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Tools for Robustness in Dynamic Dial-a-Ride Problem

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Abstract. The Dial-a-Ride Problem (DARP) models an operation research problem related to the on demand transport. This paper introduces one of the fundamental features of this type of transport: the robustness. This paper solves the Dial-a-Ride Problem by integrating a measure of insertion capacity called *Insertability*. The technique used is a greedy insertion algorithm based on time constraint propagation (time windows, maximum ride time and maximum route time). In the present work, we integrate a new way to measure the impact of each insertion on the other not inserted demands. We propose its calculation, study its behavior, discuss the transition to dynamic context and present a way to make the system more robust.

1 Introduction

Today, the Dial-a-Ride Problems are used in transportation services for elderly or disabled people. Also, the recent evolution in the transport field such as connected cars, autonomous transportation, and the emergence of the shared service might need to use this type of problem at much larger scales. But this type of transport is expensive and the management of the vehicles requires as much efficiency as possible, however the number of requests included in the vehicles planning can vary depending on the resolution used.

In [15], we solve the DARP by using constraint propagation in a greedy insertion heuristic. This technique obtains good results, especially in a reactive context, and is easily adaptable to a dynamic context. But, each demand is inserted one after another and the process does not take into account the impact of each insertion on the other not inserted demands, and so, in a dynamic context, the future demands. In this work, we present a measure of an insertion capacity named *Insertability*.

We introduce its calculation by integrating indirectly the impact of an insertion on the time constraints (time windows, maximum route time and maximum ride time) of the other demands.

This measure may be used in different steps of the resolution: selection of the demand to insert, selection of the insertion parameters, taking the decision to exclude a demand, and in the process making the service time. These four uses may be related to static as well as dynamic contexts. The main goal is to insert the current demand in a way to create routes which will be able to insert the future demands.

This paper is organized in the following manner: after a literature review, the next section will propose a model of the classic DARP. Then, we will review how to handle the temporal constraints with a heuristic solution based on insertion techniques using propagation constraints. We will continue by explaining the way to measure the *Insertability*, a calculation based on the evolution of the time windows after an insertion. Then, we will give some other uses of this measure including the setting of the service times which minimize the time windows (cf. Figure 1). In the last part of the paper, the computational results will show the efficiency of our *Insertability*'s measure and we will report the evolution of the number of demands inserted in a resolution of some instances' sets.

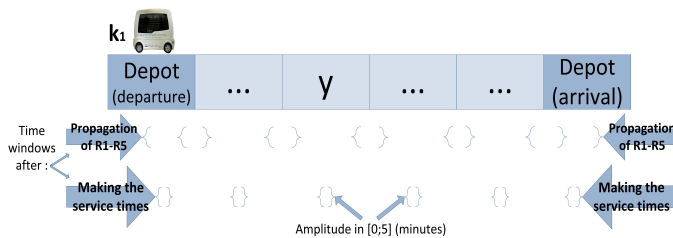


Fig. 1. Times windows contraction

2 Literature review

The first works of the transportation optimization problem are related to the Traveling Salesman Problem ([9]). Since that time, other transportation problems have emerged as the vehicle routing and scheduling problems, and also the Pick-up and Delivery Problem (PDP). Based on the PDP, the DARP which has been studied since the 1970's. The Dial-a-Ride problem is related to the on demand transportation which is more often manage in a dynamic context, unlike the PDP.

There are a number of integer linear programmings [6], but the problem complexity is too high for using it in a real context. Indeed, it is an NP-Hard because it also generalizes the Traveling Salesman Problem with Time Windows (TSPTW). The optimal solution can be find only for small instances [8]. Therefore, and more specially for the dynamic context, the problem must be handled through heuristic techniques.

For the static DARP, [4] is an important study on the subject. In this work, they use the Tabu search metaheuristic to solve the problem. Starting from an initial solution, the resolution process moves from a solution to another in a neighborhood. The operator which executes these moves will take a demand already assigned to vehicle k and insert it in another vehicle k' . The Tabu list saves each application of the operator in order to avoid cycling. The insertion

parameters of the demand (for the origin and the destination) are chosen in order to minimize the total distance. They don't take in account all the constraints of the DARP during the process. They reach a feasible solution by integrating penalties in the performance criteria.

Other techniques are efficient for the static DARP. Dynamic programming (e.g. [10] and [3]) or variable neighborhood searches (VNS) (e.g. [11] and [12]) gived good solutions.

A basic feature of DARP is that it usually derives from a dynamic context. Therefore, algorithms for static DARP should be designed in order to take into account the fact that they will have to be adapted to dynamic and reactive contexts, which means synchronization mechanisms, interactions between the users and the vehicles, and uncertainty about for-coming demands ([7]).

[13], and [14] later, developed the most used technique in dynamic context or in a real exploitation is heuristics based on insertion techniques. These techniques give good solutions even if the people's requests have to be taken into account in a very short period of time.

