Efficient Market Hypothesis, Eugene Fama and Paul Samuelson: A reevaluation

Thomas Delcey

Efficient Market Hypothesis, Eugene Fama and Paul Samuelson: A reevaluation.

Thomas Delcey

Working Paper

June 2017

Abstract:

Two main claims are associated with the Efficient Market Hypothesis (EMH). First of all, the price changes are nearly random in the financial markets. Secondly, the prices reflect the economic fundamentals. The relation between these two claims remains unclear in the actual literature. The purpose of this article is to show that this confusion is not new but began during the theoretical construction of EMH in the 1960s. The analysis is based on the reading of their 1965 papers and on the archives of Paul Samuelson from the *Paul A. Samuelson Papers, David M. Rubenstein Rare Book Manuscript Library, Duke University*. The authorship of the EMH is attributed to Paul A. Samuelson and Eugene F. Fama. In two independent articles, published in 1965, they both reacted to empirical studies showing the random character of stock prices. Indeed, both Fama and Samuelson interpreted the random character of prices as the consequence of rational markets. In this paper, I argue that the apparent similarity between the two authors hides a strong opposition, and show that they conclude very differently about the accuracy of the stock market prices determined by the concurrence mechanism. I find that two different senses are granted to the EMH. Indeed, Fama and Samuelson explain the randomness of price variation, and yet both develop a very different explanation of this phenomenon. According to Fama, the EMH is defined as a competitive market, where the random character of price is explained by the fact that prices converge to the Fundamental value. call this definition “Fama’s EMH.” According to Samuelson though, randomness of price variation, and unpredictability can be simply explained by the competition between investors, with no regard to the FV. I call this definition “Samuelson’s EMH”.

1. Introduction

The History of Efficient Market Hypothesis (EMH) can be divided in three steps. The first step is the construction of the theory in the 1960s. In the second one, the establishment of an empirical corroboration made consensual the theory in the 1970s. Finally, the third step is defined by the increase of empirical studies challenging the theory since the 1980s. This last step leads to the production of alternative approaches such as the Behavioral Finance

(Thaler 1999; Shiller 2003), or more recently the Adaptive Market Hypothesis (Lo 2004). One striking characteristic of these alternatives is that they give two really different meanings to EMH. Andrew Lo defends an evolutionary framework in order to explain some predictability in price fluctuation. On the contrary, Robert Shiller defends the unpredictability of price fluctuation. His contribution tries to challenge the claim that price evaluate accurately the economic fundamental, an issue ignored by the contribution of Lo.

This article intends to produce an historical analysis of this confusion. More specifically the aim of this article is to show that this confusion is not new but began during the theoretical construction of EMH in the 1960s. During this period, the behavior of the price in financial market was one of the most discussed issues in the new field of Financial Economics. Since the 1930s, numerous empirical studies showed the random character of prices (Working 1934; Kendall 1953). These results followed another study casting doubts on the capacity of financial analysists to forecast the price (Cowles 1933; Cowles et Jones 1937; Cowles 1944).[1]. Answering *a posteriori* to these empirical studies, EMH was an economist explanation of this phenomenon (Walter 1996, 891; Jovanovic 2009, 51).

According to the theoretical and historical literature (Merton 2006; Bernstein 1992; Brian et Walter 2007; Mignon 2008), EMH's authorship has to be attributed to the works of Eugene Fama (1965a; 1965b) and Paul Samuelson (1965a). Both, Fama and Samuelson explain the random character of prices as the consequence of rational markets. The only difference between the two authors though resides in the probabilistic model they used to describe the random variation. While Fama chooses the already known Random Walk Model,[2] Samuelson introduces for the first time the Martingale model.[3]

Our lecture of these contributions leads us to nuance these claims. Indeed, Fama and Samuelson both explain the randomness of price variation, and yet they both produce a very different explanation of this phenomenon. According to Fama, EMH is a competitive market, where price converges to the Fundamental Value (FV), explaining the random character of price. I call this definition the "Fama's EMH". According to Samuelson, randomness of price variation can be simply explained by the competition between investors with no regard to the FV. I call this definition the "Samuelson's EMH". We do not argue that the understanding of this crucial periods in the History of EMH can be limited to an analysis of the theoretical differences between Fama and Samuelson.[4] However, these theoretical differences are ignored by the literature and deserved to be firstly highlighted given the importance of their contributions.

The article is organized as follow. In a first part, I introduce some element of contextualization of the 1965's articles (Section 2). In a second part, I focus on the two

[1] See (Bernstein 1992)and (Walter 2013)

[2] A random variable X_t follow a random walk if, and only if, the increments are independent and identically distributed. However, in Financial Economics , and mainly on the empirical test of EMH, Random Walk is used to describe less restrictive process, respecting uniquely the independence for instance (Campbell, Lo, et MacKinlay 1997, 29).

