



**HAL**  
open science

# A multiscale computational strategy for assembly of structures in dynamics

David Odièvre, Pierre-Alain Boucard, Fabrice Gatuingt

► **To cite this version:**

David Odièvre, Pierre-Alain Boucard, Fabrice Gatuingt. A multiscale computational strategy for assembly of structures in dynamics. Computational Methods in Structural Dynamics and Earthquake Engineering -COMPADYN 2007, 2007, Rethymno, Greece. hal-01625209

**HAL Id: hal-01625209**

**<https://hal.science/hal-01625209>**

Submitted on 27 Oct 2017

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# A MULTISCALE COMPUTATIONAL STRATEGY FOR ASSEMBLY OF STRUCTURES IN DYNAMIC

D. ODIÈVRE, P.-A. BOUCARD AND F. GATUINGT

LMT-Cachan (ENS Cachan/CNRS/Université Paris 6/PRES UniverSud Paris), France

## Abstract

The aim of this work is to develop a multi-scale strategy for the simulation of structures assemblies subjected to transient dynamic loadings. The approach is based on the LATIN method with domain decomposition. The multi-scale LATIN method is a mixed method which deals at the same time with the forces and velocities at the interfaces of the different sub-domains. Already successfully applied to a variety of static problems, an extension of the method to dynamics is proposed.

We first show how to adapt the multi-scale strategy to dynamics and in particular how to construct a macroscopic problem in space. Second, a specific treatment of the interfaces which leads to the continuities of the forces and velocities is developed. At last, we illustrate the performances of the method on a 1D wave propagation problem.

## 1 INTRODUCTION

The modelisation/simulation takes an important place in the engineering and design departments. The issues are multiple, in particular for modelling large assemblies with connections in dynamics. However, these connections play a major role in the design process, because of strongly nonlinear local phenomena: contact, friction and plasticity. These phenomena are more important in fast transient dynamic problems which require very fine meshes to be correctly represented [1]. Therefore, the choice of an appropriate and efficient computational method is of vital importance.

The aim of the present work is to build a specialized method to answer the problems previously described due to the fact that the non-linearities are localised in the connections. Our approach is based on a decomposition of the assembly into sub-structures and interfaces. The problem is solved in each sub-structure by the finite element method and an iterative scheme based on the multi-scale LArge Time INcrement (LATIN) method developed at the LMT Cachan [2, 3] is used for the global resolution. This approach allows for a very important reduction of the computation costs in quasi-statics.

Among the methods usually used to deal with such problems in dynamics, one can quote the two-level FETI method (often qualified dual Schur method) [4], which is based on an inter-sub-structure field continuity enforced via Lagrange multipliers applied at the sub-structure interface. The multi-scale LATIN method is a mixed method which deals simultaneously with velocities and forces on the interfaces. Already largely developed in statics and quasi-statics, the objective of the work suggested here relates to its extension to dynamics.

We will show first how to adapt the multi-scale LATIN strategy to the dynamic case, in particular the construction of the "macroscopic" problem in space, which has in this case a less conventional interpretation

than in statics. In the second phase, the specific treatment of the interfaces ensuring the efforts and speeds continuities will be detailed. To finish, the performances of the method will be illustrated on a 1D wave propagation example.

## 2 THE MULTI-SCALE LATIN METHOD FOR DOMAIN DECOMPOSITION

Our multi-scale domain decomposition method is based on three ingredients: a space decomposition of the domain, a separation of the scales and an algorithm of resolution. These three ingredients are developed below.

### 2.1 Two scale domain decomposition problem

**- Decomposition into substructures and interfaces.** The basic idea is to describe the structure as an assembly of simple components, i.e. substructures and interfaces, each with their own variables and equations (admissibility, equilibrium and behavior, see Figure 1). Each substructure  $\Omega_E$  and  $\Omega_{E'}$  of  $\Omega$  are subjected through its boundary  $\partial\Omega_E$  and  $\partial\Omega_{E'}$  to the action of its environment (the neighboring interface  $\gamma_{EE'}$  for instance), defined by a velocity distribution ( $W_E, W_{E'}$ ) and a force distribution ( $F_E, F_{E'}$ ). So, in this description, the interfaces contain all the informations of the nonlinear behavior of the structure (contact, friction, ...). Then, the interface concept can be easily extended to the boundary, where either the displacements, the velocities or the forces are prescribed data of the problem.