[1] adapt the Tabu search of [4] to the dynamic DARP and a parallel environment. Indeed, each replication is independent in order to be executed on different CPU cores ([1]). In fact, the operator moving from a solution to another uses simple insertion technique. Other recent works are made on the dynamic Dial-a-Ride problem, and in particular about the problems of reactivity. For instance, in [2], they develop an algorithm able to take in account requests in real time. The requests are given directly to the driver, thus the decision is made immediately. Therefore, they describe the resolution process launched each time a new request appears in the system.

3 Mathematical model

We first introduce the standard mathematical model for a single criterion DARP. The objective here is the minimization of the total distance. The DARP is defined by a transit network $G = (V, E)$, which contains at least two nodes *Depot* (departure and arrival), and whose arcs $e \in E$ are endowed with riding times equal to the Euclidean distance between two nodes of V , i and j , $DIST(i, j) \geq 0$. A fleet VH of K vehicles k with a capacity CAP and a Demand set $D = (D_i, i \in I)$. Any demand (or request) D_i is defined as a 6-uple $D_i = (o_i, d_i, D_i, F(o_i), F(d_i), Q_i)$, where:

- $o_i \in V$ is the origin node of the demand D_i and the set of all the o_i is DE .
- $d_i \in V$ is the destination node of the demand D_i and all the d_i are gathered in AR ;
- $F(o_i)$ and $F(d_i)$ are respectively the time windows for o_i and d_i of the demand D_i . $F.MIN$ and $F.MAX$ are the two bounds of a window;
- $\Delta_i \geq 0$ is an upper bound on the ride time of the demand i ;
- Q_i is the D_i 's load such that $q_{o_i} = Q_i = -q_{d_i}$.

We denote by t_x^k the time at which the vehicle k begins service in $x, x \in V$. Also, $\delta_j, j \in V$, is the non-negative service time necessary at the node j and Δ^k is the maximum route time for the vehicle k .

Then, we consider in $G = (V, E)$ all the nodes corresponding to the $o_i \in V$ and $d_i \in V$ such that $V = DE \cup AR \cup \{0, 2|D| + 1\}$ with $\{0, 2|D| + 1\}$ the two depot nodes respectively for the departure and the arrival, $o_i \in \{1..|D|\}$, and $d_i \in \{(|D| + 1)..(2|D|)\}$. Moreover we denote by V' the set V without the depot nodes (i.e. $V' = DE \cup AR$) and by ζ_j^k the total load of the vehicle k leaving the node $j, j \in V$.

Dealing with such an instance means planning the handling demands of D , by the fleet VH , while taking into account the constraints which derive from the technical characteristics of the network G , of the vehicle fleet VH , and of the 6-uples $D_i = (o_i, d_i, D_i, F(o_i), F(d_i), Q_i)$, and while optimizing some performance criterion which is usually minimizing the total distance or a mix of an economical cost (point of view of the fleet manager) and of QoS criteria (point of view of the users).

Let x_{ij}^k a boolean equals to 1 if the vehicle k travels from the node i to the node j . Then, based on [5], the mathematical formulation is the following mixed-integer program :

$$\text{Min} \sum_{k \in K} \sum_{i \in V} \sum_{j \in V} \text{DIST}(i, j) x_{ij}^k \quad (1)$$

subject to

$$\sum_{k \in K} \sum_{j \in V'} x_{ij}^k = 1, \forall i \in V' \quad (2)$$

$$\sum_{j \in V'} x_{ij}^k - \sum_{j \in V'} x_{|D|+i, j}^k = 0, \forall i \in DE, k \in K \quad (3)$$

$$\sum_{j \in DE} x_{0j}^k = 1, \forall k \in K \quad (4)$$

$$\sum_{j \in V'} x_{ji}^k - \sum_{j \in V'} x_{ij}^k = 0, \forall i \in V', k \in K \quad (5)$$

$$\sum_{i \in AR} x_{i, 2|D|+1}^k = 1, \forall k \in K \quad (6)$$

$$t_j^k \geq (t_i^k + \delta_i + \text{DIST}(i, j)) x_{ij}^k, \forall i \in V, j \in V, k \in K \quad (7)$$

$$\zeta_j^k \geq (\zeta_i^k + q_j) x_{ij}^k, \forall i \in V, j \in V, k \in K \quad (8)$$

$$\text{DIST}(i, |D| + i) \leq t_{|D|+i}^k - (t_i^k + \delta_i) \leq \Delta_i, i \in DE \quad (9)$$

$$F.\text{MIN}_i \leq t_i^k \leq F.\text{MAX}_i, \forall i \in V, k \in K \quad (10)$$

$$t_{2|D|+1}^k - t_0^k \leq \Delta^k, \forall k \in K \quad (11)$$

$$\zeta_i^k \leq CAP^k \quad (12)$$

$$x_{ij}^k \in \{0; 1\}, t \in R^+ \quad (13)$$

The program above is a three index formulation (report to [5] for more explanations about the objective function (1) and the constraints (2)-(13)), it exists in literature several other mathematical formulations for the DARP, even some with two index formulation [6]. But, the complexity of all these linear programs doesn't allow finding an exact solution with a solver, the operation is too time consuming.

