\mathbf{y}_1 | \mathbf{v}), \quad (122b)$$

$$\leq \left| \mathcal{T}_{VUU_1S_1U_2Y_1}^{(N, \epsilon)} \right| 2^{-N(H(U, U_1, U_2, S_1 | V) + H(Y_1 | V) + H(V) - 4\epsilon)} \quad (122c)$$

$$\leq 2^{-N(H(U, U_1, U_2, S_1 | V) + H(Y_1 | V) + H(V) - H(U, U_1, U_2, S_1, Y_1, V) - 4\epsilon)} \quad (122d)$$

$$= 2^{-N(H(U, U_1, U_2, S_1 | V) + H(Y_1 | V) - H(U, U_1, U_2, S_1, Y_1 | V) - 4\epsilon)} \quad (122e)$$

$$= 2^{-N(I(U, U_1, U_2, S_1; Y_1 | V) - 4\epsilon)} \quad (122f)$$

$$= 2^{-N(I(U, U_2, X_1; Y_1 | V) - 4\epsilon)}, \quad (122g)$$

where (122f) follows from (44); and (122g) follows from the fact that  $X_i = \theta_i(U, U_i, S_i, V)$ . Note that the probability operator  $\Pr[\cdot]$  in (122a) applies with a probability distribution  $P_{VUU_1S_1U_2Y_1}$  that factorizes as  $P_{\mathbf{V}}P_{\mathbf{U}, \mathbf{U}_1, \mathbf{S}_1, \mathbf{U}_2 | \mathbf{V}}P_{\mathbf{Y}_1 | \mathbf{V}}$  given that all the codewords  $\mathbf{u}$ ,  $\mathbf{u}_1$ ,  $\mathbf{s}_1$ , and  $\mathbf{u}_2$  are independent from the output of the channel output  $\mathbf{y}_1$ .

For all  $r \in \{2, 3, \dots, 2^{NR_2, C}\}$  and  $l \in \{2, 3, \dots, 2^{NR_1, P}\}$ , the following holds

$$\Pr[E_{1rl}] = \Pr\left[(\mathbf{V}, \mathbf{U}, \mathbf{U}_1, \mathbf{S}_1, \mathbf{U}_2, \mathbf{Y}_1) \in \mathcal{T}_{VUU_1S_1U_2Y_1}^{(N, \epsilon)}\right] \quad (123a)$$

$$= \sum_{(\mathbf{v}, \mathbf{u}, \mathbf{u}_1, \mathbf{s}_1, \mathbf{u}_2, \mathbf{y}_1) \in \mathcal{T}_{VUU_1S_1U_2Y_1}^{(N, \epsilon)}} P_{\mathbf{V}}(\mathbf{v}) P_{\mathbf{U}, \mathbf{U}_1, \mathbf{S}_1, \mathbf{U}_2 | \mathbf{V}}(\mathbf{u}, \mathbf{u}_1, \mathbf{s}_1, \mathbf{u}_2 | \mathbf{v}) P_{\mathbf{Y}_1 | \mathbf{V}}(\mathbf{y}_1 | \mathbf{v}), \quad (123b)$$

$$\leq \left| \mathcal{T}_{VUU_1S_1U_2Y_1}^{(N, \epsilon)} \right| 2^{-N(H(U, U_1, U_2, S_1 | V) + H(Y_1 | V) + H(V) - 4\epsilon)} \quad (123c)$$

$$\leq 2^{-N(H(U, U_1, U_2, S_1 | V) + H(Y_1 | V) + H(V) - H(U, U_1, U_2, S_1, Y_1, V) - 4\epsilon)} \quad (123d)$$

$$= 2^{-N(H(U, U_1, U_2, S_1 | V) + H(Y_1 | V) - H(U, U_1, U_2, S_1, Y_1 | V) - 4\epsilon)} \quad (123e)$$

$$= 2^{-N(I(U, U_1, U_2, S_1; Y_1 | V) - 4\epsilon)} \quad (123f)$$

$$= 2^{-N(I(U, U_2, X_1; Y_1 | V) - 4\epsilon)}, \quad (123g)$$

where (123f) follows from (44); and (123g) follows from the fact that  $X_i = \theta_i(U, U_i, S_i, V)$ . Note that the probability operator  $\Pr[\cdot]$  in (123a) applies with a probability distribution  $P_{\mathbf{V}UU_1\mathbf{s}_1U_2\mathbf{Y}_1}$  that factorizes as  $P_{\mathbf{V}}P_{U,U_1,\mathbf{s}_1,U_2|\mathbf{V}}P_{\mathbf{Y}_1|\mathbf{V}}$  given that all the codewords  $\mathbf{u}, \mathbf{u}_1, \mathbf{s}_1$ , and  $\mathbf{u}_2$  are independent from the output of the channel output  $\mathbf{y}_1$ .

For all  $s \in \{2, 3, \dots, 2^{NR_{1,C}}\}$ ,  $r \in \{2, 3, \dots, 2^{NR_{2,C}}\}$  and  $l \in \{2, 3, \dots, 2^{NR_{1,P}}\}$ , the following holds

$$\Pr[E_{srl}] = \Pr\left[(\mathbf{V}, U, U_1, \mathbf{s}_1, U_2, \mathbf{Y}_1) \in \mathcal{T}_{VUU_1S_1U_2Y_1}^{(N,\epsilon)}\right] \quad (124a)$$

$$= \sum_{(\mathbf{v}, \mathbf{u}, \mathbf{u}_1, \mathbf{s}_1, \mathbf{u}_2, \mathbf{y}_1) \in \mathcal{T}_{VUU_1S_1U_2Y_1}^{(N,\epsilon)}} P_{\mathbf{V}}(\mathbf{v})P_{U,U_1,\mathbf{s}_1,U_2|\mathbf{V}}(\mathbf{u}, \mathbf{u}_1, \mathbf{s}_1, \mathbf{u}_2|\mathbf{v})P_{\mathbf{Y}_1|\mathbf{V}}(\mathbf{y}_1|\mathbf{v}), \quad (124b)$$

$$\leq \left| \mathcal{T}_{VUU_1S_1U_2Y_1}^{(N,\epsilon)} \right| 2^{-N(H(U,U_1,U_2,S_1|V)+H(Y_1|V)+H(V)-4\epsilon)} \quad (124c)$$

$$\leq 2^{-N(H(U,U_1,U_2,S_1|V)+H(Y_1|V)+H(V)-H(U,U_1,U_2,S_1,Y_1,V)-4\epsilon)} \quad (124d)$$

$$= 2^{-N(H(U,U_1,U_2,S_1|V)+H(Y_1|V)-H(U,U_1,U_2,S_1,Y_1|V)-4\epsilon)} \quad (124e)$$

$$= 2^{-N(I(U,U_1,U_2,S_1;Y_1|V)-4\epsilon)} \quad (124f)$$

$$= 2^{-N(I(U,U_2,X_1;Y_1|V)-4\epsilon)}, \quad (124g)$$

where (124f) follows from (44); and (124g) follows from the fact that  $X_i = \theta_i(U, U_i, S_i, V)$ . Note that the probability operator  $\Pr[\cdot]$  in (124a) applies with a probability distribution  $P_{\mathbf{V}UU_1\mathbf{s}_1U_2\mathbf{Y}_1}$  that factorizes as  $P_{\mathbf{V}}P_{U,U_1,\mathbf{s}_1,U_2|\mathbf{V}}P_{\mathbf{Y}_1|\mathbf{V}}$  given that all the codewords  $\mathbf{u}, \mathbf{u}_1, \mathbf{s}_1$ , and  $\mathbf{u}_2$  are independent from the output of the channel output  $\mathbf{y}_1$ .

