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# DOA estimation in fluctuating environments: an approximate message-passing approach

## technical report

Guillaume Beaumont, Ronan Fablet, Angélique Drémeau

### Abstract

In this work, we propose a new algorithm in order to perform direction-of-arrival (DOA) estimation considering waves propagating through a fluctuating medium. In this particular context, the measurements are affected by a phase noise, which can mislead the estimation. It is then important to take into account these phase uncertainties in the estimation procedures. In this paper, we propose to model the phase noise by some random distribution and combine it with a sparse-enforcing distribution on the directions of arrival. These prior models are exploited through a message passing algorithm. Our method relates to previous works in phase recovery and can be seen in particular as an extension of the so-called “prSAMP” algorithm. Once implemented, we evaluated the performance of our algorithm in terms of quality of the reconstruction in comparison with other state-of-the-art algorithms. In particular, the method proves to be more robust to additive noise and present a better behavior than conventional beamforming and recent variational Bayesian algorithms.

### Index Terms

DOA estimation, sparse representation, Bayesian estimation, variational Bayesian approximations, message passing algorithms

## I. INTRODUCTION

Common to many applications such as SONAR, RADAR, and telecommunications, the task of direction-of-arrival (DOA) estimation aims at locating one or more sources emitting in some propagation

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media. Many various methods dealing with this problem have been proposed in the literature. They can roughly be distinguished by the assumptions put on the propagating medium and the sources. Thus, the beamforming approach [1], which constitutes probably the most famous approach, exploits a simple assumption of Gaussian additive noise, while the so-called “high-resolution” techniques consider in addition some assumptions over the number or the nature of the sources. This is the case of the well-known MUSIC method [2], based on strong assumptions on the number of sources and the separability of the sub-spaces where noise and signal are assumed to live. More recently, we can also mention the “compressive” beamforming approaches proposed in [] that benefit from an explicit sparse model on the sources.

In addition to these assumptions, recent work has focused on the integration of phase noise models. The objective of such approaches is to take into account the wave front distortion occurring when waves travel through fluctuating media. This case study is observed in many application areas as in acoustics [3], space observation or communications [4].

These contributions mainly relate to recent advances in phase recovery (see e.g. [5]) and extend them in the sense that they put informative prior on the missing phases. In this respect, we can mention the work [] which proposes a method inspired by the PhaseCut algorithm [] and the Bayesian approach [] based on a mean-field approximation.

Part of the continuity of this latter work, we propose here a novel approach, based on the “swept approximate message passing” (SAMP) algorithm introduced in [].

## II. PROBLEM STATEMENT

In this section, we recall the Bayesian modeling introduced in [], which we shall follow in the remaining of the paper, and introduce the estimation problem we propose to solve.

### A. Observation Model

We consider an antenna composed of  $N$  sensors. Then, the received signal at sensor  $n$  can be expressed as

$$y_n = e^{j\theta_n} \left( \sum_{m=1}^M d_{nm} x_m + \omega_n \right), \quad (1)$$

where  $\theta_n$  stands for the phase noise due to the propagation through the fluctuating medium and  $\omega_n$  is an additive noise. The scalar  $d_{nm}$  is the  $n$ -th element of the steering vector  $\mathbf{d}_m =$

$[e^{j\frac{2\pi}{\lambda}\Delta\sin(\phi_m)} \dots e^{j\frac{2\pi}{\lambda}\Delta N\sin(\phi_m)}]^T$  where the  $\phi_m$ 's are some potential angles of arrival,  $\Delta$  is the distance between two adjacent sensors and  $\lambda$  is the wavelength of the propagation waves.

