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Torque ripple reductions for non-sinusoidal BEMF Motor : An observation based control approach ^{*}

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Abstract: Torque ripples are usually found in brushless DC (BLDC) motor control applications. This article presents the design of non-sinusoidal back electromotive forces (BEMF) observer for a BLDC motor in order to reduce electromechanical torque ripples. For their robustness quality, a high order sliding mode observer is used to observe the non-sinusoidal BEMF which allows to determine magnetic field angular relative position leading to a so-called extended Park's transformation. Using this extended transformation, speed and torque vector control can be made with better performances than the classical Park's transformation. Simulation results are performed to show the effectiveness of the proposed approach.

Keywords: BLDC, non-sinusoidal BEMF, extended Park's transformation, torque ripple diminution, high order sliding mode observer, vector control.

1. INTRODUCTION

Brushless DC (BLDC) motors are non-sinusoidal back electromotive forces (BEMF) synchronous motors. They are widely used due to their higher power density and lower cost than sinusoidal BEMF synchronous motors (or permanent magnet synchronous motors: PMSM). BLDC motors can thus be advantageous in some embedded applications. Generally, BLDC motors are controlled by six step control (Firmansyah et al., 2014; Chern et al., 2010) or Direct Torque Control (DTC) (Shao et al., 2015; Ebin Joseph and Sreethumol, 2015) using Hall sensor (Pillay and Krishnan, 1991). The drawback of these controls are low position or speed accuracy and high torque ripples. The well know vector control for PMSM can be used for BLDC.

The vector control for BLDC can obtain better performances than six steps or DTC, but the torque ripple is important due to non-sinusoidal BEMF shape. In (Grenier and Louis, 1995), an extended Park's transformation is introduced to model non-sinusoidal PMSM. The vector control using this model is proposed for a BLDC in (Lidozzi et al., 2008). It shows that the torque ripple can be considerably reduced compared to the vector control using classical Park's transformation, but requires the knowledge of non-sinusoidal BEMF which can be determined by a look-up table obtained from BEMF measurements or BEMF waveform Fourier transform (Angelo et al., 2005). In parallel, intensive research has focused on senseless control for PMSM. In this case, sensorless refers to systems that do not have a position sensor nor velocity, although current sensors are still assumed to be available. Among

the approach found in the literature, BEMF observers based approach is an important one such as Extended Kalman Filter (Bolognani et al., 2001), linear observer (Son et al., 2002), nonlinear observer (Ortega et al., 2011), adaptive interconnected observer (Hamida et al., 2013) and sliding mode observers (Ezzat et al., 2010; Delpoux and Floquet, 2014). Although not exhaustive, this list gives an idea of the issues. These approaches have been extended to non-sinusoidal BEMF sensorless approach in (Baratieri and Pinheiro, 2014a,b, 2016).

This paper treats the case where position and velocity measures are available. Indeed, for some industrial applications, the speed sensor is mandatory for security issue and the torque quality is crucial. For this reason, the extended Park's transformation is exploited. The difference with (Grenier and Louis, 1995) is that the BEMF waveform is observed and do not require a priori knowledge. Moreover the proposed approach uses a model based on sinusoidal BEMF to estimate the displacement angle between a sinusoidal BEMF and the actual non-sinusoidal BEMF. With respect to (Baratieri and Pinheiro, 2016), the proposed approach, although the use of the position measurement, benefits from a model with low frequency variables. Moreover, it circumvents the problem of zero speed observability, which remains a challenge in sensorless control. The proposed observer is based on the electrical equation of the motors like what is done traditionally in the sensorless control, however here the objective is not to estimate the position and velocity but the non-sinusoidal BEMF to reduce torque ripple. Due to non linearities, sliding mode observer theory developed in (Davila et al., 2005) is applied using current measurement and mechanical sensor. The proposed sliding mode observer is inspired from the works presented in (Ezzat et al., 2010; Delpoux and Floquet, 2014). The estimated extended rotating frame allows to

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preserve the principle of sinusoidal PMSM vector control with less torque ripples.

The paper is organized as follows: in section 2, different models of non-sinusoidal BLDC are given to show the problem statement. The proposed observer is detailed in section 3 and observer convergence is given. In section 4, the simulation results of the proposed observer and the vector control in the new extended rotating frame compared with the classical vector control show the effectiveness of the proposed solution. Finally, conclusions are summarized in section 5.