For all  $l \in \{2, 3, \dots, 2^{NR_{1,P}}\}$ , the following holds

$$\Pr[E_{1ll}] = \Pr\left[(\mathbf{V}, U, U_1, \mathbf{s}_1, U_2, \mathbf{Y}_1) \in \mathcal{T}_{VUU_1S_1U_2Y_1}^{(N,\epsilon)}\right] \quad (125a)$$

$$= \sum_{(\mathbf{v}, \mathbf{u}, \mathbf{u}_1, \mathbf{s}_1, \mathbf{u}_2, \mathbf{y}_1) \in \mathcal{T}_{VUU_1S_1U_2Y_1}^{(N,\epsilon)}} P_{\mathbf{V}}(\mathbf{v})P_{U,U_1,U_2,\mathbf{Y}_1|\mathbf{V}}(\mathbf{u}, \mathbf{u}_1, \mathbf{u}_2, \mathbf{y}_1|\mathbf{v})P_{\mathbf{s}_1|\mathbf{V}}(\mathbf{s}_1|\mathbf{v}), \quad (125b)$$

$$\leq \left| \mathcal{T}_{VUU_1S_1U_2Y_1}^{(N,\epsilon)} \right| 2^{-N(H(U,U_1,U_2,Y_1|V)+H(S_1|V)+H(V)-4\epsilon)} \quad (125c)$$

$$\leq 2^{-N(H(U,U_1,U_2,Y_1|V)+H(S_1|V)+H(V)-H(U,U_1,U_2,S_1,Y_1,V)-4\epsilon)} \quad (125d)$$

$$= 2^{-N(I(S_1;U,U_1,U_2,Y_1|V)-4\epsilon)} \quad (125e)$$

$$= 2^{-N(I(X_1;Y_1|U,U_1,U_2,V)-4\epsilon)}, \quad (125f)$$

where (125e) follows from (44); and (125f) follows from the fact that  $X_i = \theta_i(U, U_i, S_i, V)$ . Note that the probability operator  $\Pr[\cdot]$  in (125a) applies with a probability distribution  $P_{\mathbf{V}UU_1\mathbf{s}_1U_2\mathbf{Y}_1}$  that factorizes as  $P_{\mathbf{V}}P_{U,U_1,U_2,\mathbf{Y}_1|\mathbf{V}}P_{\mathbf{s}_1|\mathbf{V}}$  given that the codewords  $\mathbf{s}_1$  is independent from the output of the channel  $\mathbf{y}_1$ . Plugging (119) into (118) yields:

$$\begin{aligned} P_e &\leq 2^{N(R_{1C}+R_{2C}+R_{2P}-I(U,X_1,U_2;Y_1|V)+4\epsilon)} + 2^{N(R_{1C}+R_{2C}-I(U,X_1,U_2;Y_1|V)+4\epsilon)} \\ &\quad + 2^{N(R_{1C}+R_{1P}-I(U,X_1,U_2;Y_1|V)+4\epsilon)} + 2^{N(R_{1C}-I(U,X_1,U_2;Y_1|V)+4\epsilon)} \\ &\quad + 2^{N(R_{2C}+R_{1P}-I(U,X_1,U_2;Y_1|V)+4\epsilon)} + 2^{N(R_{2C}-I(U,X_1,U_2;Y_1|V)+4\epsilon)} \\ &\quad + 2^{N(R_{1P}-I(X_1;Y_1|U,U_1,U_2,V)+4\epsilon)}. \end{aligned} \quad (126)$$

The same analysis of the probability of error holds for transmitter-receiver pair 2. Hence in general, from (117) and (126), reliable decoding holds under the following conditions:

$$R_{2C} \leq I(U_2; Y_1 | X_1, U, V), \quad (127a)$$

$$R_{1P} \leq I(X_1; Y_1 | U_1, U_2, U, V), \quad (127b)$$

$$R_{1C} + R_{2C} + R_{1P} \leq I(U, X_1, U_2; Y_1 | V), \quad (127c)$$

$$R_{1C} \leq I(U_1; Y_2 | X_2, U, V), \quad (127d)$$

$$R_{2P} \leq I(X_2; Y_2 | U_1, U_2, U, V), \quad (127e)$$

$$R_{1C} + R_{2C} + R_{2P} \leq I(U, X_2, U_1; Y_2 | V). \quad (127f)$$

The proof continues by applying a Fourier-Motzkin elimination process on (127). Set  $R_{1P} = R_1 - R_{1C}$ ,  $R_{2P} = R_2 - R_{2C}$ . Eliminate  $R_{1P}$ ,  $R_{2P}$  from the inequalities in (127) to obtain:

$$R_{2C} \leq a_1, \quad (128a)$$

$$R_1 - R_{1C} \leq a_2, \quad (128b)$$

$$R_1 + R_{2C} \leq a_3, \quad (128c)$$

$$R_{1C} \leq b_1, \quad (128d)$$

$$R_2 - R_{2C} \leq b_2, \quad (128e)$$

$$R_2 + R_{1C} \leq b_3, \quad (128f)$$

$$-R_{1C} \leq 0, \quad (128g)$$

$$-R_1 + R_{1C} \leq 0, \quad (128h)$$

$$-R_{2C} \leq 0, \quad (128i)$$

$$-R_2 + R_{2C} \leq 0. \quad (128j)$$

Collect the inequalities in (128) that do not include  $R_{1C}$  among the above inequalities to obtain:

$$R_{2C} \leq a_1, \quad (129a)$$

$$R_1 + R_{2C} \leq a_3, \quad (129b)$$

$$R_2 - R_{2C} \leq b_2, \quad (129c)$$

$$-R_{2C} \leq 0, \quad (129d)$$

$$-R_2 + R_{2C} \leq 0. \quad (129e)$$

Collect the inequalities in (128) that include  $R_{1C}$  with positive coefficients to obtain:

