Control by Interconnection and Energy-Shaping Methods of Port Hamiltonian Models. Application to the Shallow Water Equations

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Abstract— **In this paper a control algorithm for the reduced port-Controlled Hamiltonian model (PCH) of the shallow water equations (PDEs) is developed. This control is developed using the Interconnection and Damping Assignment Passivity Based Control (IDA-PBC) method on the reduced PCH model without the natural dissipation. It allows to assign desired structure and energy function to the closed loop system. The same control law is then derived using an energy shaping method based on Casimir's invariants, associated with a particular conservative interconnection between the boundary variables. This gives a physical interpretation for the designed controller. Finally, a stability analysis of the dissipative system in closed loop with the designed control is done using LaSalle's invariance principle. Simulation results and an experimental validation of the control algorithm on an a micro-canal platform are presented showing the effectiveness of the control law.**

I. INTRODUCTION

The port Hamiltonian formulation of distributed parameter is based on a generalization of the finite dimensional Dirac structure used to model power conserving interconnections [4]. This structure, in the case of distributed parameter systems, is called a Stokes-Dirac structure. It expresses the coupling between physical domains of the systems, like potential and kinetic energies in mechanical systems or electric and magnetic energies in electromagnetic systems modelled by Maxwell's equations (see [15] for a detailed presentation).

A port Hamiltonian formulation of the shallow water equations (called also Saint-Venant equations, see [1]) has been proposed in [2] considering infinitisimal volumes and momentum densities as state variables and the sum of global kinetic and potential energies as the total Hamiltonian of the system. This formulation leads to define the boundary fluid flows and hydrodynamic pressures as the natural interconnection boundary variables for the energy exchanges with the external environment [15], [20]. This model has the advantage to be stated independently of the specific geometry of the reaches, boundary conditions may be chosen in a-causal way between the natural boundary variables of the system and some of its dynamic properties are trivial to establish (stability, passivity, stored energy, dissipation map or entropy production).

B. Hamroun and E. Mendes are with the Laboratoire de Conception et d'Intégration des Systèmes (LCIS), Grenoble Institute of technology, France boussad.hamroun@lcis.grenoble-inp.fr ; eduardo.mendes@lcis.grenoble-inp.fr

L. Lefèvre is with the Laboratoire d'Automatique et de Génie des Procédés (LAGEP UMR 5007), Université Lyon 1 ,Villeurbanne, France lefevre@lagep.univ-lyon1.fr

We proposed in [3] a reduced port-controlled Hamiltonian model for the shallow water equations which is obtained using a geometric reduction scheme based on a mixed finite elements method [5], [6]. This reduction scheme preserves both the interconnection structure and the energetic properties of the original equations. The obtained reduced model also exhibits some interesting spectral properties. Namely the finite spectrum of the linearized reduced model converges to the infinite-dimensional spectrum of the shallow water equations [3]. The convergence of the imaginary part is similar to the one observed for other traditional reference numerical schemes for such systems (orthogonal collocation or Preissmann implicit finite differences schemes) but the real part of the spectrum of the reduced PCH system is the theoretical one. The proposed reduction scheme does not add any supplementary numerical dissipation since it preserves the power conserving interconnection structure [3].

Many control algorithms for fluid flow through open-air channels have been developed. Most of them are based on reduced models of the shallow water equations. Some works are based on continuous time reduced model obtained by the orthogonal collocation method as in [7] (input-output linearization), in [8] (backstepping) or in [9] (robust optimal control). Other are developed on discrete time models obtained using the Preissmann implicit finite differences scheme [10] as in [11] (predictive control) or in [12] (optimal control).

In this paper we intend to use the structured PCH form of the reduced model to design a control law which makes the closed loop system passive with respect to desired storage function. To achieve this result we use the interconnection and damping assignment passivity based control (IDA-PBC) developed in [18] which also allows us to assign prescribed interconnection and damping structures to the closed loop. We claim that in our application case this technique provides a robust control law based on the model structure and not on some particular parameters values. The IDA-PBC is applied to the reduced PCH system without the natural dissipation. This allows to shape simultaneously the kinetic and potential energies. We then show how to design the same controller using the Casimir's invariant method associated with a specific conservative boundary interconnection. This second method allows us to give a physical interpretation of the control action from an interconnection point of view. The regulation problem addressed in this paper is to achieve a desired water flow and a *uniform* water level on the reach, i.e. an equal fluid volume in each cell. This is the case when the reach is assumed to provide some defined demand while ensuring a safe operating of the hydraulic works. The natural dissipation is firstly not considered in the control design. As pointed out in [16], if it is not taken into account this may result in a destabilizing effect on the closed loop dynamic due to over compensation. Using the LaSalle's invariance principle, we give a stability analysis of the closed loop in presence of this natural dissipation. We show by this stability analysis that the desired fluid flow is achieved even in the presence of natural dissipation and that the corresponding volume or kinetic momentum could be given by solving a static equation.

The paper is organized as follows; in section II we recall the port Hamiltonian formulation for shallow water equations, in section III we recall also the reduced port Hamiltonian model already developed in [3]. In section IV we design a control law for the regulation problem given above using the interconnection and damping assignment passivity based control (IDA-PBC) method. In section V we design the same control law using the Casimir's invariants based energy shaping method associated with a conservative boundary operator. In section VIII we define an integral action which allows to conserve the Hamiltonian form of the closed loop system and to cancel the steady errors due to some empirical parameter errors (Manning-Strickler parameter and gates parameters). In section VI we take into account the natural dissipation of the system and we analyse its stability using LaSalle's invariance principle. The control law designed in this paper depends on the mass center of the system, this mass center have to be estimated. In section VII we discuss the robustness of the control towards the mass center estimation error. In section IX we give some simulation results and an experimental validation of the control law on a micro-canal platform available in the Laboratory. Finally, we give some conclusion remarks and some perspectives.

II. PORT HAMILTONIAN FORMULATION FOR SHALLOW WATER EQUATIONS

We consider the rectangular open channel of fig. (1) with a single reach of slope I, with length L and width B. It is delimited by upstream and downstream gates and terminated by an hydraulic outfall. This reach configuration is one of the most common ones in open-air irrigation channels. The flow dynamic within the reach is modelled by the well known shallow water equations.

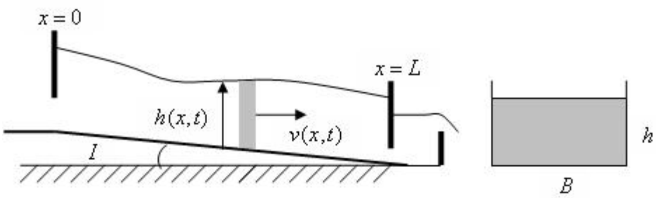

Fig. 1. Longitudinal(left) and lateral (right) sights of an open rectangular hydraulic channel

Its port-based model has been developed in [2] and is here only briefly recalled. By choosing the infinitesimal volume ($q(x,t)$) and kinetic momentum density ($p(x,t)$) as energy (state) variables along the spatial domain $Z = [0, L]$, we can write the port Hamiltonian formulation of the shallow water equations as follows:

$$q(x,t) = Bh(x,t)dx, \;\; p(x,t) = \rho v(x,t)dx \tag{1}$$

$$\begin{bmatrix} -\frac{\partial q}{\partial t} \\ -\frac{\partial p}{\partial t} \end{bmatrix} = \begin{bmatrix} 0 & d \\ d & 0 \end{bmatrix} \begin{bmatrix} \delta_q H \\ \delta_p H \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & G(q,p) \end{bmatrix} \begin{bmatrix} \delta_q H \\ \delta_p H \end{bmatrix} \tag{2}$$

$$\begin{array}{rclc} e_\partial^0(t) & = & -\delta_q H_{|x=0} & f_\partial^0(t) = \delta_p H_{|x=0} \\ e_\partial^L(t) & = & -\delta_q H_{|x=L} & f_\partial^L(t) = \delta_p H_{|x=L} \end{array} \tag{3}$$

where d is the exterior derivative which maps k-differential forms on $(k+1)$-differential forms and where H denotes the total energy of the fluid. From the kinetic and potential energies, computed on an "elementary" length of reach, it is easy to obtain the total energy:

$$H(h,v,x) = \frac{1}{2}\int_0^L (\rho g B h^2 - 2\rho B I h g x + \rho B h v^2)dx \tag{4}$$