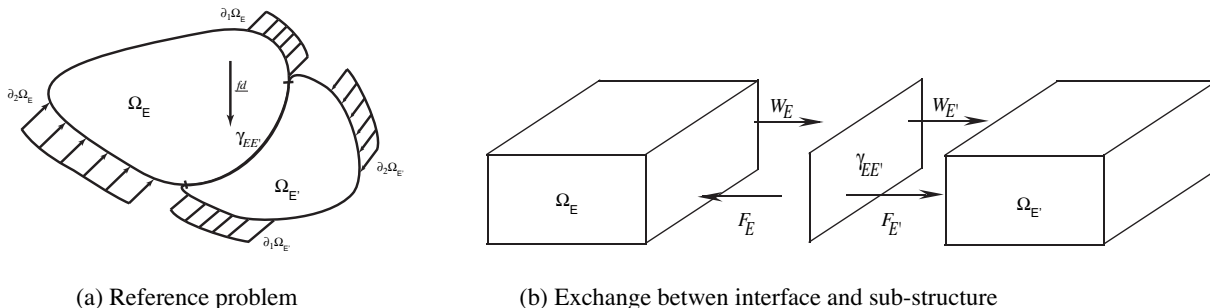


Figure 1: Decomposition into substructures and interfaces for a problem of reference.

**- Micro and macroscopic unknown separation.** In the micro/macro strategy used, the interfaces play a major role in the scale separation: the definition of the microscopic and macroscopic fields relates to the interfaces quantities of the sub-structured problem and is proposed before any discretization.

Let us consider the velocities and forces fields defined on an interface  $\gamma_{EE'}$ . We define then the macroscopic part of these fields,  $(F_E^M, F_{E'}^M, W_E^M, W_{E'}^M)$  as a projection of  $(F_E, F_{E'}, W_E, W_{E'})$ . The separation of the two scales is obtained by means of the projection operator  $\Pi_{\gamma_{EE'}}$ , defined for the interface  $\gamma_{EE'}$ . The choice of the macroscopic projector influences the efficiency of the algorithm. The choice of an optimal projector has been studied in [3]. It contains four basic functions per interface in 2D and the macroscopic kinematics which results from this choice consists of two translations, one rotation and one deformation.

## 2.2 Strategy of resolution: LATIN method

The LATIN method (Large Time INcrement method [5]) is a general, mechanics-based computational strategy for the resolution of time-dependent nonlinear problems, which operates over the entire time-space domain. It has been successfully applied to a variety of problems [6, 7, 8, 3].

- **Separation of the difficulties:** The method split the equations into two independent subspaces: the space  $Ad$  of the global linear equations (defined on the structure level) and the space  $\Gamma$  of the local nonlinear equations (defined on the point level). The solution of the problem  $s_{ex}$  is the intersection of these two subspaces  $s_{ex} = Ad \cap \Gamma$ . For a linear behavior, the spaces  $Ad$  and  $\Gamma$  are defined as:

$$s \in Ad \Leftrightarrow \forall \Omega_E \left\{ \begin{array}{l} \text{Kinematic admissibility} \\ \text{Dynamic admissibility} \\ \text{Elastic behavior for } \Omega_E \\ \text{Macroscopic variables admissibility: } (F^M, W^M) \end{array} \right. \quad (1)$$

$$s \in \Gamma \Leftrightarrow \forall \gamma_{EE'} \mid \text{Behavior for } \gamma_{EE'} \quad (2)$$

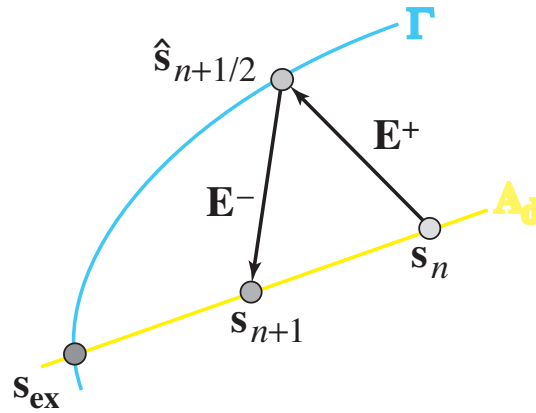


Figure 2: One iteration of the LATIN method.