$$R_{1C} \leq b_1, \quad (130a)$$

$$R_2 + R_{1C} \leq b_3, \quad (130b)$$

$$-R_1 + R_{1C} \leq 0. \quad (130c)$$

Collect the inequalities in (128) that include  $R_{1C}$  with negative coefficients to obtain:

$$R_1 - R_{1C} \leq a_2, \quad (131a)$$

$$-R_{1C} \leq 0. \quad (131b)$$

Eliminate  $R_{1C}$  by adding each inequality from (130) and each inequality from (131) to obtain:

$$R_1 \leq b_1 + a_2, \quad (132a)$$

$$R_2 \leq b_3, \quad (132b)$$

$$R_2 + R_1 \leq b_3 + a_2, \quad (132c)$$

$$-R_1 \leq 0, \quad (132d)$$

$$R_{2C} \leq a_1, \quad (132e)$$

$$R_1 + R_{2C} \leq a_3, \quad (132f)$$

$$-R_2 + R_{2C} \leq 0, \quad (132g)$$

$$R_2 - R_{2C} \leq b_2, \quad (132h)$$

$$-R_{2C} \leq 0. \quad (132i)$$

Collect the inequalities in (132) that do not include  $R_{2C}$  to obtain:

$$R_1 \leq b_1 + a_2, \quad (133a)$$

$$R_2 \leq b_3, \quad (133b)$$

$$R_2 + R_1 \leq b_3 + a_2, \quad (133c)$$

$$-R_1 \leq 0. \quad (133d)$$

Collect the inequalities in (132) that include  $R_{2C}$  with positive coefficients to obtain:

$$R_{2C} \leq a_1, \quad (134a)$$

$$-R_2 + R_{2C} \leq 0, \quad (134b)$$

$$R_1 + R_{2C} \leq a_3. \quad (134c)$$

Collect the inequalities in (132) that include  $R_{2C}$  with negative coefficients to obtain:

$$R_2 - R_{2C} \leq b_2, \quad (135a)$$

$$-R_{2C} \leq 0. \quad (135b)$$

Eliminate  $R_{2C}$  by adding each inequality in (134) with each inequality in (135) to obtain:

$$R_1 \leq a_3, \quad (136a)$$

$$R_1 \leq b_1 + a_2, \quad (136b)$$

$$R_2 \leq b_3, \quad (136c)$$

$$R_2 \leq a_1 + b_2, \quad (136d)$$

$$R_1 + R_2 \leq b_3, \quad (136e)$$

$$R_1 + R_2 \leq a_3 + b_2. \quad (136f)$$

The proof of Theorem 4 continues as follows, let  $k \in \{0, 1\}$  be fixed and consider the following Gaussian input distribution for transmitter  $k$ :

$$V \sim \mathcal{N}(0, 1); U \sim \mathcal{N}(0, \rho); U_k \sim \mathcal{N}(0, \lambda_{kc}); \text{ and } S_k \sim \mathcal{N}(0, \lambda_{kp}), \quad (137)$$

where  $X_k = \sqrt{P_k}U + \sqrt{P_k}X_{kp} + \sqrt{P_k}U_k + \sqrt{\lambda_{ke}P_k}V$ ; and  $(\rho, \lambda_{kp}, \lambda_{kc}, \lambda_{ke}) \in [0, 1]^4$  and  $\rho + \lambda_{kp} + \lambda_{kc} + \lambda_{ke} \leq 1$ . By symmetry, it suffices to prove (17a), (17b) and (17e). The choice of the

Gaussian input distribution in (136) yields:

$$\begin{aligned}
I(U, X_1, U_2; Y_1|V) &= h(Y_1|V) - h(Y_1|U, X_1, U_2, V) \\
&= \frac{1}{2} \log \left( 2\pi e \text{Var}[Y_1|V] \right) - \frac{1}{2} \log \left( 2\pi e \text{Var}[Y_1|U, X_1, U_2, V] \right) \\
&= \frac{1}{2} \log \left( 2\pi e (\sigma_1^2 + (1 - \lambda_{1e})P_1 h_{1,1}^2 + (1 - \lambda_{2e})P_2 h_{1,2}^2) \right) \\
&\quad - \frac{1}{2} \log \left( 2\pi e (\sigma_1^2 + \lambda_{2p}P_2 h_{1,2}^2) \right) \\
&= \frac{1}{2} \log \left( \frac{1 + (1 - \lambda_{1e})\text{SNR}_1 + (1 - \lambda_{2e})\text{INR}_1 + 2\rho\sqrt{\text{SNR}_1\text{INR}_1}}{1 + \lambda_{2p}\text{INR}_1} \right), \quad (138a)
\end{aligned}$$

which proves (17a). With the same power setting in (137), the following holds

$$\begin{aligned}
I(U_1; Y_2|U, X_2, V) &= h(Y_2|U, X_2, V) - h(Y_2|U, X_2, U_1, V) \\
&= \frac{1}{2} \log \left( 2\pi e \text{Var}[Y_2|U, X_2, V] \right) - \frac{1}{2} \log \left( 2\pi e \text{Var}[Y_2|U, X_2, U_1, V] \right) \\
&= \frac{1}{2} \log \left( 2\pi e (\sigma_2^2 + (1 - (\rho + \lambda_{1e}))P_1 h_{2,1}^2) \right) \\
&\quad - \frac{1}{2} \log \left( 2\pi e (\sigma_2^2 + \lambda_{1p}P_1 h_{2,1}^2) \right) \\
&= \frac{1}{2} \log \left( \frac{1 + (1 - (\rho + \lambda_{1e}))\text{INR}_2}{1 + \lambda_{1p}\text{INR}_2} \right), \text{ and} \quad (138b)
\end{aligned}$$

$$\begin{aligned}
I(X_1; Y_1|U, U_1, U_2, V) &= h(Y_1|U, U_1, U_2, V) - h(Y_1|U, U_1, U_2, X_1, V) \\
&= \frac{1}{2} \log \left( 2\pi e \text{Var}[Y_1|U, U_1, U_2, V] \right) - \frac{1}{2} \log \left( 2\pi e \text{Var}[Y_1|U, U_1, U_2, X_1, V] \right) \\
&= \frac{1}{2} \log \left( 2\pi e (\sigma_1^2 + \lambda_{1p}P_1 h_{1,1}^2 + \lambda_{2p}P_2 h_{1,2}^2) \right) \\
&\quad - \frac{1}{2} \log \left( 2\pi e (\sigma_1^2 + \lambda_{2p}P_2 h_{1,2}^2) \right) \\
&= \frac{1}{2} \log \left( \frac{1 + \lambda_{1p}\text{SNR}_1 + \lambda_{2p}\text{INR}_1}{1 + \lambda_{2p}\text{INR}_1} \right). \quad (138c)
\end{aligned}$$

This proves (17b). Finally, using (138b) and (138c), yields the proof of (17e).

