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Efficient Market Hypothesis, Eugene Fama and Paul Samuelson: A reevaluation

Thomas Delcey*

Abstract

Two main claims are associated with the Efficient Market Hypothesis (EMH). First of all, the price changes are nearly random in the financial markets. Secondly, the prices reflect economic fundamentals. The relation between these two claims remains unclear in the actual literature. The purpose of this article is to show that this confusion is not new but began during the theoretical construction of EMH in the 1960s. The authorship of the EMH is attributed to Paul A. Samuelson and Eugene F. Fama. In two independent articles, published in 1965, they both reacted to empirical studies showing the random character of stock prices. Fama and Samuelson both interpret random fluctuations of prices as the consequence of rationality behaviors. The analysis is based on the reading of their 1965 articles. This corpus is extended with the archives of Paul Samuelson from the *Paul A. Samuelson Papers, David M. Rubenstein Rare Book Manuscript Library*, at Duke University.

Our analysis argues that apparent similarity between the two authors hides a strong opposition, and show that they conclude very differently about the accuracy of the stock market prices determined by the concurrence mechanism. Two different senses are granted to the EMH. According to Fama, the EMH is defined as a competitive market, where the random character of the fluctuation is explained by the fact that price converges to the fundamental value. We call this

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definition “Fama’s EMH.” According to Samuelson though, randomness of price variation, and unpredictability can be simply explained by the competition between investors, with no regard to the fundamental value. We call this definition “Samuelson’s EMH”. The end of the article, we suggest to reformulate the theoretical difference as an epistemological difference. Fama reduces the random fluctuation to a deterministic relation whereas Samuelson takes the randomness of the fluctuation as a phenomenon in itself. We conclude that both interpretations rise fruitful but distinct questions. Recent oppositions in debates about EMH should be read through this interpretative issue.

1 Introduction

The History of Efficient Market Hypothesis (EMH) can be divided in three steps. The first step is the construction of the theory in the 1960s. In the second one, the establishment of an empirical corroboration made consensual the theory in the 1970s. Finally, the third step is defined by the increase of empirical studies challenging the theory since the 1980s. This last step leads to the production of alternative approaches such as the Behavioral Finance (Thaler 1999; Shiller 2003), or more recently the Adaptive Market Hypothesis (Lo 2004). One striking characteristic of these alternatives is that they give two really different meanings to EMH. Andrew Lo defends an evolutionary framework in order to explain some predictability in price fluctuation. On the contrary, Robert Shiller defends the unpredictability of price fluctuation. His contribution tries to challenge the claim that price evaluate accurately the economic fundamental, an issue ignored by the contribution of Lo.

This article intends to produce an historical analysis of this confusion. More specifically the aim of this article is to show that this confusion is not new but began during the theoretical construction of EMH in the 1960s. During this period, the behavior of the price in financial market was one of the most discussed issues in the new field of Financial Economics. Since the 1930s, numerous empirical studies showed the random character of prices (Working 1934; Kendall 1953). These results followed another study casting

doubts on the capacity of financial analysts to forecast the price (Cowles 1933; Cowles et Jones 1937; Cowles 1944).¹ Answering a posteriori to these empirical studies, EMH is an economist explanation of this phenomenon (Walter 1996, 891; Jovanovic 2009, 51). According to the theoretical and historical literature (Merton 2006; Bernstein 1992; Brian et Walter 2007; Mignon 2008), EMH’s authorship has to be attributed to the works of Eugene Fama (1965a; 1965b) and Paul Samuelson (1965a). Both, Fama and Samuelson explain the random character of prices as the consequence of rational behaviors.²

We extend this corpus with the *Paul A. Samuelson Papers, David M. Rubenstein Rare Book Manuscript Library* from *Duke University*.³ Based on these materials, the claim of this article is to nuance strongly the theoretical proximity between Fama and Samuelson. Indeed, Fama and Samuelson both explain the randomness of price variation, and yet they both produce a very different explanation of this phenomenon. According to Fama, EMH is a competitive market composed of rational agents, where price converges to the Fundamental Value (FV), explaining the random character of price. We call this definition the “Fama’s EMH”. According to Samuelson, randomness of price variation can be simply explained by the competition between rational agents with no regard to the FV. We call this definition the “Samuelson’s EMH”. We do not argue that the understanding of this crucial periods in the history of EMH can be limited to an analysis of the theoretical differences between Fama and Samuelson. However, these theoretical differences are largely ignored by the literature and deserved to be firstly highlighted.