[3] P_t, a random variable, follow a martingale if: $E[P_{t+1}|P_t, P_{t-1} \dots] = P_t$

[4] Some others contributions and authors from this period deserve to be analyzed (Working 1949; Roberts 1959; Osborne 1959; Cootner 1962). The contextualization during the periods deserves also to be developed, notably the importance of the institution beside Samuelson and Fama in the development of financial economics, respectively the *Massachusetts Institute of Technology* and the *Chicago University*.

articles written by Fama (Section 3). In the following section, which presents the work of Samuelson, I make a comparison between his definition of EHM and the one of Fama (Section 4).

2. Elements of contextualization

In this section, a first subsection which presents the Chartism and Fundamentalism (2.1) is then followed by a presentation of the two authors, Samuelson (2.3) and Fama (2.2) in order to explain how they were led to write their articles.

2.1. Chartists and Fundamentalists

If Fama and Samuelson react to empirical studies, their contributions challenge existent theories and practices. Before the development of financial economics, two methods of trading dominated: the chartist analysis and the fundamentalist analysis.[5]

The first assume that the stock market price follow repeated trends, which investors can exploit to make profits. Randomness of price variation denies the possibility of such trends. Consequently, this method will be directly challenged by Samuelson's and Fama's EMH. The main founder of this methods is Charles Dow [1851 – 1902], chief editor of the *Wall Street Journal*. William Peter Hamilton [1867 – 1929], the following chief editor, was one of the main proponent of the "Dow Theory"[6]. The performance of his financial analysis will be studied and attacked directly by Alfred Cowles (1933).

The fundamentalist analysis assumes the existence of an intrinsic value for any asset equivalently to the modern concept of FV. The estimation of the FV is based on the firms features. From this estimation, investors determine if the stocks are over-valuated, well valuated or under-valuated. From an academic perspective, the expected cash flow valuation goes back to Irving Fisher and his book *Theory of interest* published in 1930. The fundamentalist analysis will be more systematically developed by John Burr Williams [1900 – 1989] in his thesis, published in a book *The Theory of Investment Value* in 1938. Williams calculated the FV by the discounted expected dividend of the firms. However, this method will be mainly popularized by a non-academic work in the very famous investor manual *Security analysis* written by Benjamin Graham [1894 – 1976] and David Todd [1895 – 1988] in 1934 (Bernstein 1992, 195). According to Graham and Todd, the FV is not determined by a formalized discounted calculus but by all kinds of information useful to estimate the earning of the firms.

[5] For more details, see chapter 1 and chapter 8 of (Bernstein 1992).
[6] The term "Dow Theory" was not used by Hamilton himself. It was introduced by Robert Rhea (1932), another famous chartist's proponent, and reused by Alfred Cowles (1933, 1937).

2.2 Paul Samuelson

Paul Samuelson is known to have contributed to almost all fields of the Economics including the Financial Economics (Merton 2006). The first interaction of Samuelson with financial issues was about the warrants – a security like option. Around 1950s, while he was already professor at the *Massachusetts Institute of Technology* (*MIT*), he subscribed to a financial analysis service – "The RHM warrant and low-price stock survey". He was looking for profit opportunities in future market (Bernstein 1992, 174), but quickly became sceptic about the consistence of such services. This first experience will lead him to work on two different research projects.

First, he was interested by the future contract. He supervised the thesis of Richard Kruizenga about the pricing of options in the 1950s. Around 1956 (MacKenzie 2008, 310), he rediscovered Louis Bachelier (1900) with the help of Leonard J. Savage. He encouraged his translation by his colleague Paul Cootner (Samuelson 2002, 42). In 1965, he proposed a model of option pricing before the famous Black-Scholes-Merton model (Samuelson 1965). He was also the advisor of Robert Merton and incited him to continue this research program[7].

Secondly, Samuelson was also concerned by the structure of the spot price. Samuelson had a correspondence with Hendrick Houthakker, a microeconomist who developed the *Strong Axiom* of revealed preferences. Houthakker played a key role encouraging Samuelson to initiate this research program in 1953, giving him the name of two of the three authors that observed empirically the randomness of price: Maurice Kendall and Holbrook Working. Houthakker, who had assisted to the Kendall conference (Kendall 1953), was not at all satisfied with the pure statistical work that was presented though. However, he reminded to Samuelson the name of Working, already known in the field for his empirical studies (Working 1934) but also for his theoretical contribution (Working 1949).[8] As we will see (section 4.3), Working had a great influence on Samuelson, particularly, in one of his article named "Proof That Properly Anticipated Prices Fluctuate Randomly", in 1965 in the *Industrial Management Review*, the ancestor of the current *Sloan Management Review* of *MIT.* In the present article, I will focus on this article and make a comparison with the contributions of Fama.