## C.2 Proof of (17g)

The choice of the channel input in (137) guarantee that the random variables  $Y_{3,1}, \dots, Y_{3,n}$  are independently and identically distributed. For all  $n \in \{1, 2, \dots, N\}$ ,  $Y_{3,n}$  follows a zero-mean Gaussian distribution with variance  $\bar{B}$  given by

$$\begin{aligned}
\bar{B} &= \mathbb{E} [Y_{3,n}^2] \\
&= \mathbb{E} [(h_{3,1}X_{1,n} + h_{3,2}X_{2,n} + Z_{3,n})^2] \\
&= h_{3,1}^2 \mathbb{E} [X_{1,n}^2] + h_{3,2}^2 \mathbb{E} [X_{2,n}^2] + 2h_{3,1}h_{3,2} \mathbb{E} [X_{1,n}X_{2,n}] + \sigma_3^2 \\
&= h_{3,1}^2 P_1 + h_{3,2}^2 P_2 + 2h_{3,1}h_{3,2} \mathbb{E} [X_{1,n}X_{2,n}] + \sigma_3^2 \\
&\leq h_{3,1}^2 P_1 + h_{3,2}^2 P_2 + 2h_{3,1}h_{3,2} \sqrt{P_1 P_2} (\rho + \sqrt{\lambda_{1e}\lambda_{2e}}) + \sigma_3^2, \quad (139)
\end{aligned}$$

By the weak law of large numbers, it holds that

$$\lim_{n \rightarrow \infty} \Pr [B^{(N)} < \bar{B}] = 0. \quad (140)$$

From (140), it holds that for any energy  $B$  which satisfies  $0 < B \leq \bar{B}$ , it holds that

$$\lim_{n \rightarrow \infty} \Pr [B^{(N)} < B] = 0 \quad (141)$$

This proves (17g) and completes the proof of Theorem 4.

## D Proof of Theorem 5

Fix an information-energy rate triplet  $(R_1, R_2, B)$  achievable with a given coding scheme (Definition 1). Denote by  $\mathbf{X}_1$  and  $\mathbf{X}_2$  the channel inputs resulting from transmitting the independent messages  $(W_1, \Omega)$  and  $(W_2, \Omega)$  using such coding scheme. Denote by  $\mathbf{Y}_1$  and  $\mathbf{Y}_2$  the corresponding channel outputs. Define the following random variables:

$$S_1 = h_{2,1}X_1 + Z_2 \text{ and} \quad (142)$$

$$S_2 = h_{1,2}X_2 + Z_1, \quad (143)$$

where,  $Z_1$  and  $Z_2$  are real Gaussian random variables independent of each other with zero means and variances  $\sigma_1^2$  and  $\sigma_2^2$ , respectively. Using assumption (13a), considering Fano's inequality and following similar steps as in [6], it can be shown that the information rates  $R_1$  and  $R_2$  must satisfy the following inequalities

$$NR_1 \leq \sum_{n=1}^N [h(Y_{1,n}) - h(Z_{1,n})] + o(N), \quad (144a)$$

$$NR_1 \leq \sum_{n=1}^N [h(Y_{2,n}|X_{2,n}) - h(Z_{2,n}) + h(Y_{1,n}|X_{2,n}, S_{1,n}) - h(Z_{1,n})] + o(N), \quad (144b)$$

$$NR_2 \leq \sum_{n=1}^N [h(Y_{2,n}) - h(Z_{2,n})] + o(N), \quad (144c)$$

$$NR_2 \leq \sum_{n=1}^N [h(Y_{1,n}|X_{1,n}) - h(Z_{1,n}) + h(Y_{2,n}|X_{1,n}, S_{2,n}) - h(Z_{2,n})] + o(N), \quad (144d)$$

$$N(R_1 + R_2) \leq \sum_{n=1}^N [h(Y_{1,n}|S_{1,n}, X_{2,n}) - h(Z_{1,n}) + h(Y_{2,n}) - h(Z_{2,n})] + o(N), \quad (144e)$$

$$N(R_1 + R_2) \leq \sum_{n=1}^N [h(Y_{2,n}|S_{2,n}, X_{1,n}) - h(Z_{2,n}) + h(Y_{1,n}) - h(Z_{1,n})] + o(N). \quad (144f)$$

Using assumption (13b), for any  $\eta > 0$  there exists  $N_0(\eta)$  such that for any  $n \geq N_0(\eta)$  it holds that

$$\Pr (B^{(N)} < B) < \eta. \quad (145)$$

Equivalently,

$$\Pr (B^{(N)} \geq B) \geq 1 - \eta. \quad (146)$$



Using Markov's inequality, the probability in (146) can be upper-bounded as follows:

$$B \Pr(B^{(N)} \geq B) \leq \mathbb{E}[B^{(N)}]. \quad (147)$$

Combining (146) and (147) yields:

$$B(1 - \eta) \leq \mathbb{E}[B^{(N)}], \quad (148)$$

which can be written as

$$(B - \delta^{(N)}) \leq \mathbb{E}[B^{(N)}], \quad (149)$$

for some  $\delta^{(N)} > 0$  (for sufficiently large  $N$ ). The bounds in (144) and (149) are evaluated assuming that the channel inputs  $X_{1,n}$  and  $X_{2,n}$  are arbitrary correlated random variables with

$$\mu_{i,n} \triangleq \mathbb{E}[X_{i,n}], \quad (150)$$

$$\gamma_{i,n}^2 \triangleq \text{Var}[X_{i,n}], \quad (151)$$

$$\lambda_n \triangleq \text{Cov}[X_{1,n}X_{2,n}], \quad (152)$$

for all  $n \in \{1, \dots, N\}$  and for all  $i \in \{1, 2\}$ . The input sequence must satisfy the input power constraint which can be written, for  $i \in \{1, 2\}$ , as follows:

$$\frac{1}{N} \sum_{n=1}^N \mathbb{E}[X_{i,n}^2] = \left( \frac{1}{N} \sum_{n=1}^N \gamma_{i,n}^2 \right) + \left( \frac{1}{N} \sum_{n=1}^N \mu_{i,n}^2 \right) \leq P_i. \quad (153)$$

