# Additional file 1: Description of the used within herd model of Map Spread

## A Equations for the within-herd dynamic

The description of the model we used corresponds to the within-herd model described in [2] and based on [11]. The set of parameters used are described in tables 1, 2 and 3.

### Notations

In the following equations, $X_{(t,a)}$ is the number of animals in health state $X$ at time $t$ and age $a$. Age is given in weeks until the entry in the adult stage (from 1 to $cal$, with intermediary stages $w$ for weaning age, $y$ for young heifers age, $h$ for heifers age and $u$ for maximal age in the susceptible compartments), then by age group ($P_1$ to $P_{5+}$). Some variables can have a prefix: "$b$" for births in health states $X$ ($bX$), "N" for animals transiting between two health states (N$X$) and "$s$" for exits (mortality and culling) ($sX$). After entering the adult stage, flows corresponding to aging are noted using a superscript $ng$, whereas those remaining in the same age group are noted using a superscript $sg$. $N_{(t,a)}$ is the number of animals of age $a$ at time $t$. Average duration in health states are noted by $v_X$. The remaining terms used are defined when introduced.

### Equations for the updating of variables describing health states

In this section we introduce the equations for the updating of variables corresponding to the health states, for a given herd $i$.

#### Susceptible ($S$) and No more Susceptible ($R$)

$$S_{(t+1,a=1)} = bS_{(t)} - \text{N}T_{(t+1,a)}$$

$$S_{(t+1,a\in[2;52])} = \left[S_{(t,a-1)} - sS_{(t,a-1)}\right] - \text{N}T_{(t+1,a)}$$

$$R_{(t+1,53)} = S_{(t,52)} - sS_{(t,52)}$$

$$R_{(t+1,a\in[54;cal])} = R_{(t,a-1)} - sR_{(t,a-1)}$$

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$$R_{(t+1,P_1)} = R^{sg}_{(t,P_1)} - sR^{sg}_{(t,P_1)} + R_{(t,cal)} - sR_{(t,cal)}$$

$$R_{(t+1,P_i\in[P_2;P_4])} = R^{sg}_{(t,P_i)} - sR^{sg}_{(t,P_i)} + R^{ng}_{(t,P_{i-1})} - sR^{ng}_{(t,P_{i-1})}$$

$$R_{(t+1,P_{5+})} = R_{(t,P_{5+})} - sR_{(t,P_{5+})} + R^{ng}_{(t,P_4)} - sR^{ng}_{(t,P_4)}$$

**Transiently infected ($T$)**

$$T_{(t+1,a=1)} = \; bT_{(t)} + \text{N}T_{(t+1,a)}$$

$$T_{(t+1,a\in[2;52])} = \left[T_{(t,a-1)} - sT_{(t,a-1)}\right] - \text{N}L_{(t+1,a)} + \text{N}T_{(t+1,a)}$$

$$T_{(t+1,a\in[53;cal])} = \left[T_{(t,a-1)} - sT_{(t,a-1)}\right] - \text{N}L_{(t+1,a)}$$

**Latently infected ($L$)**

$$L_{(t+1,a=2)} = \; \text{N}L_{(t+1,a)}$$

$$L_{(t+1,a\in[3;h])} = \; \left[L_{(t,a-1)} - sL_{(t,a-1)}\right] + \text{N}L_{(t+1,a)}$$

$$L_{(t+1,a\in[h+1;cal])} = \; \left[L_{(t,a-1)} - sL_{(t,a-1)}\right] - \text{N}Is_{(t+1,a)} + \text{N}L_{(t+1,a)}$$

$-$

$$L_{(t+1,P_1)} = \; \left[L^{sg}_{(t,P_1)} - sL^{sg}_{(t,P_1)} + L_{(t,cal)} - sL_{(t,cal)} + T_{(t,cal)} - sT_{(t,cal)}\right] - \text{N}Is_{(t+1,P_1)}$$

$$L_{(t+1,P_i\in[P_2;P_4])} = \left[L^{sg}_{(t,P_i)} - sL^{sg}_{(t,P_i)} + L^{ng}_{(t,P_{i-1})} - sL^{ng}_{(t,P_{i-1})}\right] - \text{N}Is_{(t+1,P_i)}$$

$$L_{(t+1,P_{5+})} = \left[L_{(t,P_{5+})} - sL_{(t,P_{5+})} + L^{ng}_{(t,P_4)} - sL^{ng}_{(t,P_4)}\right] - \text{N}Is_{(t+1,P_{5+})}$$

**Moderate shedding ($Is$)**

$$Is_{(t+1,a=h+1)} = \; \text{N}Is_{(t+1,a)}$$

$$Is_{(t+1,a\in[h+2;cal])} = \; \left[Is_{(t,a-1)} - sIs_{(t,a-1)}\right] - \text{N}Ic_{(t+1,a)} + \text{N}Is_{(t+1,a)}$$