2. PROBLEM STATEMENT

2.1 Motor model in the fixed (a, b, c) frame

The model in the fixed (a, b, c) frame for the case of non-salient poles is given as:

$$L \frac{di_{abc}}{dt} = v_{abc} - Ri_{abc} - e_{abc}, \quad (1a)$$

$$J \frac{d\omega}{dt} = \tau_{em} - \tau_{ext}, \quad (1b)$$

where i_{abc} , v_{abc} and e_{abc} represent respectively the phase currents, voltages and BEMF. Here and in the rest of the article, a vector $x_{ij\dots}$ denotes the vector made from variables x_i, x_j, \dots such that $x_{ij\dots} := [x_i, x_j, \dots]^T$. The variables θ and ω are the mechanical angular position and speed. The parameters L and R are the motor phase inductance and resistance, J is the total moment of inertia and τ_{ext} the external torque, including load and friction torque. Due to non-sinusoidal BEMF, the variable e_{abc} is expressed as

$$e_{abc} = p\omega\Phi_f \begin{bmatrix} f(p\theta) \\ f\left(p\theta - \frac{2\pi}{3}\right) \\ f\left(p\theta + \frac{2\pi}{3}\right) \end{bmatrix}, \quad (2)$$

where Φ_f is the rotor magnetic flux, p is the number of pole pairs and $f(p\theta)$ refers to the $\frac{2\pi}{p}$ periodic non-sinusoidal function describing BEMF normalized waveform. Finally, the electromechanical torque τ_{em} is given as:

$$\tau_{em} = \frac{1}{\omega} e_{abc}^T i_{abc}. \quad (3)$$

Remark 1. In this article, star-connected BLDC motors are considered such that

$$\sum_k i_k = 0, k \in \{a, b, c\}.$$

2.2 Equivalent two-phase (α, β) representation

Three motor phase electrical equations can be projected onto a two-dimensional system of axes. In this article one consider the Clarke's transformation given by

$$x_{\alpha\beta} = \frac{2}{3} C_{23} x_{abc}, \quad C_{23} = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}. \quad (4)$$

The Clarke's transformation is considered because the balanced three-phase quantities are converted into balanced

two-phase orthogonal quantities by keeping the amplitude of the variables. This implies that the power is not kept. The model in this frame is given by :

$$L \frac{di_{\alpha\beta}}{dt} = v_{\alpha\beta} - Ri_{\alpha\beta} - e_{\alpha\beta}, \quad (5a)$$

$$J \frac{d\omega}{dt} = \tau_{em} - \tau_{ext}, \quad (5b)$$

where

$$e_{\alpha\beta} = p\omega\Phi_f f_{\alpha\beta}(\theta), \quad (6)$$

and

$$\tau_{em} = -\frac{3}{2\omega} e_{\alpha\beta}^T i_{\alpha\beta}. \quad (7)$$

Note that $f_{\alpha\beta}(\theta)$ remain a $\frac{2\pi}{p}$ periodic vector.

2.3 Rotating frame transformation

The Rotating frame transformation, called (d, q) frame, based on the Park's transformation is commonly used for synchronous motor in general. It is defined as :

$$x_{dq} = P(\theta)x_{\alpha\beta}, \quad P(\theta) = \begin{bmatrix} \cos(p\theta) & \sin(p\theta) \\ -\sin(p\theta) & \cos(p\theta) \end{bmatrix}. \quad (8)$$

For PMSM, case where sinusoidal BEMF, it results in constant voltages and currents at constant speed (instead of the high-frequency phase variables). Also, the model highlights the role of the quadrature current i_q in the torque determination. In the case of non-sinusoidal BEMF, the resulting equations in this frame are:

$$L \frac{di_{dq}}{dt} = v_{dq} - Ri_{dq} - Lp\omega\mathcal{J}i_{dq} - e_{dq}, \quad (9a)$$

$$J \frac{d\omega}{dt} = \tau_{em} - \tau_{ext}, \quad (9b)$$

where the matrix \mathcal{J} denotes the rotation matrix with $\pi/2$ angle given by $\mathcal{J} := \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$,

$$e_{dq} = p\omega \cdot \Phi_f f_{dq}(\theta), \quad (10)$$

and

$$\tau_{em} = \frac{3}{2\omega} e_{dq}^T i_{dq}. \quad (11)$$

One remarks that in (9a), unlike the sinusoidal modeling, e_d is nonzero. It is a periodic function dependent on the rotor position θ and the non-sinusoidal BEMF (Lidozzi et al., 2008). In order to preserve the advantage of Park's transformation in the case of a non-sinusoidal BEMF, the Park's transformation (8) is extended with a parameter μ leading to