At each sensor of the antenna, we thus assume that the received field is a combination of a few waves arriving from different angles  $\phi_m$ . The DOA estimation problem consists then in identifying the positions of the non-zero coefficients in  $\mathbf{x} \triangleq [x_1 \dots x_m]^T$ . To model the sparse assumption on the number of the sources, we adopt a Bernoulli-Gaussian model

$$p(x_m) = \rho_m \mathcal{CN}(x_m; 0, \sigma_x^2) + (1 - \rho_m)\delta_0(x_m), \quad (2)$$

where  $\rho_m$  is the Bernoulli parameter and equals the probability for  $x_m$  to be non-zero,  $\mathcal{CN}(x_m; 0, \sigma_x^2)$  stands for the centered circular complex Gaussian distribution with variance  $\sigma_x^2$  and  $\delta_0(x_m)$  for a Dirac distribution. The Bernoulli-Gaussian model is now widely used when considering Bayesian inference methods for sparsity-constrained problems (see *e.g.* []).

To model spatial fluctuations all along the antenna, we assume in addition that the phase noise obeys the following Markov model

$$p(\theta_n|\theta_{n-1}) = \mathcal{N}(a\theta_{n-1}, \sigma_\theta^2), \quad \forall n \in \{2, \dots, N\}, \quad (3)$$

$$p(\theta_1) = \mathcal{N}(0, \sigma_1^2), \quad (4)$$

with  $a \in \mathbb{R}_+$ . Note that variance  $\sigma_\theta^2$  allows us to take into account the strength of the fluctuations.

Finally, the additive noise  $\omega_n$  is assumed to be circular complex Gaussian of variance  $\sigma_n^2$ .

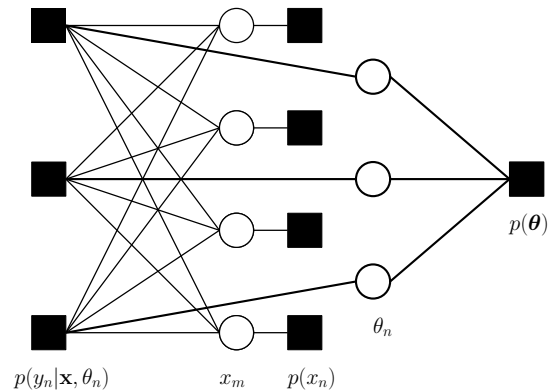


Fig. 1. Factor graph of the observation model

To illustrate model (1)-(4), Figure 1 gives the corresponding factor graph. White circles denote random variables when black squares are for the probability distribution factors.

### B. Bayesian formulation of the problem

Within model (1)-(4), we focus on the following Minimum Mean Square Error (MMSE) problem:

$$\hat{\mathbf{x}} = \underset{\tilde{\mathbf{x}}}{\operatorname{argmin}} \int_x \|\mathbf{x} - \tilde{\mathbf{x}}\|_2^2 p(\mathbf{x}|\mathbf{y}) d\mathbf{x} \quad (5)$$

where  $\mathbf{y} \triangleq [y_1 \dots y_N]^T$  and

$$p(\mathbf{x}|\mathbf{y}) = \int_{\boldsymbol{\theta}} p(\mathbf{x}, \boldsymbol{\theta}|\mathbf{y}) d\boldsymbol{\theta} \quad (6)$$

with  $\boldsymbol{\theta} \triangleq [\theta_1 \dots \theta_N]^T$ .

To solve this problem, we propose to resort to a Bayesian inference strategy, that approximates the posterior distributions by distributions having suitable factorizations. In [], a mean-field approximation  $p(\mathbf{x}, \boldsymbol{\theta}|\mathbf{y}) \simeq q(\boldsymbol{\theta}) \prod_{m=1}^M q(x_m)$  was considered. In this paper, we are interested in a different type of factorization, relating to the ‘‘approximate message passing’’ (AMP) algorithms []. We motivate and detail our approach in the next section.

## III. PROPOSED PA-SAMP APPROACH

### A. Motivation and main principles of the approach

AMP algorithms have been considered since a few years as a serious solution to linear problems under sparsity-constraints. First considered in the sole case of random matrices<sup>1</sup>, they have been recently improved by the so-called ‘‘swept approximate message passing’’ (SAMP) approach, which helps alleviate the convergence issues of AMP.