- **A two steps iterative strategy:** As shown in Figure 2, the LATIN method consist in building fields of  $\Gamma$  and  $Ad$  alternatively giving search direction  $E^+$  et  $E^-$ , an iterative process which converges to the solution  $s_{ex}$ . One iteration involves two stages called local and linear:

- The local stage at iteration  $n + 1/2$

This stage consists in building  $\hat{s}_{n+1/2} \in \Gamma$  knowing  $s_n \in Ad$ . Then  $(\hat{s}_{n+1/2} - s_n)$  must follow the search direction  $E^+$  defined as follow:

$$(\hat{F}_E - F_E) - \mathbf{k}^m(\hat{W}_E - W_E) = 0 \quad (3)$$

For a perfect interface, the behavior of  $\gamma_{E'E}$  is:

$$\hat{F}_E + \hat{F}_{E'} = 0 \quad \hat{W}_E = \hat{W}_{E'} \quad (4)$$

The solution of these systems is:

$$\widehat{W}_E = \widehat{W}_{E'} = \frac{1}{2}(W_E + W_{E'}) - \frac{1}{2\mathbf{k}^m}(F_E + F_{E'}) \quad (5)$$

$$\widehat{F}_E = -\widehat{F}_{E'} = \frac{1}{2}(F_E - F_{E'}) - \frac{\mathbf{k}^m}{2}(W_E - W_{E'}) \quad (6)$$

where  $\mathbf{k}^m$  is a parameter of the method which can be interpreted as a "micro" rigidity of the interface. The local stage consists of solving local problems on the interface. The case of more complex interfaces (contact, friction, ...) has been developed in [3].

- The linear stage at iteration  $n + 1$

This stage consists in building  $s_{n+1} \in Ad$  knowing  $\widehat{s}_{n+1/2} \in \Gamma$ . Then  $(s_{n+1} - \widehat{s}_{n+1/2})$  must follow the search direction  $E^-$  defined as follow:

$$(F_E - \widehat{F}_E) + \mathbf{k}^m(W_E - \widehat{W}_E) = 0 \quad (7)$$

The equations associated to the sub-structures and the "micro" search direction lead to a formulation of independent problems per substructure, called "micro" problems. In order to ensure the admissibility conditions of the macro variables, we introduce a Lagrange multiplier  $\widetilde{W}_E^M$  on each interface. If we use a classical finite element discretization associated to an implicit time integration scheme, the linear system to be solved at each time step has the form:

$$Av_E(t + \Delta t) = [\widehat{F}_E + \mathbf{k}^m(\widehat{W}_E + \widetilde{W}_E^M)](t + \Delta t) + f(a_E(t), v_E(t), u_E(t)) \quad (8)$$

where  $u_E, v_E, a_E$  are respectively the displacement, velocity and acceleration fields for the sub-structure  $\Omega_E$ .  $A$  and  $f(\cdot)$  are linear combination of the mass and rigidity matrixes of the sub-structures and the mass matrix of the interfaces. We define the fields  $v_E^1$  et  $v_E^2$  with their restrictions on the interfaces  $W_E^1$  and  $W_E^2$  as :  $v_E = v_E^1 + v_E^2$ . The macroscopic parts  $W_E^{1,M}$  and  $W_E^{2,M}$  are then obtained using the projector  $\Pi_{\gamma_{EE'}}$ . Then, we can also calculate the macro part  $F_E^{1,M}$  and  $F_E^{2,M}$  using the search direction (equation 7) and  $F_E = F_E^{1,M} + F_E^{2,M}$ . These fields are solution of the following equations:

$$Av_E^1(t + \Delta t) = [\widehat{F}_E + \mathbf{k}^m\widehat{W}_E](t + \Delta t) + f(a_E(t), v_E(t), u_E(t)) \quad (9)$$

$$Av_E^2(t + \Delta t) = \mathbf{k}^m\widetilde{W}_E^M(t + \Delta t) \quad (10)$$