$-$

$$Is_{(t+1,P_1)} = \; \left[Is^{sg}_{(t,P_1)} - sIs^{sg}_{(t,P_1)} + Is_{(t,cal)} - sIs_{(t,cal)}\right] - \text{N}Ic_{(t+1,P_1)} + \text{N}Is_{(t+1,P_1)}$$

$$Is_{(t+1,P_i\in[P_2;P_4])} = \left[Is^{sg}_{(t,P_i)} - sIs^{sg}_{(t,P_i)} + Is^{ng}_{(t,P_{i-1})} - sIs^{ng}_{(t,P_{i-1})}\right] - \text{N}Ic_{(t+1,P_i)} + \text{N}Is_{(t+1,P_i)}$$

$$Is_{(t+1,P_{5+})} = \left[Is_{(t,P_{5+})} - sIs_{(t,P_{5+})} + Is^{ng}_{(t,P_4)} - sIs^{ng}_{(t,P_4)}\right] - \text{N}Ic_{(t+1,P_{5+})} + \text{N}Is_{(t+1,P_{5+})}$$

**High shedding and clinically affected ($Ic$)**

$$Ic_{(t+1,a=h+2)} = \; \text{N}Ic_{(t+1,a)}$$

$$Ic_{(t+1,a\in[h+3;cal])} = \; \left[Ic_{(t,a-1)} - sIc_{(t,a-1)}\right] + \text{N}Ic_{(t+1,a)}$$

$-$

$$Ic_{(t+1,P_1)} = \; \left[Ic^{sg}_{(t,P_1)} - sIc^{sg}_{(t,P_1)} + Ic_{(t,cal)} - sIc_{(t,cal)}\right] + \text{N}Ic_{(t+1,P_1)}$$

$$Ic_{(t+1,P_i\in[P_2;P_4])} = \left[Ic^{sg}_{(t,P_i)} - sIc^{sg}_{(t,P_i)} + Ic^{ng}_{(t,P_{i-1})} - sIc^{ng}_{(t,P_{i-1})}\right] + \text{N}Ic_{(t+1,P_i)}$$

$$Ic_{(t+1,P_{5+})} = \left[Ic_{(t,P_{5+})} - sIc_{(t,P_{5+})} + Ic^{ng}_{(t,P_4)} - sIc^{ng}_{(t,P_4)}\right] + \text{N}Ic_{(t+1,P_{5+})}$$

### Equations describing flows

New incoming (and outgoing) flows in each health states are mainly drawn using binomial laws.

#### Births ($bX$)

At each time step $t$, births are calculated with regards to the health state of the dam. These births are then distributed into $S$ and $T$ states

$$bS_{(t)} = \ bS^{R}_{(t)} + bS^{L}_{(t)} + bS^{Is}_{(t)} + bS^{Ic}_{(t)} \qquad \text{et} \qquad bT_{(t)} = \ bT^{L}_{(t)} + bT^{Is}_{(t)} + bT^{Ic}_{(t)},$$

where $bS^{X}_{(t)}$ et $bT^{X}_{(t)}$ $(X \in R, L, Is, Ic)$ represent the number of births at time $t$ from cows in health state $X$:

$$\begin{aligned}
bS^{R}_{(t)} &\sim Bin\left(R_{(t,cal)} + \sum_{i=1}^{i=5}\left[R^{ng}_{(t,P_i)}\right]; 1-\sigma_B\right)\\
bS^{L}_{(t)} &= nbV^{L}_{(t)} - bT^{L}_{(t)}, \quad bT^{L}_{(t)} \sim Bin\left(nbV^{L}_{(t)}; p_L * \varphi_{p_{LIs}}\right)\\
bS^{Is}_{(t)} &= nbV^{Is}_{(t)} - bT^{Is}_{(t)}, \quad bT^{Is}_{(t)} \sim Bin\left(nbV^{Is}_{(t)}; p_{Is} * \varphi_{p_{LIs}}\right)\\
bS^{Ic}_{(t)} &= nbV^{Ic}_{(t)} - bT^{Ic}_{(t)}, \quad bT^{Ic}_{(t)} \sim Bin\left(nbV^{Ic}_{(t)}; p_{Ic} * \varphi_{p_{Ic}}\right)
\end{aligned}$$

In equations above, $nbC^{X}_{(t)}$ is the number of female calves alive born at time $t$. It is obtained from $nbV^{X}_{(t)}$, the number of female calves born at time $t$, from cows in the health state $X$:

$$\begin{aligned}
nbV^{L}_{(t)} &\sim Bin\left(L_{(t,cal)} + T_{(t,cal)} + \sum_{i=1}^{i=5}\left[L^{ng}_{(t,P_i)}\right]; 1-\sigma_B\right)\\
nbV^{Is}_{(t)} &\sim Bin\left(Is_{(t,cal)} + \sum_{i=1}^{i=5}\left[Is^{ng}_{(t,P_i)}\right]; 1-\sigma_B\right)\\
nbV^{Ic}_{(t)} &\sim Bin\left(Ic_{(t,cal)} + \sum_{i=1}^{i=5}\left[Ic^{ng}_{(t,P_i)}\right]; 1-\sigma_B\right)
\end{aligned}$$

We note that at the age of moving in the adult group ($cal$), all the animals in the health state $T$ enter the health state $L$ ($L_{(t,cal)} = L_{(t,cal)} + T_{(t,cal)}$). $\varphi_{p_x}$ is a factor, varying from 100% to 0%, of decrease in probability of in utero transmission where $x$ correspond to the dam giving birth in health states L and Is ($\varphi_{p_{LIs}}$) or Ic ($\varphi_{p_{Ic}}$).