Both AMP and SAMP have been extended to generalized but component-wise measurement models []. They have been then successfully applied to the phase recovery task where  $\theta_n \sim \mathcal{U}[0, 2\pi]$ ,  $\forall n \in \{1, \dots, N\}$ , given raise to the prGAMP [] and the prSAMP [] algorithms. In particular, the latter has proved to outperform other state-of-the-art algorithms among which the mean-field approximation [].

The prSAMP algorithm constitutes thus a promising approach for our DOA estimation problem (5). However, in our study case, the phases  $\theta_n$ 's are considered as dependent on each other (as highlighted by figure 1). This prevents us from a direct adaptation of prSAMP.

We thus propose an iterative algorithm based on the two following mathematical derivations:

- i) extend prSAMP to a *i.i.d.* Gaussian prior on the phases,
- ii) use a mean-field approximation to estimate the (Gaussian) posterior distribution on the phases.

We detail both operations in the next two sub-sections.

<sup>1</sup>Namely, the elements  $d_{nm}$  are assumed to be *i.i.d.* randomly distributed.

### B. Extension of prSAMP to i.i.d. Gaussian phases

For a sake of clarity and due to space limitation, we will adopt and refer the reader to the notations of paper [] which introduced the SAMP algorithm.

Letting  $z_n = \sum_{m=1}^M d_{nm}x_m, \forall n \in \{1, \dots, N\}$  and assuming that the  $z_n$ 's follow Gaussian distributions of mean  $\omega_n$  and variance  $V^2$  as mixtures of Bernoulli-Gaussian distributions, we can write the following version of Bayes Formula

$$p(z_n|y_n, \omega_n, V) = \frac{p(y_n|z_n)\mathcal{CN}(z_n; \omega_n, V)}{\int_{z'_n} p(y_n|z'_n)\mathcal{CN}(z'_n; \omega_n, V)}. \quad (7)$$

By integrating over the realizations of  $z_m$  and then over  $\theta_m$  and with the use of Von Mises approximations [6] to compute the momentums of the distribution, we can obtain the following formulas :

$$E_{Z|Y}\{z_n|y_n, \omega_n, V\} = \frac{y_n V e^{-j\mu_{\theta_n}^z}}{\sigma_n^2 + V} \mathbf{R}_0\left(\frac{1}{\Sigma_{\theta}^z}\right) + \frac{\omega_n \sigma^2}{\sigma^2 + V}, \quad (8)$$

$$\text{var}_{Z|Y}\{z_n|y_n, \omega_n, V\} = \frac{|y_n V^2 e^{-j\mu_{\theta_n}^z} + \omega_n \sigma^2|^2}{|\sigma^2 + V|^2} \mathbf{R}_0\left(\frac{1}{\Sigma_{\theta}^z}\right) + \frac{V \sigma^2}{\sigma^2 + V} - E_{Z|Y}\{z_n|y_n, \omega_n, V\}^2, \quad (9)$$

with

$$\frac{1}{\Sigma_{\theta}^z} = \frac{1}{\alpha} + \frac{1}{\Sigma_{\theta_n}}, \quad \mu_{\theta_n}^z = \frac{-\frac{\arg(y_n^* \omega_n)}{\alpha} + \frac{\mu_{\theta_n}}{\Sigma_{\theta_n}}}{\frac{1}{\alpha} + \frac{1}{\Sigma_{\theta_n}}}, \quad \alpha = \frac{V + \sigma^2}{|y_n| |\omega_n|},$$

$\mu_{\theta_n}$  (resp.  $\Sigma_{\theta_n}$ ) is the marginalized posterior mean (resp. variance) of the phase noise  $\theta_n$  as discussed in the next section, and  $\mathbf{R}_0(\cdot) = \frac{I_1(\cdot)}{I_0(\cdot)}$  where  $I_n(\cdot)$  is the modified Bessel function of the first kind at order  $n$ . We refer the reader to our technical report [] which details the derivations of the computations.

Another distribution we have to compute is  $p_{x|R}(x|\hat{r}; \mu^r)$ , is the *a-posteriori* distribution of  $\mathbf{x}$  regarding propagation of the Gaussian fields propagated by the model, the calculation of the momentum defined as  $g_{in}$  and  $g'_{in}$  will follow the works presented in [7].