This problem has a unique solution. We can then write:

$$W_E^{2,M} = \mathbf{L}_E^{-1}\widetilde{W}_E^M \quad (11)$$

$\mathbf{L}_E$  represents an homogeneous behavior operator for the sub-structure  $\Omega_E$ . It is compute once at the beginning of the algorithm for all the sub-structures.

Using the admissibility of the macro quantities (corresponding to the continuity of the velocities and to the equilibrium of the macroscopic efforts) for all the perfect interfaces and equation 11 for all the sub-structures we obtain the definition of the "macro" problem (written here for the interface  $\gamma_{EE'}$ ):

$$\begin{bmatrix} \mathbf{L}_E & -\mathbf{L}_{E'} \\ \mathbf{k}^m(1 + \mathbf{L}_E) & \mathbf{k}^m(1 + \mathbf{L}_{E'}) \end{bmatrix} \begin{bmatrix} \widetilde{W}_E^M \\ \widetilde{W}_{E'}^M \end{bmatrix} = \begin{bmatrix} -W_E^{1,M} + W_{E'}^{1,M} \\ -F_E^{1,M} - F_{E'}^{1,M} \end{bmatrix} \quad (12)$$

This problem couple all the macro variables of the structure and define the Lagrange multiplier  $\tilde{W}_E^M$  for all the sub-structures.

In order to improve the strategy, we also tested an explicit time integration scheme. In this case, equation 8 becomes:

$$Bv_E(t + \Delta t) = [\hat{F}_E + \mathbf{k}^m(\hat{W}_E + \tilde{W}_E^M)](t + \Delta t) + g(a_E(t), v_E(t), u_E(t)) \quad (13)$$

where  $g(\cdot)$  is always a linear combination of the mass and rigidity matrixes of the sub-structures and the mass matrix of the interfaces but where  $B$  only depends on the lumped mass matrixes. The same decomposition presented before can then be applied. We thus recover all the numerical advantages of an explicit method.

## 2.3 Algorithm

The LATIN method consists in processing linear and local stages alternatively. In algorithm 1, we show the principal points of an iteration of the multi-scale method.

---

**Algorithm 1:** micro/macro LATIN (velocity approach)

---

### - Linear stage

- Loop on the sub-structures:  
Micro problem: Determination of  $(v_E^1, W_E^1)$  knowing  $(\hat{W}_E, \hat{F}_E)$  (eq. 9). Compute  $W_E^{1,M} = \Pi_{\gamma_{EE'}} W_E^1$  then  $F_E^{1,M}$  using the search direction and admissibility conditions of macro quantities.
- Macro problem:  
Determination of  $\tilde{W}_E^M$  knowing  $W_E^{1,M}$  and  $F_E^{1,M}$  (eq. 12)
- Loop on the sub-structures:  
Micro problem: Determination of  $(v_E^2, W_E^2)$  knowing  $\tilde{W}_E^M$  (eq. 10)

Computation of  $v_E = v_E^1 + v_E^2$

### - Local stage

- Loop on the interfaces:  
Determination of  $(\hat{W}_E, \hat{F}_E)$  knowing  $(W_E, F_E)$  (eq. 5 and eq. 6)

**Iterate until convergence**

---

## 3 APPLICATION TO A 1D PROBLEM

In order to test the method previously developed, we implemented it in Matlab for a 1D case. The test deal with the propagation of a compressive wave in a bar. The structure is divided in five sub-structures with perfect interfaces (see Figure 3). The bar has a Young modulus of 200 *GPa*, mass density of 7800 *kg/m<sup>3</sup>* and one meter length. The input velocity has a "period"  $T$  varying from 20 to 200  $\mu s$  which lead to a frequency of the signal ( $f = 1/T$ ) varying from 500 to 50000 *Hz* with  $V_{max} = 1m/s$ .