2.3 Eugene Fama

The first interaction of Eugene Fama with Financial Economics was during his graduate studies in the university of Tuft at the end of the 1950s. Fama worked for one of his economics professor, Harry Ernst, who had a service forecasting price securities (Fama 2011, 2). A part of his job was to find trends in the fluctuation of prices. If he would found such

[7] For instance, helped by Henry P. McKean, Samuelson introduces to Merton the Ito Lemme (MacKenzie 2008, 122-23).

[8] "Correspondence with Kendall" (February, 1953), Paul A. Samuelson Papers, *David M. Rubenstein Rare Book Manuscript Library,* Duke University.

trends, they were not exploitable profitably speaking. The trends never resisted to an out-of-sample test (Fama 2011, 2).

Later on, Fama moved to the University of Chicago, where he participated intensively to the Econometrics workshop, with, among others, Harry Robert, Lester Telser, Merton Miller and occasionally Benoit Mandelbrot. All of them were particularly focused on the behavior of the stock market prices. Supervised by Miller, he began a Ph.D using the sample of data of his chartist experience at Tuft. His dissertation, submitted in 1964, has two conclusions: the probabilistic distribution of the stock prices has fat-tailed and stocks price variations are nearly independent. In 1965, using the result of his thesis, he published a long article in the *Journal of Business* named "Behavior of Stock Market Prices" (Fama 1965a), and although it was not the main purpose of the article, he introduces for the first time the notion of "Efficient Market". It is only In a second article, published the same year, named "Random Walk in Stock Market Price" in the same journal (Fama 1965b), that Fama focuses on EMH. The present paper will be concentrated on these two fundamental contributions in the history of EMH.

In the rest of his career, Fama will publish only a few papers about the non-normal distribution (MacKenzie 2008, 115‑16)[9]. However, he will play a central role in the empirical research about fluctuations of financial markets, and be a figure of the *Center for Research in Security Prices* (*CRSP*)[10]. The center has been created by James Lorie et Lawrence Fisher in the *Graduate School of business* of Chicago University in the beginning of the 1960s (Jovanovic 2008, 63). The CRSP was funded by Merry Lynch. The bank wanted to prove scientifically the legitimacy of investment in stock market (Fox 2011, 98).

Although, the Fama's these and the two articles of 1965 were not based on the CRSP data (Fama 2011, 4), he was deeply involved in this project. The empirical corroboration of the EMH will be one of his main research topic leading him to change many times his EMH *testing* formulation (Fama 1970, 1976a, 1991).

3. Fama's EMH

In this section devoted to the Fama's contributions, I first begin by presenting the theoretical elements of his first article (1965a) about the independence assumption (Section 3.1) and the fat-tailed (Section 3.2°). In the last section, I focus on his second article (1965b) where he reformulates his EMH (Section 3.3).

3.1 The sophisticated traders

[9] To my knowledge, Fama will published about non-normal distribution in only two articles with a general purpose: "Some Properties of Symmetric Stable Distributions" (with Richard Roll), *Journal of the American Statistical Association* (September 1968) and "Parameter Estimates for Symmetric Stable Distributions" (with Richard Roll), *Journal of the American Statistical Association* (June 1971).

[10] Still today, the *CRSP*'s provides one of the larger database for researchers in Financial Economics.

As Fama reminds it, the term "efficient market" didn't appear in his thesis (Fama 2011, 3) but in one of this first article "Behavior of Stock Market Prices". This article summarizes the main results of his thesis: stock market variations are independent and the distribution have fat-tailed. In a first section, Fama introduces (1) the assumption of independence and, (2) the Levy distribution law describing the fat-tailed (see section 2.2). The two next sections treat of the empirical validation of fat-tailed distributions. Finally, in a last section Fama presents empirical tests of independence. The term "efficient market" only appears in the conclusion of the article:

> We […] saw that a situation where successive price changes are independent is consistent with the existence of an "efficient" market for securities, that is, a market, where given the available information, actual price at every point in time represent very good estimates of intrinsic values" (Fama 1965a)

We will call this definition, the Fama's EMH. It was already developed implicitly in the first section of his paper where Fama try to explain economically the independence of successive price variations.[11]

Because price variations are nearly independent, Fama defends Random Walk as good description of fluctuations in the stock market. Random Walk is only a good approximation of the price behavior. The statistic independence is not strictly verified: empirical observations show that past variations influence present and future variations. However, because of transaction costs, these little dependences cannot be used to make profits, even if investors spot them (Fama 1965a, 35-36). Thereby, even if statistically speaking independence is not verified, his financial consequence can be. Consequently, chartist methods will fail. But the consequences of independence on investors were understood since the contribution of Working (1934) and Cowles and Jones (1937). The innovation brought by Fama is to explain economically the formation of such dependences. Fama introduces two distinct set of traders, the "sophisticated traders" and the others:

> For example, let us assume that there are many sophisticated traders in the stock market and that sophistication can take two forms: (1) some traders may be much better at predicting the appearance of new information and estimating its effects on intrinsic values than others, while (2) some may be much better at doing statistical analyses of price behavior. (Fama 1965a, 37).