$$x_{d'q'} = P(\theta + \mu)x_{\alpha\beta} = P(\mu)x_{dq}. \quad (12)$$

This transformation is known as extended Park's transformation, introduced in (Grenier and Louis, 1995). The frame (d', q') is called extended rotating frame. The parameter μ represents the displacement angle between a sinusoidal BEMF and the actual non-sinusoidal BEMF and is used to force $e'_d = 0$. In the extended rotating frame, the model is defined as:

$$L \frac{di_{d'q'}}{dt} = v_{d'q'} - Ri_{d'q'} - Lp \left(\omega + \frac{d\mu}{dt} \right) \mathcal{J}i_{d'q'} - e_{d'q'}, \quad (13a)$$

$$J \frac{d\omega}{dt} = \tau_{em} - \tau_{ext}, \quad (13b)$$

where

$$e_{d'q'} = p\omega \Phi_f \begin{bmatrix} 0 \\ f'_{q'} \end{bmatrix}, \quad (14)$$

and

$$\tau_{em} = \frac{3}{2\omega} i_{q'} e_{q'} = \frac{3}{2} p \Phi_f i_{q'} f_{q'}. \quad (15)$$

The electromechanical torque highlights in this case the role of $i_{q'}$ in the torque determination. From (12) and (14) one has:

$$e_{d'} = 0; \quad e_{q'} = \sqrt{e_d^2 + e_q^2}, \quad (16)$$

$$\sin(p\mu) = -\frac{e_d}{\sqrt{e_d^2 + e_q^2}}; \quad \cos(p\mu) = \frac{e_q}{\sqrt{e_d^2 + e_q^2}}. \quad (17)$$

From (17), the extended Park's transformation $P(\mu)$ can be expressed as:

$$P(\mu) = \begin{bmatrix} \frac{e_q}{\sqrt{e_d^2 + e_q^2}} & -\frac{e_d}{\sqrt{e_d^2 + e_q^2}} \\ \frac{e_d}{\sqrt{e_d^2 + e_q^2}} & \frac{e_q}{\sqrt{e_d^2 + e_q^2}} \end{bmatrix}, \quad (18)$$

$$\mu = \begin{cases} \frac{1}{p} \arctan\left(-\frac{e_d}{e_q}\right) & \text{if } \cos(p\mu) > 0, \\ \frac{\pi}{p} & \text{if } \sin(p\mu) = 1, \\ -\frac{\pi}{p} & \text{if } \sin(p\mu) = -1, \\ \frac{1}{p} \left(\arctan\left(-\frac{e_d}{e_q}\right) - \pi \right) & \text{if } \cos(p\mu) < 0, \\ & \text{and } \sin(p\mu) > 0, \\ \frac{1}{p} \left(\arctan\left(-\frac{e_d}{e_q}\right) + \pi \right) & \text{if } \cos(p\mu) < 0, \\ & \text{and } \sin(p\mu) < 0, \end{cases} \quad (19)$$

and depends only on variables e_d and e_q .

The different frames presented in this section are summarized in Fig. 1.

2.4 Problem definition

One remarks that the control of motor with non-sinusoidal BEMF can use classical vector control, since the model is expressed in the extended rotating frame. However, the expression of this frame requires the knowledge of the BEMF.

In the literature, there are many articles dealing with the observation of BEMF for mechanical sensorless control. In this paper, the mechanical position being known, the objective is to estimate the difference between the non-sinusoidal BEMF and a theoretical sinusoidal BEMF. This difference can be estimated from (9) through the expression of e_{dq} to build the extended Park's transformation. It

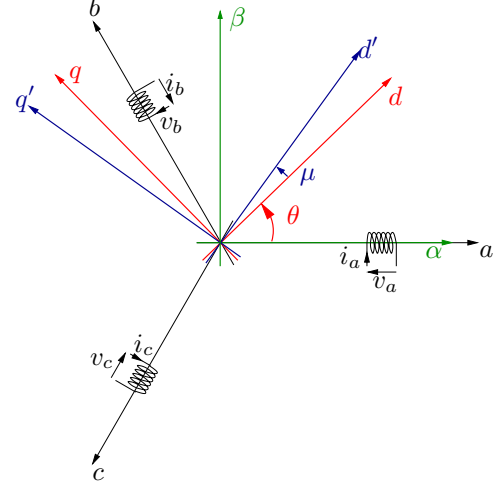


Fig. 1. Different frame representations.

results the model in the (d', q') frame. Due to the nonlinearities in (9), observers will be designed based on sliding mode observer theory described in the following section.

