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Abstract. This work is devoted to the numerical analysis of saturated porous media, taking into account the damage phenomena on the solid skeleton. The porous media is taken into poro-elastic framework, in full-saturated condition, based on the Biot's Theory. A scalar damage model is assumed for this analysis. An implicit boundary element method (BEM) formulation, based on time-independent fundamental solutions, is developed and implemented to couple the fluid flow and two-dimensional elastostatic problems. The integration over boundary elements is evaluated by using a numerical Gauss procedure. A semi-analytical scheme for the case of triangular domain cells is followed to carry out the relevant domain integrals. The non-linear problem is solved by a Newton-Raphson procedure. Numerical examples are presented, in order to validate the implemented formulation and to illustrate its efficiency.

Introduction

The study of porous materials is extremely relevant in several areas of knowledge, such as soil and rock mechanics, contaminant diffusion, biomechanics and petroleum engineering. The mechanics of porous media deals with materials where the mechanical behavior is significantly influenced by the presence of fluid phases. The response of the material is highly dependent on the fluids that flow through the pores. Biot [1] was the first to propose a coupled theory for three-dimensional consolidation, based on the Terzaghi's studies on soil settlement [2]. This thermodynamically consistent theory is described in the book by Coussy [3], who improved significantly the knowledge on poromechanics. Cleary [4] presented the fundamental solutions to porous solids, representing the first contributions on integral equations dedicated to this kind of problems. Among others pioneers BEM works applied to porous media, the ones from Cheng and his collaborators [5-7] are well-known, using the direct BEM formulation.

In the field of material mechanics, we note the modelling of nonlinear physical processes, as damage and fracture. Processes of energy dissipation and consequent softening have been extensively studied, so that one can count on a wide range of models already developed. Continuum Damage Mechanics (CDM) deals with the load carrying capacity of solids whose material is damaged due to the presence of micro-cracks and micro-voids. CDM was originally conceived by Kachanov [8], to analyze uniaxial creeping of metals subjected to high-order temperatures. Several authors studied and developed models related to CDM. Lemaitre and colleagues [9-10] contributed significantly to the field. In this work, we use the model of Marigo [11], who presented a scalar isotropic model for brittle and quasi-brittle materials. The first applications of BEM to damage mechanics reported in the literature are Herding & Kuhn [12] and Garcia et al [13]. Recently, we can cite the works of Sladek et al. [14], Botta et al. [15] and Benallal et al. [16]. These works include non-local formulations to treat strain localization phenomena and associated numerical instabilities. Some aspects on the numerical analysis of porous media experiencing damage are found in Cheng & Dusseault [17] and Selvadurai [18].

Due to the increasing complexity of models developed for engineering problems, robust numerical models capable to provide accurate results with the least possible computational effort are looked for. In this scenario, BEM appears as an interesting choice for obtaining numerical solutions in various engineering applications.

In this paper, a non-linear set of transient BEM equations is developed, based on Betti's reciprocity theorem, to deals with isotropic-damaged porous media. The description of porous solid is done in a Lagrangean approach. Marigo's damage model is applied with a local evaluation of the thermodynamic force associated to damage.

Regarding the BEM numerical procedure, the integration over boundary elements is evaluated by using a numerical Gauss procedure. A semi-analytical scheme for the case of triangular domain cells is followed to carry

out the relevant domain integrals. A Newton-Raphson procedure is applied to solve the non-linear system, with a consistent tangent operator. This is done in the light of the procedure introduced by Simo and Taylor [19] for finite elements.

Governing Equations

The following free energy potential is considered,

$$\rho \psi(\varepsilon_{kj}, D, \phi - \phi_0) = \frac{1}{2}(1-D)\varepsilon_{kj} E^d_{kjlm} \varepsilon_{lm} + \frac{1}{2}b^2 M \left[Tr(\varepsilon_{kj}) \right]^2$$
$$+ \frac{1}{2} M (\phi - \phi_0)^2 - bM(\phi - \phi_0) Tr(\varepsilon_{kj})$$

(1)

where the constants M and b represent the Biot modulus and Biot coefficient of effective stress, respectively. In the case of saturated media, filled by an incompressible fluid, the Biot coefficient assumes unit value. In full-saturated conditions, the lagrangian porosity ϕ measures the variation of fluid content per unit volume of porous material. The bulk density is described by ρ. E^d_{jklm} represents the isotropic drained elastic tensor. ε_{jk} denotes the strains in the solid skeleton. Assuming isotropic case, the damage is represented by the scalar-valued internal variable D, which defines the internal state of the material, taking values between zero (sound material) and one (complete degradation). The initial porosity field is indicated by ϕ_0.