Using this assumptions, the following holds:

$$\begin{aligned} h(Y_{1,n}) &\leq \frac{1}{2} \log \left( 2\pi e \text{Var}[Y_{1,n}] \right) \\ &= \frac{1}{2} \log \left( 2\pi e (h_{1,1}^2 \gamma_{1,n}^2 + h_{1,2}^2 \gamma_{2,n}^2 + 2h_{1,1}h_{1,2}\lambda_n + \sigma_1^2) \right), \end{aligned} \quad (154a)$$

$$\begin{aligned} h(Y_{2,n}|X_{2,n}) &\leq \frac{1}{2} \log \left( 2\pi e \text{Var}[Y_{2,n}|X_{2,n}] \right) \\ &= \frac{1}{2} \log \left( 2\pi e \left( \gamma_{2,n}^2 + \frac{h_{2,1}^2}{\sigma_2^2} (\gamma_{1,n}^2 \gamma_{2,n}^2 - \lambda_n^2) \right) \right) - \frac{1}{2} \log(2\pi e \gamma_{2,n}^2) + \frac{1}{2} \log(2\pi e \sigma_2^2) \\ &= \frac{1}{2} \log \left( 1 + \gamma_{1,n}^2 \left( 1 - \frac{\lambda_n^2}{\gamma_{1,n}^2 \gamma_{2,n}^2} \right) \frac{h_{2,1}^2}{\sigma_2^2} \right) + \frac{1}{2} \log(2\pi e \sigma_2^2), \end{aligned} \quad (154b)$$

$$\begin{aligned} h(Y_{1,n}|X_{2,n}, S_{1,n}) &\leq \frac{1}{2} \log(2\pi e \text{Var}[Y_{1,n}|X_{2,n}, S_{1,n}]) \\ &\leq \frac{1}{2} \log \left( 1 + \frac{(\gamma_{1,n}^2 \gamma_{2,n}^2 - \lambda_n^2) \frac{h_{1,1}^2}{\sigma_1^2}}{\gamma_{2,n}^2 + (\gamma_{1,n}^2 \gamma_{2,n}^2 - \lambda_n^2) \frac{h_{2,1}^2}{\sigma_2^2}} \right) + \frac{1}{2} \log(2\pi e \sigma_1^2) \\ &= \frac{1}{2} \log \left( 1 + \frac{\gamma_{1,n}^2 \left( 1 - \frac{\lambda_n^2}{\gamma_{1,n}^2 \gamma_{2,n}^2} \right) \frac{h_{1,1}^2}{\sigma_1^2}}{1 + \gamma_{1,n}^2 \left( 1 - \frac{\lambda_n^2}{\gamma_{1,n}^2 \gamma_{2,n}^2} \right) \frac{h_{2,1}^2}{\sigma_2^2}} \right) + \frac{1}{2} \log(2\pi e \sigma_1^2). \end{aligned} \quad (154c)$$

Given  $(X_{2,n}, S_{1,n})$ , the variance of  $Y_{1,n}$  is upper-bounded by

$$\text{Var}[Y_{1,n}|X_{2,n}, S_{1,n}] \leq K_{Y_{1,n}} - K_{Y_{1,n}(X_{2,n}, S_{1,n})} K_{(X_{2,n}, S_{1,n})}^{-1} K_{Y_{1,n}(X_{2,n}, S_{1,n})}^\top, \quad (155)$$

where

$$\begin{aligned} K_{Y_{1,n}} &= \sigma_{1,n}^2 h_{1,1}^2 + \sigma_{2,n}^2 h_{1,2}^2 + 2\lambda_n h_{1,1} h_{1,2} + \sigma_1^2, \\ K_{Y_{1,n}(X_{2,n}, S_{1,n})} &= [\lambda_n h_{1,1} + h_{1,2} \gamma_{2,n}^2, h_{2,1} h_{1,1} \gamma_{1,n}^2 + \lambda_n h_{2,1} h_{1,2}], \text{ and} \\ K_{(X_{2,n}, S_{1,n})} &= \begin{bmatrix} \gamma_{2,n}^2 & \lambda_n h_{2,1} \\ \lambda_n h_{2,1} & \gamma_{1,n}^2 h_{2,1}^2 + 1 \end{bmatrix}, \end{aligned}$$

Finally, the bounds in (144) can be rewritten as follows:

$$NR_1 \leq \sum_{n=1}^N \frac{1}{2} \log \left( \frac{h_{1,1}^2 \gamma_{1,n}^2}{\sigma_1^2} + \frac{h_{1,2}^2 \gamma_{2,n}^2}{\sigma_1^2} + \frac{2h_{1,1} h_{1,2} \lambda_n}{\sigma_1^2} + 1 \right) + o(N), \quad (156a)$$

$$\begin{aligned} NR_1 &\leq \sum_{n=1}^N \frac{1}{2} \log \left( 1 + \gamma_{1,n}^2 \left( 1 - \frac{\lambda_n^2}{\gamma_{1,n}^2 \gamma_{2,n}^2} \right) \frac{h_{2,1}^2}{\sigma_2^2} \right) \\ &\quad + \sum_{n=1}^N \frac{1}{2} \log \left( 1 + \frac{\gamma_{1,n}^2 \left( 1 - \frac{\lambda_n^2}{\gamma_{1,n}^2 \gamma_{2,n}^2} \right) \frac{h_{1,1}^2}{\sigma_1^2}}{1 + \gamma_{1,n}^2 \left( 1 - \frac{\lambda_n^2}{\gamma_{1,n}^2 \gamma_{2,n}^2} \right) \frac{h_{2,1}^2}{\sigma_2^2}} \right) + o(N), \end{aligned} \quad (156b)$$

$$NR_2 \leq \sum_{n=1}^N \frac{1}{2} \log \left( \frac{h_{2,2}^2 \gamma_{2,n}^2}{\sigma_2^2} + \frac{h_{2,1}^2 \gamma_{1,n}^2}{\sigma_2^2} + \frac{2h_{2,2} h_{2,1} \lambda_n}{\sigma_2^2} + 1 \right) + o(N), \quad (156c)$$

$$\begin{aligned} NR_2 &\leq \sum_{n=1}^N \frac{1}{2} \log \left( 1 + \gamma_{2,n}^2 \left( 1 - \frac{\lambda_n^2}{\gamma_{1,n}^2 \gamma_{2,n}^2} \right) \frac{h_{2,1}^2}{\sigma_1^2} \right) \\ &\quad + \sum_{n=1}^N \frac{1}{2} \log \left( 1 + \frac{\gamma_{2,n}^2 \left( 1 - \frac{\lambda_n^2}{\gamma_{1,n}^2 \gamma_{2,n}^2} \right) \frac{h_{2,2}^2}{\sigma_2^2}}{1 + \gamma_{2,n}^2 \left( 1 - \frac{\lambda_n^2}{\gamma_{1,n}^2 \gamma_{2,n}^2} \right) \frac{h_{2,1}^2}{\sigma_1^2}} \right) + o(N), \end{aligned} \quad (156d)$$