The complete approach is described in the scheme of Fig. 2.

3. HIGH ORDER SLIDING MODE OBSERVER DESIGN

3.1 Observer design

In the following the notation \hat{x} describes the estimate of the variable x . The observer design is based on the model of the electrical equations in the (d, q) frame. In order to estimate the necessary BEMF e_{dq} used to compute the estimated extended Park's transformation the current vector i_{dq} needs to be estimated. From this estimation, the BEMF e_{dq} can be reconstruct. The model (9a) with (9b) and (10) is rewritten with an augmented form, where the vector

$$\varepsilon_{dq} = -\frac{1}{L} e_{dq}, \quad (20)$$

are the augmented state variables to be estimated:

$$\frac{di_{dq}}{dt} = \frac{1}{L} v_{dq} - \frac{R}{L} i_{dq} - p\omega \mathcal{J}i_{dq} + \varepsilon_{dq} \quad (21a)$$

$$\frac{d\varepsilon_{dq}}{dt} = -\frac{p\Phi_f}{L} \left(\frac{1}{J} (\tau_{em} - \tau_{ext}) f_{dq}(p\theta) + \omega \frac{df_{dq}(p\theta)}{dt} \right). \quad (21b)$$

The observer is designed based on the assumption that the BEMF derivative is bounded :

Assumption 1. It is assumed that it is possible to find an upper bound such that

$$\left| -\frac{p\Phi_f}{L} \left(\frac{1}{J} (\tau_{em} - \tau_{ext}) f_i(p\theta) + \omega \frac{df_i(p\theta)}{dt} \right) \right| < m_i^+, \quad i \in d, q. \quad (22)$$

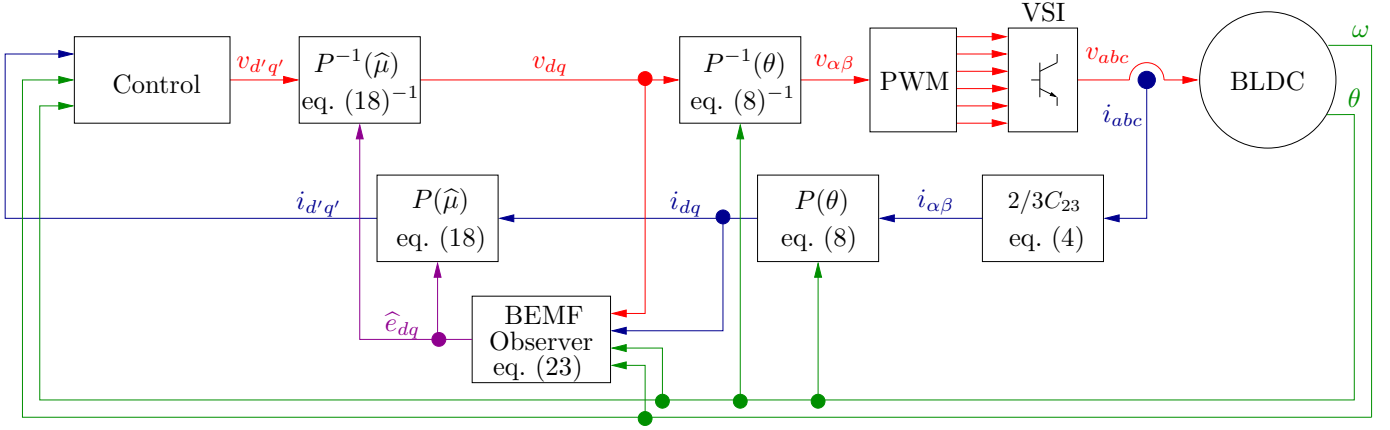


Fig. 2. Proposed complete scheme.

Consider the Super Twisting based observer proposed in (Davila et al., 2006), to match with this synthesis one defines the observer model (21) as:

$$\frac{d\hat{i}_{dq}}{dt} = \hat{\varepsilon}_{dq} + \chi(i_{dq}, u) + \alpha_2 \lambda(\tilde{i}_{dq}) \text{sign}(\tilde{i}_{dq}), \quad (23a)$$

$$\frac{d\hat{\varepsilon}_{dq}}{dt} = \alpha_1 \text{sign}(\tilde{i}_{dq}), \quad (23b)$$

