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Convex Optimization for the Synthesis of Matching Filters

G. Bose* F. Ferrero† L. Lizzi† D. Martinez* F. Seyfert*

***Abstract* — In this work we study a particular filter synthesis problem in order to minimize the reflection coefficient of the global system consisting of filter and antenna. The matching problem is formulated as an optimization problem involving the minimization of a pseudo hyperbolic distance and the solution to this problem using H^∞ approach yields a lower bound for the matching criterion related to the computation of a matching filter, with prescribed finite degree, under selectivity constraints.**

1 INTRODUCTION

In classical communication systems, antennas used in the reception or emission chain are often associated with a matching network followed by a bandpass filter so as to select a proper frequency channel, to reject unwanted signals and to maximize the power transfer. In the last years, to fulfill the constraints imposed by modern applications (e.g., the internet-of-things (IoT), wireless sensor networks, e-health, etc.), wireless communicating devices are required to be compact and energy autonomous to enable easy integration and long-lasting operation. In order to improve the energy efficiency as well as the footprint of the transmission chain we propose to design, in a single circuit, the filter and the matching network. This matching filter has to be design accordingly to the antenna. This codesign approach amounts to tackle a particular filter synthesis problem where one of the filter's port is loaded by a frequency varying load, namely the antenna (Fig. 1).

The matching problem, is a rather old one, and goes back to the foundational work of Fano and Youla in the fifties [1]. This theory develops a synthesis procedure for matching networks, based on the derivation of a 2 port loss-less scattering matrix representing the filter chained to the antenna. However unlike in the Darlington's insertion-loss synthesis, this framework does not translate into a convex or quasi-convex optimization problem of Zolotariov type yielding an optimal response. This is mainly due to the interpolation constraints that

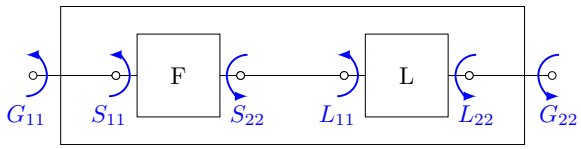

Figure 1: Matching Filter (S) and Load (L)

needs to be added to the two port's description to take into account the presence of the antenna. For this reason, this theory was progressively abandoned to the benefit of a non-convex optimization method called real frequency technique and originally introduced by Carlin [2]. Although this heuristical approach seems to give reasonable results in practice, no result is known about the global optimality or near-optimality of the so obtained matching network. More recently J.W Helton proposed an H^∞ approach to the problem where an infinite dimensional matching network is being sought for [3]. The matching problem is recast as a quasi-convex optimization problem involving the minimization of a pseudo hyperbolic distance. The absence of any degree constraint on the circuital response is here traded for the guaranteed global optimality of the obtained response. The relative mathematical complexity of this procedure coupled to the impossibility to realize in practice an infinite dimensional H^∞ response have severely limited its impact in electronics.

In this work we study a practical implementation of J.W Helton's approach based on the resolution of a bounded extremal problem in H^∞. Details are in particular given about the effective computation of the best approximation operator from $L^\infty \to H^\infty$ via Nehari's theory. We show that the solution to this problem furnishes a lower bound for the matching criterion related to the computation of a matching filter, with prescribed finite degree, under selectivity constraints. When the antenna's reflection parameter admits a rational approximation of degree one in the frequency band of interest, we have recently shown that the best matching filter synthesis problem admits a convex formulation. This approximation well applies to miniature and narrowband antennas suitable for compact IoT devices requiring the transmission of a limited amount of data. We studied on concrete antenna examples

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the computation and behaviour of the best finite dimensional matching filter response using Helton's approach.

2 MATCHING PROBLEM

We consider the synthesis of a matching filter for a given frequency varying load. Given a passband B the matching filter is designed so as to minimize, when plugged on the load, the power reflected by the load. The global system (G) consisting of filter together with the load is represented in Fig. 1. The parameters G_{11}, S_{11} and L_{11} denote the input reflection coefficient of the global system, matching filter and load respectively whereas G_{22}, S_{22} and L_{22} represent the output reflection coefficient of the same respectively.