These two specific skills refer obviously to chartist and fundamentalist practitioners (discusses in section 1.1). Fama was indeed very concerned to be heard by investors, who were very sceptic about the research on Random Walk (Bernstein 1992, 202). The superior analysis of sophisticated traders is double. It is in the same time statistic – on the dependences – and economics – on the FV valuation. Fama assumes that the "noise" that represents discrepancies between FV and observed price, is dependent:

[11] The theoretical explanation of randomness variations made by Fama are purely literal (Samuelson will use the axiomatic methodology).

> Suppose now that the noise generating process in the stock market is dependent" (Fama 1965a, 38)

With this assumption, if there are discrepancies between FV and price, the two kind of traders will be able to spot them and avoid them. Whereas the fundamentalist will estimate if the price overestimates or underestimates the FV, the chartist, without knowing anything about FV, will spot dependencies generated by these discrepancies. Thereby, with this assumption, a fundamentalist *and* a chartist make the price closer to FV. If the market is composed mainly by sophisticated traders in competition, they will avoid profit opportunities they are looking for. With this assumption, the random character of price defended by the literature and contested by investors can be explained by the behavior of the investors themselves.

The force of the Fama's assumptions on the dependences of discrepancies is that randomness doesn't appear as the description of a turbulent world, but as a sign of stability. Fama's explanation doesn't just explain origins of randomness, it reduces this empirical fact to an indirect and a secondary consequence of a stable relation between goods and services markets and financial market. Interestingly, in his next *theoretical* contributions on EMH, Fama will focus less and less on the random character of price variation. As highlighted by Jovanovic (2009, 82), the last formulation of EMH by Fama using the rational expectation (Fama 1976a, 1976b), doesn't refer at all to a specific random process.

This representation of a stable is also apparent when Fama presents the importance of extreme variations. This is the topic of the next subsection.

3.2 The explanation of fat-tailed

The other innovation of "Behavior of Stock Market" is the Fama's discussion of fat-tailed. This contribution is mostly influenced by the work of Benoit Mandelbrot, a French mathematician famous for his work on the concept of Fractal. In finance, he is mostly known to have criticized the utilization of the normal law, whose has tails smaller than the empirical observations. He advocated the utilization of a more general class of distribution, the Levy distribution, for which normal law is just a particular case (Mandelbrot 1963). As I already discussed in the first section, Benoit Mandelbrot was a regular participant of the econometrics seminar in Chicago University where Fama met him during the writing of his thesis (Fama 2011, 2).

Fama had already defended Mandelbrot in the *Journal of Business* (Fama 1963). Again in "Behavior of Stock Market Prices", Fama advocates the utilization of the Levy distributions to describe the stock prices variations. Fama criticizes the use of the normal law, especially in (Bachelier 1900) and (Osborne 1959). If statistician can be focused only on the more likely variations, investors cannot ignore a larger dispersion in the price distribution:

> The classic approach to this problem has been to assume that the extreme values are generated by a different mechanism than most the observations. Consequently, one tries a posteriori to find "causal" explanations for the large observations and thus to rationalize their exclusion from any tests carried out on the body of the data. Unlike the statistician, however, the investor cannot ignore the possibility of large price changes before committing is funds, and once he has made his decision to invest, he must consider their effects on his wealth. (Fama 1965a, 42)

The discussion is purely technical and empirical. In a first time, Fama presents the formalized law of Levy (Fama 1965a, 40-45), then followed by 2 sections of empirical tests (Fama 1965, 45-68). Contrary to the independence discussion, Fama doesn't discuss theoretically the fat-tailed phenomenon. Fama's concerns about fat-tailed are mainly on the investors point of views. Nevertheless, he argues also that this issue is important for "an academic point of views" (Fama 1965a, 41) because:

> For example, if very large price occurs quite frequently, it may be safe to infer that the economic structure that is the source of the prices changes is itself subject to frequent and sudden shifts over time. (Fama 1965a, 41)

The academic issues about empirical fat-tailed concern the macro-economists. Extreme random variations are explained by a proportional change of the "economic structure". It is likely that Fama had in mind the “sophisticated traders” explanation developed in the first section of his article. Thereby, he assumes implicitly that constant deviation from FV cannot explain extreme variations because it would be already exploited by sophisticated traders. The crucial point of this subsection is that Fama's knowledge of Levy distribution does not impact the way he explains economically the randomness of price variation.

The editor of the *Financial Analysts Journal* invited Fama to write for his journal a shorter version of the first article. The latter was too long and too technical to catch the attention of investors (Bernstein 1992, 200). Named "Random Walk of Stock Market prices" (Fama 1965b), Fama’s second article proposed a second formulation of EMH studied in the next section.