The effort variables (thermodynamics forces) are derived from the energy expression as the variational derivatives:

$$\begin{array}{rcl} e_q(x,t) & = & \delta_q \mathcal{H} = \frac{1}{2}\rho v^2(x,t) + \rho g(h(x,t) - Ix) \\ e_p(x,t) & = & \delta_p \mathcal{H} = Bh(x,t)v(x,t) \end{array} \tag{5}$$

These efforts are functions on the spatial domains (also called 0-differential forms). The first effort ($e_q(x,t)$) is generally called the hydrodynamic pressure and the second ($e_p(x,t)$) represents the water flow in the channel. In (2), $G(q,p)$ is the momentum dissipated by friction forces. It is usually modelled by the empirical nonlinear Manning strikler constitutive formula:

$$G_d = \frac{\rho g |v|}{K^2 Bh(\frac{Bh}{B+2h})^{\frac{4}{3}}} dx \tag{6}$$

The dynamical system (2) admits an infinity of uniform (constant) water flow equilibrium profiles and spatially varying equilibrium water levels profiles. For a constant equilibrium water flow we can obtain uniform, accumulation or drying equilibrium profiles. The uniform equilibrium profile is obtained when the friction forces equal the gravity ones.

III. A REDUCED PORT HAMILTONIAN MODEL FOR THE SHALLOW WATER EQUATIONS

The reduced port-controlled Hamiltonian model has been developed in [3] using a mixed finite element method. It is based on a subdivision of the total length of the open channel into n elementary cells. The whole model is obtained by series interconnections of the adjacent cells and may be written in the following explicit form:

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \left[\begin{bmatrix} 0 & M \\ -M^T & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & G(q,p) \end{bmatrix}\right] \begin{bmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial H}{\partial p} \end{bmatrix} + g_u \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{7}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = {g_u}^T \begin{bmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial H}{\partial p} \end{bmatrix} \tag{8}$$

where $q = [q_1 \dots q_n]^T$ and $p = [p_1 \dots p_n]^T$ form the state vector of cell volumes and kinetic momentums. The global interconnection structure sub-matrix $M (\in \Re^{n \times n})$ is given by:

$$M = \begin{bmatrix} -1 & 0 & \dots & \dots & 0 \\ 1 & -1 & \ddots & & \vdots \\ 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & 1 & -1 \end{bmatrix} \tag{9}$$

The input matrix g_u is defined as:

$$g_u = \begin{bmatrix} 0 & 1 \\ 0_{(2n-2)\times 1} & 0_{(2n-2)\times 1} \\ -1 & 0 \end{bmatrix} \tag{10}$$

The dissipation matrix $G(q,p) \in \Re^{n \times n}$ is obtained using the Manning-Strickler dissipation formula and is given as:

$$G(q,p) = \begin{bmatrix} G_1(q,p) & 0 & \dots & \dots & 0 \\ 0 & G_2(q,p) & \ddots & & \vdots \\ \vdots & & \ddots & \ddots & \ddots & \vdots \\ \vdots & & & \ddots & \ddots & 0 \\ 0 & & \dots & \dots & 0 & G_n(q,p) \end{bmatrix} \tag{11}$$

with

$$G_i(q,p) = g \frac{p_i}{K^2 q_i} \left(\frac{B^2(b-a) + 2q_i}{Bq_i} \right)^{\frac{4}{3}} (b-a) > 0, \forall q_i > 0 \tag{12}$$

where K is the Manning-Strickler dissipation parameter. The total energy of the system is given as the sum of the individual energies of the cells as:

$$H(q,p) = \sum_{i=1}^{n} \left(\frac{1}{2}\frac{q_i(t)^2}{C_{ab}} + \frac{1}{2}\frac{q_i(t)p_i(t)^2}{L_{ab}} - \rho g I K^i_{ab} q_i(t)\right) \tag{13}$$

with the following expressions for the reduced elements on a cell $[a,b]$:

$$C_{ab} = \frac{B}{\rho g}(b-a), L_{ab} = \rho(b-a)^2, K^i_{ab} = \frac{b+a}{2} \tag{14}$$

From the reduced energy function we derived the reduced internal efforts as:

$$\frac{\partial H}{\partial q_i}(q,p) = \frac{q_i}{C_{ab}} + \frac{p_i^2}{2L_{ab}} - \rho g I K^i_{ab} \tag{15}$$

$$\frac{\partial H}{\partial p_i}(q,p) = \frac{q_i p_i}{L_{ab}} \tag{16}$$

IV. Interconnection control method

A. IDA-PBC control method

Hereafter we recall the basic principles of the IDA-PBC methodology as developed in [18]. Consider a port Hamiltonian system in general form that we want to stabilize around a desired equilibrium point $x_d \in \Re^n$:

$$\dot{x} = (J(x) - R(x))\frac{\partial H}{\partial x} + g_u(x)u \tag{17}$$

If we may find a control law $\beta(x)$, matrices $J_a(x) = -J_a^T(x)$ and $R_a = R_a^T(x) \geq 0$ and an efforts vector $K(x)$ such that:

$$[J(x) + J_a(x) - (R(x) + R_a(x))]K(x) = -[J_a(x) - R_a(x)]\frac{\partial H}{\partial x}(x) + g_u(x)\beta(x) \tag{18}$$

where the efforts vector $K(x)$ satisfies the following conditions:

$$\text{Integrability}: \quad \frac{\partial K(x)}{\partial x} = [\frac{\partial K(x)}{\partial x}]^T \tag{19}$$

$$\text{Equilibrium point:} \quad K(x_d) = -\frac{\partial H}{\partial x}(x_d) \tag{20}$$

$$\text{Lyapunov stability}: \quad \frac{\partial K}{\partial x}(x_d) > -\frac{\partial^2 H}{\partial x^2}(x_d) \tag{21}$$

The closed loop system with the feedback control $u = \beta(x)$ will be written as follows:

$$\dot{x} = (J_d(x) - R_d(x))\frac{\partial H_d}{\partial x} \tag{22}$$

with $J_d(x) = J(x) + J_a(x)$, $R_d(x) = R(x) + R_a(x)$ and $H_d(x) = H(x) + H_a(x)$ where $\frac{\partial H_a}{\partial x}(x) = K(x)$. Thus we obtain a closed loop port controlled-Hamiltonian system with assigned interconnection and dissipation structures and a shaped energy function which admits a minimum at the desired equilibrium point. Moreover the equilibrium x_d is (locally) stable. It is *asymptotically stable* if, in addition, x_d is an isolated minimum of H_d and if the largest invariant manifold of $\phi = \left\{x \in \Re^n | [\frac{\partial H_d}{\partial x}(x)]^T R_d(x) \frac{\partial H_d}{\partial x}(x) = 0\right\}$ is $\{x_d\}$ (see [18] for details).