### C. Mean-field approximation for the phase noise

To simplify the computation of the posterior distribution of the phase noise  $\theta$ , we propose in this step to resort to a mean-field approximation. Thus, we define

$$q(\theta) \propto p(\theta) \exp\left(\int_{\mathbf{x}} p(\mathbf{x}|\hat{r}; \mu^r) \log p(\mathbf{y}|\mathbf{x}, \theta) d\mathbf{x}\right). \quad (10)$$

Particularized to the previous derivations, we get, in a similar way as in []

$$q(\boldsymbol{\theta}) = \mathcal{N}(\boldsymbol{\theta}; \boldsymbol{\mu}_\theta, \Sigma_\theta), \quad (11)$$

$$\text{where } \Sigma_\theta^{-1} = \Lambda_\theta^{-1} + \text{diag}\left(\frac{2}{\sigma^2}|\boldsymbol{\eta}|\right), \quad (12)$$

$$\boldsymbol{\mu}_\theta = \Sigma_\theta \left( \text{diag}\left(\frac{2}{\sigma^2}|\boldsymbol{\eta}|\right) \text{arg}(\boldsymbol{\eta}) \right), \quad (13)$$

$\boldsymbol{\eta} \triangleq [\eta_1 \dots \eta_N]^T$  with  $\eta_n = y_n E_{Z|Y}^* \{z_n | y_n, \omega_n, V^2\}$ , and  $\Lambda_\theta^{-1}$  is the precision matrix attached to the prior distribution (4) on  $\boldsymbol{\theta}$ , *i.e.*

$$\Lambda_\theta^{-1} = \begin{pmatrix} \frac{1}{\sigma_1^2} + \frac{a^2}{\sigma_\theta^2} & -\frac{a}{\sigma_\theta^2} & 0 & 0 \\ -\frac{a}{\sigma_\theta^2} & \frac{1+a^2}{\sigma_\theta^2} & \ddots & 0 \\ 0 & \ddots & \ddots & -\frac{a}{\sigma_\theta^2} \\ 0 & 0 & -\frac{a}{\sigma_\theta^2} & \frac{1}{\sigma_\theta^2} \end{pmatrix}. \quad (14)$$

Note that the distribution  $q(\boldsymbol{\theta})$  being Gaussian, the marginals  $q(\theta_n)$  used in the prSAMP-step of the algorithm come straightforwardly as

$$q(\theta_n) = \mathcal{N}(\theta_n; \mu_{\theta_n}, \Sigma_{\theta_n}) \quad (15)$$

where  $\mu_{\theta_n}$  (resp.  $\Sigma_{\theta_n}$ ) is the  $n$ th element in  $\boldsymbol{\mu}_\theta$  (resp. in the diagonal of  $\Sigma_{\theta_n}$ ).

#### D. Additive noise estimation

In order to refine our estimation we propose to estimate  $\sigma^2$ , the second order momentum of the additive noise, according to the maximum likelihood criterion of the posterior distribution, we finally have to find :

$$\hat{\sigma}^2 = \underset{\sigma^2}{\text{argmax}} \int_{\mathbf{z}, \boldsymbol{\theta}} p(\mathbf{z}, \boldsymbol{\theta} | \mathbf{y}) \log(p(\mathbf{y}, \mathbf{z}, \boldsymbol{\theta}; \sigma^2)) d\mathbf{z} d\boldsymbol{\theta} \quad (16)$$

After some calculus<sup>2</sup>, we obtain

$$\hat{\sigma}^2 = \frac{1}{N} \left( \mathbf{y}^H \mathbf{y} - 2\mathcal{R}(\bar{\mathbf{y}}^H \mathbf{E}_{Z|Y}) + \mathbf{E}_{Z|Y}^H \mathbf{E}_{Z|Y} + \mathbf{var}_{Z|Y}^H \mathbf{var}_{Z|Y} \right) \quad (17)$$

with

$$\bar{\mathbf{y}} = \left[ y_n e^{-j\mu_{\theta_n}} \mathbf{R}_0 \left( \frac{1}{\Sigma_\theta} \right) \right]_{n=\{1, \dots, N\}}, \quad (18)$$

$$\mathbf{E}_{Z|Y} = [E_{Z|Y}^* \{z_1 | y_1, \omega_1, V^2\} \dots E_{Z|Y}^* \{z_N | y_N, \omega_N, V^2\}]^T.$$