3.3 A second formulation of EMH

The article "Random Walk in Stock Market Price" will have a notable success in the analysts’ world. Published first in the *Financial Analysts Journal* in 1965, it will be reprinted in *The Analysts Journal,* in 1966, and finally in The *Institutional Investor* in 1968 (Bernstein 1992, 200-201). This second article mainly points out the implication of Random Walk on chartist and fundamentalist analyses. From this point of view, it is basically a resume of the first section of "Behavior of Stock Market Price" without the discussion on fat-tailed. The article is divided in tree parts. Fama begins by presenting the Random Walk and his economic

explanation, the EMH. Then, Fama presents a short review of empirical works on this topic. Finally, in a last section, he discusses the implications of Random Walk *and* EMH on chartist and fundamentalist analyses. Significant changes could be noted compared to the original article though. First, though it appears only marginally in "Behavior of Stock Market Price", Fama primarily focuses on the EMH. Second, the EMH formulation is quite different and does not use the concept of "sophisticated trader".

As in "Behavior of Stock Market Price", in his second article, Fama states that a market is "efficient" when stock price is a good estimator of his FV (Fama 1965a, 90-94; Fama 1965b, 76). Now this statement is only a *consequence* of the definition. According to Fama, a market is efficient when he is composed of rational agents, that is, profit-maximizers agents in competition:

> An "efficient" market is defined as a market where there are large numbers of rational profit-maximizers actively competing, with each trying to predict future market values of individual securities, and where important current information is almost freely available to all participants. (Fama 1965b, 76).

The "Rational profit-maximizer" assumption substitutes the "sophisticated traders", the characterization of the investor's behavior of his first article. This formulation is by far more general. The two kinds of Sophisticated traders only characterized two ways of making profit. The maximization behavior is focused on the finality: all ways driving to maximization are taken in account. From this new assumption, he directly deducts that:

> *In an efficient market,* competition among the many intelligent participants leads to a situation where, at any point in time, actual prices of individual securities already reflect the effects of information based both on events that have already occurred and on events which, as of now, the market expects to take place in the future. *In other words,* in an efficient market at any point in time the actual price of a security will be a good estimate of its intrinsic value. (Fama 1965b, 76, I emphasize)

The argument can be recall as follow: if investors have a maximization behavior and symmetric information, prices should converge to their FV. This reasoning is based on the same statistical assumption that appears in the first article, discrepancies between FV and observed prices follow some patterns traceable and exploitable:

> If the discrepancies between actual prices and intrinsic values are systematic rather than random in nature (Fama 1965b, 76).

Maximization behavior of investors drive them to exploit and avoid such patterns in price. The arguments between the two articles are nearly the same except a different formulation of the investors behaviors. In the first article, Fama is very careful in the relation between EMH and the random character of price. If EMH was a possible explanation, it was not the

only one. Trying to explain the consistency of fundamentalist analysis and EMH, Fama assumes the dependences of the discrepancies. From an historical perspective, it is easy to understand why Fama introduces the sophisticated traders and the dependences of the discrepancies. He adopts the analytical framework of the investors to convince them. However, from a theoretical perspective, the independence of discrepancies is not justified by Fama. Thereby, in the second article, the link between the rationality of the investors and the good valuation of FV is far from obvious. The exploitation of economic information refers directly indeed to the FV and can explain the efficiency of the market. But without the dependences of discrepancies, there is no reason to rely efficiency of the market (Fama's EMH) and independence of the variations (and more generally the random character of price variation). It is interesting to see that this assumption will rarely be challenged in the literature on EMH (Shiller 1990).[12]

In the next section, I will show that the same economic assumptions on the investors behavior- interested investors in competition - lead Samuelson to a very different explanation of the random variation observed.

4. Samuelson's EMH.

In this section, I am focusing the Samuelson article (1965a). In a first part, I will lay out the Samuelson's argument against random walk (Section 5.1). Then, I will present the Martingale model (Section 5.1). The last section is devoted to the implication of the Samuelson's model in term of efficiency (Section 5.3).

4.1 Martingale versus Random Walk

"Proof That Properly Anticipated Price Fluctuate Randomly" is one the most influential paper Samuelson wrote in Finance (Merton 2006, 269). As described in the first section of his article (see Section 2.2), Samuelson was perfectly aware of the recent research dynamic about the Random Walk model used to describe randomness of the price variation. In "Proof", Samuelson introduces a new probabilistic model describing randomness, the Martingale Model.[13] His methodology is strongly different than that laid out in Fama's contributions. On the one hand, Fama had a literal reasoning explaining *a posteriori* the

[12] Robert Shiller reversed the argument in order to highlight this point: "It would seem peculiar to argue that irrational markets should display regular and lasing patterns" (Shiller 2003, 102). This assumption is also challenged in (Summers 1985, 1986). Other authors are more focused on the (1) the lack of sophisticated traders in the market (Delong et al. 1993) or (2) the lack of sophisticated traders funding (Shleifer et Vishny 1997).