If the filter is considered lossless, the output reflection coefficient of the global system (at each frequency ω) can be computed as [5] :

$$\begin{aligned} G_{22} &= L_{22} + \frac{L_{12}L_{21}S_{22}}{1 - S_{22}L_{11}} \\ &= \frac{L_{22} - S_{22}\det(\mathrm{L})}{1 - S_{22}L_{11}} \\ &= \det(\mathrm{L})\frac{L_{11}^* - S_{22}}{1 - S_{22}L_{11}} \end{aligned} \tag{1}$$

Hence the modulus of the output reflection coefficient of the global system is obtained as the pseudo hyperbolic distance between S_{22} and L_{11}^* :

$$|G_{22}| = \left| \frac{S_{22} - L_{11}^*}{1 - S_{22}L_{11}} \right| = \delta\left(S_{22}, L_{11}^*\right) \tag{2}$$

2.1 Optimization Problem

In the infinite dimensional setting, where the filter's scattering parameters are sought for in the Hardy space H^∞ of bounded analytic functions of the lower half plane [6], the matching problem over a frequency band B can be formulated as follows.

Problem 1 : Minimize the pseudo hyperbolic distance between S_{22} and L_{11}^*.

$$\min \max_{\omega \in B} \delta(S_{22}(\omega), L_{11}^*(\omega)) \tag{3}$$

$$\text{subject to: } S_{22}(\omega) \in H^\infty \text{ and } |S_{22}(\omega)|_{\omega\in\mathbb{R}} \leq 1 \tag{4}$$

where B is the desired matching band (passband).

2.1.1 Practical Algorithm

We suppose that we possess a passive rational model f of the antenna's reflection parameter L_{11}, obtained for example via rational approximation

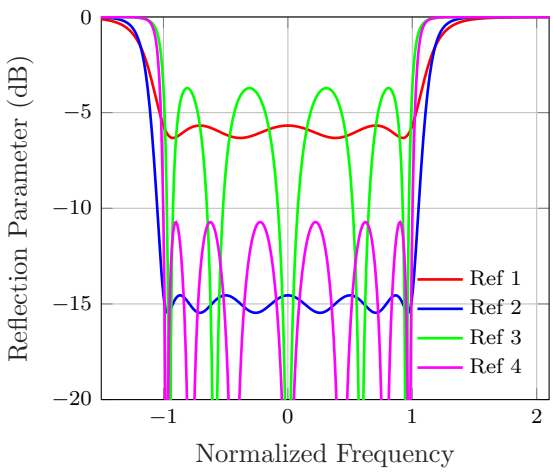

Figure 2: Reference Functions

techniques at hand of scattering measurements. We form a family of rational reference functions $\{k_\alpha\}$ parametrized by $\alpha \in \mathbb{R}^+$, the modulus of which mimic an ideal step function k_{opt}:

$$k_{opt}(\omega) = \begin{cases} L & \omega \in B \quad 0 \leq L \leq 1 \\ 1 & \omega \notin B \end{cases} \tag{5}$$

There are multiple ways to approach rationally a step function. We choose here to follow the classical Darlington insertion loss synthesis for filters. Consider the Belevitch form of a general loss-less rational scattering matrix,

$$G = \frac{1}{q}\begin{bmatrix} \epsilon p^* & -\epsilon r^* \\ r & p \end{bmatrix} \tag{6}$$

where ϵ is a unimodular constant, q satisfies the spectral equation $qq^* = pp^* + rr^*$. The modulus square of G_{22} can be expressed as :

$$|G_{22}|^2 = \frac{pp^*}{qq^*} \tag{7}$$

$$= \frac{pp^*}{pp^* + rr^*} = \frac{1}{1 + \frac{rr^*}{pp^*}} \tag{8}$$

For simplicity, we will consider here the case where r is of degree zero. We fix $\alpha = rr^*$, and define k_α to be the rational outer factor verifying:

$$|k_\alpha|^2 = \frac{1}{1 + \alpha|\frac{1}{p}|^2} \tag{9}$$

If we take for p, $p = T_N$ the Tchebyschev polynomial of degree N in the interval B, $\{k_\alpha\}$ forms a family of Tchebyschev rational reference functions whose modulus can be varied monotonously using the parameter α.

Alternatively, if we take $p = T_N + c$, where c is a positive constant the $\{k_\alpha\}'s$ form a family of rational functions with monotonously decreasing moduli, that equi-oscillate on B between the values $1/(1+\alpha/(c+1)^2)$ and $1/(1+\alpha/(c)^2)$. Fig. 2 shows some examples of reference functions where Ref 1 and Ref 2 are equi-oscillating references whereas Ref 3 and Ref 4 are pure Tchebyschev references.