$$\begin{aligned} N(R_1 + R_2) &\leq \sum_{n=1}^N \frac{1}{2} \log \left( \frac{h_{2,2}^2 \gamma_{2,n}^2}{\sigma_2^2} + \frac{h_{2,1}^2 \gamma_{1,n}^2}{\sigma_2^2} + \frac{2h_{2,2} h_{2,1} \lambda_n}{\sigma_2^2} + 1 \right) \\ &\quad + \sum_{n=1}^N \frac{1}{2} \log \left( 1 + \frac{\gamma_{1,n}^2 \left( 1 - \frac{\lambda_n^2}{\gamma_{1,n}^2 \gamma_{2,n}^2} \right) \frac{h_{1,1}^2}{\sigma_1^2}}{1 + \gamma_{1,n}^2 \left( 1 - \frac{\lambda_n^2}{\gamma_{1,n}^2 \gamma_{2,n}^2} \right) \frac{h_{2,1}^2}{\sigma_2^2}} \right) + o(N), \end{aligned} \quad (156e)$$

$$\begin{aligned} N(R_1 + R_2) &\leq \sum_{n=1}^N \frac{1}{2} \log \left( \frac{h_{1,1}^2 \gamma_{1,n}^2}{\sigma_1^2} + \frac{h_{1,2}^2 \gamma_{2,n}^2}{\sigma_1^2} + \frac{2h_{1,1} h_{1,2} \lambda_n}{\sigma_1^2} + 1 \right) \\ &\quad + \sum_{n=1}^N \frac{1}{2} \log \left( 1 + \frac{\gamma_{2,n}^2 \left( 1 - \frac{\lambda_n^2}{\gamma_{1,n}^2 \gamma_{2,n}^2} \right) \frac{h_{2,2}^2}{\sigma_2^2}}{1 + \gamma_{2,n}^2 \left( 1 - \frac{\lambda_n^2}{\gamma_{1,n}^2 \gamma_{2,n}^2} \right) \frac{h_{2,1}^2}{\sigma_1^2}} \right) + o(N). \end{aligned} \quad (156f)$$

The average received energy rate is given by

$$\begin{aligned} \mathbb{E} \left[ \frac{1}{N} \sum_{n=1}^N Y_{3,n}^2 \right] &= \sigma_3^2 + h_{3,1}^2 \left( \frac{1}{N} \sum_{n=1}^N (\gamma_{1,n}^2 + \mu_{1,n}^2) \right) + h_{3,2}^2 \left( \frac{1}{N} \sum_{n=1}^N (\gamma_{2,n}^2 + \mu_{2,n}^2) \right) \\ &\quad + 2h_{3,1} h_{3,2} \left( \frac{1}{N} \sum_{n=1}^N (\lambda_n + \mu_{1,n} \mu_{2,n}) \right). \end{aligned} \quad (157)$$

Using Cauchy-Schwarz inequality, the expected value on the energy rate in (157) can be upper-bounded as follows:

$$\begin{aligned} \mathbb{E} \left[ \frac{1}{N} \sum_{n=1}^N Y_{3,n}^2 \right] &\leq \sigma_3^2 + h_{3,1}^2 \left( \frac{1}{N} \sum_{n=1}^N (\gamma_{1,n}^2 + \mu_{1,n}^2) \right) + h_{3,2}^2 \left( \frac{1}{N} \sum_{n=1}^N (\gamma_{2,n}^2 + \mu_{2,n}^2) \right) \\ &\quad + 2h_{3,1}h_{3,2} \left( \left| \frac{1}{N} \sum_{n=1}^N \lambda_n \right| + \left( \frac{1}{N} \sum_{n=1}^N \mu_{1,n}^2 \right)^{1/2} \left( \frac{1}{N} \sum_{n=1}^N \mu_{2,n}^2 \right)^{1/2} \right). \end{aligned} \quad (158)$$

Combining (149) and (158) yields the following upper-bound on the energy rate  $B$ :

$$\begin{aligned} B &\leq \sigma_3^2 + h_{3,1}^2 \left( \frac{1}{N} \sum_{n=1}^N (\gamma_{1,n}^2 + \mu_{1,n}^2) \right) + h_{3,2}^2 \left( \frac{1}{N} \sum_{n=1}^N (\gamma_{2,n}^2 + \mu_{2,n}^2) \right) \\ &\quad + 2h_{3,1}h_{3,2} \left( \left| \frac{1}{N} \sum_{n=1}^N \lambda_n \right| + \left( \frac{1}{N} \sum_{n=1}^N \mu_{1,n}^2 \right)^{1/2} \left( \frac{1}{N} \sum_{n=1}^N \mu_{2,n}^2 \right)^{1/2} \right) + \delta_N. \end{aligned} \quad (159)$$

In order to obtain a single-letterization of the upper-bound given by constraints (156) and (159), define also

$$\mu_i^2 \triangleq \frac{1}{N} \sum_{n=1}^N \mu_{i,n}^2, \quad i \in \{1, 2\}, \quad (160)$$

$$\gamma_i^2 \triangleq \frac{1}{N} \sum_{n=1}^N \gamma_{i,n}^2, \quad i \in \{1, 2\}, \quad (161)$$

$$\rho \triangleq \frac{\left( \frac{1}{N} \sum_{n=1}^N \lambda_n \right)}{|\gamma_1| |\gamma_2|}. \quad (162)$$

With this notation, the input power constraint in (153) can be rewritten as

$$\gamma_i^2 + \mu_i^2 \leq P_i, \quad i \in \{1, 2\}. \quad (163)$$

Note that

$$\frac{1}{N} \sum_{n=1}^N \frac{1}{2} \log \left( 1 + \frac{\gamma_{i,n}^2 \left( 1 - \frac{\lambda_n^2}{\gamma_{i,n}^2 \gamma_{j,n}^2} \right) \frac{h_{i,i}^2}{\sigma_i^2}}{1 + \gamma_{i,n}^2 \left( 1 - \frac{\lambda_n^2}{\gamma_{i,n}^2 \gamma_{j,n}^2} \right) \frac{h_{j,i}^2}{\sigma_j^2}} \right) \leq \frac{1}{2} \log \left( 1 + \frac{\gamma_i^2 (1 - \rho^2) \frac{h_{i,i}^2}{\sigma_i^2}}{1 + \gamma_i^2 (1 - \rho^2) \frac{h_{j,i}^2}{\sigma_j^2}} \right). \quad (164)$$