All along this work, we are going to deal with homogeneous fleets and with nominal demands, and we shall limit ourselves to static points of view but our insertion process allows flexibility for using it in a dynamic context. Still, we shall pay special attention to cases when temporal constraints are tight.

4 A greedy insertion algorithm

This section explains the techniques used to solve a static DARP instance. In our framework, the routes are saved in lists Γ_k surrounding by two depot nodes (one for the departure on the other for the arrival) with k a vehicle of the fleet VH (cf. Figure 1). For any sequence (or list) Γ_k we set:

- for any z in Γ_k , $\text{Succ}(\Gamma_k, z) =$ Successor of z in Γ_k and $\text{Pred}(\Gamma_k, z) =$ Predecessor of z in Γ_k ;
- for any z, z' in Γ_k , $z \ll_k z'$ if z is located before z' in Γ_k and $z \ll_k z'$ if $z \ll_k z'$ or $z = z'$.

We also defined the *Twin* function such that, for any node o_i (respectively d_i) which appears in Γ_k , the node $\text{Twin}(o_i) = d_i$ (and $\text{Twin}(d_i) = o_i$). The twin of a depot node is the other depot node in the same list.

In [15], we present an insertion greedy algorithm based on constraint propagation in order to contract time windows according to the time constraints of the DARP.

An insertion which does not imply constraint violation is said *valid* if $\Gamma = \cup_{k \in K} \Gamma_k$, the resultant collection of routes, is *load-valid* and *time-valid*. A route is *load-valid* if the capacity is not exceeded, so, the *load-validity* is obtained if $\text{ChT}_k(x) \leq CAP$ with $\text{ChT}_k(x) = \sum_{y \ll_k x} q_y$, x and y nodes in the route k . The *time-validity* is obtained if there is no violation of the time constraints modeling by, for each demand i , $i \in D$, Δ_i the maximum ride time, $\Delta^k, k \in K$ the maximum route time and the constraints modeled by each time window $F(o_i) = [F.\text{min}(o_i), F.\text{max}(o_i)]$ and $F(d_i) = [F.\text{min}(d_i), F.\text{max}(d_i)]$. The service time δ_j of each node $j \in V$ and origin of an arc $e \in E$ is added in the DIST matrix, but without considering the departure depot nodes. Checking the *load-validity* on $\Gamma = \cup_{k \in K} \Gamma_k$ is easy, and we show the efficiency of the constraint propagation in order to prove to *time-validity* after each planned insertion once the *load-validity* is proved. According to a current time window set $FP = \{FP(x) = [FP.\text{min}(x),$

$FP.max(x)$], $x \in \Gamma_k, k = 1..K$ the *time-validity* may be performed through propagation of the five following inference rules **R**_{*i*}, *i* = 1..5 in a given route Γ_k :

for each (x,y) pair of nodes such that $y = Succ(\Gamma_k, x)$:

$$\begin{aligned}
 & FP.min(x) + DIST(x,y) > FP.min(y) \\
 - \mathbf{R1} : & \quad \quad \quad \models \\
 & (FP.min(y) \leftarrow FP.min(x) + DIST(x,y)); (FNodeModif \leftarrow y) \\
 & FP.max(y) - DIST(x,y) < FP.max(x) \\
 - \mathbf{R2} : & \quad \quad \quad \models \\
 & (FP.max(x) \leftarrow FP.max(y) - DIST(x,y)); (FNodeModif \leftarrow x)
 \end{aligned}$$

if $(y = Twin(x))$ and $(x \ll_{\Gamma} y)$:

$$\begin{aligned}
 & FP.min(x) < FP.min(y) - \Delta_i \\
 - \mathbf{R3} : & \quad \quad \quad \models \\
 & (FP.min(x) \leftarrow FP.min(y) - \Delta_i); (FNodeModif \leftarrow x) \\
 & FP.max(y) > FP.max(x) + \Delta_i \\
 - \mathbf{R4} : & \quad \quad \quad \models \\
 & (FP.max(y) \leftarrow FP.max(x) + \Delta_i); (FNodeModif \leftarrow y)
 \end{aligned}$$

and for each $x, x \in \Gamma_k, k = 1..K$:

$$- \mathbf{R5} : FP.min(x) > FP.max(x) \models REJET \leftarrow true.$$

These 5 rules are propagated in a loop while there no time windows exists FP modified at the last iteration, i.e. once $FNodeModif$ is empty. In the R3 and R4 rules, the ' Δ_i ' is replaced by ' Δ_k ' if the pair of nodes (x,y) are depots. The tour $\Gamma_k, k = 1..K$, is *time-valid* according to the input time window set FP if and only if the $REJET$ Boolean value is equal to *false* as initialized at the beginning of the process. In such a case, any *valid-time* value set t related to Γ_k and FP is such that: for any x in $\Gamma_k, t(x)$ is the service time in $FP(x)$.