#### Change in age group ($X^x$)

$$\begin{aligned}
X_{(t,P_i)} &= X^{sg}_{(t,P_i)} + X^{ng}_{(t,P_i)}\\
X^{ng}_{(t,P_i)} &\sim Bin\left(X_{(t,P_i)}; \frac{1}{\tau_{aa}}\right) \quad ; \quad \text{where } X = \{R, L, Is, Ic\},
\end{aligned}$$

and $\tau_{aa}$ is the average time spent in each of age group $P_1$ to $P_4$.

**Exits ($sX$)**

The mortality of calves during the first week of life is applied at birth and defined above in the section concerning births.
From age 1 to $cal$, mortality and culling rates $\sigma_x$ are defined as:

- $a \in \{1;2\} \rightarrow \sigma_x = \sigma_{c1}$
- $a \in [3;w] \rightarrow \sigma_x = \sigma_{c2}$
- $a \in [w+1;cal] \rightarrow \sigma_x = \sigma_{c3}$

Then, exits following death and culling write as:
for $a = 1$ :

$$sX_{(t,a)} \sim Bin\left(X_{(t,a)};\sigma_x\right), \quad \text{where } X = \{S,T\},$$

for $a \in [2;4]$ :

$$sX_{(t,a)} = sale + death, \quad \text{where } X = \{S,T,L\}$$
$$sale \sim Bin\left(X_{(t,a)};\sigma_m\right)$$
$$death \sim Bin\left(X_{(t,a)} - sale;\sigma_x\right)$$

for $a \in [5;cal-11]$ :

$$sX_{(t,a)} \sim Bin\left(X_{(t,a)};\sigma_x\right) \quad ; \quad X = \{S,R,T,L\}$$

for $a = cal - 10$, we consider management by heifers (safe management, keep all female calves):

$$Cows_{(t)} = \sum_{i=1}^{5} R_{(t,P_i)} + \sum_{i=1}^{5} L_{(t,P_i)} + \sum_{i=1}^{5} Is_{(t,P_i)} + \sum_{i=1}^{5} Ic_{(t,P_i)}$$
$$Heifers_{(t)} = \sum_{a=1}^{u} S_{(t,a)} + \sum_{a=u+1}^{cal} R_{(t,a)} + \sum_{a=1}^{cal} T_{(t,a)} + \sum_{a=1}^{cal} L_{(t,a)} + \sum_{a=1}^{cal} Is_{(t,a)} + \sum_{a=1}^{cal} Ic_{(t,a)}$$

If the number of heifers, $Heifers_{(t)}$, is greater than $K_g$ or that the number of cows, $Cows_{(t)}$, is greater than $K_v$, we consider the sale of heifers:

$$sX_{(t,a)} = sale + death \quad ; \quad X = \{R,T,L,Is,Ic\}$$
$$sale \sim Bin\left(X_{(t,a)};\exp\left(-\sigma_h.(Cows_{(t)}/K_v)^6\right).((Heifers_{(t)}/K_g)^6)\right)$$
$$death \sim Bin\left(X_{(t,a)} - sale;\sigma_x\right)$$

where $K_v$ is the capacity of the holding in number of cows (see Table 1) and $K_g = \sigma_P * K_v * (cal - h)$ is the capacity of the holding in number of heifers.

Otherwise, we do not consider the sale of heifers:

$$sX_{(t,a)} \sim Bin\left(X_{(t,a)};\sigma_x\right)$$

for $a \in [cal-9; cal]$:

$$sX_{(t,a)} = Bin\left(X_{(t,a)}; \sigma_x\right)$$

for $a \in [P_1; P_{5+}]$:

$$sX^x_{(t,P_i)} \sim Bin\left(X^x_{(t,P_i)}; \sigma_{Pi}\right), \quad \text{where } X = \{R, L, Is\},$$
$$sIc^x_{(t,P_i)} \sim Bin\left(Ic^x_{(t,P_i)}; \frac{1}{v_{Ic}}\right)$$

where $v_{Ic}$ corresponds to the average time spent in the health state $Ic$.

**New infections (S $\rightarrow$ T, except for *in-utero* transmission)**

We have:

$$\text{N}T_{(t+1,a)} = inf^c_{(t+1,a)} + inf^m_{(t+1,a)} + inf^l_{(t+1,a)} + inf^g_{(t+1,a)}$$

Superscripts correspond to different possible routes of transmission, respectively colostrum ($c$), milk ($m$), local environment ($l$) and global environment ($g$). New infections by *in-utero* transmission are accounted for through births.