This distinction between two different claims belonging to EMH has been already mentioned in the literature in different ways (Thaler 2016; Charron

1. See (Bernstein 1992)and (Walter 2013).

2. The only difference between the two authors though resides in the probabilistic model they used to describe the random variation. While Fama chooses the already known Random walk Model, Samuelson introduces for the first time the Martingale model. A random variable X_t follow a random walk if, and only if, the increments are independent and identically distributed. P_t , a random variable, follow a martingale if: $E[P_{t+1}P_t, P_{t-1}...] = P_t$. See (Campbell, Lo, et MacKinlay 1997, 85).

3. We use the acronym ARC ahead reference to archives in the body of text The exact reference of each archives used in this article can be found in the bibliography. We are really thankful to the History of Political Economy Center and the David M. Rubenstein Rare Book Manuscript Library from Duke University to have given access to these archives.

2016). However, no discussions of Fama’s and Samuelson’s claims have been proposed yet excepted by (Guerrien et Gun 2011). The present article discusses the implication of these theoretical differences. Fama and Samuelson draw two different interpretations to the random character of the fluctuation. The theoretical difference can be reformulated as an epistemological difference: whereas Fama reduce the random fluctuation to a deterministic relation (and so non-random), Samuelson take the randomness of the fluctuation as a phenomenon in itself. We conclude that both interpretations rise fruitful but distinct questions. Recent oppositions in the debate about EMH should be read through this interpretative issue.⁴

The article is organized as follow. In a first part, we introduce element of contextualization of the 1965’s articles (Section 2). In a second part, we focus on the two articles written by Fama (Section 3). In the following section, which presents the work of Samuelson, we make a comparison between his definition of EHM and the one of Fama (Section 4). Finally, in a last section, we discuss the issue about the interpretation of the concept of fluctuation (Section 5).

2 The context of the writing

2.1 Paul Samuelson

Paul Samuelson is known to have contributed to almost all fields of the Economics including the Financial Economics (Merton 2006). The first interaction of Samuelson with financial issues is about the warrants – a security like option. Around 1950s, while he is already professor at the *Massachusetts Institute of Technology (MIT)*, he subscribed to a financial analysis service – “The RHM warrant and low-price stock survey”. Until the end of the 1950s, he currently speculates in financial market notably with his colleague Hendrick Houthakker. He is convinced of his comparative advantage as macroe-

4. Some others contributions and authors from this period deserve to be analyzed (Working 1949; Roberts 1959; Osborne 1959; Cootner 1962). The contextualization during the periods deserves also to be developed, notably the importance of the institution beside Samuelson and Fama in the development of financial economics, respectively the Massachusetts Institute of Technology and the Chicago University.

conomic:

Since I saw you last –can it be three years—I have been learning a lot about practical aspects of speculations. These days an economist with a good knowledge of macroeconomics seems to be able to do very well as practical speculator; what it is hard to understand is the workings of supply and demand: I suppose that takes real specialized study. (ARC, letter from Samuelson to Houthakker, Feb 6th 1953, we emphasize)

Novice in this field, Samuelson looks for researches about statistical structure of price. Houthakker gives him the name of two of the three authors that observed empirically the randomness of price, Maurice Kendall and Holbrook Working. Houthakker, who had assisted to the Kendall conference (Kendall 1953), is not at all satisfied with the pure statistical work that was presented though.⁵ However, he reminded to Samuelson the name of Working, already known in the field for his random fluctuation studies (Working 1934, 1949) and for his work on the early econometrics (Morgan 1995). As we will see (section 4.2), Working had a great influence on Samuelson particularly on the writing of “Proof That Properly Anticipated Prices Fluctuate Randomly”, published in 1965 in the *Industrial Management Review*, the ancestor of the current *Sloan Management Review* of MIT. In the present article, we will focus on this article and make a comparison with the contributions of Fama.

Samuelson advocates the spreading of stochastic tools in finance (MacKenzie 2008, 310). He supervised the thesis of Richard Krueger about the pricing of options in the 1950s. Around 1956, he rediscovered Louis Bachelier (Bachelier 1900) with the help of Leonard J. Savage. He encouraged his translation by his colleague Paul Cootner (Samuelson 2002, 42). In 1965, he proposed a model of option pricing really closed the famous formulation developed by Black-Scholes-Merton model (Samuelson 1965). He is also the

5. Houthakker evokes Kendall’s conference really roughly to Samuelson: “Last December M. G. Kendall read a paper on this subject to the Royal Statistical Society in London, which was received as badly as it deserved. If these studies give any indication to the sophisticated statistician it is that he had better study some economics.”. (ARC, Letter from Houthakker to Samuelson, 12th February 1953).

advisor of Robert Merton, one of the main advocate in the introduction of continuous stochastic process in finance (MacKenzie 2008, 122-23).