The greedy insertion algorithm includes this propagation constraint technique in order to evaluate each possible insertion. Each iteration of the algorithm selects one demand according to the number of vehicle able to integrate it. Once a demand is selected, the process chooses the best insertion's parameters that are the vehicle and the location of the *origin* and *destination* nodes. These parameters are chosen according to the smaller evolution of the routes costs (like the total distance in the mathematical model). The selection of the demand and

the parameters are made in a non-deterministic way (among a small set of the best variables). Then, we are allowed to include our process in a Monte Carlo scheme. The Figure 2 summarizes the general algorithm with P the number of replications resolving a DARP instance.

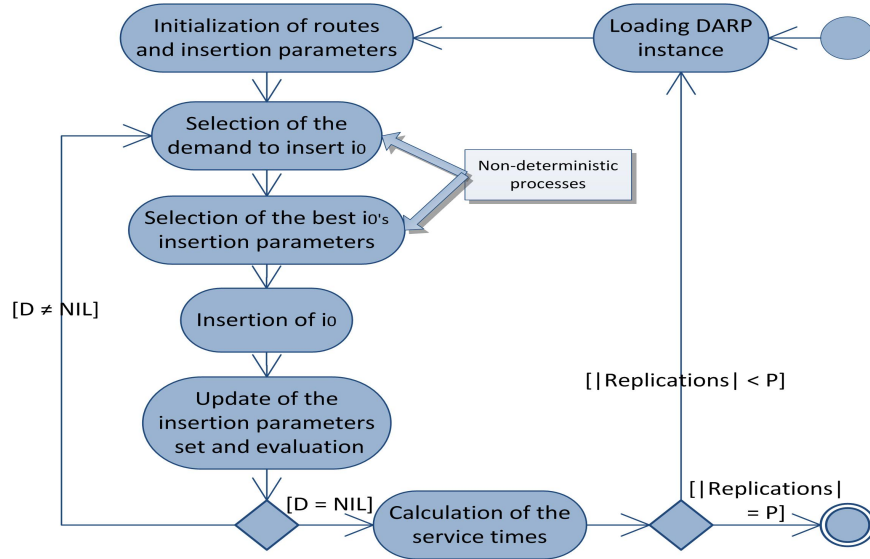


Fig. 2. The Monte-Carlo Process

5 *Insertability* optimization

In the above algorithm, each iteration selects a demand, and then, it finds the way to insert while minimizing the total cost. This greedy algorithm does not take in account the impact of this actual insertion on the future demands integration, but only the effect on the demands already inserted. In this section, we introduce a *Insertability* calculation by integrating this impact of an insertion related to the time constraints (time windows, maximum ride time and maximum route time).

During the insertion process, the state of the system is given by:

- a set of demands $D - D1$ already integrated in the routes, and $D1$ is the set of demands not inserted,
- a collection $\Gamma = \cup_{k \in K} \Gamma_k$ of routes including a list of nodes related to the *Depot*, *origin* and *destination* nodes,

- a exhaustive list of insertion's parameters sets. Each set gathers 5 elements : k the vehicle, i the demand, (x, y) the pair of insertion nodes (locating respectively o_i between x and the successor of x , and d_i between y and the successor of y), and v the evolution of the collection $\Gamma = \cup_{k \in K} \Gamma_k$'s cost.

5.1 The *Insertability* measure

Given that the difficulty of the instances' problem is linked to the time constraints, we introduce an *Insertability* calculation related to the times windows contractions. During an insertion's assessment, these reductions appear once the inference rules are propagated. Here, we try to find a good triple (k, x, y) , the vehicle and the location of the *origin/destination* nodes, in order to give enough "time space" to the future demands (which have to be integrated in $\Gamma = \cup_{k \in K} \Gamma_k$).

We set $INSER(i, \Gamma)$ the *Insertability* measure of the demand I . The quantity $U_n^k(z)$ denotes the vehicle k time windows' amplitude of the node n once it has been inserted to the right of node z . $INSER$ is calculated as follows:

- $INSER(i, \Gamma) = \sum_{k \in K} INSER1(i, \Gamma_k)$;
- $INSER1(i, \gamma) = Max_{(x,y)} INSER2(i, \gamma, x, y)$, γ a tour of Γ ;
- $INSER2(i, \gamma, x, y) = U_{o_i}^\gamma(x) \cdot U_{d_i}^\gamma(y)$.

$INSER2$ is the product of the time windows amplitude resulting to the insertion of the demand i in the scheduling of the tour γ . The origin o_i is inserted to the right of x and the destination d_i to the right of y . The product was chosen in order to penalize the *Insertability* if the time windows of the origin or the destination is tight and also to have $INSER2$ equals to zero if one of the two nodes can not be inserted.