The derivatives of free energy potential with respect to the internal variables lead to the associate variables, that are the total stress σ_{jk}, the pore-pressure p and the thermodynamical force Y conjugated to D.

$$\sigma_{jk} = \rho \frac{\partial \psi}{\partial \varepsilon_{jk}} = (1-D)E^d_{jklm}\varepsilon_{lm} + bM \left[bTr(\varepsilon_{jk}) - (\phi - \phi_0) \right]\delta_{jk}$$

(2)

$$(p - p_0) = \rho \frac{\partial \psi}{\partial (\phi - \phi_0)} = M \left[(\phi - \phi_0) - bTr(\varepsilon_{jk}) \right]$$

(3)

$$Y = -\rho \frac{\partial \psi}{\partial D} = \frac{1}{2}\varepsilon_{jk} E^d_{jklm}\varepsilon_{lm}$$

(4)

Using equations (2) and (3) the total stress tensor is written as

$$\sigma_{jk} = E_{jklm}\varepsilon_{lm} - DE_{jklm}\varepsilon_{lm} - b(p - p_0)\delta_{jk}$$

(5)

from which it is seen that it includes three different contributions, being the first one the effective stress σ^{ef}_{jk}, acting on the grains of the solid matrix, and the second one the stress due to damage σ^d_{jk}.

In addition to the state laws given above, it is necessary to define a damage criterion. In Marigo's model it takes the form:

$$F(Y, D) = Y - \kappa(D)$$

(6)

The term $\kappa(D)$ represents the maximum value of Y reached during the loading history, and is adopted here in its simple linear form $\kappa(D) = Y_0 + AD$, where parameters Y_0 and A are material dependent. The damage evolution becomes from the consistency condition $\dot{F}(Y, D) = 0$, resulting in:

$$\dot{D} = \dot{Y}/A \tag{7}$$

The fluid flow through the porous space can be described by Darcy's law. Assuming a laminar flow, this law considers a linear relationship between the flow rate and the pressure gradient:

$$v_k = k\left[-p_{,k} + f_k\right] \tag{8}$$

In this simple version, it is assumed isotropic, with $k = \dfrac{k}{\mu}$ the scalar permeability coefficient, defined as a function of the intrinsic permeability k and the fluid viscosity μ. The fluid body force is represented by f_k. The fluid mass balance equation, assuming no external fluid sources, is written as:

$$\frac{d(\rho_f \phi)}{dt} + (\rho_f v_k)_{,k} = 0 \tag{9}$$

The following equilibrium and compatibility relations, added to appropriate boundary conditions complete the set of equations that describes the poro-elasto-damage problem, in quasi-static conditions:

$$\sigma_{jk,k} + b_j = 0 \tag{10}$$

$$\varepsilon_{jk} = \frac{1}{2}(u_{k,j} + u_{j,k}) \tag{11}$$

Integral Equations

In order to couple the behaviour of the solid and fluid phases, two sets of integral equations are derived. The first one is related to the elastostatics problem, for which a pore-pressure field is distributed over the domain, while the other equation refers to the pore-pressure itself.

In order to obtain the integral equations one can use Betti's reciprocity theorem, which can only be applied to elastic fields. Thus, in the case of elasticity, assuming the effective stress definition:

$$\int_\Omega \sigma_{jk}^{ef}(q)\varepsilon_{ijk}^*(s,q)d\Omega = \int_\Omega \varepsilon_{jk}(q)\sigma_{ijk}^*(s,q)d\Omega \tag{12}$$

$$\int_\Omega \left(\sigma_{jk}(q) + \sigma_{jk}^d(q) + b\delta_{jk}p(q)\right)\varepsilon_{ijk}^*(s,q)d\Omega = \int_\Omega \varepsilon_{jk}(q)\sigma_{ijk}^*(s,q)d\Omega \tag{13}$$

where s and q represent the source and field points, and X^* is the fundamental solution for the variable X, from now on. The direction i refers to the application of the unit load on the source point into the fundamental domain. In elastostatics, one applies the well-known Kelvin fundamental solutions. By applying the divergence theorem to equation (13), and considering the transient nature of the problem, one obtains the following integral equation for displacements on the boundary points S:

$$\begin{aligned}
C_{ik}\dot{u}_k(S) = &\int_\Gamma \dot{T}_k(Q)u_{ik}^*(S,Q)d\Gamma - \int_\Gamma T_{ik}^*(S,Q)\dot{u}_k(Q)d\Gamma \\
&+ \int_\Omega b\delta_{jk}\dot{p}(q)\varepsilon_{ijk}^*(S,q)d\Omega + \int_\Omega \dot{\sigma}_{jk}^d(q)\varepsilon_{ijk}^*(S,q)d\Omega
\end{aligned} \tag{14}$$