2.2 Eugene Fama

The first interaction of Eugene Fama with Financial Economics is during his graduate studies in the university of Tuft at the end of the 1950s. Fama works for one of his economics professor, Harry Ernst, who had a service forecasting price securities (Fama 2011, 2). A part of his job is to find trends in the fluctuation of prices. If he would found such trends, they are not exploitable profitably speaking. The trends never resist to an out-of-sample test (Fama 2011, 2).

Later on, Fama moves to the University of Chicago, where he participates intensively to the econometrics workshop, with, among others, Harry Robert, Lester Telser, Merton Miller and occasionally Benoit Mandelbrot. All of them are particularly focus on the behavior of the stock market prices. Supervised by Miller, he began a Ph.D using the sample of data of his Tuft experience at Tuft. His dissertation, submitted in 1964, has two conclusions: the probabilistic distribution of the stock prices has fat-tailed and stocks price variations are nearly independent. In 1965, using the result of his thesis, he publishes a long article in the Journal of Business named “Behavior of Stock Market Prices” (Fama 1965a), and although it is not the main purpose of the article, he introduces for the first time the notion of “Efficient Market”. It is only In a second article, publishes the same year, entitled “Random walk in Stock Market Price” in the same journal (Fama 1965b), that Fama focuses on EMH. The present article will be concentrated on these two fundamental contributions in the history of EMH.

Fama arrives at Chicago during the development of the *Center for Research in Security Prices (CRSP)*.⁶ He will be one the main figure of the center. The center has been created by James Lorie et Lawrence Fisher in the Graduate School of Business (today called the Chicago Booth School of Business) of Chicago University in the beginning of the 1960s (Jovanovic

6. Still today, the CRSP’s provides one of the larger database for researchers in Financial Economics.

2008, 63). The CRSP has been funded by Merry Lynch. The bank promotes research on stock price in order to prove scientifically the legitimacy of investment in stocks market (Fox 2011, 98). Although, the Fama's these and the two articles of 1965 are not based on the CRSP data (Fama 2011, 4), he is already deeply involved in this project. We should understand his contribution as an attempt to demonstrate the legitimacy of stocks market. The empirical corroboration of the EMH will be one of his main research topic leading him to change many times his EMH testing formulation (Fama 1970, 1976a, 1991).

3 Fama's EMH

In this section devoted to the Fama's contributions, we first begin by presenting the theoretical elements of his first article (1965a) about the independence assumption (Section 3.1). The last section focuses on his second article (1965b) where he reformulates his EMH (Section 3.2).

3.1 The sophisticated traders

As Fama reminds it, the term "efficient market" didn't appear in his thesis (Fama 2011, 3) but in one of this first article "Behavior of Stock Market Prices". This article summarizes the main results of his thesis: stock market variations are independent and the distribution have fat-tailed. In a first section, Fama introduces (1) the assumption of independence and, (2) the Levy distribution law describing the fat-tailed (see section 2.2). The two next sections treat of the empirical validation of fat-tailed distributions. Finally, in a last section Fama presents empirical tests of independence. The term "efficient market" only appears in the conclusion of the article:

We [...] saw that a situation where successive price changes are independent is consistent with the existence of an "efficient" market for securities, that is, a market, where given the available information, actual price at every point in time represent very good estimates of intrinsic values" (Fama 1965a)

We call this definition, the Fama's EMH. It was already developed implicitly in the first section of his paper where Fama try to explain economically the independence of successive price variations.⁷

Because price variations are nearly independent, Fama defends Random walk as good description of fluctuations in the stock market. Random walk is only a good approximation of the price behavior. The statistic independence is not strictly verified: empirical observations show that past variations influence present and future variations. However, because of transaction costs, these little dependences cannot be used to make profits, even if investors spot them (Fama 1965a, 35-36). Thereby, even if statistically speaking independence is not verified, his financial consequence can be. Consequently, chartist methods will fail. But the consequences of independence on investors are understood since the contribution of Working (1934) and Cowles and Jones (1937). The innovation brought by Fama is to explain economically the formation of such dependences. Fama introduces two distinct set of traders, the "sophisticated traders" and the others:

For example, let us assume that there are many sophisticated traders in the stock market and that sophistication can take two forms: (1) some traders may be much better at predicting the appearance of new information and estimating its effects on intrinsic values than others, while (2) some may be much better at doing statistical analyses of price behavior. (Fama 1965a, 37).

These two specific skills refer obviously to chartist and fundamentalist practitioners (discusses in section 1.1). Fama is indeed very concerned to be heard by investors, who are very sceptic about the research on random walk at this time (Bernstein 1992, 202). The superior analysis of sophisticated traders is twofold. It is in the same time statistic – on the capacity to spot dependences – and economics – on the capacity to estimate the FV. Fama assumes that the "noise" that represents discrepancies between FV and observed price, is dependent:

7. The theoretical explanation of randomness variations made by Fama are purely literal (Samuelson will use the axiomatic methodology).