<sup>2</sup>We refer here again the reader to our technical report []

In the following, we refer to the proposed two-procedure as “paSAMP” (ou PARMESAN) for “phase-aware SAMP algorithm”. The pseudo-code of paSAMP is presented in **Algorithm 1** with the references to the intermediary variables ( $\{\omega_n\}_n, V^2 \dots$  A COMPLETER) introduced above.

With the two function phase. estimation and additive.noise. estimation which follows the previously evoked calculations (see 3.2 & 3.3).

#### IV. NUMERICAL EXPERIMENT

After these calculations we proceeded to a MATLAB implementation. Now we will compare its performances regarding other algorithms which propose to solve the same phase-retrieval problem with bayesian approaches.

##### A. Testing environment

In order to evaluate those performances, we fixed model parameters. We will consider a 256 sensors (N) array measuring random projections of  $\mathbf{x}$  of size 64. For this sparse vector, we consider two cases. The first vector considers only two active sources, known as a critical cases for DOA techniques. The second case consider five active sources to check on the benefits of pa-SAMP with the structured noise model.

We also represent the structured phase noise model of parameters  $\sigma_{\theta_1} = 10^6$  et  $\sigma_{\theta} = 1$  and an interdependence factor  $a = 0.8$  to represent the high uncertainty of angle measurement and also the physical link between two space-consecutive angle measurements.

In order to characterize the performances of this algorithm we consider the normalized correlation between the true vector  $\mathbf{x}$  and it’s estimation  $\hat{\mathbf{x}}$  regarding to the SNR of the received signal after 500 iterations of each algorithm.

We consider 100 reconstruction results per noise value and the momentum of the normalized correlation which are resulting.

We chose to compare this results regarding the the original AMP algorithm; i.e pr-SAMP[5], an EM-type algorithm based on a similar model, pa-VBEM[8], and the performances of conventional beamforming.

##### B. Results

After applying the previous testing environment on our algorithm we finally obtain the following results:



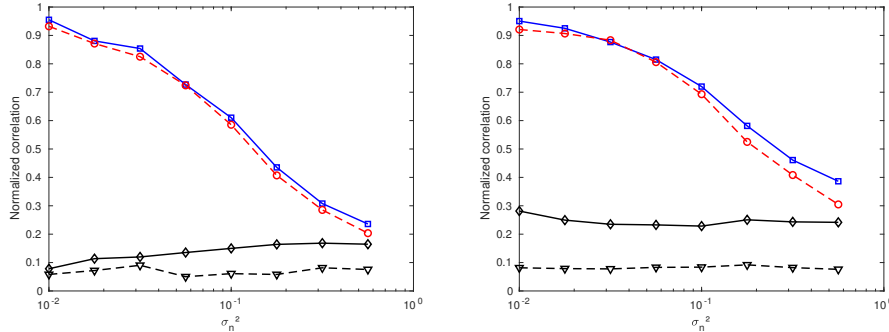


Fig. 2. Normalized correlation of  $\mathbf{x}$  estimation regarding additive noise, with 2 sources (left) and 5 sources (right)

The performance illustrated in **Fig. 2** shows us that the conventional beamforming algorithm and the Pr-SAMP algorithm fail to reconstruct  $\mathbf{x}$  properly as the linear model is no longer used. Which underline the importance of the new "physically-based" DOA estimation algorithms in order to tackle with the fluctuation issue.

We can see that for the critical case of two sources, Pa-SAMP (squares blue) and pa-VBEM (circles red) offers similar performances.