The stresses at internal points are obtained by differentiating equation (14), now written for internal points, and applying Hooke's law, which leads to

$$\dot{\sigma}_{ij}(s) = -\int_{\Gamma} S_{ijk}(s,Q)\dot{u}_k(Q)d\Gamma + \int_{\Gamma} D_{ijk}(s,Q)\dot{T}_k(Q)d\Gamma + \int_{\Omega} R_{ijkl}(s,q)\dot{\sigma}^d_{kl}(q)d\Omega$$
$$+TL_{ij}\left[\dot{\sigma}^d_{kl}(s)\right] + \int_{\Omega} b\delta_{kl}R_{ijkl}(s,q)\dot{p}(q)d\Omega + TL_{ij}\left[b\delta_{kl}\dot{p}(s)\right]$$

(15)

where S_{ijk}, D_{ijk} and R_{ijkl} are the derivatives of the fundamental solutions, and TL_{ij} are the free-terms coming from differentiation.

The integral equation for the pore-pressure can be obtained in a similar way, defining the proportional flow vector $v^{pr}_k = v_k - kf_k = -kp_{,k}$ in order to apply Betti's Theorem

$$\int_{\Omega}\left[v_k - kf_k\right]p^*_{,k}(s,q)d\Omega = \int_{\Omega} v^*_k(s,q)p_{,k}(q)d\Omega$$

(16)

from what the divergence theorem leads to write:

$$p(s) = -\int_{\Gamma} v^*_\eta(s,Q)p(Q)d\Gamma + \int_{\Gamma} p^*(s,Q)v_\eta(Q)d\Gamma - \int_{\Omega} p^*(s,q)v_{k,k}(q)d\Omega - \int_{\Omega} p^*_{,k}(s,q)kf_k(q)d\Omega$$

(17)

η indicates the outward normal direction to the boundary. Assuming $v_{k,k} = -\dot{\phi}$ (see (9)) and, neglecting the body force f_k, we get:

$$p(s) = -\int_{\Gamma} v^*_\eta(s,Q)p(Q)d\Gamma + \int_{\Gamma} p^*(s,Q)v_\eta(Q)d\Gamma + \int_{\Omega} p^*(s,q)\dot{\phi}(q)d\Omega$$

(18)

For convenience, it is possible to take the derivative $\dot{\phi}(q)$ from (3), so that the pore-pressure is given by the following equation:

$$p(s) = -\int_{\Gamma} v^*_\eta(s,Q)p(Q)d\Gamma + \int_{\Gamma} p^*(s,Q)v_\eta(Q)d\Gamma + \int_{\Omega} p^*(s,q)\left[\frac{1}{M}\dot{p}(q) + b\,\mathrm{Tr}\left(\dot{\varepsilon}(q)\right)\right]d\Omega$$

(19)

Considering a finite time step $\Delta t_n = t_{n+1} - t_n$ and a corresponding variable increment $\Delta X = X_{n+1} - X_n$, one can integrate equations (14), (15) and (19) along the interval Δt, leading to the following set of equations, in terms of the variable increments:

$$C_{ik}\Delta u_k(S) = \int_{\Gamma}\Delta T_k(Q)u^*_{ik}(S,Q)d\Gamma - \int_{\Gamma} T^*_{ik}(S,Q)\Delta u_k(Q)d\Gamma + \int_{\Omega} b\delta_{jk}\Delta p(q)\varepsilon^*_{ijk}(S,q)d\Omega$$
$$+\int_{\Omega}\Delta\sigma^d_{jk}(q)\varepsilon^*_{ijk}(S,q)d\Omega$$

(20)

$$\Delta\sigma_{ij}(s) = -\int_{\Gamma} S_{ijk}(s,Q)\Delta u_k(Q)d\Gamma + \int_{\Gamma} D_{ijk}(s,Q)\Delta T_k(Q)d\Gamma + \int_{\Omega} R_{ijkl}(s,q)\Delta\sigma^d_{kl}(q)d\Omega$$
$$+TL_{ij}\left[\Delta\sigma^d_{kl}(s)\right] + \int_{\Omega} b\delta_{kl}R_{ijkl}(s,q)\Delta p(q)d\Omega + TL_{ij}\left[b\delta_{kl}\Delta p(s)\right]$$

(21)

$$c(s)p(s) = -\int_{\Gamma} v_\eta^*(s,Q)p(Q)d\Gamma + \int_{\Gamma} p^*(s,Q)v_\eta(Q)d\Gamma$$

$$+ \frac{1}{\Delta t}\int_{\Omega}\frac{1}{M}p^*(s,q)\Delta p(q)d\Omega + \frac{1}{\Delta t}\int_{\Omega}bp^*(s,q)\mathrm{Tr}\big(\Delta\varepsilon(q)\big)d\Omega \tag{22}$$