Suppose now that the noise generating process in the stock market is dependent” (Fama 1965a, 38)

With this assumption, if there are discrepancies between FV and price, the two kind of traders will be able to spot them and avoid them. Whereas the fundamentalist will estimate if the price overestimates or underestimates the FV, the chartist, without knowing anything about FV, will spot dependencies generated by these discrepancies. Thereby, with this assumption, a fundamentalist and a chartist make the price closer to FV. If the market is composed mainly by sophisticated traders in competition, they will avoid profit opportunities they are looking for. With this assumption, the random character of price defended by the literature and contested by investors can be explained by the behavior of the investors themselves.

The force of the Fama’s assumptions on the dependences of discrepancies is that randomness doesn’t appear as the description of a turbulent world, but as a sign of stability. Fama’s explanation doesn’t just explain origins of randomness, it reduces this empirical fact to an indirect and a secondary consequence of a stable relation between goods and services markets and financial market. Interestingly, in his next theoretical contributions on EMH, Fama will focus less and less on the random character of price variation. As highlighted by Jovanovic (2009, 82), the last formulation of EMH by Fama, using the rational expectation (Fama 1976a, 1976b), doesn’t refer at all to a specific random process. We come back at this element in section 5. The next subsection presents the second article of Fama published the same years that reformulate his formulation of EMH with rationality.

3.2 Introduction of rationality

The article ”Random walk in Stock Market Price” will have a notable success in the analysts’ world. Published first in the *Financial Analysts Journal* in 1965, it will be reprinted in *The Analysts Journal*, in 1966, and finally in *The Institutional Investor* in 1968 (Bernstein 1992, 200-201). This second article mainly points out the implication of Random walk on chartist and fundamentalist analyses. From this point of view, it is basically a resume of the first section of ”Behavior of Stock Market Price” without the discussion

on fat-tailed. The article is divided in three parts. Fama begins by presenting the Random walk and his economic explanation, the EMH. Then, Fama presents a short review of empirical works on this topic. Finally, in a last section, he discusses the implications of Random walk and EMH on chartist and fundamentalist analyses. Significant changes could be noted compared to the original article though. First, though it appears only marginally in "Behavior of Stock Market Price", Fama primarily focuses on the efficiency. Second, the EMH formulation itself is quite different and does not use the concept of "sophisticated trader".

Like in "Behavior of Stock Market Price", Fama states that a market is "efficient" when stock price is a good estimator of its FV (Fama 1965a, 90-94; Fama 1965b, 76). Equivalently a market is efficient when it is composed of rational agents, that is, profit-maximizers agents in competition:

An "efficient" market is defined as a market where there are large numbers of *rational profit-maximizers* actively competing, with each trying to predict future market values of individual securities, and where important current information is almost freely available to all participants. (Fama 1965b, 76, we emphasize).

The "Rational profit-maximizer" assumption substitutes the "sophisticated traders", the characterization of the investor's behavior of his first article. This formulation is by far more general. The two kinds of sophisticated traders only characterized two ways of making profit. The maximization behavior is focused on the finality: all ways driving to maximization are taken in account. From this new assumption, he directly deduces that an exact relation between FV and price:

In an efficient market, competition among the many intelligent participants leads to a situation where, at any point in time, actual prices of individual securities already reflect the effects of information based both on events that have already occurred and on events which, as of now, the market expects to take place in the future. In other words, in an efficient market at any point in time the actual price of a security will be a good estimate of its intrinsic value. (Fama 1965b, 76)

The argument can be recall as follow: if investors have a maximization behavior, prices should converge to their FV. This reasoning is based on the same statistical assumption that appears in the first article: discrepancies between FV and observed prices follow some patterns traceable and exploitable:

If the discrepancies between actual prices and intrinsic values are systematic rather than random in nature (Fama 1965b, 76).

Maximization behavior of investors drive them to exploit and avoid such patterns in price. The arguments between the two articles are nearly the same except a different formulation of the investors behaviors. In the first article, Fama is very careful in the relation between EMH and the random character of price. If EMH is a possible explanation, it is not the only one. Trying to explain the consistency of fundamentalist analysis and EMH, Fama assumes the dependences of the discrepancies. From an historical perspective, it is easy to understand why Fama introduces the sophisticated traders and the dependences of the discrepancies. He adopts the analytical framework of the investors at this time, to convince them of the consistency of random walk model. In the second article however, the link between the rationality of the investors and the good valuation of FV is far from obvious. The dependence of discrepancies is not justified by Fama. The exploitation of economic information refers directly indeed to the FV and can explain the efficiency of the market. But without the dependences of discrepancies, there is no reason to rely efficiency of the market (in sense of Fama's EMH) and independence of the variations (and more generally the random character of price variation). It is interesting to see that this assumption will rarely be challenged in the literature on EMH (Shiller 1990).⁸

The next section shows that the same economic assumptions on the investors behavior- interested investors in competition- lead Samuelson to a very different explanation of the random variation observed.