By age, the possible routes of infection are:

$$0 \underbrace{\longrightarrow}_{cmlg} 1 \underbrace{\rightarrow \cdots \rightarrow}_{mlg} \text{weaning} \underbrace{\rightarrow \cdots \rightarrow}_{lg} \text{grazing allowed} \underbrace{\rightarrow \cdots \rightarrow}_{\substack{\text{outdoor: } l \\ \text{indoor: } lg}} \text{limit of susceptibility}$$

**Transmission through colostrum**

It is consideblackcol that calves drink colostrum from their mothers during the first three days before drinking milk:

$$inf^c_{(t+1,a=1)} = \sum_1^{bS^{Is}_{(t)}} \left[Bern\left(1 - exp\left(-\frac{\beta_l \; q_c^{Is}}{\alpha}\right)\right)\right] + \sum_1^{bS^{Ic}_{(t)}} \left[Bern\left(1 - exp\left(-\frac{\beta_l \; q_c^{Ic}}{\alpha}\right)\right)\right],$$

with

$$q_c^{Is} \sim Bern(sh_{Is}) \times \left[3 \times b\left(10^5.Beta(8;8) + 1 + 10^3.Beta(1;25)\right)\right] * \varphi Milk_{Is},$$
$$q_c^{Ic} \sim Bern(sh_{Ic}) \times \left[3 \times b\left(10^5.Beta(8;8) + 10^{(3+10.Beta(50;200))}\right)\right] * \varphi Milk_{Ic}.$$

Where $\varphi Milk_x$ is a factor, varying from 100% to 0%, of decrease in quantity of bacteria shed in colostrum and milk by an animal in health state $x$ (here $x$ could be $Is$ or $Ic$)

**Transmission through milk**

Regarding the age, we have:

$$a = 1: \quad inf^{m}_{(t+1,a)} \sim Bin\left(bS; 1 - exp\left(-\frac{\beta_l \; q_l \; \frac{4}{7}}{\alpha}\right)\right),$$

$$a \in \{2; w\}: \quad inf^{m}_{(t+1,a)} \sim Bin\left(\left[S_{(t,a-1)} - sS_{(t,a-1)}\right]; \left[1 - exp\left(-e^{(-\gamma(a-1))}.\frac{\beta_l \; q_l}{\alpha}\right)\right]\right),$$

with

$$q_l = \frac{7 \times d \times \left(Q^{milk}_{Is(t)} + Q^{milk}_{Ic(t)}\right)}{MilkTot_{(t)}},$$

where

$$Q^{milk}_{Is(t)} = \; 7 \times \varepsilon \times g_{Is} \times \left(\sum_{1}^{nbExcr^{Is}_{(t)}} (10^5.Beta(8;8)) + \sum_{1}^{nbLac^{Is}_{(t)}} (1 + 10^3.Beta(1;25))\right) * \varphi Milk_{Is},$$

$$Q^{milk}_{Ic(t)} = \; 7 \times \varepsilon \times g_{Ic} \times \left(\sum_{1}^{nbExcr^{Ic}_{(t)}} (10^5.Beta(8;8)) + \sum_{1}^{nbLac^{Ic}_{(t)}} 10^{(3+10.Beta(50;200))}\right) * \varphi Milk_{Ic},$$

$$MilkTot_{(t)} = \; 7 \times \varepsilon \times \left(nbLac^{R}_{(t)} + g_L.nbLac^{L}_{(t)} + g_{Is}.nbLac^{Is}_{(t)} + g_{Ic}.nbLac^{Ic}_{(t)}\right).$$

with

$$nbExcr^{Is}_{(t)} \sim Bin\left(nbLac^{Is}_{(t)}; sh_{Is}\right),$$

$$nbExcr^{Ic}_{(t)} \sim Bin\left(nbLac^{Ic}_{(t)}; sh_{Ic}\right),$$

$$nbLac^{R}_{(t)} \sim Bin\left(\sum_{i=1}^{i=5} R_{(t,P_i)}, prop\right),$$

$$nbLac^{L}_{(t)} \sim Bin\left(\sum_{i=1}^{i=5} L_{(t,P_i)}, prop\right),$$

$$nbLac^{Is}_{(t)} \sim Bin\left(\sum_{i=1}^{i=5} Is_{(t,P_i)}, prop\right),$$

$$nbLac^{Ic}_{(t)} \sim Bin\left(\sum_{i=1}^{i=5} Ic_{(t,P_i)}, prop\right).$$

**Local transmission (in collective pens, $a \in [1; u]$)**

$$inf^{l}_{(t+1,a)} \sim Bin\left(S_{(t,a-1)} - sS_{(t,a-1)}; p^{i}_{inf(t+1,a)}\right),$$

where

during housing period:

$$p^i_{inf(t+1,a)} = 1 - exp\left(-e^{(-\gamma(a-1))}.\frac{\beta_c E^i_{(t+1)}}{\alpha N^i_{(t+1)}}\right), \text{ and}$$

during grazing period, regarding the age,

$$a \in [1;26]: \quad p^i_{inf(t+1,a)} = 1 - exp\left(-e^{(-\gamma(a-1))}.\frac{\beta_c E^i_{(t+1)}}{\alpha N^i_{(t+1)}}\right),$$

$$a \in [27;u]: \quad p^i_{inf(t+1,a)} = 1 - exp\left(-e^{(-\gamma(a-1))}.\frac{\beta_o E^i_{(t+1)}}{\alpha N^i_{(t+1)}}\right).$$

$N^i_{(t+1)}$ is the total number of animals in environment $i$, across all health states, and $E^i$ represents the quantity of bacteria in the environment, with $i$ corresponding to the specific area.