$INSER1$ gives us the maximum of $INSER2$ over the possible insertion positions x and y in the route γ . When $INSER1$ is equal to 0, the new route γ resulting to the new insertion is not *time-valid*.

Finally, $INSER$ makes the sum of all the $INSER2$ for each vehicle of the fleet VH . If $INSER$ equals to zero, the demand can not be inserted in all cases.

We set $Inserted(\Gamma, i_0, k, x, y)$ the updated collection of tours Γ with the insertion of the selected demand i_0 at the locations x and y in the vehicle k .

The $INSER(i, \Gamma)$ measure allows us to write the *Optimization Insertability Problem* which consists to find the best insertion parameters in order to keep the vehicles' scheduling more flexible (in a sense to let enough possibilities of insertions for the future demands):

Optimization Insertability Problem.

Find the optimal parameters (k, x, y) inserting i_0 and maximizing the value $Min_{i \in D1 - i_0} INSER(i, Inserted(\Gamma, i_0, k, x, y))$.

For instance, the value $\text{Min}_{i \in D1-i_0} \text{INSER}(i, \text{Inserted}(\Gamma, i_0, k, x, y))$ may be used if all the demands have to be inserted. Another optimization may be process as the maximization of the sum $\sum_{i \in D1-i_0} \text{INSER}(i, \text{Inserted}(\Gamma, i_0, k, x, y))$. The choice is made according to the homogeneity of the demands and if the problem requires to insert all the set D .

This problem only optimizes the variation of the *Insertability* values and does not include other performance (or costs) criteria like the minimization of the ride times, waiting times or distances. The *Insertability* criterion can be integrated in a mix of economical cost (point of view of the fleet manager) and of QoS criteria (point of view of the users). Then, the process maximizes the function $\text{Perf} = \mu \cdot \sum_{i \in D1-i_0} \text{INSER}(i, \text{Inserted}(\Gamma, i_0, k, x, y)) - v(\text{Inserted}(\Gamma, i_0, k, x, y))$ with μ a criterion coefficient and v the cost value function mixing the costs related to the both points of view.

5.2 Other uses of the *Insertability* measure

So far, we select the demand i_0 according to the number of vehicles available (taking in account all the time and load constraints). The *Insertability* measure $\text{INSER}(i_0, \Gamma)$ may be also used in order to select the next request i_1 to insert. This application could be used in a context where all the demands of D have to be integrated. The selection is based on the smallest *Insertability* measure. Once a demand is selected, the problem may solve the *Optimization Insertability Problem*. Here, the two steps may be written in a non-deterministic way. The demand may be selected randomly through a set of $N1$ elements with the smallest *INSER* value. The same scheme may be applied on a set of a insertion parameters of $N2$ elements with a best (k, x, y) elements maximizing the quantity $\text{Min}_{i \in D1-i_0} \text{INSER}(i, \text{Inserted}(\Gamma, i_0, k, x, y))$.

Also, $\text{INSER}(i_0, \Gamma)$ may be useful for a larger set D . If the instance does not have any solution integrating all the set D , it is preferable to identify requests to exclude as soon as possible. The exclusion of a demand i_0 may be set up if its insertion results in Γ not enough flexible to include the other elements of $D1$. In other words, the demands excluded will be those that will have the most impact of future insertions. The difference $\sum_{i \in D1-i_0} (\text{INSER}(i, \Gamma) - \text{INSER}(i, \text{Inserted}(\Gamma, i_0, k, x, y)))$ of the inequality (14) takes in account the *Insertability* measure of $D1 - i_0$ before and after the insertion of i_0 in the routes of Γ . If this difference is larger than the threshold ξ , the demand is excluded. In the experimentation' section, we will discuss the fact this threshold should be dynamic and decreases over the execution.

$$\sum_{i \in D1-i_0} (\text{INSER}(i, \Gamma) - \text{INSER}(i, \text{Inserted}(\Gamma, i_0, k, x, y))) > \xi \quad (14)$$

5.3 The *Insertability* optimization suited to the greedy insertion algorithm

The calculation of $INSER(i, \Gamma)$, $i \in D$, begins to be time consuming starting from a medium size of D , while the $INSER2$ value is based on the time windows amplitude obtained after the propagation of the time constraints. Therefore, this is important to spot each step of the process where the *Insertability* measure does not have to be updated. When i_0 is selected, $INSER2(i, \Gamma_k, x, y)$, $INSER1(i, \Gamma_k)$ and $INSER(i, \Gamma)$ are known for all demand in $D1 - i_0$ and all $k = 1..K$. Once i_0 is about to be inserted, the process computed the value $H(i)$, $i \in D1 - i_0$ (cf. formulation (15)). Then, the algorithm tries the insertion of each i from $D1 - i_0$ in $Inserted(\Gamma, i_0, k, x, y)$ and deduce the value $K(i)$ given in formula (16) for all $i \in D1 - i_0$ and ultimately the quantity $Val(k, x, y) = Min_{i \in D1 - i_0} (K(i) + H(i))$.

