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A simple 1D 1 group model to study uniaxial Xenon instabilities
in a large PWR core and their control.

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Abstract

We study a 1D 1 group model, independently considered in [3] and [4], to study uniaxial Xenon instabilities.

Such a simple model is useful for educational purpose. It shows the existence and the shape of Xenon oscillations which are at the basis of the well known axial offset anomaly.

With a suitable choice of the migration area M^2 , we can obtain oscillations with a 28 or 30 hours period which correspond to what is expected (see [1]).

We also show that, with such a model, it is possible to simulate lightly absorbing control rods and implement a feedback control to damp the oscillations out.

We have implemented the Shimazu feedback control law [6].

We show that the Shimazu method, which relies on a simple 2 zones lumped parameter model, is an efficient one.

Taking into account the Doppler effect is possible with our model, as has been shown in [3].

Our model is a nonlinear one. A linear stability analysis has been carried out in [5]

The 1 group diffusion model

For an homogeneous reactor core the one group diffusion model can be written as :

$$(1) \quad -M^2 \Delta \Phi + \Phi = k_{\infty} \Phi, \quad x \in \Omega$$

where Φ is the (thermal) neutron flux in $n/cm^2/s$, M^2 the migration area (in cm^2), and k_{∞} multiplication factor that would be reached in the case of an infinite core.

The uniaxial case

Actually the core of a PWR is not homogeneous, it is made of vertical cells with height L and step δ in the two horizontal directions : each cell is a parallelepiped with volume $\delta \times \delta \times L$ that contains a cylindrical fuel pin, the height of which is also equal to L .

The moderation ratio being equal to 2, the section area of the fuel pin may be chosen equal to $\frac{\delta^2}{3}$.

The light water used as moderator will be located in the complementary part with section $2 \frac{\delta^2}{3}$.

The well known self-shielded phenomenon makes that the neutron flux is smaller in the fuel than in the moderator.

MC simulations show that the thermal neutron flux is 1.2 times larger in the moderator, but what is important for us is the thermal neutron flux in the fuel pins, since it leads to the actual number of fissions per fuel cm^3 per second and then to the core power.

In the 1D case equation (1) will be replaced by

$$(2) \quad -M^2 \Phi'' + \Phi = k_{\infty} \Phi, \quad 0 < z < L$$

and completed by some boundary conditions (BC). In the following we shall choose the Dirichlet BC :

$$(3) \quad \Phi(0) = \Phi(L) = 0$$

We refer the reader to [3] for the Fourier BC.

Fast neutrons vs thermal neutrons

Even though it is somewhat arbitrary, neutrons with energy $E > 0,625 \text{ eV}$ are said to be *fast* and neutrons with energy $E < 0,625 \text{ eV}$ are called *thermal neutrons*.

In a PWR, the thermal neutron flux is equal to $3.7 \cdot 10^{13} \text{ n/cm}^2/\text{s}$ in average in the fuel pins (see [2] p.382).

The fast neutron flux is about 7 times higher : it is then approximatively equal to $2.7 \cdot 10^{14} \text{ n/cm}^2/\text{s}$ in average in the fuel.

On the contrary, there are about 5 times more fissions in the thermal domain than in the fast domain. The fuel volume being equal to 11.1 m^3 on a 1300 MWe PWR, it means that there are about $1.1 \cdot 10^{13} \text{ fissions/cm}^3/\text{s}$ in the fuel at nominal power 3800 MWth ($1.1 = 0.19 \text{ fast} + 0.91 \text{ thermal}$).

In a one group model, the migration area is evaluated by taking into account both fast and thermal neutrons.

However, only thermal neutrons can be captured by Xenon 135 and this explains why 2 group models have been considered by CEA in the CRONOS software or by EDF in the internal codes COCCINELLE and LIBELLULE.

In the present paper which is dedicated to the one group model, the *solution Φ to equation (1) or (2) will be assumed to be the thermal neutron flux in the fuel.*

We remind the reader that (2) (3) is an eigenvalue problem and then that Φ is defined up to a multiplicative factor.

In our one group model, the *fast neutron flux* and *the thermal neutron flux* are assumed to have the same spatial dependence.