8. Robert Shiller reversed the argument in order to highlight this point: "It would seem peculiar to argue that irrational markets should display regular and lasting patterns" (Shiller 2003, 102). This assumption is also challenged for long-time intervals in (Summers 1985, 1986). Other critical authors are more focused on the lack of rational traders and funds in the market (Delong et al. 1993; Shleifer et Vishny 1997).

4 Samuelson's EMH

The section is focusing the Samuelson article (1965a). In a first part, we present the martingale model (Section 4.1). The last section is devoted to the implication of the Samuelson's model in term of efficiency (Section 5.3).

"Proof That Properly Anticipated Price Fluctuate Randomly" is one the most influential paper Samuelson wrote in Finance (Merton 2006, 269). As described in the first section of his article (see Section 2.1), Samuelson is aware of the recent research dynamic about the random walk model used to describe randomness of the price variation. In "Proof", Samuelson introduces a new probabilistic model describing randomness, the martingale.⁹

4.1 Samuelson's model

"Proof That Properly Anticipated Price Fluctuate Randomly" is one the most influential paper Samuelson wrote in Finance (Merton 2006, 269). As described in the first section of his article (see Section 2.1), Samuelson is aware of the recent research dynamic about the random walk model used to describe randomness of the price variation. In "Proof", Samuelson introduces a new probabilistic model describing randomness, the martingale.¹⁰

P_{t+T} an estimation in t of the spot price in T

P_{t+T} is representable by a given distribution law.

Suppose now a future market. The price of the future contract for the same asset is noted Y_{t+T} with t the valuation moment and T the time before the contract maturity. For n period, we can write $Y_{t+n, T-n}$. At $T+1$ the future price is noted $Y_{t+1, T-1}$. $T+2$, the future price is $Y_{t+2, T-2}$ etc. At the $t+T$ period, the price of the future is noted $Y_{t+T, 0}$.

Samuelson aims to characterized the relation between the sequence P_{t+T} and the sequence Y_{t+T} . In a first step, he uses the arbitrage reasoning (Samuelson 1965a, 43) introduced by the famous theorem of Modigliani and Miller (Modigliani et Miller 1958). With the arbitrage reasoning, it is pos-

9. The martingale model will be introduce independently the same year by Mandelbrot (Mandelbrot 1965).

10. The martingale model will be introduce independently the same year by Mandelbrot (Mandelbrot 1965).

sible to characterize the relation between P_{t+T} and the Y_{t+T} for a particular case. At the $t+T$ period, by definition, P_{t+T} is known with certainty. At this period, the spot price must be equal to future price. If not, an arbitrage opportunity will exist and investors will avoid it.

$$\text{At } t + T, Y_{t+T} = P_{t+T}$$

But before the $t + T$ period, no one know with certainty P_{t+T} . The arbitrage reasoning is not enough to characterized a relation between P_{t+T} and the Y_{t+T} . Samuelson proposes another assumption he named "Mathematically Excepted Price Formation". This axiom asserts that investors know and use the law distribution describing the sequence P_{t+T} to valuate Y_{t+T} . by the expected value:

$$Y_{t,T} = E[P_{t+T}|I_t]$$

Samuelson extends the reasoning by arbitrage considering that, because of competition, investors valuate Y_{t+T} by the expected value of the random variable P_{t+T} conditionally to the information of past price ($P_t, P_{t-1}...$), noted here I_t to simplify. The best estimation of the tomorrow spot price is the actual price of the future contract. In a competitive marker, the valuation of future price by investors take in account the past sequences P_{t+T} . Thereby, no systematic profits can be made by using the relationship between future and spot price. This is not a strict arbitrage reasoning since - P_{t+T} and so the profits - is not certain. The economic justification of this hypothesis is based on competition and maximization:

it is tempting to assume that people in the market place make as full use as they can of the posited probability distribution of next period's price and $Y_{t,T}$ bid by supply and demand to the mean or the mathematically expected level of tomorrow's price.
(Samuelson 1965b, 42)

Future Market is interpreted by Samuelson as a place where anticipations of tomorrow spot price are priced. The future price, is the concrete observations of the spot price anticipations. Samuelson try to understand the unpredictability of the price variation by the characterization of the sequence $Y_{t,T}$.