Algebraic Equations and Solution Procedure

The numerical solution of the boundary value problem requires both the time and space discretizations. It should represent the system of equations in a discrete way along the linear boundary elements and into the triangular domain cells in order to obtain the approximate values of the variables of interest. One defines the number of boundary points by N_n and the number of internal nodes by N_i. The appropriate discretization of the integrals on (20)-(22), followed by some algebraic manipulations inherent to BEM, leads to the following system:

$$[H]\{\Delta u\} = [G]\{\Delta T\} + [Q]\{\Delta\sigma^d\} + b[Q][IK]\{\Delta p\} \tag{23}$$

$$\{\Delta\sigma\} = -[HL]\{\Delta u\} + [GL]\{\Delta T\} + [QL]\{\Delta\sigma^d\} + b[QL][IK]\{\Delta p\} \tag{24}$$

$$\{p_{(i)}\} = -[HP_{(i)}]\{p\} + [GP_{(i)}]\{V\} + \frac{1}{M\Delta t}[QP_{(i)}]\{\Delta p_{(i)}\} + \frac{b}{\Delta t}[QP_{(i)}][Tr]\{\Delta\varepsilon\} \tag{25}$$

The subscript (i) refers to internal points. The influence matrices represented by $[\ ]$ come from the integration of the fundamental solutions and its derivatives. The variables represented by $\{\ \}$ are prescribed or unknown variables along the boundary or over the domain. After some arrangements, the system given above is written as

$$[E]\{\Delta\varepsilon\} = \{\Delta Ns\} + \big[[QS]+[I]\big]\{\Delta\sigma^d\} + b\big[[QS]+[I]\big][IK]\{\Delta p_{(i)}\} \tag{26}$$

$$\Big[[I] - \frac{1}{M\Delta t}\big[\overline{QP}_{(i)}\big]\Big]\{\Delta p_{(i)}\} = \{\overline{Np}\} + \frac{b}{\Delta t}\big[\overline{QP}_{(i)}\big][Tr]\{\Delta\varepsilon\} \tag{27}$$

where $\{\Delta Ns\}$ and $\{\overline{Np}\}$ are vectors containing prescribed values and $[E]$ the drained elastic tensor. Finally, arranging the two equations in a single one, in terms of $\{\Delta\varepsilon\}$ only, leads to

$$[\overline{E}]\{\Delta\varepsilon\} = [\Delta Ns] + \{\overline{\overline{Np}}\} + [\overline{QS}]\{\Delta\sigma^d\} \tag{28}$$

which contains the new terms:

$$\{\overline{\overline{Np}}\} = b[\overline{QS}][IK]\Big[[I] - \frac{1}{M\Delta t}\big[\overline{QP}_{(i)}\big]\Big]^{-1}\{\overline{Np}\} \tag{29}$$

$$[\overline{E}] = \Big[[E] - \frac{b^2}{\Delta t}[\overline{QS}][IK]\Big[[I] - \frac{1}{M\Delta t}\big[\overline{QP}_{(i)}\big]\Big]^{-1}\big[\overline{QP}_{(i)}\big][Tr]\Big] \tag{30}$$

Due to the presence of correction terms associated with damage, equation (28) is non-linear at each time increment, and can be written:

$$\{Y(\{\Delta\varepsilon_n\})\} = -\left[\overline{E}\right]\{\Delta\varepsilon_n\} + [\Delta Ns] + \{\overline{\overline{Np}}\} + \left[\overline{QS}\right]\{\Delta\sigma^d_n\} = 0 \tag{31}$$

The solution is carried out by a Newton-Raphson's scheme. An iterative process is required to reach equilibrium. Then, from iteration i, the next try $i+1$ is given by $\{\Delta\varepsilon_n^{i+1}\} = \{\Delta\varepsilon_n^i\} + \{\delta\Delta\varepsilon_n^i\}$. The correction $\{\delta\Delta\varepsilon_n^i\}$ is calculated from the first term of the Taylor expansion, as follows:

$$\{Y(\{\Delta\varepsilon_n^i\})\} + \frac{\partial\{Y(\{\Delta\varepsilon_n^i\})\}}{\partial\{\Delta\varepsilon_n^i\}}\{\delta\Delta\varepsilon_n^i\} = 0 \tag{32}$$

where the derivative $\dfrac{\partial\{Y(\{\Delta\varepsilon_n^i\})\}}{\partial\{\Delta\varepsilon_n^i\}}$ is the consistent tangent operator.