$$H(i) = INSER(i, \Gamma) - INSER1(i, \Gamma_k) \quad (15)$$

$$\begin{aligned} K(i) &= INSER(i, Inserted(\Gamma, i_0, k, x, y)) \\ &= H(i) + INSER1(i, Inserted(\Gamma, i_0, k, x, y)_k) \end{aligned} \quad (16)$$

Other calculations may be avoided. First, we set the variable W_1 such that $W_1 = Min_{i \in D1 - i_0} INSER(i, \Gamma)$. If the quantity $INSER(i, \Gamma) - INSER1(i, \Gamma_k)$ is larger than W_1 , there is no need to test the impact of the insertion of i_0 on i .

Finally, we are able to use $INSER(i, \Gamma)$ once we integrate the future demands presented in the next section. In a dynamic context, the *Insertability* measure helps the routes to be enough flexible for the next insertion process. Moreover, the service time have to be set with the same purpose and $INSER(i, \Gamma)$ is able to help to do it.

6 Future demands in the Dynamic DARP

The problem may have to be handled in a dynamic way and the greedy insertion algorithm is easily adaptable to this context. Once the *Insertability* measure is included in the performance criteria, the system may increase its robustness and we need to exploit knowledge about future demands for that. In our case, this knowledge is related to the type of on demand transportation service. In this paper, we will use a simple extrapolation of this probable demand based on the demand already broadcasted.

We will not take into account the way the system supervises its various communication components with the users. In reality, there are eventual divergences between the data which were used during the planning phases and the situation of the system.

We set $D-V$ the virtual demands, $D-R$ the real demands, and $D-Rejet$ the set of the ones excluded from the insertion algorithm. The $D-V$ formulation is

given in (17). p_i gives us the number of times the future demand D_f will appear for each period of each discrete planning horizon.

$$D-V = \sum_{i \in D_f} D_i \cdot p_i \quad (17)$$

Then, we are able to update the formula (18) giving the performance function $Perf$.

$$Perf = \alpha \cdot \sum_{i \in D_f} p_i INSER(i, Inserted(\Gamma, i_0, k, x, y)) \\ + \mu \cdot \sum_{i \in DI - i_0} INSER(i, Inserted(\Gamma, i_0, k, x, y)) - v(Inserted(\Gamma, i_0, k, x, y)) \quad (18)$$

As in the previous sections, the process may exclude some demands taking in account the future requests. We updated the inequality (14) by the (19). α is a coefficient based on the importance of the future demands.

$$\alpha \cdot \sum_{i \in D_f} p_i \cdot (INSER(i, \Gamma) - INSER(i, Inserted(\Gamma, i_0, k, x, y))) \\ + \sum_{i \in DI - i_0} (INSER(i, \Gamma) - INSER(i, Inserted(\Gamma, i_0, k, x, y))) > \xi \quad (19)$$

7 Discussion about the service times and the dynamic context

Most works on vehicle scheduling problems including time window studies how to integrate a set of demands in the vehicle planning. Making a service time anticipating the future is especially rare.

Once routes are built and integrated a first set D , the users expect the date when the vehicle selected will pick them up. In the lists forming the K routes, each node has a time window. After the service time is set, each time window becomes tight with zero amplitude or equals a very small delay. How the service times are made is very important for the next insertion's process.

For instance, we consider a fleet of 2 vehicles with two schedules made by a first resolution of the DARP minimizing the total distance. The date of each service time has already given to the users. Once these dates are created, the system has to reduce the time windows to 0 like in the Figure 3. While the vehicles are not already left from the depots, the system needs to integrate one more demand in a second resolution process. Then, each new demand such as (o6,d6) will not be able to be inserted. Indeed, for the origin node we have: $t(DepotD2) + DIST(DepotD2, o5) = t(o5)$ which make the insertion of o6 impossible.

One more time, the $INSER(i, \Gamma)$ values may be used in order to set the service time without to have the problem above. The service time may be calculated

once the process have inserted the virtual demands $D-V$ and the real demands $D-R$.