Since Φ is assumed to be the thermal neutron flux in the fuel, we shall have to replace the fission (thermal) macroscopic cross section Σ_f in the fuel by Σ_{ff} for $\Sigma_{ff}\Phi$ to be equal to the number of (fast + thermal) fissions/ cm^3/s .

From reference [2] (p.382) we see that when $\Sigma_f = 0.246 \text{ cm}^{-1}$ we should take $\Sigma_{ff} = 0,297 \text{ cm}^{-1}$.

Introduction of Iodine and Xenon

We have the following evolution equations:

$$(4) \quad \frac{dY}{dt} = \gamma \Sigma_{ff} \Phi - \lambda Y$$

$$(5) \quad \frac{dX}{dt} = \gamma' \Sigma_{ff} \Phi + \lambda Y - (\mu + \sigma \Phi) X$$

where σ is the microscopic capture cross section for Xenon in the thermal domain (evaluated in barns).

As explained above, Φ is *the thermal neutron flux in the fuel*.

In the following, we neglect γ' since $\gamma' \ll \gamma$.

In (4) and (5) X and Y are evaluated in at/cm^3 , Σ_{ff} in cm^{-1} and Φ in $\text{n/cm}^2/\text{s}$.

Obviously, we have $Y = Y(z, t)$, $X = X(z, t)$ and $\Phi = \Phi(z, t)$; $Y(z, t)$ (resp. $X(z, t)$) denotes the number of Iodine (resp. Xenon) atoms per cm^3 of fuel at elevation z , and $\Phi(z, t)$ the *thermal neutron flux* at elevation z in the fuel.

To take into account the (large) ^{135}Xe capture cross section, equation (2) should be replaced by

$$(6) \quad -M^2 \frac{\partial^2}{\partial z^2} \Phi + (1 + \alpha X) \Phi = k_\infty \Phi, \quad 0 < z < L$$

where the parameter α should be evaluated in such a way that, at the nominal flux $\Phi = \Phi_N$, the negative reactivity induced by ^{135}Xe be $\beta = 0.03$, which corresponds to 3000 pcm.

Evaluation of parameter α .

By doing $\Phi(z, t) = \Phi_N(z)$ in equations (4) and (5) and making $t \rightarrow \infty$, we find :

$$0 = \gamma \Sigma_{ff} \Phi_N(z) - \lambda Y(z)$$

$$0 = \lambda Y(z) - (\mu + \sigma \Phi_N(z)) X(z)$$

hence

$$X(z) = \frac{\gamma \Sigma_{ff} \Phi_N(z)}{\mu + \sigma \Phi_N(z)}$$

In view of equation (6), in average, the product $\alpha \cdot X(z)$ should be equal to β .

Let $\Phi_M = \frac{1}{L} \int_0^L \Phi_N(z) dz$, a rough approximation is to choose $\alpha = \beta \frac{\mu + \sigma \Phi_M}{\gamma \Sigma_{ff} \Phi_M}$.

Rescaling

Following [4] we let

$$\Phi(z, t) = \Phi_* \varphi(z, t)$$

$$X(z, t) = X_* x(z, t)$$

$$Y(z, t) = Y_* y(z, t)$$

We also replace t by $\tau = \lambda t$ where λ is the half-life time for ^{135}I β -decay. Then we obtain

$$(7) \quad -M^2 \frac{\partial^2}{\partial z^2} \varphi + (1 + \alpha X_* x) \varphi = k_\infty \varphi, \quad 0 < z < L$$

$$(8) \quad \lambda Y_* \frac{dy}{d\tau} = \gamma \Sigma_{ff} \Phi_* \varphi - \lambda Y_* y$$

$$(9) \quad \lambda X_* \frac{dx}{d\tau} = \lambda Y_* y - (\mu + \sigma \Phi_* \varphi) X_* x$$

so that, if we choose $Y_* = X_*$, we get

$$(10) \quad \lambda \frac{dy}{d\tau} = \frac{1}{X_*} \gamma \Sigma_{ff} \Phi_* \varphi - \lambda y$$

$$(11) \quad \lambda \frac{dx}{d\tau} = \lambda y - (\mu + \sigma \Phi_* \varphi) x$$