Numerical Example

To illustrate the BEM formulation applied to poro-elastic media we first analyze the consolidation of a semi-infinite plane, under a strip uniform load of width $2a$. Due to the symmetry, we consider only the half-plane, as shown in Fig. 1. The load is applied instantaneously at $t = 0$. An analytical solution was proposed by Schiffman et al. [20], in terms of an adjusted time factor τ, and the dimensionless values of pore-pressure, total and effective stresses: $\tau = \dfrac{(1-\upsilon)}{(1-2\upsilon)}\dfrac{2Gk}{\gamma_f}t$, $\overline{p} = \dfrac{p(\tau)}{p(0)}$, $\overline{\sigma} = \dfrac{(\sigma_{11}(\tau)+\sigma_{22}(\tau))}{(\sigma_{11}(0)+\sigma_{22}(0))}$ and $\overline{\sigma^{ef}} = \dfrac{(\sigma_{11}^{ef}(\tau)+\sigma_{22}^{ef}(\tau))}{(\sigma_{11}^{ef}(\infty)+\sigma_{22}^{ef}(\infty))}$.
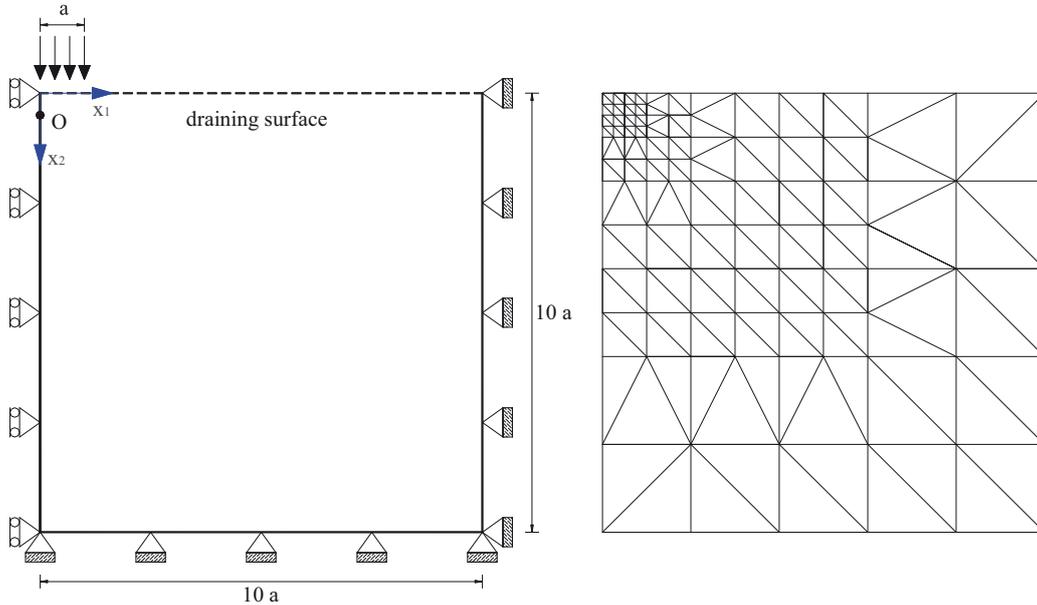

Figure 1. Problem definition and internal cells mesh adopted

From Fig. 2 one can observe the response on the point $O(0,a/2)$ (see Fig.1). The results obtained with the proposed model and the ones presented in [20] are compared. In the early-time response, just after the loading, fluid flow is slow, inducing the highest pore-pressures. One should note the occurrence of Mandel-Cryer effect, which is characterized in this plane strain consolidation by an increase in the pore-pressure at early times compared to the initial pore-pressure. With time, the drainage process leads to an increase in effective stress field, accompanied by a proportional pore-pressure reduction, until its vanishing.
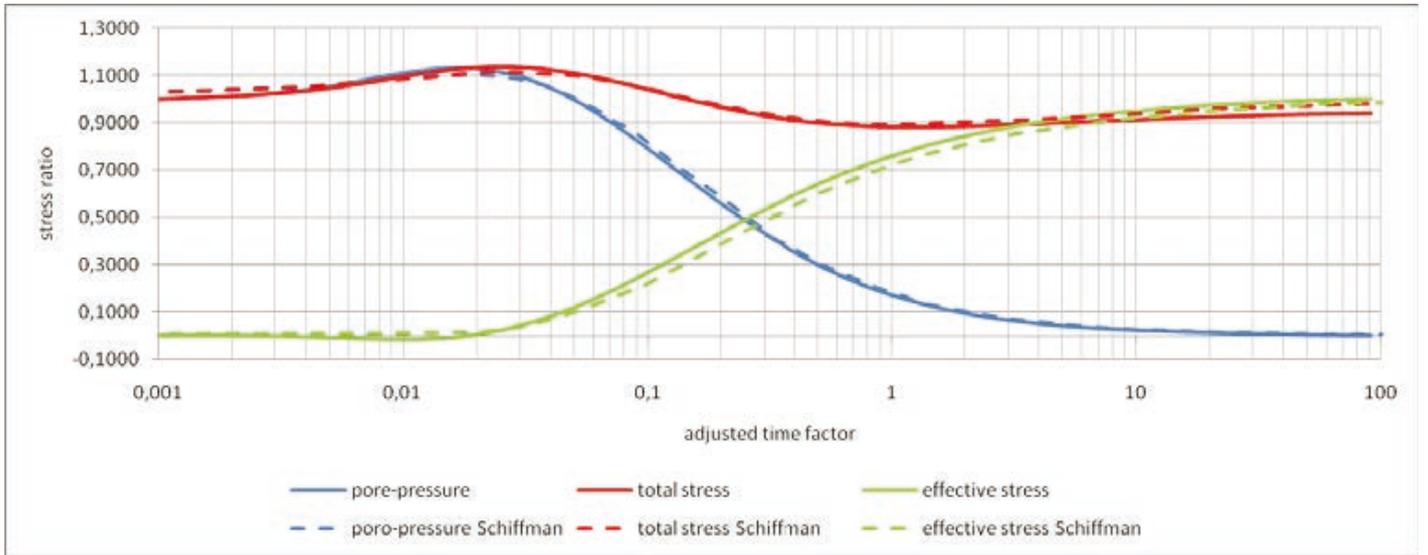
Figure 2. Normalized pore-pressures, total and effective stresses