B. Control Design

We use in this section the IDA-PBC approach to design a control law for the model detailed in section III. It is well known that a condition on the natural dissipation induces strong restrictions on the energy shaping possibilities [18]. For this reason, we consider first the model without natural dissipation ($R(x) = 0$) and a control without any additional damping $R_a(x) = 0$. Let the skew symmetric matrix $J_a(x)$ be such that $J_a(1, 2n) = -J_a(2n, 1) = \delta \in \Re^+$ with all other elements being set to 0. It will be shown later that this parametrization of J_a simplify the resolution of the matching equation (18). In order to find the set of possible energy modulation $H_a(q,p)$, as proposed in [18], the equation (18) is projected onto the orthogonal space of the input matrix g_u:

$$g_u^{\perp}(x)[J(x) + J_a(x)]\frac{\partial H_a}{\partial x} = -g_u^{\perp} J_a(x)\frac{\partial H}{\partial x}(x) \tag{23}$$

where $g_u^\perp$ is a left orthogonal matrix of g_u ($g_u^\perp g_u = 0$). This matrix is assigned to the following canonical form:

$$g_u^\perp(x) = \begin{bmatrix} 0 & \lambda_1(x) & 0 & & & 0 & 0 \\ \vdots & 0 & \ddots & \ddots & & & \vdots \\ & \vdots & \ddots & \lambda_i(x) & \ddots & & \\ \vdots & & & \ddots & \ddots & 0 & \vdots \\ 0 & 0 & & \dots & 0 & \lambda_{2n-2}(x) & 0 \end{bmatrix} \tag{24}$$

With this specific choice of $g_u^\perp$ and $J_a(x)$, we have $g_u^\perp J_a(x) = 0$, hence $g_u^\perp(x) J(x) \frac{\partial H_a}{\partial x} = 0$, or in developed form:

$$\begin{cases} \dfrac{\partial H_a}{\partial p_i} - \dfrac{\partial H_a}{\partial p_{i+1}} = 0 \\ \dfrac{\partial H_a}{\partial q_i} - \dfrac{\partial H_a}{\partial q_{i+1}} = 0 \end{cases} \quad \text{for } i = 1, \cdots, n-1 \tag{25}$$

These equations lead to a solution for $H_a(q,p)$ in the form of an arbitrary function of the total volume ($\sum_{i=1}^n q_i$) and the total momentum ($\sum_{i=1}^n p_i$). In order to deal with the equilibrium condition (20) which corresponds in our case to a desired water flow Q_d and a desired cells volumes q_d, we choose the following particular solution for the energy function:

$$\begin{aligned} H_a(q,p) = -(\frac{q_d}{C_{ab}} \quad &+ \quad \frac{L_{ab}}{2}(\frac{Q_d}{q_d})^2 - \rho g I M_c) \sum_{i=1}^n q_i \\ &+ \frac{q_d^2}{C_{ab}} - Q_d \sum_{i=1}^n p_i \end{aligned} \tag{26}$$

where M_c denotes the center of mass:

$$M_c = \frac{\sum_{i=1}^n K_{ab}^i q_i}{\sum_{i=1}^n q_i} \tag{27}$$

This energy function $H_a(q,p)$ satisfies the equilibrium point condition. The Lyapunov stability condition (21) may be written :

$$\frac{p_i^2 C_{ab}}{q_i L_{ab}} < 1 \quad \text{for} \quad i = 1, \dots, n \tag{28}$$

This is the well known subcritical (or fluvial) flow condition [1] expressed with the Froude number $\left(\dfrac{v^2}{gh} < 1\right)$. The closed loop energy function is:

$$\begin{aligned} H_d(q,p) \;&=\; \frac{1}{2C_{ab}} \sum_{i=1}^n (q_i - q_d)^2 + \sum_{i=1}^n p_i (\frac{q_i p_i}{2L_{ab}} - Q_d) \\ &\quad - \frac{L_{ab}}{2}(\frac{Q_d}{q_d})^2 \sum_{i=1}^n q_i \end{aligned} \tag{29}$$

It is important to note that the mass center term allows to compensate the slope potential energy of the system (see equation 13). Figure (2) shows the level sets of $H_d(q,p)$ with q_d and Q_d fixed to achieve the desired water level ($h_d = 0.1(m)$) and velocity ($v_d = 0.2(m/s)$). This desired water level and velocity correspond to a desired water flow $Q_d = B h_d v_d$ and desired cells volume $q_d = B h_d (b-a)$. The first

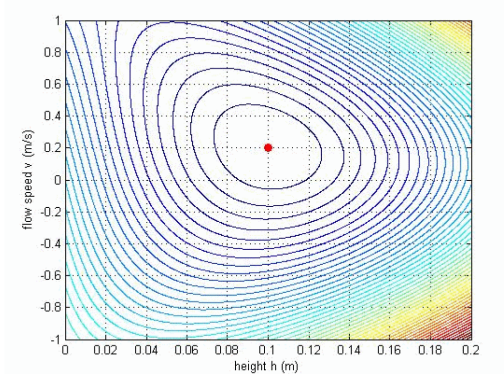

Fig. 2. Level sets of $H_d(q,p)$ with $h_d = 0.1(m)$ and $v_d = 0.2(m/s)$

and the last row of the IDA-PBC matching equations (18) lead to the following control laws:

$$\begin{aligned} u_1 \;&=\; -\frac{\partial H_a}{\partial q_n} + \delta[\frac{\partial H}{\partial q_1} + \frac{\partial H_a}{\partial q_1}] \\ &=\; \frac{q_d}{C_{ab}} + \frac{L_{ab}}{2}(\frac{Q_d}{q_d})^2 - \rho g I M_c + \frac{\delta}{C_{ab}}(q_1 - q_d) + \\ &\quad\; \delta(\frac{p_1^2}{2L_{ab}} - \frac{L_{ab}}{2}(\frac{Q_d}{q_d})^2) \end{aligned} \tag{30}$$

$$\begin{aligned} u_2 \;&=\; -\frac{\partial H_a}{\partial p_1} + \delta[\frac{\partial H}{\partial p_n} + \frac{\partial H_a}{\partial p_n}] \\ &=\; Q_d + \delta[\frac{q_n p_n}{L_{ab}} - Q_d] \end{aligned} \tag{31}$$

The obtained control laws are thus the sum of a feed-forward action related to the desired equilibrium point (q_d, Q_d) and a proportional closed loop one related to downstream water flow error and upstream hydrodynamic pressure error.

V. An energy shaping approach using Casimir's invariants

We now derive a control law for the considered reduced port-controlled Hamiltonian model (without dissipation) by using an energy shaping method based on Casimir's invariants and an additional external power conserving interconnection. As suggested in [16] we split the control action $u = u_{es} + u_{st}$ into an energy shaping action u_{es} which aims at shaping the energy and a structural action u_{st} which modifies the structure of the system. This last action may be called a damping action when it modifies the damping structure $R(x)$ or a symplectic action when it modifies the interconnection structure $J(x)$. Consider the following controller given in PCH form:

$$\begin{aligned} \begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{bmatrix} &= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H_c(\xi_1,\xi_2)}{\partial \xi_1} \\ \frac{\partial H_c(\xi_1,\xi_2)}{\partial \xi_2} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_{c_1} \\ u_{c_2} \end{bmatrix} \\ \begin{bmatrix} y_{c_1} \\ y_{c_2} \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\partial H_c}{\partial \xi_1}(\xi_1,\xi_2) \\ \frac{\partial H_c}{\partial \xi_2}(\xi_1,\xi_2) \end{bmatrix} \end{aligned} \tag{32}$$

with the controller state vector $\xi \in \Re^2$, the input-output port variables (u_c, y_c) and the energy of the controller $H_c(\xi)$. The system is connected to the controller through the feedback interconnection $u_{c_i} = y_i$, $u_i = -y_{c_i}$ which is power conserving since $u_{c_i} y_{c_i} - u_i y_i = 0$. Thus the extended system can be written in the PCH form:

$$\begin{bmatrix} \dot{x} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} J(x) & -g(x)I_{2\times 2} \\ I_{2\times 2}g^T(x) & J_c \end{bmatrix} \begin{bmatrix} \frac{\partial H_{cl}}{\partial x}(x,\xi) \\ \frac{\partial H_{cl}}{\partial \xi}(x,\xi) \end{bmatrix} \quad (33)$$

where J_c is the interconnection structure of the controller (32) and $H_{cl}(x, \xi)$ represent the closed loop energy function given by:

$$H_{cl}(x,\xi) = H(x) + H_c(\xi) \quad (34)$$

In order to shape the energy of the closed system we have to relate the controller state ξ to the system state x. We follow [19] and look for Casimir's functions which are invariant quantities along the dynamic of the system (33). As adopted in [19] and without loss of generality this Casimir's functions are chosen in the following form:

$$C(x,\xi) = F(x) - \xi \quad (35)$$

These Casimir's functions are invariant quantities along the system trajectories independant from the energy function $H_{cl}(x, \xi)$. Hence they satisfy:

$$\frac{dC}{dt} = \begin{bmatrix} \frac{\partial^T C}{\partial x} & \frac{\partial^T C}{\partial \xi} \end{bmatrix}^T \begin{bmatrix} J(x) & -g(x)I_{2\times 2} \\ I_{2\times 2}g^T(x) & J_c \end{bmatrix} \begin{bmatrix} \frac{\partial H_{cl}}{\partial x}(x,\xi) \\ \frac{\partial H_{cl}}{\partial \xi}(x,\xi) \end{bmatrix} = 0 \quad (36)$$

$$\Rightarrow \begin{bmatrix} \frac{\partial^T F_i}{\partial x} & -e_i^T \end{bmatrix} \begin{bmatrix} J(x) & -g(x)I_{2\times 2} \\ I_{2\times 2}g^T(x) & J_c \end{bmatrix} = 0$$

$$\Rightarrow \begin{cases} \frac{\partial^T F_i}{\partial x} J(x) - I^i_{2\times 2} g^T(x) = 0 \\ -\frac{\partial^T F_i}{\partial x} g(x) I_{2\times 2} - J^i_c = 0 \end{cases} \quad (37)$$

where e_i represents the i-th basis vector. The solutions of equation (37) are given by:

$$\begin{cases} F_1(q,p) = \sum_{i=1}^n q_i \\ F_2(q,p) = \sum_{i=1}^n p_i \end{cases} \quad (38)$$

finally we obtain the following invariants:

$$C_1(q,p,\xi) = \sum_{i=1}^n q_i - \xi_1, \quad C_2(q,p,\xi) = \sum_{i=1}^n p_i - \xi_2 \quad (39)$$

We observe that these invariant quantities are related to the total volume and total kinetic momentum of the system. The controller energy function is fixed as follows:

$$\begin{aligned} H_c(\xi_1,\xi_2) &= -\left(\frac{q_d}{C_{ab}} + \frac{L_{ab}}{2}\left(\frac{Q_d}{q_d}\right)^2 - \rho g I M_c\right)\xi_1 \\ &\quad + \frac{q_d^2}{C_{ab}} - Q_d \xi_2 \end{aligned}$$

This expression of H_c allows to shape the energy H_{cl} and to assign the desired equilibrium point (q_d, Q_d) to the closed loop system (33) as it will be shown hereafter. Considering $C_1 \equiv C_2 \equiv 0$ (which are obviously invariant quantities), The controller energy function is rewritten as follows:

$$\begin{aligned} H_c(\xi_1,\xi_2) &= -\left(\frac{q_d}{C_{ab}} + \frac{L_{ab}}{2}\left(\frac{Q_d}{q_d}\right)^2 - \rho g I M_c\right)\sum_{i=1}^n q_i \\ &\quad + \frac{q_d^2}{C_{ab}} - Q_d \sum_{i=1}^n p_i \end{aligned} \quad (40)$$

We finally obtain the following energy shaping control laws:

$$\begin{aligned} u_{1es} &= -\frac{\partial H_c}{\partial q_n} \\ &= \frac{q_d}{C_{ab}} + \frac{L_{ab}}{2}\left(\frac{Q_d}{q_d}\right)^2 - \rho g I M_c \quad (41) \\ u_{2es} &= -\frac{\partial H_c}{\partial p_1} \\ &= Q_d \quad (42) \end{aligned}$$

The closed loop system will be given by:

$$\begin{aligned} \dot{x} &= J(x)\frac{\partial H_s}{\partial x}(x) \quad (43) \\ y^{'} &= g_u^T \frac{\partial H_s}{\partial x}(x) \end{aligned}$$

where y' represents the passive output of the closed loop system which may be different from the natural output of the system. The shaped energy function $H_s(q, p)$ is given as:

$$\begin{aligned} H_s(q,p) &= H(q,p) + H_c(\xi = F(q,p))) \\ &= \frac{1}{2C_{ab}}\sum_{i=1}^n (q_i - q_d)^2 + \sum_{i=1}^n p_i\left(\frac{q_i p_i}{2L_{ab}} - Q_d\right) \\ &\quad - \frac{L_{ab}}{2}\left(\frac{Q_d}{q_d}\right)^2 \sum_{i=1}^n q_i \end{aligned} \quad (44)$$

We would like now to assign a desired structure to the closed loop system using the following power conserving interconnection:

$$\begin{cases} u_{1st} = \delta y_2^{'} \\ u_{2st} = -\delta y_1^{'} \end{cases} \Rightarrow u_{1st} y_1^{'} + u_{2st} y_2^{'} = 0 \quad (45)$$

This operator corresponds to the so-called *gyrator* in the *Bond Graph* theory. This leads to the following port Hamiltonian system:

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & M_d \\ -M_d^T & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H_s}{\partial q}(q,p) \\ \frac{\partial H_s}{\partial p}(q,p) \end{bmatrix} \quad (46)$$

where the assigned matrix M_d is given as follows:

$$M_d = \begin{bmatrix} -1 & 0 & \dots & 0 & \delta \\ 1 & -1 & \ddots & & 0 \\ 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & 1 & -1 \end{bmatrix} \quad (47)$$

Finally the obtained control laws are given by:

$$\begin{aligned} u_1 &= -\frac{\partial H_a}{\partial q_n} + \delta[\frac{\partial H}{\partial q_1} + \frac{\partial H_a}{\partial q_1}] \\ &= \frac{q_d}{C_{ab}} + \frac{L_{ab}}{2}(\frac{Q_d}{q_d})^2 - \rho g I M_c + \frac{\delta}{C_{ab}}(q_1 - q_d) + \\ &\quad \delta(\frac{p_1^2}{2L_{ab}} - \frac{L_{ab}}{2}(\frac{Q_d}{q_d})^2) \qquad (48) \\ u_2 &= -\frac{\partial H_a}{\partial p_1} + \delta[\frac{\partial H}{\partial p_n} + \frac{\partial H_a}{\partial p_n}] \\ &= Q_d + \delta[\frac{q_n p_n}{L_{ab}} - Q_d] \qquad (49) \end{aligned}$$

The obtained control laws are the same than the ones obtained using the IDA-PBC approach. However this second method allows us to give a physical interpretation of the control action. The feed-forward action shape the energy of the system and stabilize the total volume and momentum of the system corresponding to the desired equilibrium point. The external power conserving interconnection allows to modify the skew symmetric structure of the system and introduce a proportional correction for the closed loop.

VI. Introduction of the natural dissipation

In the control design, the natural dissipation of the system $R(x)$ was not taken in account. Thus the system corresponds to a cascade of oscillators very sensitive to perturbations. We will see hereafter the effect of the natural dissipation on the closed loop system with the designed control. This system may be written:

$$\begin{aligned} \begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} &= \begin{bmatrix} 0 & M_d \\ -M_d^T & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H_d}{\partial q} \\ \frac{\partial H_d}{\partial p} \end{bmatrix} \\ &\quad - \begin{bmatrix} 0 & 0 \\ 0 & G(q,p) \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial H}{\partial p} \end{bmatrix} \qquad (50) \end{aligned}$$

The energy balance of the system is given by:

$$\frac{dH_d}{dt}(q,p) = -\frac{\partial^T H_d}{\partial p} G(q,p) \frac{\partial H}{\partial p} \qquad (51)$$

including the dissipation term $G(q,p) > 0$. The energy balance may be written:

$$\begin{aligned} \frac{dH_d}{dt}(q,p) &= -\frac{\partial^T (H + H_c)}{\partial p} G(q,p) \frac{\partial H}{\partial p} \\ &= -\sum_{i=1}^{n} (\frac{q_i p_i}{Lab} - Qd) G(q_i, p_i) (\frac{q_i p_i}{Lab}) \qquad (52) \end{aligned}$$

Hence we can use the LaSalle's invariance principle and analyze the sign of the energy balance. The energy plays the role of Lyapunov function for the closed loop. Due to the bijection of the dissipation matrix $G(q,p)$, the following equivalence can be directly proven:

$$\frac{dH_d}{dt}(q,p) = 0 \Leftrightarrow \begin{cases} q_i = 0 & \text{or} \\ p_i = 0 & \text{or} \\ \frac{q_i p_i}{L_{ab}} = Qd \end{cases} \qquad (53)$$

The first case $q_i = 0$ corresponds to the situation where there is no fluid in the reach. The second case $p_i = 0$ corresponds to a static fluid in the reach. It is easy to see that the sets ($p_i = 0$ or $q_i = 0$) are not invariant under the dynamic of the closed loop system (50). The last case ($\frac{q_i p_i}{L_{ab}} = Qd$) corresponds to the situation where the fluid flow in the reach equals the desired equilibrium fluid flow. Thus the LaSalle's invariant manifold is given by an hyperbola in the (q,p) plane as illustrated for one cell in figure (3).