We can also see that performances of the Pa-SAMP (squares blue) offers better performances than CB (triangles black), pr-SAMP (diamonds black) and pa-VBEM (circles red), especially when dealing with important additive noise, i.e realistic noises. Even if they both integrates the same knowledge of priors over  $\mathbf{x}$  and  $\theta$ , Pa-SAMP seems more accurate than pa-VBEM considering the normalized correlation between the true  $\mathbf{x}$  and the reconstructed  $\mathbf{x}$ .

## V. CONCLUSION

We have presented in this paper a novel Belief Propagation able to perform DOA in a corrupted phase noisy environment. This approach relied on Bethe approximations and strong assumption over the sparsity on  $\mathbf{x}$  and on the structure of the covariance matrix of the multiplicative noise. We confronted it to conventional Beamforming and pa-VBEM which is known to have good performances with this kind of model. Regarding the good performances of this algorithms, we hope to include further assessment like the estimation of the physical parameter, multi-frequential estimation and taking temporal evolution of the fluctuations into account.

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**Algorithm 1** Pa-SAMP Algorithm
 

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**Require:**  $\mathbf{y}$ ,  $\mathbf{D}$ ,  $\sigma^2$ ,  $\rho$ ,  $\sigma_x^2$ ,  $\boldsymbol{\mu}_\theta$ ,  $\Sigma_\theta$ ,  $T_{max}$ 
**Define:**

$$g_{out,m}(\omega, V) = \frac{1}{\mu_p} (E_{Z|Y,P}\{Z|y_m, \omega, V\} - \omega)$$

$$g'_{out,m}(\omega, V) = \frac{1}{V} \left( \frac{\text{var}_{Z|Y,P}\{Z|y_m, \omega, V\}}{V} - 1 \right)$$

$$p_{X|\mathbf{Y}}(x|\mathbf{y}; R, \mu^r) = \frac{p_X(x)\mathcal{N}(x; R, \mu^r)}{\int_{x'} p_X(x')\mathcal{N}(x'; R, \mu^r)}$$

$$g_{in,n}(R, \mu^r) = E_{X_n|R_n}\{X_n|R; \mu^r\}$$

$$g'_{in,n}(R, \mu^r) = \text{var}_{X_n|R_n}\{X_n|R; \mu^r\}$$

- 1: **while**  $t < T_{max}$  **do**
  - 2:     **for**  $m = 1, 2, 3 \dots M$  **do**
  - 3:          $\hat{z}_m(t) = \sum_{n=1}^N d_{mn} a_n(t)$
  - 4:          $V^2(t) = \sum_{n=1}^N |d_{mn}|^2 v_n(t)$
  - 5:          $\omega(t) = \hat{z}_m(t) - V \cdot g_{out}(t-1)$
  - 6:          $\hat{u}_m(t) = g_{out}(y_m, \omega(t), V(t))$
  - 7:          $\mu_m^u(t) = -g'_{out}(y_m, \omega(t), V(t))$
  - 8:         **for**  $n = 1, 2, 3 \dots N$  **do**
  - 9:              $\mu_n^r(t) = (\sum_{m=1}^N |d_{mn}|^2 \mu_m^u(t))^{-1}$
  - 10:              $R_n(t) = a_n(t) + \mu_n^r(t) \sum_{m=1}^M |d_{mn}| \hat{u}_m(t)$
  - 11:              $v_n(t+1) = \mu_n^r(t) g'_{in}(R_n(t), \mu_n^r(t))$
  - 12:              $a_n(t+1) = g_{in}(R_n(t), \mu_n^r(t))$
  - 13:             **for**  $m = 1, 2, 3 \dots M$  **do**
  - 14:                  $V_m^{n+1}(t+1) =$
  - 15:                  $V_m^n(t+1) + d_{mn}^2 (v_n(t+1) - v_n(t))$
  - 16:                  $\omega_m^{n+1}(t+1) =$
  - 17:                  $\omega_m^{n+1}(t) + d_{mn} (a_n(t+1) - a_n(t))$
  - 18:             **end for**
  - 19:         **end for**
  - 20:         update  $\sigma^2$  according to (20).
  - 21:         update  $[\theta_{m_m}, \Sigma_{\theta_m}]$  according to (14-15).
  - 22:     **end for**
  - 23: **end while**
-