Poro-elastic column subject to damage. Let us consider another consolidation case. It consists on a soil layer of thickness equal to 10m, resting on a rigid impermeable base. A constant unit load is progressively applied on the top surface of the layer over 1000s, under drained conditions (Fig. 3). The response is compared for the poro-elastic and elasto-damage behaviours, besides the coupled response. The results presented correspond to the bottom of the layer. The material parameters, assuming the layer made of Berea Sandstone, are defined as follows (Detournay & Cheng [21]): $E = 14400$ MPa, $\upsilon = 0.2$, $b = 0.79$, $M = 12250$ MPa, $k = 1.9 \times 10^{-13}$ m^2 and $\mu = 10^{-9}$ MPa·s. For the damage model, we adopt the parameters $Y_0 = 10^{-7}$ MPa and $A = 2 \times 10^{-5}$ MPa. The analysis involving damage are presented up to the limit load.
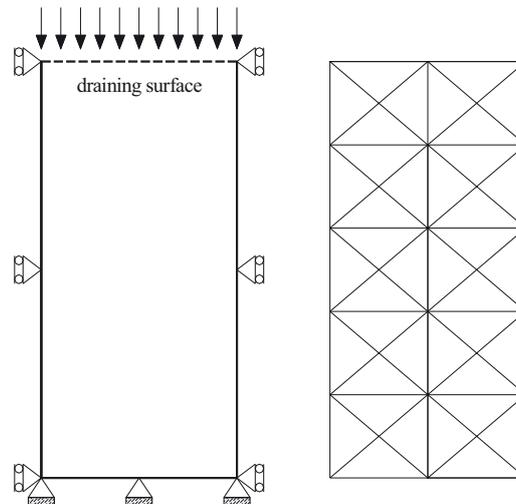

Figure 3. Problem definition and internal cells mesh adopted

The strain behaves in a similar way for the poro-elastic and the elastic materials, increasing almost linearly up to the total time (Fig. 4). The difference between the two curves results from the fluid phase flow. Taking into account the damage, the contribution of the fluid is also significant, leading to a delay in the damage process (Fig. 5b).
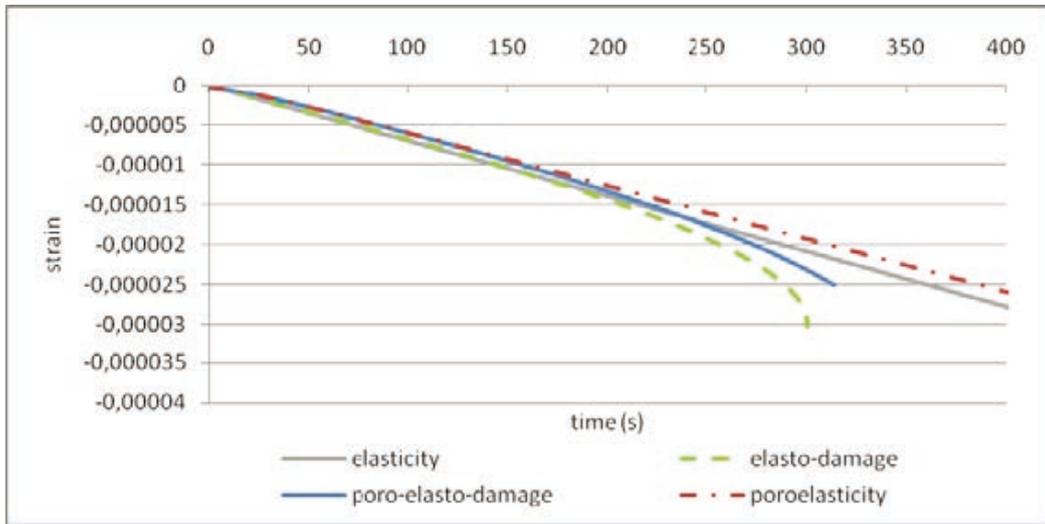
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Figure 4. Strain evolution for all the considered behaviors