[13] The martingale model will be introduce independently the same year by Mandelbrot (1965)

observed independence. On the other hand, Samuelson used a deductive approach: he asserted few axioms leading him to a theorem: "Theorem of Fair-Game Future Pricing". However, despite these methodological differences, I argue that Samuelson's and Fama's economic assumptions on *the investors behavior* are nearly the same: investors have a rational behavior and they are in competition.

Samuelson begins his article by criticizing the relation between competition and Random Walk model, a relation that Fama made in his 1965 articles, but it was also the intuition of many economists in the beginning of the 1960s (Jovanovic 2009, 66-67; Walter 2013, 115-119). Samuelson criticizes this intuition:

> And the fact [Random Walk], if it is one, is not particularly related to perfect competition or market anticipations. (Samuelson 1965a, 42)

According to Samuelson, it is possible to imagine a monopolistic market where price follows a Random Walk. Take a market with only one supplier price maker and assume then that the demand variation is composed of a numerous little and independent variation. Thereby, the quantity exchanged should variate as a Random Walk (Samuelson 1965a, 52). Retrospectively, Samuelson said that he was always skeptic about random walk:

> "From the beginning I could not believe that the "efficient market" hypothesis was dependent on a pure Brownian motion white noise or any truly random walk. [...] Taken literally, a random walk dictates with certainty that in time the price of luxury Rolls Royce relative to the price of one green pea can reach equality or any ratio you can name" (Samuelson 2009, 24).

Instead of rejecting the competition assumption, Samuelson introduces the Martingale process in order to replace the Random Walk. Samuelson focuses on the relation between future price and the spot price of an asset. The next subsection presents his model.

4.2 The model

P_{t+T} an estimation in t of the spot price in T

(1) P_{t+T} is representable by a given distribution law.

Suppose now a future market. The price of the future contract for the same asset is noted $Y_{t,T}$ with t the valuation moment and T the time before the contract maturity. For n period, we can write $Y_{t+n,T-n}$. At $T+1$ the future price is noted $Y_{t+1,T-1}$. $T+2$, the future price is $Y_{t+2,T-2}$ etc. At the $t+T$ period, the price of the future is noted $Y_{t+T,0}$.
The Samuelson's purpose is to characterized the relation between the sequence P_{t+T} and the sequence $Y_{t,T}$. In a first step, he uses the arbitrage reasoning (Samuelson 1965a, 43) introduced by the famous theorem of Modigliani and Miller. With the arbitrage reasoning, it

is possible to characterize the relation between P_{t+T} and the $Y_{t,T}$ for a particular case. At the $t+T$ period, by definition, $P_{t+T}$is known with certainty. At this period, the spot price must be equal to future price. If not, an arbitrage opportunity will exist and investors will avoid it.

(2) At $t+T$, $P_{t+T} = Y_{t,T}$

But before the $t+T$ period, no one know with certainty P_{t+T}.The arbitrage reasoning is not enough to characterized a relation between P_{t+T} and the $Y_{t,T}$. Samuelson proposes another assumption he named "Mathematically Excepted Price Formation". This axiom asserts that investors know and use the law distribution describing the sequence P_{t+T} to valuate $Y_{t,T}$. by the expected value:

(3) $Y_{t,T} = E[P_{t+T}|I_t]$

Samuelson extends the reasoning by arbitrage considering that, because of competition, investors valuate $Y_{t,T}$ by the expected value of the random variable P_{t+T} conditionally to the information of past price (P_t, P_{t-1} ...), noted here I_t to simplify. The best estimation of the tomorrow spot price is the actual price of the future contract. In a competitive marker, the valuation of future price by investors take in account the past sequences P_{t+T}. Thereby, no systematic profits can be made by using the relationship between future and spot price. This is not a strict arbitrage reasoning since - P_{t+T} and so the profits - is not certain. The economic justification of this hypothesis is based on competition and maximization:

> it is tempting to assume that people in the market place make as full use as they can of the posited probability distribution of next period's price and $Y_{t,T}$ bid by supply and demand to the mean or the mathematically expected level of tomorrow's price. (Samuelson 1965b, 42)

Future Market is interpreted by Samuelson as a place where anticipations of tomorrow spot price are priced. The future price, is the concrete observations of the spot price anticipations. Samuelson try to understand the unpredictability of the price variation by the characterization of the sequence $Y_{t,T}$.

These assumptions characterizing investors' behavior are more formalized than in Fama's assumptions. Investors share common probabilities and they use it to maximize their gains. However, beside these methodological differences, the characterization of the investors' behavior by Fama and Samuelson are close. I remind Fama's assumption: market is composed of rational profit-maximizers trying to predict price as they can, in an environment where "important current information is available" (Fama 1965b, 76, see section 3.3). In the Samuelson's model, the letter assumption is expressed by the known distribution of P_{t+T} that investors used by the expected value operator.