**Global transmission**

This occurs up to the age allowing to go to the pasture (26 weeks - 6 months) during the grazing period, and up to the age limit for sensitivity ($u$) during the housing period:

$$inf^g_{(t+1,a)} \sim Bin\left(S_{(t,a)} - sS_{(t,a)}; p^g_{inf(t+1,a)}\right),$$

where

$$p^g_{inf(t+1,a)} = 1 - exp\left(-exp[-h(a-1)].\frac{\beta_g E^g_{(t+1)}}{\alpha N^g_{(t+1)}}\right), \quad \text{with} \quad E^g_{(t+1)} = \sum_{i=1}^{i=5} E^{\text{INT}i}_{(t+1)}$$

$E^{\text{INT}i}$ represents the quantity of bacteria in the environment where INT$i$ corresponds to a specific area (see the section below about the dynamics of bacteria in the environments).

**New latently infected ($\mathbf{T} \rightarrow \mathbf{L}$)**

For $a < cal$:

$$\text{N}L_{(t+1,a\in[2;cal-1])} \sim Bin\left(T_{(t,a)} - sT_{(t,a)}; \frac{1}{v_T}\right).$$

After age $cal$, there are no more animals in $T$ state:

$$\text{N}L_{(t+1,P_1)} = T_{(t,cal)} - sT_{(t,cal)}.$$

**New subclinically infected (L → Is)**

For heifers:

$$\text{N}Is_{(t+1,a\in[h+1;cal])} \sim Bin\left(L_{(t,a)} - sL_{(t,a)}; \frac{1}{v_L}\right).$$

For cows:

$$\text{N}Is_{(t+1,P_i)} \sim Bin\left(n; \frac{1}{v_L}\right),$$

with, regarding the age,

$$\begin{aligned}
P_1 \rightarrow n &= \left[L^{sg}_{(t,P_1)} - sL^{sg}_{(t,P_1)} + L_{(t,cal)} - sL_{(t,cal)} + T_{(t,cal)} - sT_{(t,cal)}\right], \\
\{P_2; P_4\} \rightarrow n &= \left[L^{sg}_{(t,P_i)} - sL^{sg}_{(t,P_i)} + L^{ng}_{(t,P_{i-1})} - sL^{ng}_{(t,P_{i-1})}\right], \\
P_{5+} \rightarrow n &= \left[L_{(t,P_{5+})} - sL_{(t,P_{5+})} + L^{ng}_{(t,P_4)} - sL^{ng}_{(t,P_4)}\right].
\end{aligned}$$

**New clinically infected (Is → Ic)**

For heifers:

$$\text{N}Ic_{(t+1,a\in[h+1;cal])} \sim Bin\left(Is_{(t,a)} - sIs_{(t,a)}; \frac{1}{v_{Is}}\right).$$

For cows:

$$\text{N}Ic_{(t+1,P_i)} \sim Bin\left(n; \frac{1}{v_{Is}}\right),$$

with

$$\begin{aligned}
P_1 \rightarrow n &= \left[Is^{sg}_{(t,P_1)} - sIs^{sg}_{(t,P_1)} + Is_{(t,cal)} - sIs_{(t,cal)}\right], \\
\{P_2; P_4\} \rightarrow n &= \left[Is^{sg}_{(t,P_i)} - sIs^{sg}_{(t,P_i)} + Is^{ng}_{(t,P_{i-1})} - sIs^{ng}_{(t,P_{i-1})}\right], \\
P_{5+} \rightarrow n &= \left[Is_{(t,P_{5+})} - sIs_{(t,P_{5+})} + Is^{ng}_{(t,P_4)} - sIs^{ng}_{(t,P_4)}\right].
\end{aligned}$$

**New resistant (S → R)**

At age $u$, the transition from compartment S to compartment R is done in a deterministic way.

**Dynamics of bacteria in the environments (E)**

The composition of the environments according to the season is the following:

$$\text{Grazing} : a = \overbrace{1 \cdots\cdots w}^{\text{INT1}} \overbrace{\cdots\cdots 26}^{\text{INT2}} \overbrace{\cdots\cdots\cdots\cdots\cdots\cdots h}^{\text{EXT1+EXT2}} \overbrace{\cdots\cdots\cdots cal}^{\text{EXT3}}$$

$$\text{Housing} : a = \overbrace{1 \cdots\cdots w}^{\text{INT1}} \overbrace{\cdots\cdots\cdots\cdots\cdots\cdots y}^{\text{INT2}} \overbrace{\cdots\cdots\cdots\cdots h}^{\text{INT3}} \overbrace{\cdots\cdots\cdots cal}^{\text{INT4}}$$

Dynamics of bacteria in the environments ($E$) are defined below:

$E^{\text{INT1}}_{(t+1)} = E^{\text{INT1}}_{(t)}.(1-\mu^{int}_g) + Q^{\text{TNS}}_{(t)}$. If pens are empty, it becomes $E^{\text{INT1}}_{(t+1)} = E^{\text{INT1}}_{(t+1)}.(1-\mu_{cp})$, where $Q$ represents the quantity of bacteria shed.