with $\tilde{i}_{dq} = i_{dq} - \hat{i}_{dq}$, $u = [v_{dq}, \omega]^\top$ and

$$\chi(i_{dq}, u) = \frac{1}{L} v_{dq} - \frac{R}{L} i_{dq} - p\omega \mathcal{J} i_{dq}, \quad (24)$$

where gain matrices α_1 and α_2 are the correction factors designed for the convergence of estimation error defined as

$$\alpha_1 = \text{diag} \{ \alpha_{1,d}, \alpha_{1,q} \}, \quad (25a)$$

$$\alpha_2 = \text{diag} \{ \alpha_{2,d}, \alpha_{2,q} \}, \quad (25b)$$

functions $\lambda(\tilde{i}_{dq})$ and $\text{sign}(\tilde{i}_{dq})$ are defined as

$$\lambda(\tilde{i}_{dq}) = \text{diag} \left\{ |\tilde{i}_d|^{1/2}, |\tilde{i}_q|^{1/2} \right\}, \quad (26a)$$

$$\text{sign}(\tilde{i}_{dq}) = \text{diag} \left\{ \text{sign}(\tilde{i}_d), \text{sign}(\tilde{i}_q) \right\}. \quad (26b)$$

Remark 2. Note that the compensation term $\chi(i_{dq}, u)$ is nonlinear. However the variables in this function are measured and the input u is known and can thus be injected in the observer.

Theoreme 1. ((Davila et al., 2005)). Suppose that condition (22) holds for system (21), and the parameters of the observer (23) are selected according to

$$\alpha_{1,i} > m_i^+, \quad (27a)$$

$$\alpha_{2,i} > \sqrt{\frac{2}{\alpha_{1,i} - m_i^+} \frac{(\alpha_{1,i} + m_i^+)(1 + p_i)}{(1 - p_i)}}, \quad (27b)$$

where p_i are some constants to be chosen $0 < p_i < 1$, $i \in d, q$. Then the observer (23) ensures the convergence of the estimated states $(\hat{i}_{dq}, \hat{\varepsilon}_{dq})$ to the real value of the states $(i_{dq}, \varepsilon_{dq})$ after a finite time transient, and

there exists a time constant t_0 such that for all $t \geq t_0$, $(\hat{i}_{dq}, \hat{\varepsilon}_{dq}) = (i_{dq}, \varepsilon_{dq})$.

Remark on the Theorem 1. Here the author do not claim to prove Theorem 1 since the proof was proposed in (Davila et al., 2005, 2006). However missing necessary condition for the proof of the stability are added as well as sufficient informations from (Davila et al., 2005) for the reader. The solutions to the system (21) are understood in Filippov's sense (Filippov and Arscott, 1988). The functions described in (21) are Lebesgue-measurable and uniformly bounded in any compact region of the state-space $(i_{dq}, \varepsilon_{dq})$. This assumption means that we consider the space of "real" BLDC motor variables were bounded. It also implies that the injection $\chi(i_{dq}, u)$ introduced in the proposed observer (23) is bounded and that Assumption 1 is verified for this system. From (21) and (23), the error equation takes the form

$$\frac{d\tilde{i}_{dq}}{dt} = \tilde{\varepsilon}_{dq} - \alpha_2 \lambda(\tilde{i}_{dq}) \text{sign}(\tilde{i}_{dq}), \quad (28a)$$

$$\frac{d\tilde{\varepsilon}_{dq}}{dt} = -\frac{p\Phi_f}{L} \left(\frac{1}{J} (\tau_{em} - \tau_{ext}) f_{dq}(p\theta) + \omega \frac{df_{dq}(p\theta)}{dt} \right) - \alpha_1 \text{sign}(\tilde{i}_{dq}), \quad (28b)$$

where $\tilde{\varepsilon}_{dq} = \varepsilon_{dq} - \hat{\varepsilon}_{dq}$.

Here, from the proof in (Davila et al., 2005), the estimation errors \tilde{i}_{dq} and $\tilde{\varepsilon}_{dq}$ satisfy the differential inclusion

$$\frac{d\tilde{i}_{dq}}{dt} = \tilde{\varepsilon}_{dq} - \alpha_2 \lambda(\tilde{i}_{dq}) \text{sign}(\tilde{i}_{dq}), \quad (29a)$$

$$\frac{d\tilde{\varepsilon}_{dq}}{dt} \in [-m_{dq}^+, m_{dq}^+] - \alpha_1 \text{sign}(\tilde{i}_{dq}), \quad (29b)$$

The derivative of $\frac{d\tilde{i}_{dq}}{dt}$ with $\tilde{i}_{dq} \neq 0$ one gets :

$$\frac{d^2\tilde{i}_{dq}}{dt^2} \in [-m_{dq}^+, m_{dq}^+] - \left(\frac{1}{2} \alpha_2 \text{diag} \left\{ \frac{\frac{d\tilde{i}_d}{dt}}{|\tilde{i}_d|^{1/2}}, \frac{\frac{d\tilde{i}_q}{dt}}{|\tilde{i}_q|^{1/2}} \right\} \text{sign}(\tilde{i}_{dq}) + \alpha_1 \text{sign}(\tilde{i}_{dq}) \right). \quad (30)$$

which proves from (Davila et al., 2005) the observer convergence, in finite time.