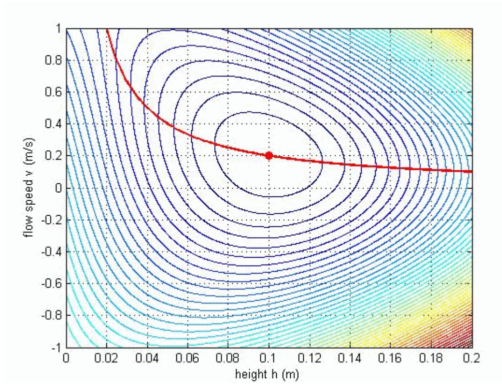

Fig. 3. Levels of energy function and LaSalle's invariant manifold

The sign of the energy balance is given as:

$$\frac{dH_d}{dt}(q,p) \Rightarrow \begin{cases} < 0 & \text{If } \frac{q_i p_i}{L_{ab}} > Qd \\ = 0 & \text{If } \frac{q_i p_i}{L_{ab}} = Qd \\ > 0 & \text{If } \frac{q_i p_i}{L_{ab}} < Qd \end{cases} \qquad (54)$$

We conclude that the fluid flow at the equilibrium will be equal to the desired one. By reducing the dynamic of the system on the LaSalle's invariant manifold we obtain the equation giving the equilibrium fluid volumes q_i in the reach:

$$-M_d^T \frac{\partial H_d}{\partial q} - G(q,p) Q_d = 0 \qquad (55)$$

Equation (55) expresses the equality of the reduced hydro-dynamic pressure gradient with the dissipated pressure by friction.

VII. Robustness towards the mass center estimation

The mass center is a parameter which depend on the values of the water level along the length of the canal. Experimentally we only have some fixed sensors to give the water level mesures. Thus the mass center should be estimated. In this section we analyse the robustness of the control towards a constant estimation $\hat{M}_c$ of the mass center.

The expression of the control action u_1 can be written using the mass center estimation as follows:

$$
\begin{aligned}
u_1 &= -\frac{\partial H_a}{\partial q_n} + \delta[\frac{\partial H}{\partial q_1} + \frac{\partial H_a}{\partial q_1}] \\
&= \frac{q_d}{C_{ab}} + \frac{L_{ab}}{2}(\frac{Q_d}{q_d})^2 - \rho g I \hat{M}_c + \frac{\delta}{C_{ab}}(q_1 - q_d) + \\
&\quad \delta(\frac{p_1^2}{2L_{ab}} - \frac{L_{ab}}{2}(\frac{Q_d}{q_d})^2) \qquad (56) \\
&= u_1^t - \rho g I(\hat{M}_c - M_c) \qquad (57)
\end{aligned}
$$

where u_1^t is the designed control action with the theoretical expression of mass center and $(\Delta M = \hat{M}_c - M_c)$ gives the estimation error of the mass center. The system in closed loop will be given as follows:

$$
\begin{aligned}
\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} &= \begin{bmatrix} 0 & M_d \\ -M_d^T & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H_d}{\partial q} \\ \frac{\partial H_d}{\partial p} \end{bmatrix} \qquad (58) \\
&\quad - \begin{bmatrix} 0 & 0 \\ 0 & G(q,p) \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial H}{\partial p} \end{bmatrix} + \begin{bmatrix} 0_{(2n-1)\times 1} \\ \rho g I \Delta M \end{bmatrix}
\end{aligned}
$$

The energy balance expression is given as follows:

$$
\begin{aligned}
\frac{dH_d}{dt}(q,p) &= -\frac{\partial^T (H + H_c)}{\partial p} G(q,p) \frac{\partial H}{\partial p} + \frac{\partial^T (H + H_c)}{\partial p_n} \rho g I \Delta M \\
&= -\sum_{i=1}^{n} (\frac{q_i p_i}{Lab} - Qd) G(q_i, p_i) (\frac{q_i p_i}{Lab}) \\
&\quad + (\frac{q_n p_n}{Lab} - Qd) \rho g I \Delta M \qquad (59)
\end{aligned}
$$

In [21] the authors derive a bound condition which has to be satisfied in order to ensure the stability of the closed loop towards an additive disturbance. Applying this result to system (58) we can prove that the stability is ensured if the following condition is satisfied:

$$
|\rho g I \Delta M| < \kappa |\frac{\partial H_d}{\partial p_n}| < \kappa |\frac{\partial H}{\partial p_n}| \quad with \quad 0 < \kappa < G(q_n, p_n) \qquad (60)
$$

Taking into account this condition the energy balance equation (59) becomes:

$$
\begin{aligned}
\frac{dH_d}{dt}(q,p) &< -\sum_{i=1}^{n-1} (\frac{q_i p_i}{Lab} - Qd) G(q_i, p_i) (\frac{q_i p_i}{Lab}) \\
&\quad - (\frac{q_n p_n}{Lab} - Qd)(G(q_n, p_n) - \kappa)(\frac{q_n p_n}{Lab}) \qquad (61)
\end{aligned}
$$

Thus the condition (60) ensures the application of the LaSalle's invariance principle as applied in section VI to prove the stability of the closed loop in presence of mass center estimation error.

VIII. Adding of an integral action

The numerical values of the Manning Strickler friction parameter and the hydraulic gate parameters are poorly known since they are issued from empirical models of bed friction within the reach and around the gates. A static error may thus appear between the desired and the real equilibrium points. In order to avoid this problem, an integral action is added on each control law [16]. For that purpose, two new states η_1 and η_2 are introduced and correspond respectively to the downstream water flow error and upstream hydrodynamic pressure error. Theirs dynamics are defined by:

$$
\begin{bmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \end{bmatrix} = -K^T g_u^T(x) \frac{\partial H_d}{\partial x} = -\begin{bmatrix} k_i^{21} & -k_i^{11} \\ k_i^{22} & -k_i^{12} \end{bmatrix} \begin{bmatrix} \frac{\partial H_d}{\partial q_1} \\ \frac{\partial H_d}{\partial p_n} \end{bmatrix} \qquad (62)
$$

With $rank(K) = 2$. The augmented system still has a PCH form:

$$
\begin{bmatrix} \dot{x} \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} J_d(x) & g_u(x)K \\ -K^T g_u^T(x) & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial W}{\partial x} \\ \frac{\partial W}{\partial \eta} \end{bmatrix} \qquad (63)
$$

with a total desired stored energy:

$$
W(q,p,v) = H_d(q,p) + \frac{1}{2}\eta^T K^{-1} \eta \qquad (64)
$$

where K is a definite positive 2x2 matrix $(K > 0)$. The new control laws of the system are then

$$
\begin{aligned}
u_1(t) &= \frac{q_d}{C_{ab}} + \frac{L_{ab}}{2}(\frac{Q_d}{q_d})^2 - \rho g I M_c + \frac{\delta}{C_{ab}}(q_1 - q_d) + \\
&\quad \delta(\frac{p_1^2}{2L_{ab}} - \frac{L_{ab}}{2}(\frac{Q_d}{q_d})^2) + \eta_1 \\
u_2(t) &= Q_d + \delta[\frac{q_n p_n}{L_{ab}} - Q_d] + \eta_2 \qquad (65)
\end{aligned}
$$

Stability is proved using a Lasalle argument [16].