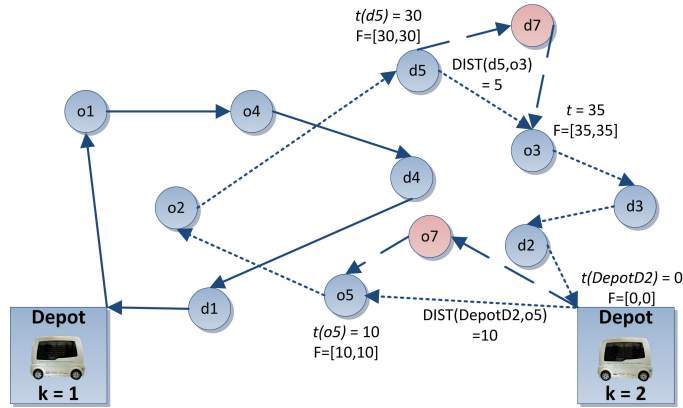


Fig. 3. New insertions after the first set of service times

The previous section shows the way to anticipate the future demands $D-V$. These demands are related to a dynamic context. Note again that our greedy algorithm is easily adaptable to this context. More specifically, the technique does not change unlike the state of each route. The first node is not a depot node anymore but a dynamic node related to the vehicle's location. The entire constraint propagation process is applied on these new routes. A simulation will be necessary to evaluate the anticipation of the future demands including in the dynamic context.

8 Computational Experiments

In this section, we study the behavior of our *Insertability* measure used in the resolution of Dial-a-Ride instances. The algorithms were implemented in C++ and compiled with GCC 4.2. In [15], we solve the [4]'s instances by our greedy insertion algorithm based on constraint propagation. We obtained good results in the majority of instances, but, only 1% of the replications gave us a feasible solution on the tenth instance (*R10a*). The CPU time was smallest or equal to the best times in the literature; we do not work on this feature for this experiment.

8.1 First experimentation: the optimization of the selection of the demand to insert

INSER used in the selection of a demand We note by R^{DARP} the rate of 100 replications which give us a feasible solution obtained by using the solution of [15]. Here, the selection of the demand is based on the lowest number of cars

which are able to accept it. R_{Rob}^{DARP} is the rate obtained with the same process except that each demand is selected at each iteration by the lowest *Insertability* value *INSER*.

The *Insertability* measure is already efficient once it's used in the selection of the demands to insert. The rate obtained for the pr08, pr09, pr10 and pr19 are clearly more interesting as shown in table 1 (e.g. for the instance pr08, the rate increases by 56% to 91% of success).

Inst.	R^{DARP}	R_{Rob}^{DARP}
pr01	99	100
pr02	100	100
pr03	97	100
pr04	100	100
pr05	100	100
pr06	100	100
pr07	90	96
pr08	56	91
pr09	18	21
pr10	1	7
pr11	100	100
pr12	100	100
pr13	99	100
pr14	100	100
pr15	100	100
pr16	100	100
pr17	98	100
pr18	99	100
pr19	64	99
pr20	43	56
Av.	83,2	88,5

Table 1. R^{DARP} Vs R_{Rob}^{DARP}

***INSER* behaviour** Each time a replication cannot integrate all the request, the *INSER* value of the demands not inserted has to be null. In Figure 4, while the resolution process applied to the R10a instance, we note the evolution of more than 4500 *INSER*'s demands not inserted. The technique used is the second approach selecting the demand by the smallest *Insertability*. The values noted are from a failed replication.

One can observe big gaps between the different *INSER* until the 4000 first values. After that, for the remaining requests, the *Insertability* values decrease strongly because the routes begin to be not flexible. Between the 2500th and the 3500th, for some demands, the values are very low at the beginning just before increasing strongly. This is explained by the fact the process inserts the demand with the lowest *INSER* but their insertion do not make a big impact on the other

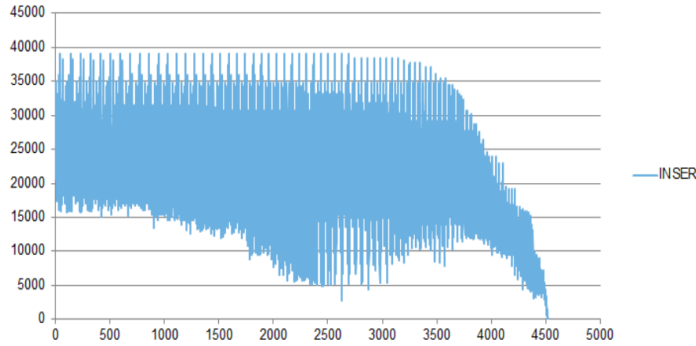


Fig. 4. INSER values on the not inserted demands

demands not inserted. This impact is related to the *Optimization Inseparability Problem* studied below.

8.2 Second experimentation: the optimization of the insertion parameters

In a second experimentation, we compare the [15]’s approach and another algorithm based on the optimization of the parameters (x, y, k) . The selection of the request to insert is the same for both solutions. For the second one, once a demand i_0 is selected, we maximize the sum $\sum_{i \in D_1 - i_0} INSER(i, Inserted(\Gamma, i_0, k, x, y))$ in order to find the best parameter (x, y, k) which will integrate i_0 in the route k . We do not optimize $Min_{i \in D_1 - i_0} INSER(i, Inserted(\Gamma, i_0, k, x, y))$ because we create instances especially with a set D too large for inserting all the requests. Therefore, the demand with the smallest value $INSER$ for a given parameters (x, y, k) could never be integrated into the routes.