3.2 Estimated extended Park's transformation

The finite time observer convergence implies that there exists a constant time $t_0 > 0$ such that for all $t \geq t_0$, $\hat{\varepsilon}_{dq} = \varepsilon_{dq}$. From this estimation, one obtains the BEMF estimation in the (d, q) frame

$$\hat{\varepsilon}_{dq} = -L\hat{\varepsilon}_{dq}. \quad (31)$$

leading to the expression of $P(\hat{\mu})$ from (18). In the extended Park's transformation, the BEMF estimation is

$$\hat{\varepsilon}_{d'q'} = P(\hat{\mu})\hat{\varepsilon}_{dq}. \quad (32)$$

Remark 3. Note that the expression $P(\hat{\mu})$ is obtained from the estimation vector $\hat{\varepsilon}_{dq}$ with (31). Although $\hat{\varepsilon}_{dq}$ is observable at zero velocity ω these expressions are not defined at this point. In practice, the extended Park's transformation is computed for $\omega > \omega_{\text{lim}}$ which can be measured since the velocity is measured. Below this limit, classical transformation is applied by imposing $\hat{\mu} = 0$. Moreover, the torque ripple at low speed can easily be compensated by the control, since this ripple depends on the velocity.

Remark 4. Unlike the mechanical sensorless control and the non-uniqueness of the solution for the position estimation, the μ estimation is well defined. Indeed, the parameter μ representing the displacement angle between a sinusoidal BEMF and the actual non-sinusoidal BEMF as mentioned before and this angle is always in the interval $\left[-\frac{\pi}{p}, \frac{\pi}{p}\right]$.

4. SIMULATION RESULTS

The theoretical results of the article are presented through Matlab/Simulink simulations. In order to be as close to reality as possible the normalized non-sinusoidal BEMF waveform $f\left(p\theta + \frac{2i\pi}{3}\right)$, $i \in \{0, 1, 2\}$ are modeled based on the harmonic content of the BEMF proposed in (Lidozzi et al., 2008). The resulting BEMF is plotted Fig. 3.

4.1 Observer based control strategy

The control objective is to track a reference speed ω_{ref} , despite an unknown external torque τ_{ext} . The controller is based on an outer Proportional-Integral (PI) control loop for the mechanical dynamic (13b) in the (d', q') frame. This loop provides a reference quadrature current $i'_{q,\text{ref}}$. The velocity profile is chosen according to industrial test trajectories (Hamida et al., 2013).

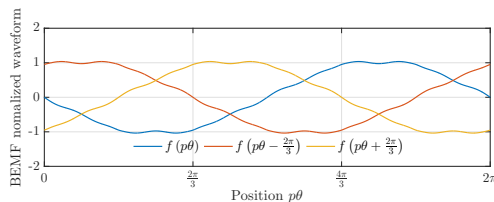


Fig. 3. Non-sinusoidal BEMF.

The electrical dynamic (13a) is linearized based on the feedback linearization proposed in (Bodson et al., 1993). The linearized model is used to track the reference currents $i'_{d,\text{ref}}$ setting to zero and $i'_{q,\text{ref}}$ using an inner current loop formed by two PI controllers.

Remark 5. The control design is out of the scope of this article. In the (d', q') frame classical control such as (Bodson et al., 1993) can be adapted. A sliding mode control such as (Nollet et al., 2008) can also fit with the framework. Here the closed loop system with the observer being nonlinear, the separation principle is then not valid. A study of the convergence of the control law and the observer together must be proven. Approach similar to the one in (Delpoux and Floquet, 2014) must be conducted.

The expression of the currents and BEMF in the extended Park's transformation are required for the control strategy to succeed. These variables are obtained from the observer described in (23). The estimated variables are injected in the control loops (see Fig. 2).

4.2 Simulation

For this simulation, the motor parameters are listed by Table 1.