During grazing period, we have:

$$\begin{aligned}
E^{\text{INT2}}_{(t+1)} &= E^{\text{INT2}}_{(t)}.(1-\mu^{int}_g) + Q^{\text{TS1}}_{(t)} \\
&\quad If \quad \sum_{a=w+1}^{a=26} SR_{t,a} + \sum_{a=w+1}^{a=26} T_{t,a} + \sum_{a=w+1}^{a=26} L_{t,a} == 0 \quad then \quad E^{\text{INT2}}_{(t+1)} = E^{\text{INT2}}_{(t+1)}.(1-\mu_{cp}) \\
E^{\text{INT3}}_{(t+1)} &= E^{\text{INT3}}_{(t)}.(1-\mu^{int}_g) \\
E^{\text{INT4}}_{(t+1)} &= E^{\text{INT4}}_{(t)}.(1-\mu^{int}_g) \\
E^{\text{INT5}}_{(t+1)} &= E^{\text{INT5}}_{(t)}.(1-\mu^{int}_g) \\
E^{\text{EXT1}}_{(t+1)} &= E^{\text{EXT1}}_{(t)}.(1-\mu^{ext}_g) + Q^{\text{TS2}}_{(t)} \\
E^{\text{EXT2}}_{(t+1)} &= E^{\text{EXT2}}_{(t)}.(1-\mu^{ext}_g) + Q^{\text{TY}}_{(t)} \\
E^{\text{EXT3}}_{(t+1)} &= E^{\text{EXT3}}_{(t)}.(1-\mu^{ext}_g) + Q^{\text{TH}}_{(t)} + Q^{\text{ISH}}_{(t)} + Q^{\text{ICH}}_{(t)}.
\end{aligned}$$

During housing period, we have:

$$\begin{aligned}
E^{\text{INT2}}_{(t+1)} &= E^{\text{INT2}}_{(t)}.(1-\mu^{int}_g) + Q^{\text{TS1}}_{(t)} + Q^{\text{TS2}}_{(t)} \\
&\quad If \quad \sum_{a=w+1}^{a=52} SR_{t,a} + \sum_{a=w+1}^{a=52} T_{t,a} + \sum_{a=w+1}^{a=52} L_{t,a} == 0 \quad then \quad E^{\text{INT2}}_{(t+1)} = E^{\text{INT2}}_{(t+1)}.(1-\mu_{cp}) \\
E^{\text{INT3}}_{(t+1)} &= E^{\text{INT3}}_{(t)}.(1-\mu^{int}_g) + Q^{\text{TY}}_{(t)} \\
E^{\text{INT4}}_{(t+1)} &= E^{\text{INT4}}_{(t)}.(1-\mu^{int}_g) + Q^{\text{TH}}_{(t)} + Q^{\text{ISH}}_{(t)} + Q^{\text{ICH}}_{(t)} \\
E^{\text{INT5}}_{(t+1)} &= E^{\text{INT5}}_{(t)}.(1-\mu^{int}_g) + Q^{\text{IS}}_{(t)} + Q^{\text{IC}}_{(t)} \\
E^{\text{EXT1}}_{(t+1)} &= 0, \quad E^{\text{EXT2}}_{(t+1)} = 0, \quad E^{\text{EXT3}}_{(t+1)} = 0.
\end{aligned}$$

Shed quantities of bacteria are defined, regarding the health states and the age, by:

unweaned calves T:

$$Q^{\text{TNS}}_{(t)} = \sum_{a=1}^{a=w} \left[ 7.f_1.10^6. \sum^{T_{(t,a)}} Beta(8.8;19) \right] * \varphi faeces_T,$$

weaned calves T, without access to grazing:

$$Q^{\text{TS1}}_{(t)} = \sum_{a=w+1}^{a=26} \left[ 7.f_2.10^6. \sum^{T_{(t,a)}} Beta(8.8;19) \right] * \varphi faeces_T,$$

weaned calves T, with access to grazing:

$$Q_{(t)}^{\mathrm{Ts2}} = \sum_{a=27}^{a=y} \left[ 7.f_2.10^6. \sum^{T_{(t,a)}} Beta(8.8;19) \right] * \varphi faeces_T,$$

young heifers T:

$$Q_{(t)}^{\mathrm{TY}} = \sum_{a=y+1}^{a=h} \left[ 7.f_Y.10^6. \sum^{T_{(t,a)}} Beta(8.8;19) \right] * \varphi faeces_T,$$