2.1.2 Bound for the reflection level using H^∞ approach

After deriving the rational function f and forming the family of rational reference functions $\{k_\alpha\}$, we approach Problem 1 as follows :

Problem 2 : Find a $g \in H^\infty$ such that:

$$\sup_{\omega\in\mathbb{R}} \delta(f^*(\omega), g(\omega)) \leq |k(\omega)| \tag{10}$$

where k runs over the family $\{k_\alpha\}$.

Solution : The hyperbolic disk, $\delta(f^*, g) \leq |k|$ with centre f^* and radius $|k|$ translates to the following euclidean disk :

$$\left| g - \frac{(1-|k|^2)}{1-|k|^2|f|^2} f^* \right| \leq \frac{(1-|f|^2)}{1-|k|^2|f|^2}|k| \tag{11}$$

Now, if we factorize $\frac{(1-|f|^2)}{1-|k|^2|f|^2} = uu^*$, where u is an outer function and consider the outer function $v = u^2k$,

$$|v| = \left| \frac{(1-|f|^2)}{1-|k|^2|f|^2} \right| |k| \tag{12}$$

Now dividing (11) by $|v|$,

$$\left| \frac{g}{v} - \frac{1}{v}\frac{(1-|k|^2)}{1-|k|^2|f|^2} f^* \right| \leq 1 \tag{13}$$

yields a classical Nehari problem. Let $G = \frac{g}{v}$ and $F = \frac{1}{v}\frac{(1-|k|^2)}{1-|k|^2|f|^2} f^*$.

The function v being invertible in H^∞, the problem reduces to

$$\min ||G - F||_\infty \tag{14}$$

subject to : $G \in H^\infty$ and finding the G at which the infimum is attained. The solution to (14) can be obtained using the classical operator theoretic approach of Nehari [6],

$$G = F - \frac{H_F(V)}{V} \tag{15}$$

where H_F is the Hankel operator with symbol F and V one of its maximizing vectors.

For numerical reasons we chose to implement Nehari's solution to the extremal problem (14) in the framework of Hardy spaces of the unit disc $\mathbb{D}$. This is done classically using the conformal map $z \to j(z-1)/(z+1)$ sending the unit disk to the lower half plane. Given a rational function F, in order to find the maximizing vector of the Hankel operator, $H_F : H_2(\mathbb{D}) \to \bar{H}_2(\mathbb{D})$, we follow the steps below:

(i) Let $\{z_j\}$ be the poles of F inside the unit disk.

(ii) $g_j = \{\frac{1}{1-z\bar{z}_j}\}$ form the basis of $(\mathrm{Ker}(H_F))^\perp$ and $h_j = \{\frac{1}{z-z_j}\}$ form a basis of the image of H_F in $\bar{H}_2$.

(iii) Let the gram matrix of the $\{g_j\}'s$ be,

$$G_1 = [a_{i,j}] =< g_i, g_j >= \left[\frac{1}{1-\bar{z}_j z_i}\right] \tag{16}$$

and the gram matrix of the $\{h_j\}'s$,

$$G_2 = [b_{i,j}] =< h_i, h_j >= \left[\frac{1}{1-\bar{z}_i z_j}\right] \tag{17}$$

(iv) Denote the matrix of the Hankel operator by A in the basis $\{g_j\}$ and $\{h_j\}$, and solve the generalized eigenvalue problem: $A^*G_2Au = \lambda G_1 u$.

The eigenvector corresponding to the largest eigenvalue will provide the maximizing vector of the Hankel operator and the square root of largest eigenvalue will provide the value of the minimum in (14). The function g is obtained multiplying back by the outer factor v, that is $g = vG$.

So for a given f, the solution to Problem 2, when one exists, provides the output reflection coefficient S_{22} of a matching filter for a given reference k_α. As shown by equation (13), there exists a solution to Problem 2, if and only if the operator norm of H_F is less-than or equal to unity. Increasing the parameter α lowers the level of reference, implying that $||H_F||$ is an increasing function of α (when $k = k_\alpha$ is set). Ruling out the case where f is a constant, and noting that an anti-analytic function can not be approached uniformly and arbitrarily close by an analytic one, implies that there exists α_0 such that $||H_f|| = 1$. For $\alpha = \alpha_0$, equality holds in equation (10). We call the so obtained reflection coefficient, an approximate solution to Problem 1 with respect to the family of rational references $\{k_\alpha\}$. A remarkable property of the latter is that its degree is comparable to that of the reference k_{α_0} (proof of this goes beyond the objective of this paper).