Parameter	Value	Parameter	Value
R (Ω)	3.25	p	5
L (mH)	18	J (gm^2)	0.417
Φ_f (Wb)	0.341	$\tau_{\text{ext}}(\omega)$ (Nm)	$3.4 \cdot 10^{-3}\omega + 0.3$

Table 1. Motor Parameters.

In order to illustrate the contribution of the proposed approach. The observer based control strategy in the estimated extended Park's transformation is compared with the same control in the classical Park's transformation, i.e. with the assumption that the BEMF is sinusoidal. In the following figures, a variable with indice 1 will be used for the variable obtained when BEMF is considered as sinusoidal. Indice 2 will refer to the case where non-sinusoidal BEMF is considered. The velocity tracking for both approaches is plotted Fig. 4. The velocity profile objective is to show the velocity tracking performances at low speed $\omega_{\text{ref}} = 20\text{rad/s}^{-1}$ and at high speed $\omega_{\text{ref}} = 100\text{rad/s}^{-1}$. On this figure the black vertical lines in dot-dash represent the limit for which the extended Park's transformation is computed. Here we choose $\omega_{\text{lim}} = 10\text{rad/s}^{-1}$. The first subplot shows a good velocity tracking for both cases. However, the second subplot shows that the torque ripple is much higher when the BEMF is considered as sinusoidal. Thereafter, are represented on the remaining figures zooms of the variables for both constant reference ω_{ref} . For $\omega_{\text{ref}} = 20\text{rad/s}^{-1}$, a time interval between 1 and 1.1s is considered. For $\omega_{\text{ref}} = 100\text{rad/s}^{-1}$, a shorter time interval between 2.5 and 2.52s is considered. Although the velocity tracking is good for both strategies at low and high speed. Fig. 5 shows that the proposed approach leads to a more accurate velocity tracking and a reduced torque ripple with the proposed approach. Fig. 6 represents the currents in the (a, b, c) frame as well as the difference between the current for both approach. Here, one poses $\Delta i_k = i_{k,2} - i_{k,1}$, $k \in \{a, b, c\}$. On this figure one can see that the current waveforms for the proposed approach oscillate at the frequency of the μ parameter.

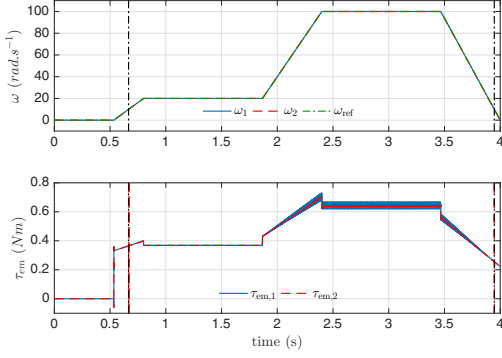


Fig. 4. Velocity tracking and electromechanical torque.

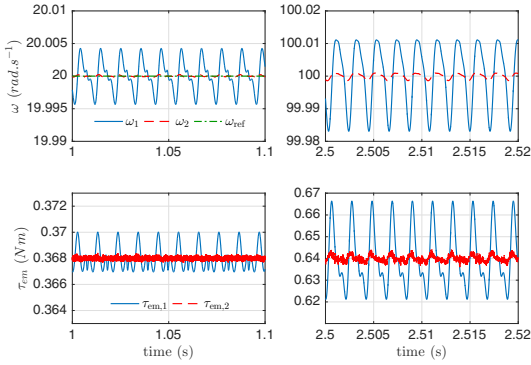


Fig. 5. Zoom on the velocity tracking and electromechanical torque, (left) at $\omega_{\text{ref}} = 20\text{rad/s}^{-1}$, (right) at $\omega_{\text{ref}} = 100\text{rad/s}^{-1}$.

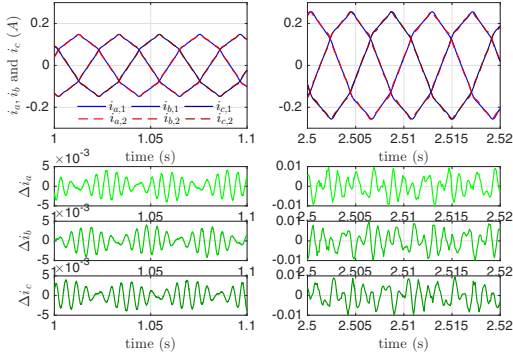


Fig. 6. Zoom on the currents in the abc -frame, (left) at $\omega_{\text{ref}} = 20\text{rad/s}^{-1}$, (right) at $\omega_{\text{ref}} = 100\text{rad/s}^{-1}$.