These assumptions characterizing investors' behavior are formalized contrary to Fama's assumptions. Investors share common probabilities and they use it to maximize their gains. However, beside these methodological differences, the characterization of the investors' behavior and market environment by Fama and Samuelson are closed. In Fama's model, market is composed of rational profit-maximizers trying to predict price as they can, in a competitive environment where "important current information is available" (Fama 1965b, 76, see section 3.3). In the Samuelson's model, the later assumption is expressed by the known distribution of P_{t+T} that investors used by the expected value operator. Using iterative exception law,¹¹ a property of probability theory independent of his model, Samuelson concludes that the $Y_{t,T}$ sequence follow a martingale:

$$E[Y_{t+1,T-1}|I_t] = Y_{t,T}$$

If a sequence of prices follows a martingale, thereby, the best estimation of the tomorrow's price, based on the information available, is the today's price. This representation respects the idea that the price is unforecastable and especially the fact that the chartist analysis is useless.¹²

It says that, within the defined model, all chart methods attempting to read out of the past sequence of known prices $P_t, P_{t-1}, Y_{t,T}, Y_{t+1,T-1}$ any profitable pattern of prediction is doomed to failure. (Samuelson 1965a, 47)

More generally, because the best estimation is the today's price, we cannot argue it is more likely to see the tomorrow's price higher or lower than today's price. Samuelson advocates the martingale process as a better stochastic process than random walk to describe competitive market:

11. The iterative exception law can be write formally as follow: $E[X|I_1] = E[E[X|I_2]|I_1]$ if and only if I_2 includes in I_1 (see (Campbell, Lo, et MacKinlay 1997, 27; LeRoy 1989). In the Samuelson's model: (3) says that $Y_{t,T} = E[P_{t+T}|I_t]$ and so $Y_{t+1,T-1} = E[P_{t+T}|I_{t+1}]$ We can conclude using the iterative exception law that $E[Y_{t+1,T-1}|I_t] = E[E[P_{t+T}|I_{t+1}]|I_t] = E[P_{t+T}|I_t] = Y_{t,T}$.

12. In the 1965's article, Samuelson is only focused on chartist analysis. In another article (Samuelson 1973b), he will show that martingale model is not at all inconsistent with presence of fundamentalist in the market. An article wrongly assimilated to Fama's EMH.

[the random walk] is not particularly related to perfect competition or market anticipations. For consider a monopolist who sells (or buys) at fixed price. If the demand (or supply) curve he faces is the resultant of numerous independent, additive, sources of variation each of which is limited or small, his resulting quantity may well behave like a random walk (Samuelson 1965a, 42).

The martingale doesn't assume independence of the price variation as random walk model does. Because of that, the martingale model has been considered as less restrictive than the Fama's random walk model. The main innovation of the "Proof" would be this new formulation of random variations (LeRoy 1989; Mignon 2008). The Random walk model will be indeed replaced by the martingale process, even by Fama (Fama 1970). It will be also a key element in the development of other research program in financial mathematics (Walter 2013, Idabouk 2010).

4.2 Efficiency of the Market

The term "efficient" is not used by Samuelson.¹³ But no more than the word, the Fama's EMH does not appear at all in the 1965's article. The Samuelson's martingale model is based on the crucial assumption of a market in competition. Thereby, like Fama, Samuelson assumes that random variations of price are the consequence of competition between interested people in financial market. While, Fama argues that today's price is the best estimation of the FV, Samuelson's conclusion is less ambitious, arguing only that today's price is the best estimation of the tomorrow's price. His conclusion is not on a good economic valuation of price, but only on a good profitable valuation of price:

This means that there is no way of making an expected profit by extrapolating past changes in the future price, by chart or any other esoteric devices of magic or mathematics. (Samuelson 1965a, 44)

13. To my knowledge, he will use it for the first time in (Samuelson 1973a).

This difference is underestimated by the literature on EMH, which does not make a distinction between Fama's EMH and Samuelson's EMH. Samuelson is however very explicit in the conclusion of his article:

It does not prove that actual competitive markets work well. It does not say that speculation is a good thing or that randomness of price changes would be a good thing. It does not prove that anyone who makes money in speculation is ipso facto deserving of the gain or even that he has accomplished something good for society or for anyone but himself. All or none of these may be true, but would require a different investigation. (Samuelson 1965b, 48)

We can give to this opposition a historical expression. On the one hand, Fama is deeply involved in the *Center for Research on Stock Prices*. The center aims to legitimate investment in stock market for the common people. The random character of price convinces that investment in stock is reachable for everyone (Fox 2011, 98), in a context where stock market was still seen suspiciously three decades after the 1929's crisis (Brisset 2017). Fama's concerns are more practical. He uses the fundamentalist framework – the sophisticated trader - in order to convince that this framework was not inconsistent with the random character of price. Then, he reformulates this explanation with the analytical economics framework - the profit-maximization (1965b). That leads him to conclude that rational behaviors implies the good evaluation of price. On the other hand, Samuelson's interest for finance is on the application of statistical and probabilistic tools. He quickly becomes closed to Working. The former gave him his empirical study (Working 1934) (Working to Samuelson, July 7th 1958) and shared with him his attempt to explain the randomness of price variation (Working 1949, 160) (Working to Samuelson, May 2nd 1961). In his theoretical article of 1949, Working is focusing the expectation of price with no regard for the accuracy of price.¹⁴ He