Now let

$$\Phi_* = \lambda / \sigma$$

$$a = \mu / \lambda$$

$$\text{and } X_* = \frac{\gamma \Sigma_{ff} \Phi_*}{\lambda} = \frac{\gamma \Sigma_{ff}}{\sigma}$$

we get

$$(12) \quad \frac{dy}{d\tau} = \varphi - y$$

$$(13) \quad \frac{dx}{d\tau} = y - (a + \varphi) x$$

Finally, let $\alpha_* = \alpha X_*$ we get

$$(14) \quad -M^2 \frac{\partial^2}{\partial z^2} \varphi + (1 + \alpha_* x) \varphi = k_\infty \varphi, \quad 0 < z < L$$

X_* has the following interpretation : it is the equilibrium value for the concentration of ^{135}Xe atoms when the neutron flux is infinitely large.

(12)(13)(14) completed with Dirichlet BC is the system which we shall solve.

Time discretization.

At time τ_n we start from the rescaled flux $\varphi(z, \tau_n)$; we solve (12) and (13) between τ_n and τ_{n+1} to obtain y and x at time τ_{n+1} . Then we compute φ at time τ_{n+1} by solving (14).

Note that (14) is an eigenvalue problem which defines φ up to a multiplicative constant. We then scale φ in such a way that the core power is equal to the nominal power.

Note that we solve (14) we actually compute k_{eff} such that

$$(15) \quad -M^2 \frac{\partial^2}{\partial z^2} \varphi + (1 + \alpha_* x) \varphi = \frac{k_\infty}{k_{eff}} \varphi, \quad 0 < z < L$$

and we tune the boron concentration in the moderator so that the core is just critical. In other words we select k_∞ so that (14) holds.

Space-discretization

We introduce a uniform grid with n cells. Let $\Delta z = L/n$.

We use the finite difference method and introduce 3 sets of discrete values : $(\varphi_i(\tau))_{i=0,\dots,n}$ for the neutron flux, $(x_i(\tau))_{i=1,\dots,n-1}$ for xenon, $(y_i(\tau))_{i=1,\dots,n-1}$ for iodine.

We replace (12)(13) (15) by

$$(16) \quad \frac{dy_i}{d\tau} = \varphi_i - y_i$$

$$(17) \quad \frac{dx_i}{d\tau} = y_i - (a + \varphi_i) x_i$$

$$(18) \quad -M^2 \frac{\varphi_{i+1} - 2\varphi_i + \varphi_{i-1}}{(\Delta z)^2} + (1 + \alpha_* x_i) \varphi_i = k_\infty \varphi_i, \quad i = 1, \dots, n-1$$

$$\varphi_0 = \varphi_n = 0$$

Numerical data

$$\lambda = 2.92 \cdot 10^{-5} \text{ s}^{-1}$$

$$\mu = 2.12 \cdot 10^{-5} \text{ s}^{-1}$$

which gives $a \cong 0.725$

$$\gamma = 0.064$$

$$\gamma' = 0 \text{ (simplification)}$$

We have selected $\sigma = 2 \cdot 10^6 \text{ barn}$, then

$$\Phi_* = 1.459 \cdot 10^{13} \text{ n/cm}^2/\text{s},$$

$$X_* = 9.504 \cdot 10^{15} \text{ at/cm}^3$$

$$\alpha_* = 0.0386$$

Solution of the eigenvalue problem by the "shooting technique"

Continuous case (14) :

Let $c = c(z) = 1 + \alpha_* x(z)$

For given $\rho > 0$, we solve the Cauchy problem : find ψ_ρ such that

$$-M^2 \psi_\rho'' + c \psi_\rho = \rho \psi_\rho \text{ pour } z \in]0, L[$$

$$\psi_\rho(0) = 0$$

$$\psi_\rho'(0) = 1$$

Obviously we shall have $\psi_\rho(L) = 0$ if and only if ρ is an eigenvalue. Since we are interested only by the smallest eigenvalue, we look for the smallest positive root of the equation

$$\psi_\rho(L) = 0$$

and for this, we use the dichotomy method.