heifers T:

$$Q_{(t)}^{\mathrm{TH}} = \sum_{a=h+1}^{a=cal} \left[ 7.f_A.10^6. \sum^{T_{(t,a)}} Beta(8.8;19) \right] * \varphi faeces_T,$$

heifers Is:

$$\text{if } \sum_{a=h+1}^{a=cal} \left( Is_{(t,a)} \right) > 0 \ : \quad Q_{(t)}^{\mathrm{ISH}} = \sum_{a=h+1}^{a=cal} \left[ 7.f_A.10^{(4+10\times\sum^{Is_{(t,a)}} Beta(2.65;17))} \right] * \varphi faeces_{Is}$$

$$\text{else} \quad Q_{(t)}^{\mathrm{ISH}} = 0,$$

cows Is:

$$\text{if } \sum_{i=1}^{i=5} \left( Is_{(t,P_i)} \right) > 0 \ : \quad Q_{(t)}^{\mathrm{IS}} = \sum_{i=1}^{i=5} \left[ 7.f_A.10^{(4+10\times\sum^{Is_{(t,P_i)}} Beta(2.65;17))} \right] * \varphi faeces_{Is}$$

$$\text{else} \quad Q_{(t)}^{\mathrm{IS}} = 0,$$

heifers Ic:

$$\text{if } \sum_{a=h+1}^{a=cal} \left( Ic_{(t,a)} \right) > 0 \ : \quad Q_{(t)}^{\mathrm{ICH}} = \sum_{a=h+1}^{a=cal} \left[ 7.f_A.10^{(8+10\times\sum^{Ic_{(t,a)}} Beta(2;17))} \right] * \varphi faeces_{Ic}$$

$$\text{else} \quad Q_{(t)}^{\mathrm{ICH}} = 0,$$

cows Ic:

$$\text{if } \sum_{i=1}^{i=5} \left( Ic_{(t,P_i)} \right) > 0 \ : \quad Q_{(t)}^{\mathrm{IC}} = \sum_{i=1}^{i=5} \left[ 7.f_A.10^{(8+10\times\sum^{Ic_{(t,P_i)}} Beta(2;17))} \right] * \varphi faeces_{Ic}$$

$$\text{else} \quad Q_{(t)}^{\mathrm{IC}} = 0.$$

$\varphi faeces_x$, in the equation above is a factor of decrease in quantity of bacteria shed by an animal in health state $x$. $x$ could be $T$, $Is$ or $Ic$.

# B Parameters ralated to population dynamics

**Table 1:** Parameters for management and population dynamics used in the *Mycobacterium avium subsp. paratuberculosis* (*Map*) infection dynamics model within a structublackcol dairy herd (reproduced from [2], Table 1).

| Notation | Value | Definition | Source |
|---|---|---|---|
| $\sigma_B$ | $0.07$ | Mortality rate of calves at birth | $*$, [17] |
| $\sigma_m$ | $0.206$ | Exit rate of male calves, weeks 2 to 4 (per week) | |
| $\sigma_{c1}$ | $0.015$ | Death rate of female calves, weeks 1 and 2 (per week) | [17] |
| $\sigma_{c2}$ | $0.0035$ | Death rate of female calves, weeks 3 to weaning (per week) | [9] |
| $\sigma_{c3}$ | $0.00019$ | Death rate of heifers from weaning to entry in adult group (per week) | † |
| $\sigma_h$ | $0.011$ | Sale rate of bblackcol heifers 10 weeks before 1st calving | † |
| $\sigma_{Ai}$ | $0.27$,$0.25$,$0.31$, $0.31$,$0.62$ | Yearly culling rate of cows in adult group $i$: 1, 2, 3, 4 and above 5 respectively (%) | $*$, [1] |
| $w$ | $10$ | Weaning age (weeks) | [12] |
| $y$ | $52$ | Age when entering the young heifer group (weeks) | |
| $h$ | $91$ | Age when entering the heifer group (weeks) | $*$ |
| $cal$ | $130$ | Age when entering the adult group (weeks) | $*$, † |
| $\tau_{aa}$ | $56.3$ | Mean time spent in adult age groups 1 to 4 (weeks) | $*$, † |
| $b$ | $5$ | Quantity of colostrum fed to calves (L/day for 3 days) | † |
| $d$ | $7$ | Quantity of milk fed to calves after 3 days (L/day/calf) | † |
| $prop$ | $0.85$ | Proportion of lactating cows | $*$ |
| $\varepsilon$ | $25$ | Quantity of milk or colostrum produced (L/day/cow) | $*$ |
| $f_1$ | $0.5$ | Quantity of feaces produced by a non-weaned calf (kg/day) | † |
| $f_2$ | $5.5$ | Quantity of feaces produced by a weaned calf (kg/day) | † |
| $f_Y$ | $10$ | Quantity of feaces produced by a heifer (kg/day) | † |
| $f_A$ | $30$ | Quantity of feaces produced by a cow (kg/day) | † |
| $Graz$ | $[14-26]$ | Grazing period (1 being the first week of the year) | † |
| $K_c$ | $110$ | Number of cows above which the heifer selling rate increases | |

$*$ Agricultural statistics.