The proof of (164) is obtained using the fact that for all  $(\alpha, \beta) \in \mathbb{R}_+^2$ ,  $\log \left( 1 + \frac{\alpha x}{1 + \beta x} \right)$  is a

concave function on  $x$ , then it follows that

$$\begin{aligned} & \frac{1}{N} \sum_{n=1}^N \frac{1}{2} \log \left( 1 + \frac{\gamma_{i,n}^2 \left( 1 - \frac{\lambda_n^2}{\gamma_{i,n}^2 \gamma_{j,n}^2} \right) \frac{h_{i,i}^2}{\sigma_i^2}}{1 + \gamma_{i,n}^2 \left( 1 - \frac{\lambda_n^2}{\gamma_{i,n}^2 \gamma_{j,n}^2} \right) \frac{h_{j,i}^2}{\sigma_j^2}} \right) \\ & \leq \frac{1}{2} \log \left( 1 + \frac{\frac{1}{N} \sum_{n=1}^N \gamma_{i,n}^2 \left( 1 - \frac{\lambda_n^2}{\gamma_{i,n}^2 \gamma_{j,n}^2} \right) \frac{h_{i,i}^2}{\sigma_i^2}}{1 + \frac{1}{N} \sum_{n=1}^N \gamma_{i,n}^2 \left( 1 - \frac{\lambda_n^2}{\gamma_{i,n}^2 \gamma_{j,n}^2} \right) \frac{h_{j,i}^2}{\sigma_j^2}} \right) \\ & \leq \frac{1}{2} \log \left( 1 + \frac{\frac{1}{N} \sum_{n=1}^N \gamma_{i,n}^2 \left( 1 - \frac{(\frac{1}{N} \sum_{n=1}^N \lambda_n)^2}{\frac{1}{N} \sum_{n=1}^N \gamma_{i,n}^2 \frac{1}{N} \sum_{n=1}^N \gamma_{j,n}^2} \right) \frac{h_{i,i}^2}{\sigma_i^2}}{1 + \frac{1}{N} \sum_{n=1}^N \gamma_{i,n}^2 \left( 1 - \frac{(\frac{1}{N} \sum_{n=1}^N \lambda_n)^2}{\frac{1}{N} \sum_{n=1}^N \gamma_{i,n}^2 \frac{1}{N} \sum_{n=1}^N \gamma_{j,n}^2} \right) \frac{h_{j,i}^2}{\sigma_j^2}} \right) \end{aligned} \quad (165)$$

$$= \frac{1}{2} \log \left( 1 + \frac{\gamma_i^2 (1 - \rho^2) \frac{h_{i,i}^2}{\sigma_i^2}}{1 + \gamma_i^2 (1 - \rho^2) \frac{h_{j,i}^2}{\sigma_j^2}} \right), \quad (166)$$

where (165) follows from the fact that  $n \in \{1, 2, \dots, N\}$ ,  $\sigma_{i,n}^2 \left( 1 - \frac{\lambda_n^2}{\gamma_{i,n}^2 \gamma_{j,n}^2} \right)$  is a concave function on  $\gamma_{1,n}^2$ ,  $\gamma_{2,n}^2$  and  $\lambda_n$ ; and for all  $(\alpha, \beta) \in \mathbb{R}_+^2$ ,  $\frac{\alpha x}{1 + \beta x}$  is an increasing function on  $x$ . By the concavity of the mutual information, applying Jensen's inequality and (164) in the bounds (156) yields, in the limit when  $N \rightarrow \infty$ ,

$$R_1 \leq \frac{1}{2} \log \left( \frac{h_{1,1}^2 \gamma_1^2}{\sigma_1^2} + \frac{h_{1,2}^2 \gamma_2^2}{\sigma_1^2} + 2\rho \sqrt{\frac{h_{1,1}^2 h_{1,2}^2 \gamma_1^2 \gamma_2^2}{\sigma_1^4}} \right), \quad (167a)$$

$$R_1 \leq \frac{1}{2} \log \left( 1 + \frac{h_{2,1}^2 \gamma_1^2 (1 - \rho^2)}{\sigma_2^2} \right) + \frac{1}{2} \log \left( 1 + \frac{\gamma_1^2 (1 - \rho^2) \frac{h_{1,1}^2}{\sigma_1^2}}{1 + \gamma_1^2 (1 - \rho^2) \frac{h_{2,1}^2}{\sigma_2^2}} \right), \quad (167b)$$

$$R_2 \leq \frac{1}{2} \log \left( \frac{h_{2,2}^2 \gamma_2^2}{\sigma_2^2} + \frac{h_{2,1}^2 \sigma_1^2}{\sigma_2^2} + 2\rho \sqrt{\frac{h_{2,2}^2 h_{2,1}^2 \gamma_1^2 \gamma_2^2}{\sigma_2^4}} \right), \quad (167c)$$

$$R_2 \leq \frac{1}{2} \log \left( 1 + \frac{h_{1,2}^2 \gamma_2^2 (1 - \rho^2)}{\sigma_1^2} \right) + \frac{1}{2} \log \left( 1 + \frac{\gamma_2^2 (1 - \rho^2) \frac{h_{2,2}^2}{\sigma_2^2}}{1 + \gamma_2^2 (1 - \rho^2) \frac{h_{1,2}^2}{\sigma_1^2}} \right), \quad (167d)$$

$$R_1 + R_2 \leq \frac{1}{2} \log \left( \frac{h_{2,2}^2 \gamma_2^2}{\sigma_2^2} + \frac{h_{2,1}^2 \gamma_1^2}{\sigma_2^2} + 2\rho \sqrt{\frac{h_{2,2}^2 h_{2,1}^2 \gamma_1^2 \gamma_2^2}{\sigma_2^4}} \right) + \frac{1}{2} \log \left( 1 + \frac{\gamma_1^2 (1 - \rho^2) \frac{h_{1,1}^2}{\sigma_1^2}}{1 + \gamma_1^2 (1 - \rho^2) \frac{h_{2,1}^2}{\sigma_2^2}} \right), \quad (167e)$$

$$R_1 + R_2 \leq \frac{1}{2} \log \left( \frac{h_{1,1}^2 \gamma_1^2}{\sigma_1^2} + \frac{h_{1,2}^2 \gamma_2^2}{\sigma_1^2} + 2\rho \sqrt{\frac{h_{1,1}^2 h_{1,2}^2 \gamma_1^2 \gamma_2^2}{\sigma_1^4}} \right) + \frac{1}{2} \log \left( 1 + \frac{\gamma_2^2 (1 - \rho^2) \frac{h_{2,2}^2}{\sigma_2^2}}{1 + \gamma_2^2 (1 - \rho^2) \frac{h_{1,2}^2}{\sigma_1^2}} \right), \quad (167f)$$

and the upper-bound on the energy rate (100) :

$$B \leq \sigma_3^2 + h_{3,1}^2 (\gamma_1^2 + \mu_1^2) + h_{3,2}^2 (\gamma_2^2 + \mu_2^2) + 2h_{2,1} h_{2,2} (|\rho| |\gamma_1| |\gamma_2| + |\mu_1| |\mu_2|). \quad (167g)$$

In the limit when  $N$  tends to infinity, any information-energy rate triplet such that  $(R_1, R_2, B) \in \mathcal{E}^F$  satisfies (167) for some  $\gamma_1, \gamma_2, \mu_1, \mu_2$  satisfying (163) and for some  $\rho \in [-1, 1]$ . Let