Discrete case

Let $c_i = 1 + \alpha_* x_i$. For given $\rho > 0$ we solve

$$(19) \quad -M^2 \frac{\varphi_{i+1} - 2\varphi_i + \varphi_{i-1}}{(\Delta z)^2} + c_i \varphi_i = \rho \varphi_i, \quad i = 1, \dots, n-1$$

$$\varphi_0 = 0 \text{ et } \varphi_1 = 1$$

By induction we obtain a value $\varphi_n = \varphi_n(\rho)$ which is not necessarily equal to zero, but we iterate on ρ so as to obtain $\varphi_n(\rho) = 0$.

Then we scale the eigenvector φ_i so obtained in such a way that its average is equal to some specified φ_{ave} . Obviously φ_{ave} depends on the nominal power which we want to consider.

Introduction of control rods

The reactivity control of the core is basically performed by tuning the Boron concentration of the moderator.

However to damp the Xenon oscillations which may appear in the core, as has been shown by Shimazu [6], it may be useful to introduce some control rods in the upper part of the core at some appropriate times and for some time.

In such a case, we just replace $c(z) = 1 + \alpha_* x(z)$ by

$$(20) \quad c(z) = 1 + \alpha_* x(z) + \beta \cdot H(z - z_0)$$

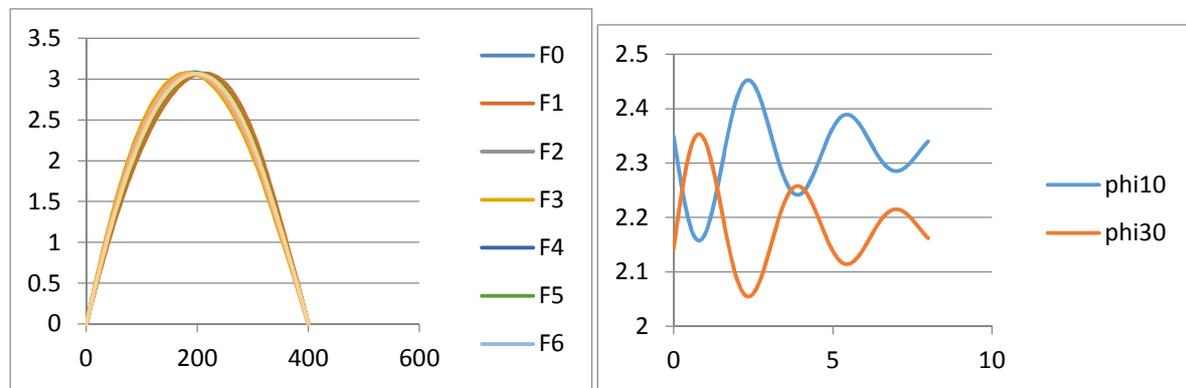
where β is the weight (in pcm) of the control rods, z_0 is the elevation of the lower end of the control rods and H the Heaviside function.

We have selected $\beta = 1100 \text{ pcm}$ which corresponds approximately to the grey control rods.