In Fig. 5a, it can be observed that the elasto-damaged material has an intermediary behaviour between the porous media and the damaged porous media. In addition, it reaches the maximum load before the poro-elasto-damaged material, with a higher deterioration level (Fig. 5b).

It is interesting to note the augmentation of the pore-pressure in the presence of damage, beyond the threshold defined on the simple poro-elastic case.
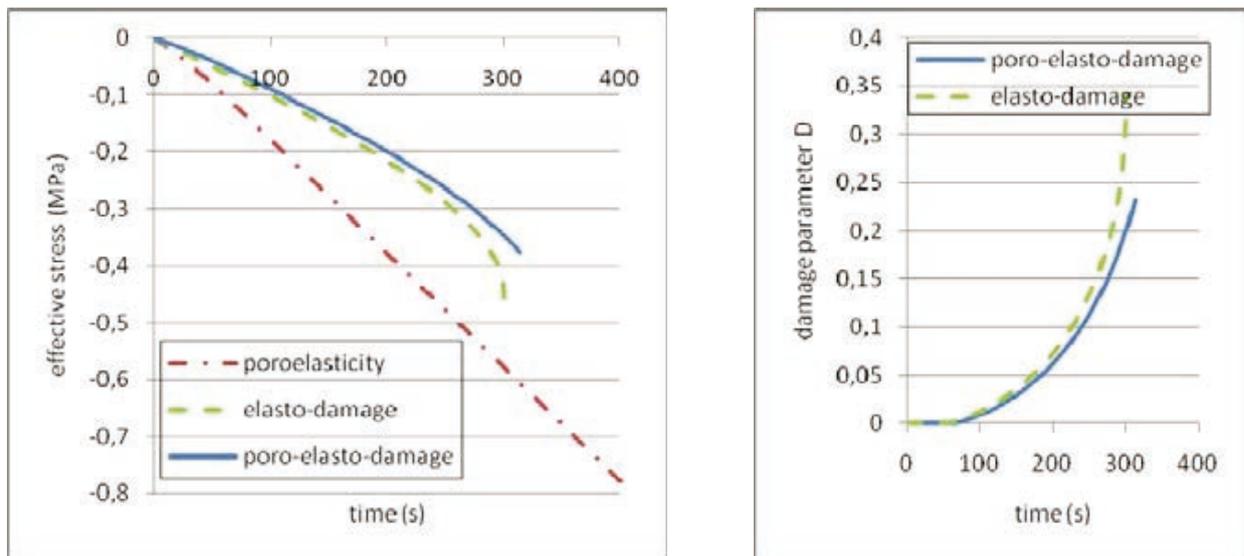

Figure 5. a) Effective stress field evolution b) Damage parameter evolution

On the numerical stability, the presented model shown to be almost independent of the time step adopted, having been tested values from 0,001 up to 10s, without any observable changes on the response.

It should be noted that, in the presence of damage, the response is represented only up to around 300s, which corresponds to the limit load as we have a load control. Besides, strain softening in the constitutive law causes localization phenomena, which leads to physically meaningless results and imposes difficulties on the numerical solution, requiring the use of regularization techniques.
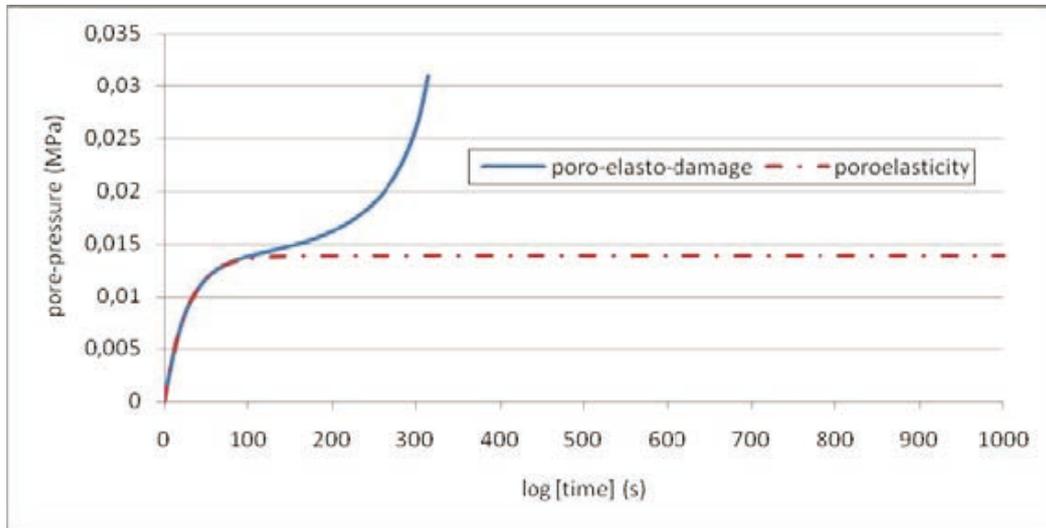
Figure 6. Pore-pressure evolution