14. Working evokes « normal backwardation » hypothesis discussed by Keynes in his *Treatise on money* (Working 1949, 151). This hypothesis says that the effective future price delivered is always below the expected spot price for the same maturity because there is a risk premium in the agent's expectation.

formulates literally the martingale claim of Samuelson by pointing out that unpredictability may be the consequence of great forecasts (Working 1949, 160). Samuelson's EMH is the formal development of the Working intuitions by the introduction of the martingale process in the story.

Interestingly, Fama (1965b) and Samuelson (1965a) assumptions are not diametrically different theoretically speaking. From a theoretical point of view, both assume somehow competition in the market (i.e., large number of participants), rational behaviors (i.e., profit maximization), and free available information. What is particularly striking is that their differences about EMH cannot be explained significantly from difference in their representation of the market. An ideal competitive market does not discriminate the two authors.¹⁵ The last section discusses these differences between Fama and Samuelson in the way they interpret the random fluctuation. The theoretical difference is thereby explained by two different epistemological approaches of price.

5 Understanding the fluctuation

In this last section, we read the contribution of Fama and Samuelson as an epistemological difference, in the way random fluctuation has been interpreted (section 5.1). Fama focuses on constant patterns and denied the random fluctuation as an object itself. Conversely, Samuelson focuses on the explanation of the random fluctuation itself. In a subsection, we develop the implications of this distinction on debate about EMH (section 5.2).

5.1 The epistemological status of randomness

The argumentation of Fama is based on the crucial assumption that discrepancies between VF and price are not "random in nature" (Fama 1965b, 76). Through this assumption Fama creates a specific representation of the fluctuation. Fama interprets the random fluctuation as errors of observation from an exact, hidden, relation. The variation of price is not understood as a

15. Thereby, we nuance the reading which claims that adhesion to competitive market discriminates *MIT* and *Chicago* research programs (Jovanovic 2009).

variation in itself, but as a deviation from a norm. If there are enough sophisticated traders in the market, the systematic nature of discrepancies from FV will be spotted, exploited and thereby, avoided by the traders. In other words, in the Fama's EMH, discrepancies from FV are possible but they can only be unspotted, and so, random. The random character of price that disturbed so much the economists is not entirely avoided, but the nature of these random fluctuations is a specific one. That is, nonsystematic causes, which have the same probability to lead the price under or over the FV. Thereby, errors vibrate randomly around the FV. From a theoretical point of view, randomness appears as secondary and insignificant, hiding the deeper relationship between price and FV. And this is what matters from a theoretical point of view.¹⁶

Samuelson (1965a) does not try to take randomness as a secondary issue, hiding a causal relation between fundamental value and the price. His main finding is what kind of behavior and market environment lead the price to behave randomly. He takes randomness as a phenomenon in itself that should be explained itself. However, this does not mean that Samuelson thought that the random character of price is not the consequence of consistent behavior. The idea that investors depended only of blind chance has been suggested provocatively by Kendall (1953), with his famous "Demon of Chance" metaphor. Like Fama, Samuelson denies the Kendall's argument. The homogenous expectation of the investors gives to the fluctuation of price a stable property, that is, to be a martingale. It is interesting to see that, further, Samuelson will stay focus on the random character of price variation in itself whereas Fama will focus less and less on the randomness of price change (see note 18 and section 3.1). For instance, he will try in a second article (Samuelson 1973b) to show that fundamentalist analyses (in the strict sense of John Burr William, see section 2.1) is not inconsistent with his previous finding of (1965b). In a third, and last theoretical article about EMH, he will still focus only the unpredictability of price (Samuelson 1990).

The purpose of the Samuelson's articles is to show that randomness of price changes can be explained by an interested speculation. As we see,

16. In the section 3.1, we suggested that the next works of Fama would be less and less interested in the randomness issue in itself.

the argument is simple since the equation (3) completely determined the theorem. Actually, Samuelson hesitates to consider his theorem as a fundamental finding or just as a triviality. Already write and given to Working in 1961, he waits for years to publish his result (ARC, letter from Samuelson to Working May 9th 1961). Later, knowing the simplicity of his finding, he will insist that this simplicity cannot be reduce to a tautology (Samuelson 1973a, 18). This hesitation of Samuelson is an illustration of the difficulty to appreciate his contribution at this time in contrast to the Fama's one. The Fama's EMH has a strong explanation based on explicit causal relationship in the fluctuation. This explanation is very comfortable for economists that sough largely for exact relations at this time (Morgan 1995). Indeed, the empirical studies showing the randomness of the price variation has been seen sometime as a denied of economic science. Random changes is a negative finding, meaning the absence of causes explaining the behavior of price. The Samuelson's argument is less easy to appreciate because there is not an explicit causal relationship between price and economic factors. His finding is a stochastic relation between prices themselves. Competition and rational behaviors lead the future price to be a martingale, that is, to be a random process and that's all.