3 Results

For $\alpha = \alpha_0$, the modulus of the global system's reflection parameter equals that of k_{α_0}. Fig. 3 shows

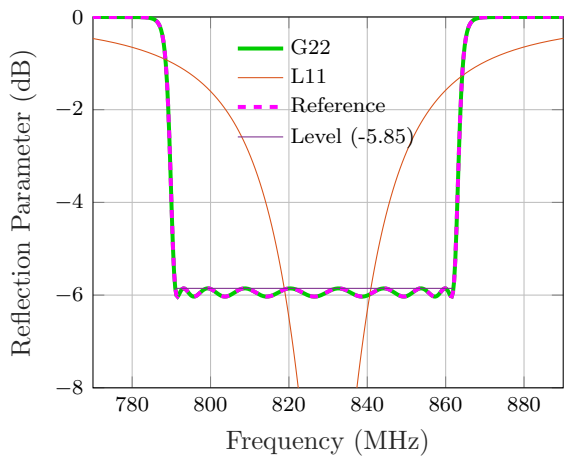

Figure 3: Optimal System reflection with degree 9 equi-oscillating reference for an antenna of degree 1 with Fano Bound = -6.20.

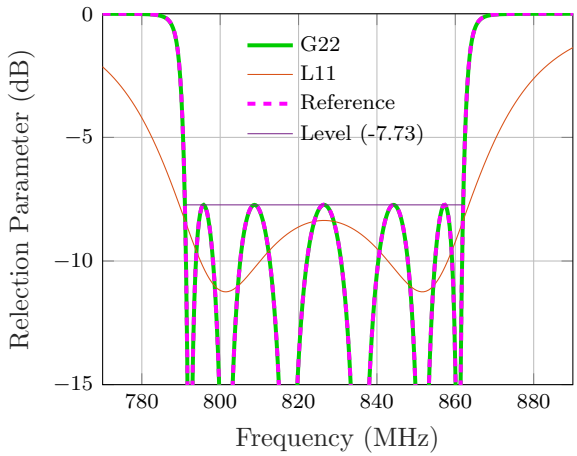

Figure 4: Optimal System reflection with degree 6 Tchebyschev reference for an antenna of degree 2 with Fano Bound = -13.64.

the optimal overall system reflection for an antenna of degree one using equi-oscillating references of degree nine, while Fig. 4 shows the same for an antenna of degree two using Tchebyschev references of degree six. Optimal system reflection level obtained using equi-oscillating references of various degrees were computed for a given antenna of degree one (Fig. 5).

4 Conclusion

Helton's H^∞ approach to matching theory, supposed to yield a guaranteed optimal response at the cost of an infinite degree matching network, can be successfully adapted to yield finite degree matching networks by using families of finite degree rational references. The procedure can therefore be put into practice to derive matching networks for mismatched antennas. Whether the one dimensional families of reference functions can be extended to broader functional manifolds, while preserving the underlying convexity of the optimization problem to solve is currently under study.

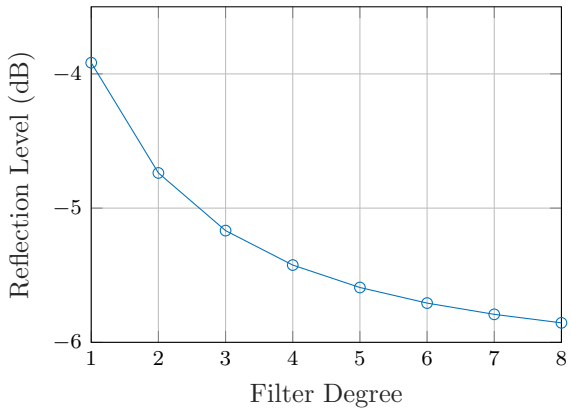

Figure 5: Optimal System reflection for different degrees of filters using equioscillating references (Degree 1 antenna with Fano Bound = -6.20).

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