Conclusions and Perspectives

A BEM formulation to poro-elasto-damaged material was presented. The model has shown a reasonable level of coupling between the damage and the fluid seepage. The literature, on theoretical and experimental levels, poses several interesting questions, among which the variations that the damage state imposes on the poro-elastic parameters. Some developments in this way are being made in the presented model, in order to improve the solid-fluid interaction.

Acknowledgements

References

[1]M.A.Biot General theory of three-dimensional consolidation *J. Appl. Physics* **12** 155-164 (1941).

[2]K.Terzaghi Die berechnung der burchlassigkeitsziffer des tones aus dem verlauf der hydrodynamischen spannungserscheinungen *Sitz. Akad. Wiss.* Abt. IIa **132** 125-138 (1923).

[3]O.Coussy *Poromechanics*, Chichester: John Wiley & Sons (2004).

[4]M.P.Cleary Fundamental solutions for a fluid-saturated porous solid *International Journal of Solids and Structures* **13** 785-806 (1977).

[5]A.H.D.Cheng and J.A.Liggett Boundary integral equation method for linear porous-elasticity with applications to fracture *Int. J. Numer. Meth. Eng.* **20** 279-296 (1984).

[6]A.H.D.Cheng and E.Detournay On singular integral equations and fundamental solutions of poroelasticity *International Journal of Solids and Structures* **35** 4521-4555 (1998).

[7]A.H.D.Cheng and M.Predeleanu Transient boundary element formulation for linear poroelasticity *International Journal of Applied Mathematical Modelling* **11** 285-290 (1987).

[8]L.M.Kachanov Time of rupture process under creep conditions *Izvestia Ak. Nauk* **8** 26-31 (1958).

[9]J.Lemaitre *A course on Damage Mechanics*, Berlin: Springer-Verlag (1992).

[10]J.Lemaitre and J.L.Chaboche *Mécanique des Matériaux Solides,* Paris: Dunod (1985).

[11]J.J.Marigo Formulation d'une loi d'endommagement d'un materiau élastique *Comptes rendus de l'académie des sciences* **292** série II 1309–1312 (1981).

[12]U.Herding and G.Kuhn A field boundary element formulation for damage mechanics *Engineering Analysis with Boundary Elements* **18** 137-147 (1996).

[13]R.Garcia, J.Florez-Lopez and M.Cerrolaza A boundary element formulation for a class of non-local damage models *International Journal of Solids and Structures* **36** 3617-3638 (1999).

[14]J.Sladek, V.Sladek and Z.P.Bazant Non-local boundary integral formulation for softening damage *International Journal for Numerical Methods in Engineering* **57** 103-116 (2003).

[15]A.S.Botta, W.S.Venturini and A.Benallal BEM applied to damage models emphasizing localization and associated regularization techniques *Eng. Anal. Bound. Elem.* **29** 814-827 (2005).

[16]A.Benallal, A.S.Botta and W.S.Venturini On the description of localization and failure phenomena by the boundary element method *Comp. Meth. Appl. Mech. Eng.* **195** 5833-5856 (2006)

[17]H.Cheng and M.B.Dusseault Deformation and diffusion behaviour in a solid experiencing damage: a continous damage model and its numerical implementation *International Journal of Rock Mechanics and Mining Science & Geomechanics Abstracts* **30** 1323-1331 (1993).

[18]A.P.S.Selvadurai On the mechanics of damage-susceptible poroelastic media *Key engineering Materials* **251-252** 363-374 (2003).

[19]J.C.Simo and R.L.Taylor Consistent tangent operators for rate-independent elastoplasticity *Computer Methods in Applied Mechanics and Engineering* **48** 101-118 (1985).

[20]R.L.Schiffman, A.T-F.Chen and J.C.Jordan An analysis of consolidation theories *Journal of the Soil Mechanics and Foundations Division* **SM 1** 285-312 (1969).

[21]E.Detournay and A.H.D.Cheng Fundamentals of poroelasticity *Comprehensive Rock Engineering: Principles, Practice and Projects* v. II, Great Britain: Pergamon Press (1993).