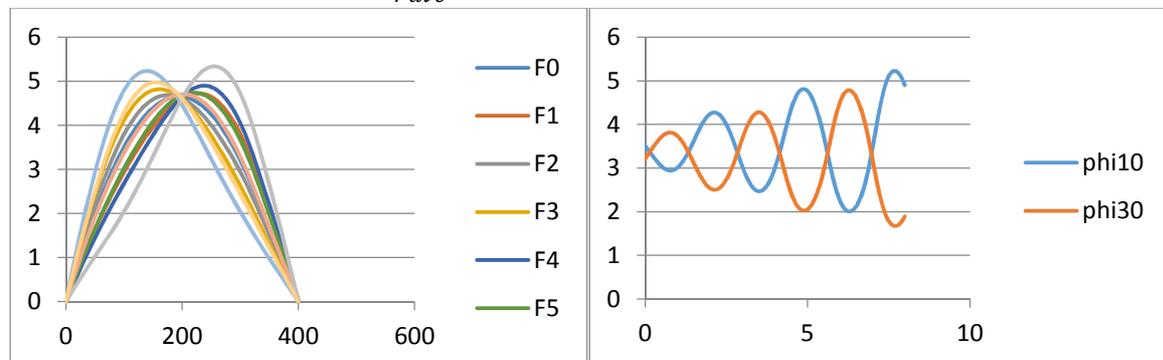
Résultats with $M^2 = 100 \text{ cm}^2$

Starting from an equilibrium solution $x(z), y(z), \varphi(z)$, with the control rods 25% inserted (that is $z_0 = 300 \text{ cm}$) we introduce a perturbation by moving the control rods upwards instantaneously from $z_0 = 300 \text{ cm}$ to $z_0 = 330 \text{ cm}$.

In the following, we plot the Xenon profiles at time $\tau = 0$; $\tau = 0.4$; $\tau = 0.8$... up to time $\tau = 4$, that is $t = 38 \text{ hours}$.



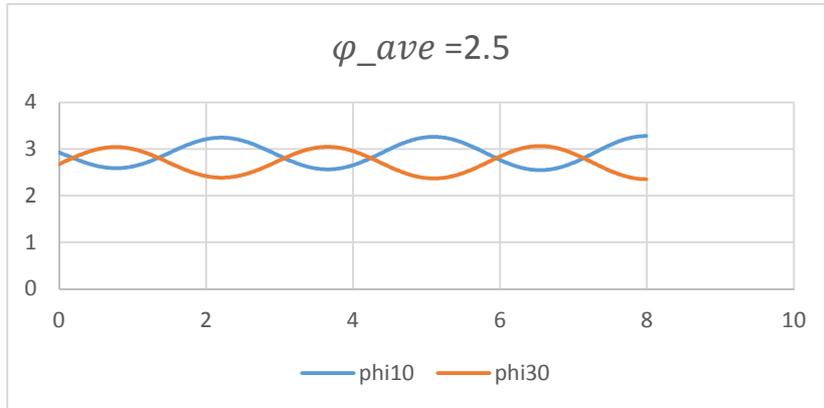
$\varphi_{ave} = 2$



$\varphi_{ave} = 3$

We observe that our system is stable for $\varphi_{ave} < 2.5$ and unstable for $\varphi_{ave} > 2.5$. The period is approximately equal to 28 hours when $\varphi_{ave} = 2$ and to 30 hours when $\varphi_{ave} = 3$ which agrees with Mathonnière's results [1].

As we can see below in the limit case $\varphi_{ave} = 2.5$ the oscillations are not amplified with time.

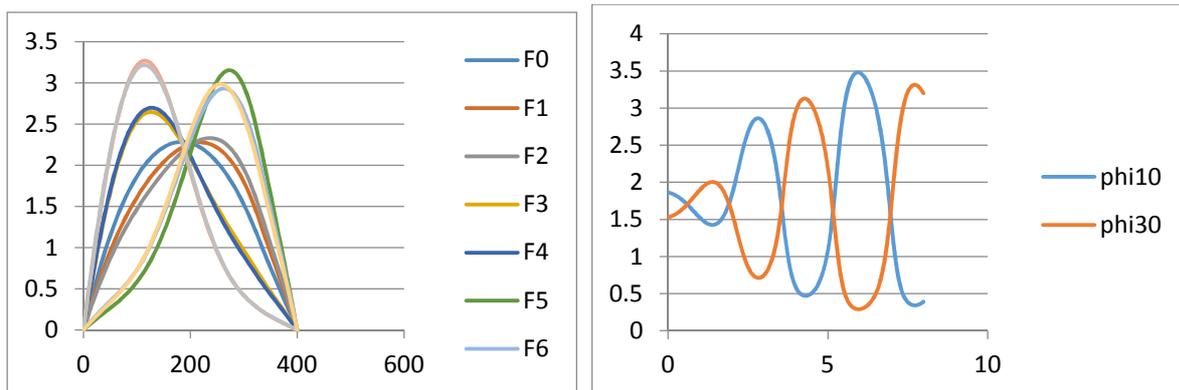


Interpretation : Since $\Phi_* = 1.459 \cdot 10^{13} \text{ n/cm}^2/\text{s}$ the case $\varphi_{ave} = 2.5$ corresponds then to $\Phi_{ave} = 3.64 \cdot 10^{13} \text{ n/cm}^2/\text{s}$ which is precisely what we have for a PWR1300 at full power.