Using iterative exception law[14], a property of probability theory independent of his model, Samuelson concludes that the $Y_{t,T}$ sequence follow a martingale:

$$E[(Y_{t+1,T-1})|I_t] = Y_{t,T}$$

If a sequence of price follows a martingale, thereby, the best estimation of the tomorrow's price, based on the information available, is the today's price. This representation respects the idea that the price is unforeseeable and especially the fact that the chartist analysis is useless.[15]

> It says that, within the defined model, all chart methods attempting to read out of the past sequence of known prices $P_t, P_{t-1}, Y_{t,T}, Y_{t+1,T-1}$ any profitable pattern of prediction is doomed to failure. (Samuelson 1965a, 47)

More generally, because the best estimation is the today's price, we cannot argue it is more likely to see the tomorrow's price higher or lower than today's price. The Samuelson's model doesn't assume independence of the price variation as Random Walk model does. Because of that, the martingale model has been considered as less restrictive than the Fama's random walk model. The main innovation of the "Proof" would be this new formulation of random variations (LeRoy 1989; Mignon 2008). The Random Walk model will be indeed replaced by the martingale process, even by Fama (1970). It will be also a key element in the development of another research program, the Financial Mathematics (Walter 2013, Idabouk 2010).

4.3 Efficiency of the market?

The term "efficient" is not used by Samuelson.[16] But no more than the word, the Fama's EMH does not appear at all in the 1965's article. The Samuelson's martingale model is based on the crucial assumption of a market in competition. Thereby, like Fama, Samuelson assumes that random variations of price are the consequence of competition between interested people in financial market. This assumption does not separate the two authors. Samuelson was indeed convinced that observed random variation and difficulty to forecast were the illustration of competition that economists already studied on the other markets:

---

[14] The iterative exception law can be write formally as follow: $E[X|I_1] = E[E[X|I_2]|I_1]$ if and only if I_2 includes in I_1 (see (Campbell, Lo, et MacKinlay 1997, 27; LeRoy 1989). In the Samuelson's model: (3) says that $Y_{t,T} = E[P_{t+T}|I_t]$ and so $Y_{t+1,T-1} = E[P_{t+T}|I_{t+1}]$ We can conclude using the iterative exception law that $E[Y_{t+1,T-1}|I_t] = E[E[P_{t+T}|I_{t+1}]|I_t] = E[P_{t+T}|I_t] = Y_{t,T}$

[15] In the 1965's article, Samuelson is only focused on chartist analysis. In another article (Samuelson 1973b), he will show that martingale model is not at all inconsistent with presence of fundamentalist in the market. An article wrongly assimilated to Fama's EMH.

[16] To my knowledge, he will use it for the first time in (Samuelson 1973a).

> Work on the other side of the street! The non-predictability of the future prices from past and present prices is the sign, not of failure of economic law, but the triumph of economic law after competition has done its best (Samuelson in Bernstein 19992, 176).

The empirical studies showing the randomness of the price variation had made react abruptly investors *but also* some economists like Houthakker, who thought that random variations were a negative result, meaning the absence of causes explaining the behavior of price.[17]

In his model, investors in competition know the probability distribution of tomorrow's price. In this perspective, the characterization of the investors behavior by Samuelson seems more restrictive than Fama. Indeed, Fama (1965b) assumes only maximization and a competition market with no specific characterization on the anticipation. In Samuelson (1965a), there is a given distribution of spot price, known and used by agents. However, while Fama argues that today's price is the best estimation of the FV, Samuelson's conclusion is less ambitious, arguing only that today's price is the best estimation of the *tomorrow's price.* His conclusion is not on a *good economic* valuation of price, but only on a *good profitable* valuation of price:

> This means that there is no way of making an expected profit by extrapolating past changes in the future price, by chart or any other esoteric devices of magic or mathematics. (Samuelson 1965a, 44)

This difference is underestimated by the literature on EMH, which does not make a distinction between Fama's EMH and Samuelson's EMH. Samuelson was however very explicit in the conclusion of his article:

> It does not prove that actual competitive markets work well. It does not say that speculation is a good thing or that randomness of price changes would be a good thing. It does not prove that anyone who makes money in speculation is *ipso facto* deserving of the gain or even that he has accomplished something good for society or for anyone but himself. All or none of these may be true, but would require a different investigation. (Samuelson 1965b, 48)

Interestingly, Fama (1965b) and Samuelson (1965a) assumptions are not diametrically different theoretically speaking. There are of course strong methodological differences as we emphasize, but from a theoretical point of view, both assume (1) competition in the market (i.e., large number of participants), (2) rational behavior of investors (i.e., profit maximization), and (3) common information. However, their conclusions on the good valuation of price are, as we saw, in total opposition.

[17] The reaction of Houthakker to the Kendall's article is particularly relevant: "Yet regression analysis will soon reveal *that the means of these apparently random differences* are functions of supply and demand factors. And can there be any doubt that the movements of share prices are connected with changes in dividends and the rate of interest?" (Kendall 1953, 32, I emphasize).