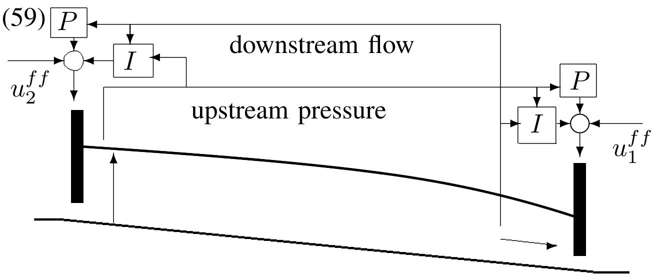

Fig. 4. Control strategy, P: proportional action, I: integral action and u_i^{ff} feedforward action

IX. Simulation results and experimental validation

A. Presentation of the experimental micro-canal

The micro channel is a technological platform completely instrumented used to reproduce the flows in irrigation canals on a reduced scale. It is spread out over seven meter length, supported by a metal lattice girder. To the channel part, comes to be added an upstream tank which acts as source and whose level is maintained constant, a downstream tank

and an intermediate tank which is used as tank of storage. The slope of the channel can be modified thanks to a mechanical jack as shown in (Fig.5). The channel has three gates; upstream, downstream and a third in the center makes it possible to experiment the case of interconnected reaches. These gates are actuated automatically using DC motors.
Ultrasonic sensors return a voltage ranging 0-10 Volts

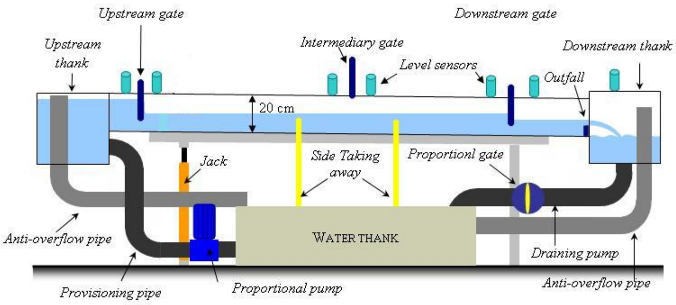

Fig. 5. Experimental Micro-Channel

which give an image of the height of water levels in the places where they are placed.

A proportional valve and a pump are intended respectively to ensure a constant water level at downstream and upstream thank of canal. They are managed by automats Crouzet of the type Millenium II. By mean of a dSPACE electronic card, all the data from the sensors are recovered on a computer in order to be analyzed and by the same device, the actuators of the micro channel are controlled. The parameter of the micro-channel are given in (Tab.I). A picture of the experimental plant may be viewed on (Fig.6)

TABLE I
MICRO-CHANNEL PARAMETERS USED IN SIMULATION

length L	7 meter
width B	0.1 meter
slope I	1.6×10^{-3}
Manning-Strickler coefficient K	97
upstream gate parameter α_1	0.66
downstream gate parameter α_2	0.73
downstream outfall height (H_{dev})	0.05 meter

B. Simulation results

Simulations presented in this section are obtained with a micro-channel simulator made with the above mentioned reduced port controlled Hamiltonian model. The total length of the channel is subdivided into ten cells. We have used for the parameters values those identified on the experimental micro-channel.
For simulations we have used two scenarios, A and B. For scenario A, we imposed a constant reference water flow value of 4 *l/s* (as shown in Fig.8) and we have varied the water level reference with two consecutive increasing steps and a final decreasing step (Fig.7). In the scenario B, we imposed

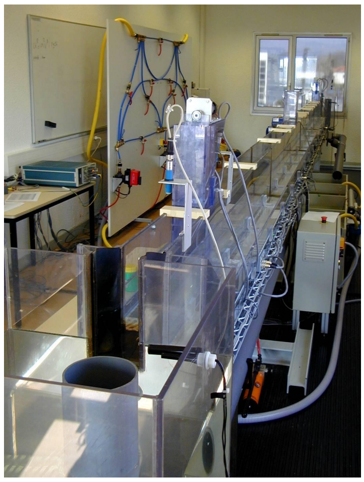

Fig. 6. Sight of the experimental Micro-Channel

a constant reference water level of 10 *cm* (Fig.12) and we have varied the water flow reference in a manner similar to that of the water level in the first scenario (Fig.11). As the considered Manning-Strickler coefficient of 98 corresponds to the very low friction case of a Plexiglas reach, all the references were given through a low pass filter, in order to reduce the oscillations. As it can be seen in Fig.9, 10, 11, 14, the control actions in both scenarios are in the admissible range of values and variations for standard hydraulic works, which make them feasible for real scale implementation. In both Fig.7 and Fig.12 we can see the existence of a steady state error in the controlled level of water along the reach. This state steady error is caused by the energy dissipation due to friction in the reach. The water levels correspond to the solution of (55).

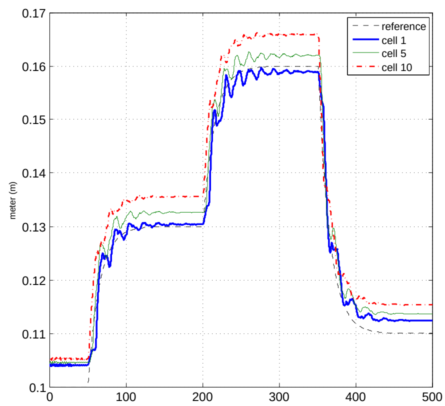

Fig. 7. Scenario A: Fluid level response in the upstream, middle and downstream of the canal (m)

C. Experimental Validation

For the experimental validation of the control law (53), we have first to rewrite it in an implementable form with taking into account the expression of the reduced elements (C_{ab}, L_{ab}, K_{ab}) given in (14). Also, for practical considerations we compute the pressure using only the hydrostatic

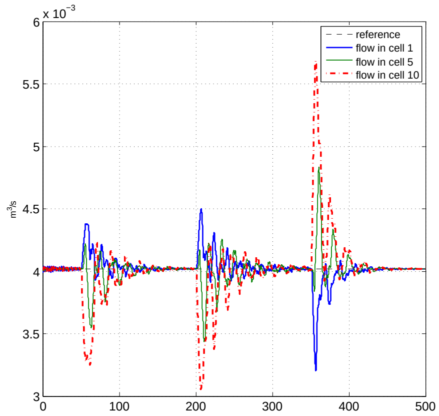

Fig. 8. Scenario A: Fluid flow response in the upstream, middle and downstream of the canal (m3/s)

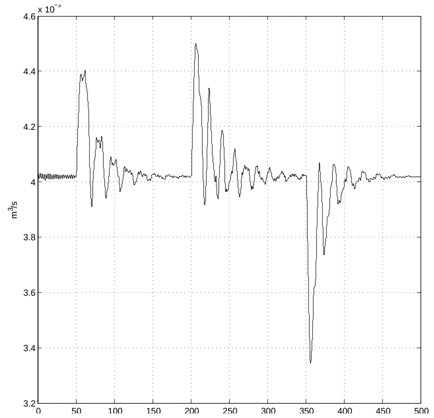

Fig. 9. Scenario A: Upstream control flow (m3/s)

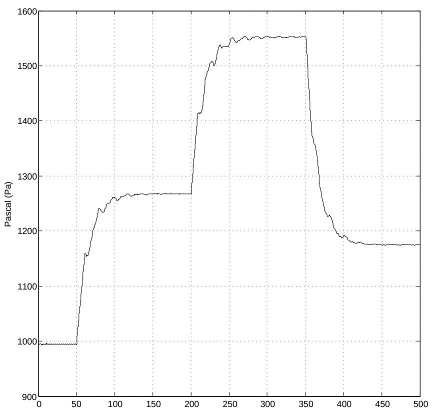

Fig. 10. Scenario A: Downstream control pressure (Pa)