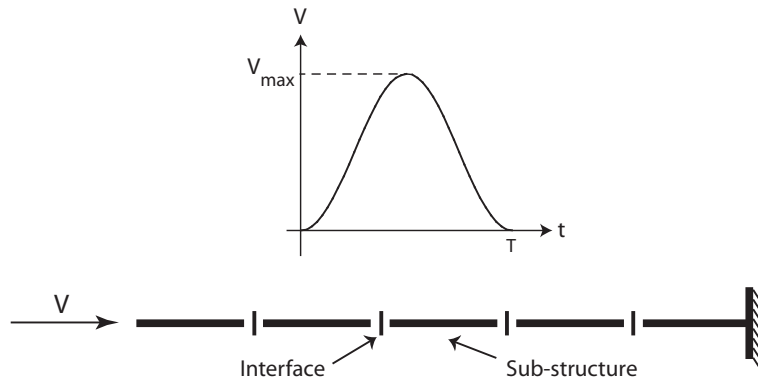


Figure 3: Problem description.

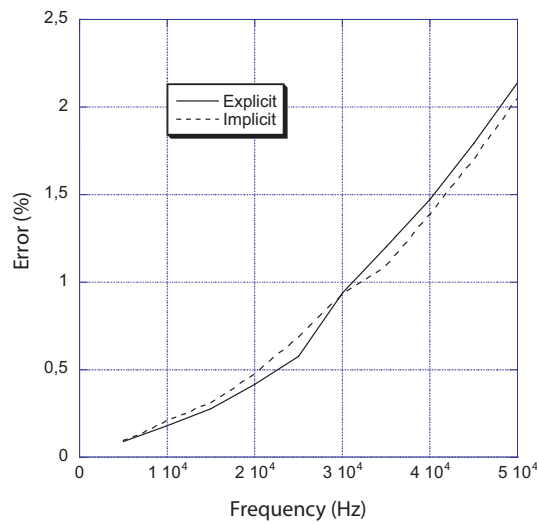


Figure 4: Error of the finite element/exact solution vs the frequency of the input velocity.

The objective of the test is to compare the efficiency of the method for an explicit time integration scheme for the multi-scale LATIN method. To do this, we propose first to adopt a finite element discretization for the explicit and implicit cases which leads to the same error compared to the exact solution of the problem (see Figure 4). For the time step, we use for the two integration schemes the Courant's condition to compute it. We obtain then 18 finite element nodes per period of the input velocity in explicit and 24 in implicit. We then keep these discretizations for the all the comparisons.

Figure 5 shows a classical result: the real profit of using a explicit time integration scheme at the same time for the multi-scale and the mono-scale strategies.

At last, if we use an explicit scheme, Figure 6 shows the advantage of the multi-scale approach in a case of a wave propagation. This is due to the fact that for the mono-scale strategy, the information propagates only in one sub-structure per iteration. On the contrary, for the multi-scale approach, the information propagates to the entire structure at the first iteration with the help of the macroscopic problem.