The two algorithms were applied to five sets of 5 randomly generated instances. All the instances have their time constraints related to the interval $[0; 400]$ and all the load was unit. We set by $e_{F(o)}$ and $e_{F(d)}$ the amplitude of the time windows at the *origin* and the *destination* given by the users, respectively. The other parameters are given in table 2.

K	$e_{F(o)}$	$e_{F(d)}$	Δ	CAP
10	35	10	∞	10

Table 2. Parameters’ instances

We generate 5 different sets of 5 instances with a variation of the number of demands $|D|$. We set by T_{Insert} and by $T_{Insert_{Rob}}$ the demand inserted’s rate

the first resolution and the second technique, respectively. Finally, Gap_{Insert} is the gap in percentage between each rate. Its calculation is given by $Gap_{Insert} = 100 \cdot (T_{Insert_{Rob}} - T_{Insert}) / T_{Insert}$. We launched 100 replications of each technique on the 5 sets. The results are provided by the table 3.

$ D $	50	75	100	150	200
T_{Insert}	100	93.2	78.9	64.2	52.6
$T_{Insert_{Rob}}$	100	96.8	85.3	66.4	54.1
Gap_{Insert}	0	3.86	8.11	3.43	2.81

Table 3. Gap between the *INSERT* rates

In future work, we could integrate the usual performance criterion (like the total distances) in the *Perf* value in order to choose each best insertion parameters. Here, we are just taken into account the *INSERT* values in order to integrate the most requests possible. The results show us that the larger of $|D|$ defines if the system needs to optimize the *Insertability* measure. For $|D| = 50$, all the requests are able to be inserted easily, so, the *INSERT* values does not have any interest. When the set is composed of 100 demands, we obtained a Gap_{Insert} of 8,11% meaning there are more than 8% more requests inserted by the second approach.

For this set of instance, we also tried to integrate a new feature in our algorithm: we've added the ability to exclude a request if the impact of one insertion involving a significant drop of the general *Insertability's* demands from $D1 - i_0$. Before that, we study the threshold which limits the variation of *Insertability*.

i_0 is excluded if $\sum_{i \in D1 - i_0} (INSERT(i, \Gamma) - INSERT(i, Inserted(\Gamma, i_0, k, x, y))) > \xi$ is true with ξ a threshold. The calculation of the threshold is not easy. In the figure 8.2, we report *Variation* which is the difference $INSERT_{av} - INSERT_{rap}$ with $INSERT_{av}$ and $INSERT_{rap}$ the values $\sum_{i \in D1 - i_0} INSERT(i, \Gamma)$ and $\sum_{i \in D1 - i_0} INSERT(i, Inserted(\Gamma, i_0, k, x, y))$, respectively. This figure shows us that the threshold ξ have to be calculated dynamically according to the average of *INSERT*.

We used this type of dynamic threshold for the third set of instances with 100 demands. We exclude an request if the current ξ is exceeded, and only this feature is added in the second approach. We obtained a gain of 1,3% in average (from 85,3% to 86,6%) meaning approximately one more demand is able to be inserted.

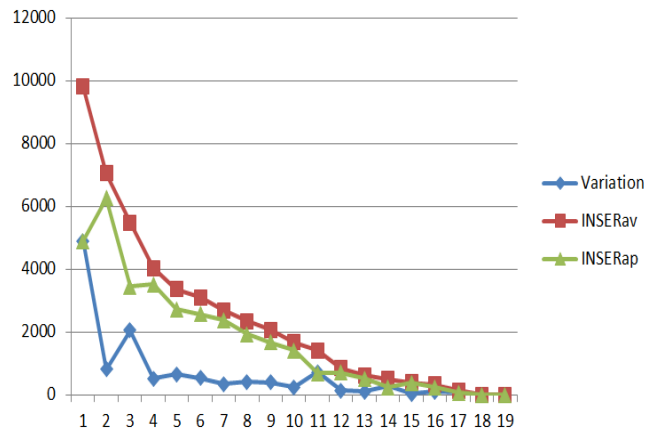


Fig. 5. Variation of the *Inseparability* values between each insertion

9 Conclusion

The Dial-a-Ride Problem is one of the transport problems with the highest number of hard constraints like time windows. The insertion techniques are able to obtain a good solution in a reasonable time. Their adaptability to a dynamic context is easy but a lack of robustness could appear once the goal is to integrate requests as much as possible.

We have introduced a way to measure the impact of each insertion on the other demands not inserted. This *Inseparability* measure could be used in order to exclude a demand, to select a demand to insert and also to calculate the best insertion parameters. This value, named *INSEr*, leads to a large amount of work opportunities. We have introduced a simple way to make the model of the future demands, and how to adapt our greedy insertion algorithm based on the constraint propagation to the dynamic context. In future work, we will develop a simulation which is necessary to show the efficiency of the demands anticipation. The final goal will be to adapt these techniques to a real case.

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