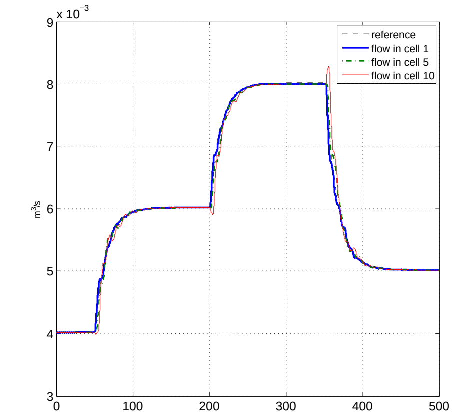

Fig. 11. Fluid flow response in the upstream, middle and downstream of the canal (m3/s)

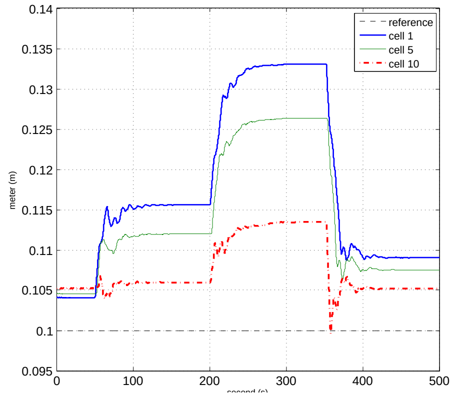

Fig. 12. Scenario B: Fluid level response in the upstream, middle and downstream of the canal (m)

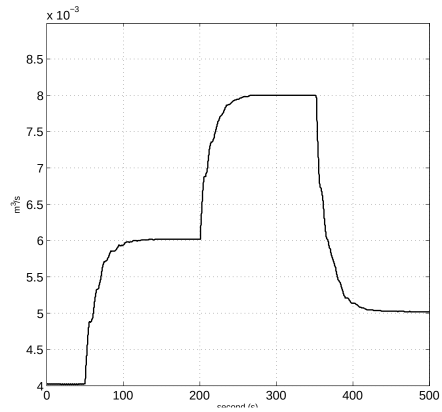

Fig. 13. Scenario B: Upstream control flow (m3/s)

term which is the predominant term for the achievable fluid flow using the available levels measures. This hypothesis could be checked numerically. Thus, we obtain the following expressions:

$$
\begin{aligned}
u_1(t) &= \rho g h_d + \rho\left(\frac{Q_d}{Bh_d}\right)^2 - \rho g I \hat{M}_c + \delta\rho g(h_1(t) - h_d) \\
&\quad + k_i^{11}\int_0^t (Q_L(\tau) - Q_d)d\tau - k_i^{21}\int_0^t \rho g(h_1(\tau) - h_d)d\tau \qquad (66)\\
u_2(t) &= Q_d(t) + \delta(Q_L(t) - Q_d(t)) - k_i^{22}\int_0^t \rho g(h_1(\tau) - h_d)d\tau \\
&\quad + k_i^{12}\int_0^t (Q_L(\tau) - Q_d)d\tau \qquad (67)
\end{aligned}
$$

Here h_d and Q_d are the desired water level and water flow

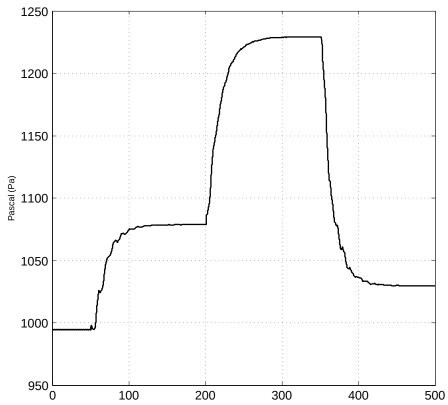

Fig. 14. Scenario B: Downstream control pressure (Pa)

in the channel, $h_1(t)$ is water level in the first upstream cell, measured with an ultrasonic sensor, $Q_L(t)$ is the downstream water flow, estimated with the help of the rectangular weir equation (68), $\hat{M}_c$ is a constant estimation of the position of the center of mass of the system, chosen as the middle of the reach, δ is the proportional action gain and $k_i^{11}, k_i^{12}, k_i^{21}, k_i^{22}$ are the integral action gains.

The hydraulic works of the system are described by equations (68), for the weir, and (69) for the sliding gates.

$$Q_w(t) = \alpha_w B \Delta h(t) \sqrt{2g\Delta h(t)} \tag{68}$$

$$Q_{sg}(t, \theta) = \alpha_{sg} B \theta \sqrt{2g(h_a(t) - h_b(t))} \tag{69}$$

Where Q_w is the water flow through a weir, α_w is an adimensional geometrical coefficient of the weir, Δh is the height of the water crest above the weir, it is given by $\Delta h = h_w - h_{w0}$, with h_w the actual water level at the weir and h_{0w} the height of the weir itself. Q_{sg} is the water flow through a sliding gate, α_{sg} is the form factor of the gate, θ is the opening of the gate and h_a, h_b are the water levels at upstream and downstream of the gate. The upstream and downstream gates are used to generate, respectively, the actions u_2 (upstream water flow) and, u_1 (downstream pressure). To ensure these computed controls, we calculate the gate openings using the inverse model of the sliding gate. This could be archived also by a local closed loop regulation of the computed actions through the gates. Using the inversion method we obtain the corresponding gates openings for the upstream and downstream gates:

$$\begin{aligned} \theta_1(u_1) &= \frac{Q_L}{\alpha_1 B \sqrt{2g(\frac{u_1}{\rho g} - h_w)}} \\ \theta_2(u_2) &= \frac{u_2}{\alpha_2 B \sqrt{2g(h_T - h_1)}} \end{aligned} \tag{70}$$

$\theta_1$is the opening of the downstream gate and θ_2 is the opening of the upstream gate, h_w is the water level after the downstream gate, corresponding to the water level at the discharge weir and h_T is the water level in the upstream tank. From these expressions we can explicitelly extract the external physical constraints on the achievable equilibrium profile of the system:

$$h_T > h_d > h_w(Q_d) \tag{71}$$

The desired water level must be inferior to the water level in the source tank and superior to the weir level corresponding the desired water flow. Some practical considerations, bring us to filter the measurements in order to attenuate the sensors noise. The experimentation procedure was similar to the one used for the simulations: the system has been given ascending and descending steps for each water level or flow, while keeping the other reference at a constant value. Before to present the results we note that for technical reasons there is sometimes some data capture errors appearing in the measures of water levels which do not correspond to real measures (see figure (15) at $t = 30s$ and figure (21) at $t = 70s$). The figure (15) shows the first experimental scenario which consists on a response to ascending filtered level step reference with a constant desired water flow. The control actions (upstream water flow, downstream pressure and gate openings) are shown in figure (16). As we can see the control objective is satisfactorily achieved with a feasible gate sollicitations.

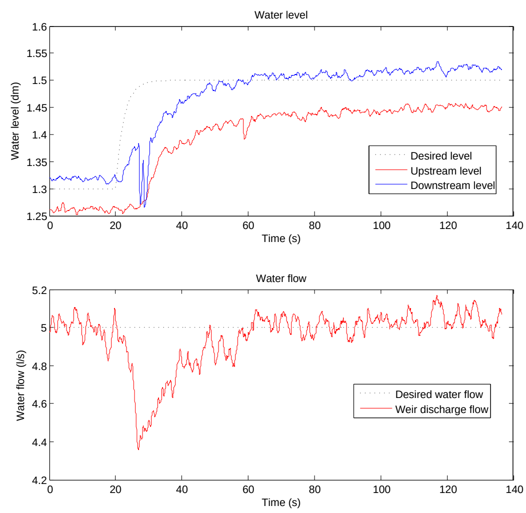

Fig. 15. Water levels and flow responses for ascending filtered step level reference

See figures (17) and (18) for a descending filtered level step reference scenario. The figure (19) shows the response to an ascending filtered water flow step reference while keeping the water level around the desired value. We can see also in (20) the obtained control actions for this scenario. See figures (21) and (22) for a descending filtered water flow step reference scenario.