Results with $M^2 = 49 \text{ cm}^2$

Our results show that the instability increases.

Here are the results for $\varphi_{ave} = 1.5$:



We note that the value $\varphi_{ave} = 1.5$ selected here is relatively low : it corresponds to $\Phi_{ave} = 2,19 \cdot 10^{13} \text{ n/cm}^2/\text{s}$. In view of our results, the value $M^2 = 100 \text{ cm}^2$ of the migration area seems more likely.

Now with such a 1D-1group model we do not pretend to get predictive results. However the trends given by our (pedagogic) model are correct : the migration area is obviously a sensitive parameter. The smaller M^2 leads to a more unstable situation. This was predictable since the most relevant parameter for the instabilities is L/M .

Remark : the grid size is not a sensitive parameter: the results with a grid of 40 cells are almost the same as with a 100 cells grid. This means that a grid of 40 cells is precise enough.

Implementing Shimazu's method for rod control

In the following we use a uniform grid with 40 cells.

Since we have $\Delta z = 10 \text{ cm}$ it makes sense to move the control rods by steps of 10 cm or a multiple of 10 cm.

Like in [6], we introduce the axial offsets

$$AO_p = (\varphi_t - \varphi_b)/(\varphi_t + \varphi_b)$$

$$AO_{pi} = (y_t - y_b)/(y_t + y_b)$$

$$AO_{px} = (x_t - x_b)/((x_t + x_b) - 2x_t x_b)$$

where

y_t, y_b are the average values for $y(z)$ (the scaled Iodine concentration) in the upper half and the lower half of the core respectively ;

x_t, x_b are the same but for the scaled Xenon concentration $x(z)$ and

φ_t, φ_b are the same but for the scaled neutron flux concentrations $\varphi(z)$

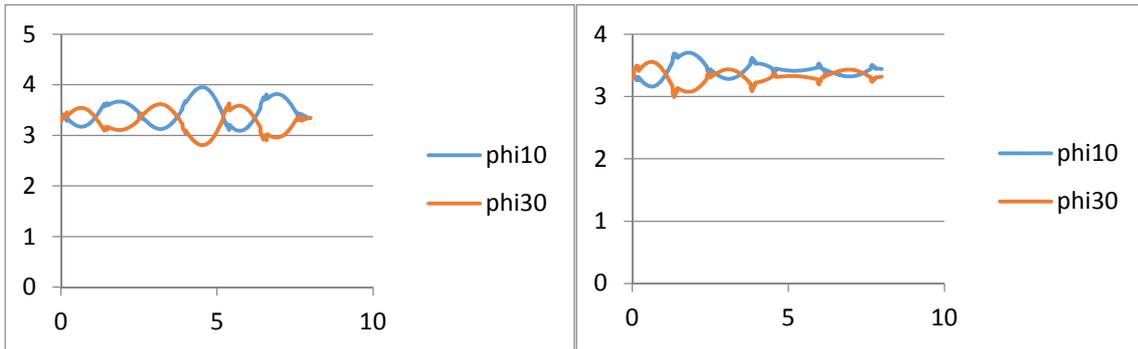
Following Shimazu, when $AO_{pi}-AO_{px}>0$ and $(AO_{pi}-AO_{px})/(AO_p-AO_{px}) < \tan(\varphi_R)$, we shift the control rods downwards for one step (10 cm).

To improve the results we introduced a new rule: in the case $AO_{pi}-AO_{px}>0$ and $(AO_{pi}-AO_{px})/(AO_p-AO_{px}) < \tan(\varphi_R)$, if $AO_{pi}-AO_{px} > 0.2$, then we shift the rods 2 steps downwards rather than 1 step.