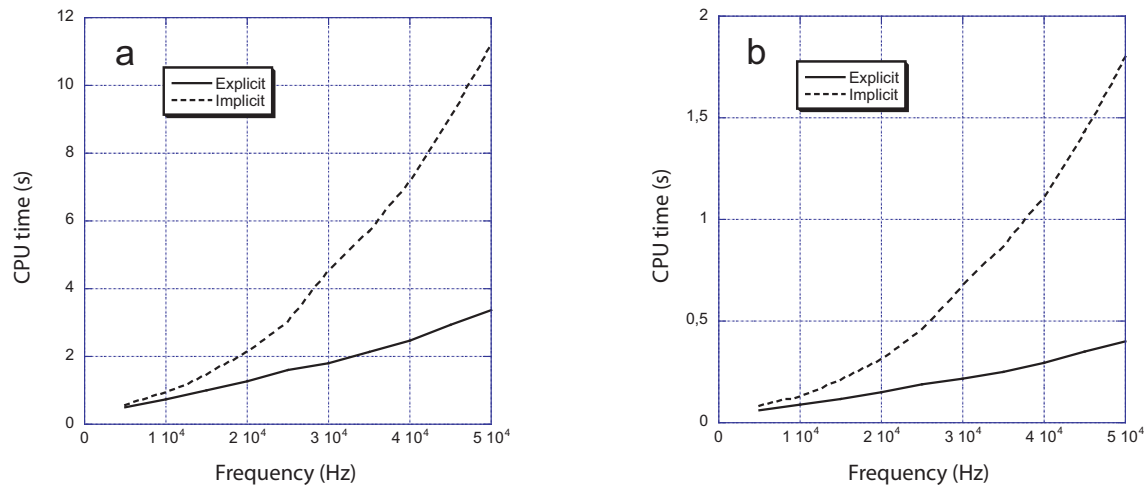


Figure 5: Comparison of the time integration efficiency for: a/ mono-scale LATIN, b/ multi-scale LATIN

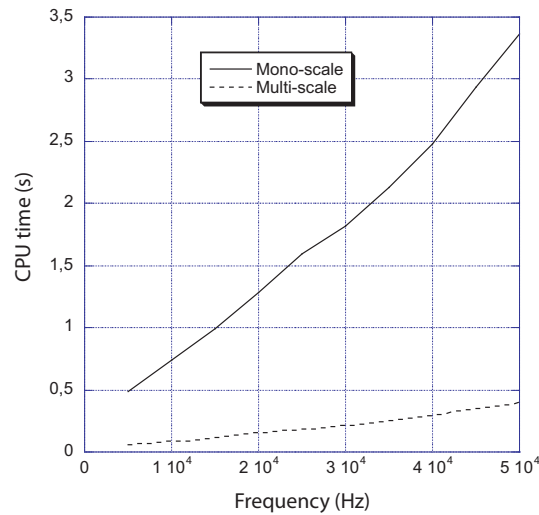


Figure 6: Comparison of the LATIN method used for explicit time integration

## 4 CONCLUSIONS

The extension to dynamics of the multi-scale LATIN method studied in this work allows to obtain a real advantage by using it compared to the mono-scale method as it was already proved in statics. Associated with an explicit description of the time integration scheme the algorithm achieved is very performant (to be confirmed in 3D).

A next step is to use the specific treatment of the equations in the case of explicit time integration to extend this work to non-linear sub-structures. Indeed, in this case we will not have to compute the rigidity matrix of the sub-structures at each time step, a feature which will allow for an efficient algorithm.



## References

- [1] J.O. Hallquist, G.L. Goudreau and D.J. Benson, *Sliding interfaces with contact-impact in large-scale lagrangien computations*, Computer Methods in Applied Mechanics and Engineering, **51**, 107–137, 1985.
- [2] P. Ladevèze, D. Dureisseix, *A micro/macro approach for parallel computing of heterogeneous structures*, Int Jal for Computational Civil and Structural Engineering, **1**, 18–28, 2000.
- [3] P. Ladevèze, A. Nouy, O. Loiseau, *A multiscale computational approach for contact problems*, Computer Methods in Applied Mechanics and Engineering Software, **191**, 4869–4891, 2002.
- [4] C. Farhat, J. Mandel, *The two-level FETI method for static and dynamic plate problems*, Computer Methods in Applied Mechanics and Engineering, **555**, 129–151, 1998.
- [5] P. Ladevèze, *Nonlinear computational structural mechanics – New approaches and non-incremental methods of calculation*, Springer-Verlag, 1999.
- [6] H. Lemoussu, P.-A. Boucard, P. Ladevèze, *A 3D shock computational strategy for real assembly and shock attenuator*, Adv. Engrg. Software, **33**, 517–526, 2002.
- [7] P.-A. Boucard, P. Ladevèze, H. Lemoussu, *A modular approach to 3D impact computation with frictional contact*, Computer & Structures, **78**, 45–52, 2000.
- [8] P. Ladevèze, A. Nouy, *On a multiscale computational strategy with time and space homogenization for structural mechanics*, Computer Methods in Applied Mechanics and Engineering, **192**, 3061–3087, 2003.