X. CONCLUSION

We developed in this paper control laws based on a reduced port-controlled Hamiltonian model for the shallow water equations, using the IDA-PBC design method. This

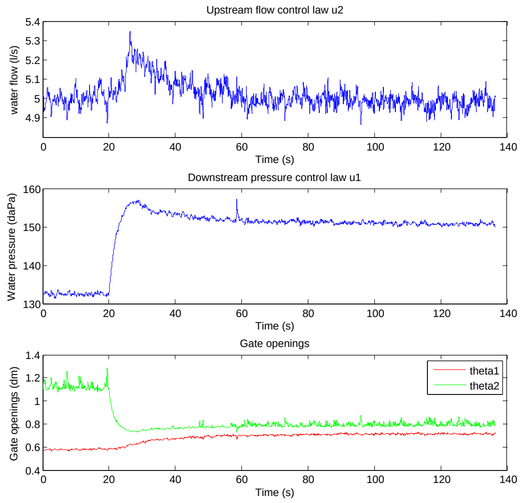

Fig. 16. Control actions for ascending filtered level step reference

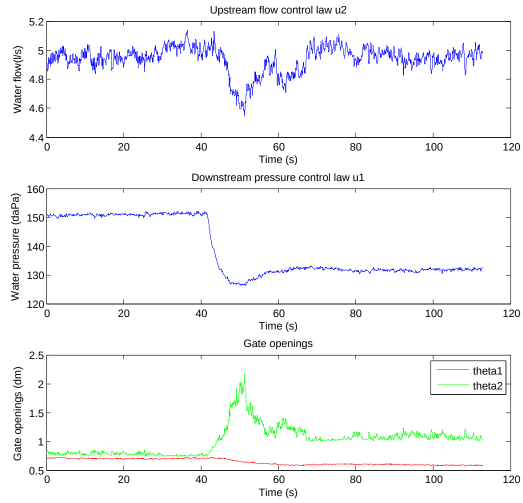

Fig. 18. Control actions for descending filtered step level reference

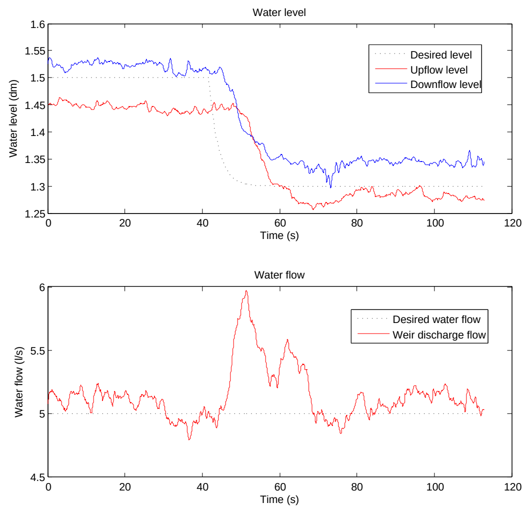

Fig. 17. Water levels and flow responses for descending filtered level step reference

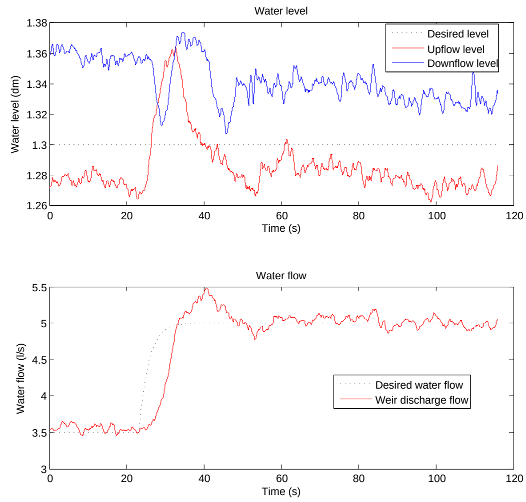

Fig. 19. Water levels and flow responses for ascending filtered water flow step reference

control design method allowed us to design a nonlinear static state feedback based on the choice of the controller interconnection structure and the desired closed shaped energy (with a prescribed minimum state). The obtained controller is composed of an energy shaping (feed-forward) action depending on the equilibrium point and a proportional closed loop action. It has to be noticed that all the choices of the controller structures and parameters values are guided by physical considerations on the structural model behavior. The application of Lasalle's invariance principle allowed us to proof the stability of the system when we take into account the natural dissipation.

We have presented its experimental testing on the Laboratory seven meter long micro-channel. The experimental conditions are taken into account like the sensors noise, the limitation on the achievable equilibriums and the nonlinear characteristic of the sliding gates. The obtained results show the effectiveness of the designed control.

We have extended this work on the distributed parameter port Hamiltonian formulation of the shallow water equations in [22] and we have designed the same controller using the casimir's invariants based energy shaping methods in the infinite dimensional case.

Among the expected developments of this work are, first to develop a good estimator of the mass center even by defining its dynamics through the shallow water equations or to define an estimator based on a simplification of its expression. Secondly, knowing that the center of mass represent the first order moment of the water level distribution along the canal, it will be interesting to look for the generalization of the high order moments to achieve some new control objectives like waves filtering.

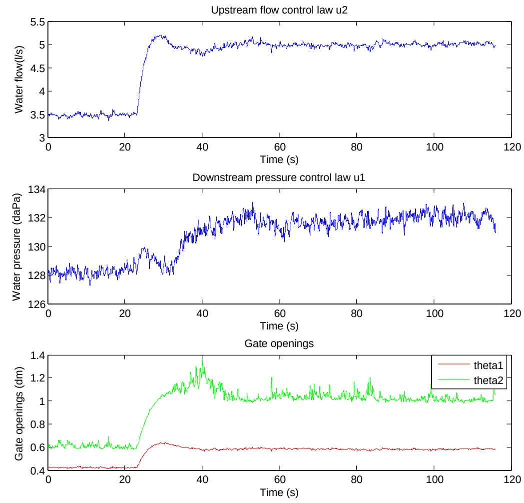

Fig. 20. Control actions for ascending filtered water flow step reference

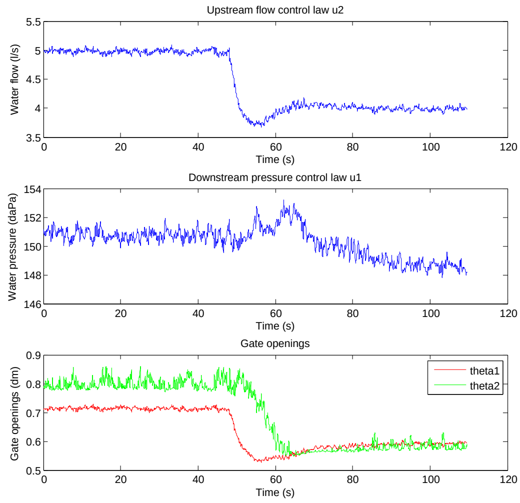

Fig. 22. Control actions for descending filtered water flow step reference

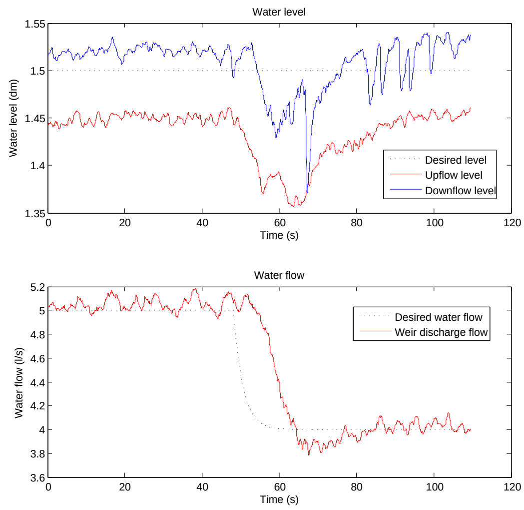

Fig. 21. Water levels and flow responses for descending filtered water flow step reference

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