In the other case $AO_{pi}-AO_{px}<0$ and $(AO_{pi}-AO_{px})/(AO_p-AO_{px}) < \tan(\varphi_R)$, if $AO_{pi}-AO_{px} < 0.2$, then we shift the rods 2 steps upwards rather than 1 step.

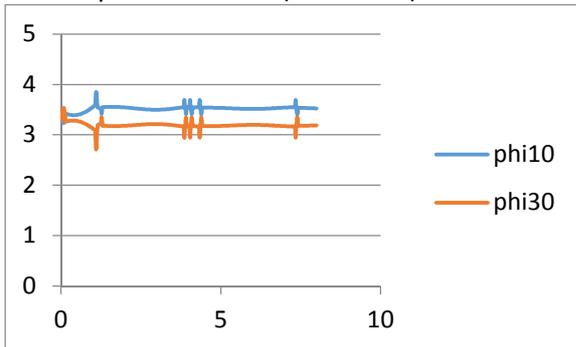
On the other hand, when $AO_{pi}-AO_{px} < 0$ and $(AO_{pi}-AO_{px})/(AO_p-AO_{px}) < \tan(\varphi_R)$ we shift the control rods upwards for one step (10 cm).

Here are some results we obtained with such a control and for the phase $\varphi_R = 0.53158$ (see remark below):



control period $\tau = 0.1$ (i-e 57 min)

control period $\tau = 0.05$ (i-e 28.5 min)



control period $\tau = 0.01$ (i-e 5.7 min)

The results show the efficiency of Shimazu's control law when the control period is sufficiently low.

Note that the control rods can move by 10 cm step, which is significant.

In the case where the control period $\tau = 0.01$, we clearly see when the control rods are moved upwards or downwards.

Remark : To compute φ_R , we proceed following Shimazu .:

$$AO_x - AO_p = (AM_p + AM_x) \exp(-\gamma t) \cos(\omega t - \varphi_p) \quad (12)$$

$$AO_i - AO_x = AM_c \exp(-\gamma t) \sin(\omega t) \quad (13)$$

$$AM_c = \left[\{AM_i \cos(\varphi_i) + AM_x \cos(\varphi_p)\}^2 + \{AM_i \sin(\varphi_i) + AM_x \sin(\varphi_p)\}^2 \right]^{1/2}, \quad (14)$$

where

AM_p, AM_i, AM_x : Amplitudes of AO_p, AO_i , and AO_x , respectively

φ_i, φ_p : Initial phase of AO_i and AO_p , respectively

γ : Stability index of the xenon oscillation

ω : Angular velocity of the oscillation.

First, we carry out a computation without control rods to evaluate the maximal oscillation amplitudes AM_p, AM_i and AM_x for AO_p, AO_i and AO_x respectively.

In our case, we have initially $AO_i = 0$ so that $\varphi_i = 0$.

Then we have

$$\varphi_p = \arccos\left[\frac{(AO_x - AO_p)}{(AM_p + AM_x)}\right]$$

$$AM_c = \left\{ [AM_i + AM_x \cos(\varphi_p)]^2 + [AM_x \sin(\varphi_p)]^2 \right\}^{1/2}$$

$$\varphi_R = 1/2 \arctan\left[2r \sin(\varphi_p) / (1 - r^2)\right]$$

with

$$r = AM_c / (AM_p + AM_x).$$

Conclusion

As a 1D 1-group model, our model is quite simple, then useful for educational purpose.

We show that it is adequate to study the Xenon oscillations, but also the control of such oscillations.

Our model is globally non-linear, but the equation for the flux φ with given Xenon distribution is linear. This is because we do not take the Doppler effect into account.

In our model the really sensitive parameter is the migration area M^2 .

We have tested values of M^2 in the range $49 \leq M^2 \leq 100 \text{ cm}^2$, and we find that the lower M^2 , the more unstable the model.

The oscillations period is approximately equal to 30 hours, in accordance with the experimental results.

We also observe that the higher the average flux the higher the oscillations are, which was expected.

As it is well known, the control laws for a PWR must take Xenon instabilities into account.

We have seen that the famous Shimazu control law is also efficient with our model.

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