† Based on expert opinion.

# C Parameters related to infection dynamics

**Table 2:** Parameters for infection and transmission used in the *Mycobacterium avium subsp. paratuberculosis* (*Map*) infection dynamics model within a structublackcol dairy herd (reproduced from [2] - Table 2).

| Notation | Value | Definition | Source |
|---|---|---|---|
| $p_X$ | | Probability of *in utero* transmission for cow in health state $X$ | [4, 26] |
| | $p_L = 0.149$ | $X$ = latently infected (L) | |
| | $p_{Is} = 0.149$ | $X$ = subclinically infected (Is) | |
| | $p_{Ic} = 0.65$ | $X$ = clinically affected (Ic) | |
| $u$ | $52$ | Maximal age in the susceptible compartment (weeks) | [6, 23] |
| $\gamma$ | $0.1$ | Susceptibility follows an exponential decrease : $exp(-\gamma(age-1))$ | [27] |
| $v_X$ | | Mean time spent in health state $X$ (weeks) | |
| | $v_T = 25$ | $X$ = transiently infectious (T) | [21] |
| | $v_L = 52$ | $X$ = latently infected (L) | [16, 14] |
| | $v_{Is} = 104$ | $X$ = subclinically infected (Is) | [13] |
| | $v_{Ic} = 26$ | $X$ = clinically affected (Ic) | † |
| $sh_X$ | | Probability of shedding in colostrum or milk for a cow in health state $X$ | [20, 19] |
| | $sh_L = 0$ | $X$ = latently infected (L) | |
| | $sh_{Is} = 0.4$ | $X$ = subclinically infected (Is) | |
| | $sh_{Ic} = 0.9$ | $X$ = clinically affected (Ic) | |
| $\alpha$ | $10^6$ | *Map* infectious dose | [3] |
| $\beta_l$ | $5 \times 10^{-4} \times 7$ | Transmission rate if ingestion of an infectious dose (per week) | ‡ |
| $\beta_c$ | $5 \times 10^{-5} \times 7$ | Transmission rate if one infectious dose is present in the local environment (per week) | [21] |
| $\beta_g$ | $9.5 \times 10^{-7} \times 7$ | Transmission rate if one infectious dose is present in the global environment (per week) | [21] |
| $\beta_o$ | $5 \times 10^{-6} \times 7$ | Transmission rate if one infectious dose is present in the pasture (per week) | ‡ |
| $g_X$ | | Decrease in milk production for cattle in health state $X$ (per week) | [15] |
| | $g_{Is} = 1 - 0.11$ | $X$ = subclinically infected (Is) | |
| | $g_{Ic} = 1 - 0.25$ | $X$ = clinically affected (Ic) | |
| $\mu_k$ | | Removal rate of Map from environment k | [7, 24] |
| | $\mu_g^{int} = 0.4$ | all the environments (per week) | |
| | $\mu_g^{ext} = 1/14$ | all the environments (per week) | |
| | $\mu_{cp} = 0.17$ | collective pens (when empty) | |

† Expert opinions.

‡ Parameters' values are assumed.

**Table 3:** Summary of published data and assumed distributions of the quantities of *Mycobacterium avium* subsp. *paratuberculosis* (*Map*) shed, depending on the health state (X) and the route of transmission (r) in the *Map* infection dynamics model within a structublackcol dairy herd (reproduced from [2] - Table 3).

| **Route of transmission (r)** | **Health state (X)** | **Literature** | | | | **Assumed distributions** |
|---|---|---|---|---|---|---|
| | | **Minimal value** | **Maximal value** | **Mean value** | **Source** | |
| Map direct shedding in milk and colostrum (Map/L) | Subclinically infected | $2.2 \times 10^4$ | $8.8 \times 10^4$ | $5 \times 10^4$ | [20] | $10^5 \times Beta(8;8)$ |
| | Clinically affected | - | - | $5 \times 10^4$ | [5] | $10^5 \times Beta(8;8)$ |
| Map indirect shedding in milk and colostrum (faecal contamination) (Map/L) | Subclinically infected | 0 | $2 \times 10^{10}$ | 40 | [10, 22] | $1 + 10^3 \times Beta(1;25)$ |
| | Clinically affected | 700 | $2 \times 10^{10}$ | $14 \times 10^4$ | [10, 22] | $10^{(3+10\times Beta(50;200))}$ |
| Map shedding in faeces (Map/kg) | Transiently infectious | $6 \times 10^4$ | $6.3 \times 10^5$ | $3 \times 10^5$ | [21] | $10^6 \times Beta(8.8;19)$ |
| | Subclinically infected | $10^4$ | $10^{15}$ | $2.6 \times 10^6$ | [18] | $10^{(4+10\times Beta(2.65;17))}$ |
| | Clinically affected | $10^8$ | $10^{15}$ | $10^{10}$ | [25, 8] | $10^{(8+10\times Beta(2;17))}$ |

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