$\mathcal{R}^F(\gamma_1, \gamma_2, \mu_1, \mu_2, \rho)$  denote the set of information-energy rate triplets satisfying (167), for some  $\gamma_1, \gamma_2, \mu_1, \mu_2$  such that (163) is true and for some  $\rho \in [-1, 1]$ . Thus, it holds that

$$\mathcal{E}^F \subseteq \bigcup_{\substack{0 \leq \gamma_1^2 + \mu_1^2 \leq P_1 \\ 0 \leq \gamma_2^2 + \mu_2^2 \leq P_2 \\ -1 \leq \rho \leq 1}} \mathcal{R}^F(\gamma_1^2, \gamma_2^2, \mu_1, \mu_2, \rho). \quad (168)$$

In this union, it suffices to consider  $0 \leq \rho \leq 1$  because for any  $-1 \leq \rho \leq 1$ ,  $\mathcal{R}^F(\gamma_1^2, \gamma_2^2, \mu_1^2, \mu_2^2, \rho) \subseteq \mathcal{R}^F(\gamma_1^2, \gamma_2^2, \mu_1^2, \mu_2^2, |\rho|)$ . Thus,

$$\mathcal{E}^F \subseteq \bigcup_{\substack{0 \leq \gamma_1^2 + \mu_1^2 \leq P_1 \\ 0 \leq \gamma_2^2 + \mu_2^2 \leq P_2 \\ -1 \leq \rho \leq 1}} \mathcal{R}^F(\gamma_1^2, \gamma_2^2, \mu_1, \mu_2, \rho) \subseteq \bigcup_{\substack{\gamma_1^2 + \mu_1^2 = P_1 \\ \gamma_2^2 + \mu_2^2 = P_2 \\ 0 \leq \rho \leq 1}} \mathcal{R}^F(\gamma_1^2, \gamma_2^2, \mu_1, \mu_2, \rho).$$

Let  $\beta_i \in [0, 1]$  be defined as follows:

$$\beta_i \triangleq \frac{\gamma_i^2}{P_i} = \frac{P_i - \mu_i^2}{P_i}, \quad i \in \{1, 2\}. \quad (169)$$

Using (169), any region  $\mathcal{R}(\gamma_1, \gamma_2, \mu_1, \mu_2, \rho)$  in the union over all  $(\mu_1, \mu_2, \gamma_1, \gamma_2)$  that satisfy  $\gamma_1^2 + \mu_1^2 = P_1$ ,  $\gamma_2^2 + \mu_2^2 = P_2$  and  $0 \leq \rho \leq 1$ , can be rewritten as follows:

$$R_1 \leq \frac{1}{2} \log \left( \frac{h_{1,1}^2 \beta_1 P_1}{\sigma_1^2} + \frac{h_{1,2}^2 \beta_2 P_2}{\sigma_1^2} + 2\rho \sqrt{\frac{h_{1,1}^2 h_{1,2}^2 \beta_1 \beta_2 P_1 P_2}{\sigma_1^4}} \right), \quad (170a)$$

$$R_1 \leq \frac{1}{2} \log \left( 1 + \frac{h_{2,1}^2}{\sigma_2^2} \beta_1 P_1 (1 - \rho^2) \right) + \frac{1}{2} \log \left( 1 + \frac{\beta_1 P_1 (1 - \rho^2) \frac{h_{1,1}^2}{\sigma_1^2}}{1 + \beta_1 P_1 (1 - \rho^2) \frac{h_{2,1}^2}{\sigma_2^2}} \right), \quad (170b)$$

$$R_2 \leq \frac{1}{2} \log \left( \frac{h_{2,2}^2 \beta_2 P_2}{\sigma_2^2} + \frac{h_{2,1}^2 \beta_1 P_1}{\sigma_2^2} + 2\rho \sqrt{\frac{h_{2,2}^2 h_{2,1}^2 \beta_1 \beta_2 P_1 P_2}{\sigma_2^4}} \right), \quad (170c)$$

$$R_2 \leq \frac{1}{2} \log \left( 1 + \frac{h_{1,2}^2}{\sigma_1^2} \beta_2 P_2 (1 - \rho^2) \right) + \frac{1}{2} \log \left( 1 + \frac{\beta_2 P_2 (1 - \rho^2) \frac{h_{2,2}^2}{\sigma_2^2}}{1 + \beta_2 P_2 (1 - \rho^2) \frac{h_{1,2}^2}{\sigma_1^2}} \right), \quad (170d)$$

$$\begin{aligned} R_1 + R_2 &\leq \frac{1}{2} \log \left( \frac{h_{2,2}^2 \beta_2 P_2}{\sigma_2^2} + \frac{h_{2,1}^2 \beta_1 P_1}{\sigma_2^2} + 2\rho \sqrt{\frac{h_{2,2}^2 h_{2,1}^2 \beta_1 \beta_2 P_1 P_2}{\sigma_2^4}} \right) \\ &\quad + \frac{1}{2} \log \left( 1 + \frac{\beta_1 P_1 (1 - \rho^2) \frac{h_{1,1}^2}{\sigma_1^2}}{1 + \beta_1 P_1 (1 - \rho^2) \frac{h_{2,1}^2}{\sigma_2^2}} \right), \end{aligned} \quad (170e)$$

$$\begin{aligned} R_1 + R_2 &\leq \frac{1}{2} \log \left( \frac{h_{1,1}^2 \beta_1 P_1}{\sigma_1^2} + \frac{h_{1,2}^2 \beta_2 P_2}{\sigma_1^2} + 2\rho \sqrt{\frac{h_{1,1}^2 h_{1,2}^2 \beta_1 \beta_2 P_1 P_2}{\sigma_1^4}} \right) \\ &\quad + \frac{1}{2} \log \left( 1 + \frac{\beta_2 P_2 (1 - \rho^2) \frac{h_{2,2}^2}{\sigma_2^2}}{1 + \beta_2 P_2 (1 - \rho^2) \frac{h_{1,2}^2}{\sigma_1^2}} \right), \end{aligned} \quad (170f)$$

$$B \leq \sigma_3^2 + h_{3,1}^2 P_1 + h_{3,2}^2 P_2 + 2h_{3,1} h_{3,2} (|\rho| \sqrt{\beta_1 P_1 \beta_2 P_2} + \sqrt{(1 - \beta_1)(1 - \beta_2) P_1 P_2}), \quad (170g)$$

for some  $(\beta_1, \beta_2) \in [0, 1]^2$  and  $\rho \in [0, 1]$ . Hence, using the definitions in (7) and (10), the region (170) contains all information-energy rate triplets  $(R_1, R_2, B)$  satisfying constraints (18), which completes the proof of Theorem 5.

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