5.2 Implications on EMH's debate

We do not aim to take a normative position about what should be the proper interpretation. We highlight however that each interpretation fits with specific and distinct set of questions. More specifically, the two contributions develop two different approach of price.

On the one hand, if we are trying to understand under what conditions a security price is forecastable, we are trying to explain fluctuation by finding a stable statistical property of the fluctuation (random walk, independences, martingale, no autocorrelation etc.). We are focusing the fluctuation as the phenomenon that matters. On the other hand, if we are trying to understand under what conditions the fundamentals are well valued, we are trying to explain fluctuation by finding a causal relation between the fluctuation and some specific factors that represent the economic fundamentals.

The fluctuation in itself is secondary, what matters is the consistency of the causal relation we are assuming a priori. From this perspective, neither interpretations are inconsistent from an epistemological point of view. They are just raising different kind of question about price assets. Samuelson only confirms deductively the intuitions of Working (1949, p.160, see Section 4.3). Fama is the first to rise a new kind of question, a really ambitious one, on the accuracy of price.

Fama rises an innovative question about the determination of price but this issue disappears through the issue of unpredictability of price. The entire empirical research before the 1980s is not focus on this question though, but only on the corroboration of the random character of price changes. As Summers (Summers 1985) emphasizes, financial economics focuses only interrelationship between different assets and ignored the fundamental question about what determine price assets. Furthermore, the claims belonging to EMH still remain ambiguous. A good illustration is to observe what claims are criticized by proponent of EMH. Yet, when Andrew Lo (Lo 2004) and Robert Shiller (Shiller 2003) attacks the EMH they are not talking at all about the same hypothesis. The first proposes an evolutionary qualitative framework to replace the EMH but his discussion never implies the question of the good valuation of price. The evolutionary framework brings by Lo attempts to explain how price variations can be less random through time or through space (between different markets). In short, he only focuses his critics on what we call the Samuelson's EMH.¹⁷ Shiller (2003) though, agrees to recognize that price changes are nearly unforecastable. What Shiller attacks is only the good valuation of price. Consequently, from a theoretical and practical point of views, the two authors who critic apparently, the same hypothesis, are diametrically different in their conclusions. Lo nuances the passive management dogma (see for instance his book about chartist analysts (Lo et Hasanhodzic 2009)) and doesn't answer to the question of the

17. This point leads Lo to challenge the passive management advocate by EMH's proponents like Samuelson and Fama: "The classical EMH suggests that certain levels of expected returns can be achieved simply by bearing a sufficient degree of risk. The AMH implies that the risk/reward relation varies through time, and that a better way of achieving a consistent level of expected returns is to adapt to changing market conditions." (Lo 2004, 23)

accuracy of price, what Shiller is actually focusing.

6 Conclusion

In this article, my aim objective is to reevaluate the work of Samuelson leading to show (1) a strong theoretical difference between the main theoreticians of the EMH. The discussion of these difference shows that (2) the fact that EMH vacillates between an explanation of the random prices changes and an explanation about the accuracy of prices, is intrinsically link to the theoretical construction of EMH in the 1960s. Noting the use of the rationality hypothesis by Fama and Samuelson, we suggest to read debates about EMH as opposition on the question asked more than on the acceptance or unacceptance of rationality.

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7.1 Archives

Name of the collection: "Paul A. Samuelson papers, 1933-2010 and undated" at *David M. Rubenstein Rare Book Manuscript Library*, Duke University, United States. Webiste: <https://library.duke.edu/rubenstein/findings/samuelsongpaul/>

Box 78: correspondence with Working, 1953-1977, Paul A. Samuelson papers, 1933-2010 and undated, *David M. Rubenstein Rare Book Manuscript Library*, Duke University.

- Letter from Working to Samuelson, July 7th 1958
- Letter from Working to Samuelson, May 2nd 1961
- Letter from Samuelson to Working, May 9th 1961

Box 38: correspondence with Houthakker, 1949-2008, Paul A. Samuelson papers, 1933-2010 and undated, *David M. Rubenstein Rare Book Manuscript Library*, Duke University.

- Letter from Samuelson to Houthakker, February 6th 1953
- Letter from Houthakker to Samuelson, February 12th 1953