Finally Fig. 7 exhibits the estimation results. On the figure the two first lines of subplots represent the current vector i_{dq} , \hat{i}_{dq} obtain from (23) as well as the current vector $\hat{i}'_{d'q'}$ obtained from the estimated transformation $P(\hat{\mu})$ (18). As expected in the classical Park frame (d, q), the direct current is non zero. However the figure shows that the observer gives good results with an accurate i_{dq} estimation. In the (d', q') frame the resulting direct current $i'_{d'}$ is indeed equal to zero. The third and the fourth lines of subplots show the BEMF estimation. On the figure e_{dq} is compared to its estimation \hat{e}_{dq} obtained from (32). Here again the BEMF reconstruction is rigorous which validates

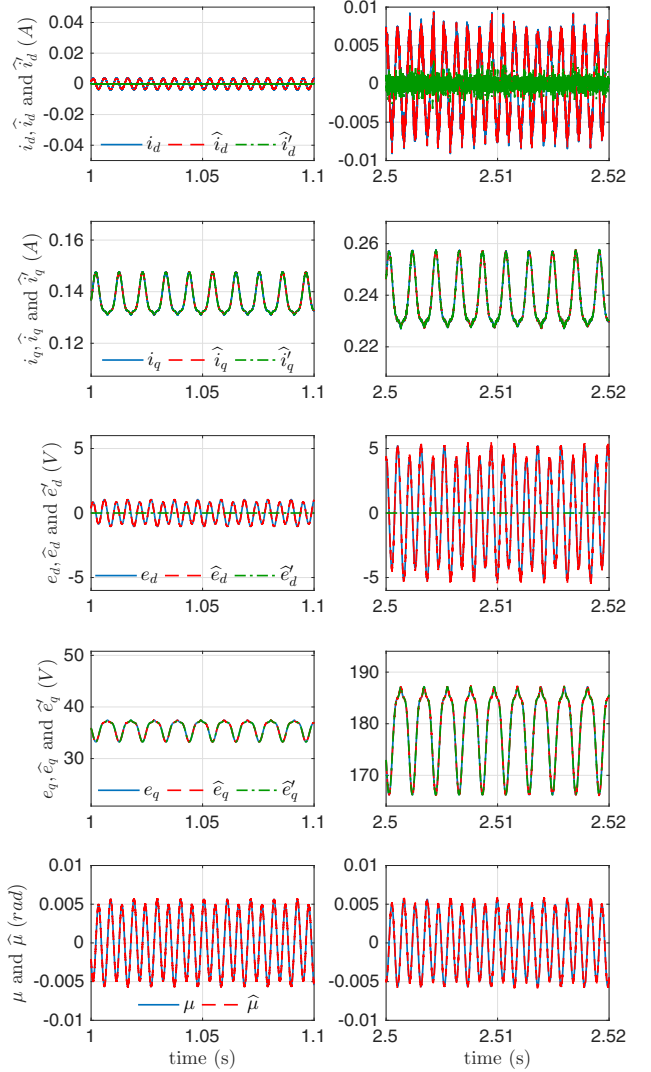


Fig. 7. Zoom on the estimated variables, (left) at $\omega_{\text{ref}} = 20\text{rad/s}^{-1}$, (right) at $\omega_{\text{ref}} = 100\text{rad/s}^{-1}$.

the proposed observer. At last, the μ parameter estimation is represented and is also very good. As mentioned in Remark it is well in the interval $\left[-\frac{\pi}{p}, \frac{\pi}{p}\right]$.

Remark 6. To improve the performance of the observers, the gain matrices α_1 and α_2 have been chosen depending on the reference velocity ω_{ref} . Note however that the smaller gain matrices ensure the stability of the observer on the entire space of the BLDC motor.

5. CONCLUSION

In this article was presented the design of a non-sinusoidal BEMF observer for a BLDC motor in order to reduce electromechanical torque ripples. For their very interesting robustness properties faced to matched perturbations, a

high order sliding mode observer was proposed for the estimation of the BEMF. It allows to determine magnetic field angular relative position leading to a so-called extended Park's transformation. Using this transformation, speed and torque vector control were applied. The simulation results have shown that the proposed observation to express the extended Park's transformation reduces considerably torque ripples.

As future work, an experimental benchmark is in progress. Moreover, it would be interesting based on this modeling to propose an approach to estimate the external torque applied to the motor in order to be compensated in the control.

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