We can give to this opposition a historical expression. On the one hand, Fama was deeply involved in the *CRSP.* The center aimed to legitimate investment in stock market for the common people. Highlighting the random character of price was a way to convince that investment in stock was reachable for everyone (Fox 2011, 98), in a context where stock market was still seen suspiciously three decades after the 1929's crisis (Brisset 2017). Fama (1965a) uses the fundamentalist framework – the sophisticated trader - in order to convince that this framework was not inconsistent with the random character of price. Then, he reformulates this explanation with the analytical economics framework - the profit-maximization (1965b). That leads him to conclude that rational market implies the good evaluation of price.

On the other hand, Samuelson studies security market under the influence of Working. When Samuelson began to focus on the behavior of security price, Working gave him his empirical study (Working 1934) and shared with him his attempt to explain the randomness of price variation (Working 1949, 160).[18] In his article of 1949, Working is focusing only on the expectation of price with no regard for the good valuation of this price.[19] He formulates the intuition that unpredictability was maybe not the consequence of an imperfect market, but rather the "evidence of the perfection of the market" (Working 1949, 160). But by "perfection of the market", Working never tried to characterize normatively the valuation of price relatively to the economic fundamentals. In his first article (1934), he warns of the consequence of his result on chartist analysis, but he also warns the *economists* against the temptation of interpreting the random character of price as "random deviation" from a "norm" (Working 1933, 11-12). Samuelson develops the Working intuitions, which lead him to conclude that rational market implies randomness of price variation and stay agnostic on the good valuation of price.

5. Conclusion

In this article, my aim objective was (1) to reevaluate the work of Samuelson leading to show (2) a strong theoretical difference between the main theoreticians of the EMH. This historical analysis show that the ambiguity about EMH, which vacillate between an explanation of the random prices changes and an explanation about the accuracy of prices, is intrinsically link to the theoretical construction of EMH in the 1960s.

In conclusion, we can ask how the random character of price has been explained in such different ways by these two authors. In the actual literature, historians take as granted that randomness of price changes was an observation unambiguously and easily accepted by economists. However, thirty years separate the first empirical observations of the random

[18] See "Correspondence with Working" (July, 1959 and May, 1961), Paul A. Samuelson Papers, *David M. Rubenstein Rare Book Manuscript Library,* Duke University.

[19] Working was focused on the « normal backwardation » hypothesis discussed by Keynes in his *Treatise on money* (Working 1949, 151). This hypothesis says that the effective future price delivered is always below the expected spot price for the same maturity because there is a risk premium in the agent's expectation.

fluctuation and the contribution of Fama and Samuelson. In this perspective, to focus the reactions and the interpretations given by economists to the randomness of price changes during these three decades, can be formative to understand the different meaning granted to EMH.

Bibliography

Bachelier, Louis. 1900. « Théorie de la spéculation ». Annales scientifiques de l'École Normale Supérieure, Sér. 3, 17, p. 21-86.

Bernstein, Peter L. 1992. *Des Idées capitales: les origines improbables du Wall Street moderne*. Paris: Quadrige/PUF.

Brian, Eric, et Christian Walter. 2007. *Critique de La Valeur Fondamentale*. [S.l.]: Springer Paris. http://dx.doi.org/10.1007/978-2-287-73070-2.

Brisset, Nicolas. 2017. « Models as Speech Acts: The Telling Case of Financial Models ». *Journal of Economic Methodology*, septembre. https://ideas.repec.org/p/gre/wpaper/2017-25.html.

Campbell, John Y., Andrew Wen-Chuan Lo, et Archie Craig MacKinlay. 1997. *The econometrics of financial markets*. Princeton, NJ: Princeton Univ. Press.

Cootner, Paul H. 1962. « Stock Prices: Random vs Systematic Changes ». *Industrial Management Review* 3: 24‑ 45.

Cowles, Alfred. 1933. « Can Stock Market Forecasters Forecasts ». *Econometrica* 1: 309‑ 24.

———. 1944. « Stock Market Forecasting ». *Econometrica* 12 (3/4): 206‑ 14. doi:10.2307/1905433.

Cowles, Alfred, et Herbert Jones. 1937. « Some a Posteriori Probabilities in Stock Market Action ». *Econometrica* 5: 289‑ 94.

Delong, Andrei Shleifer, Lawrence Summers, et Robert Waldmann. 1993. « Noise Trader Risk in Financial Markets ». *Journal of Political Economy* 98 (4).

Fama, Eugene F. 1963. « Mandelbrot and the Stable Paretian Hypothesis ». *The Journal of Business* 36. http://econpapers.repec.org/article/ucpjnlbus/v_3a36_3ay_3a1963_3ap_3a420.htm .

———. 1965a. « The Behavior of Stock-Market Prices ». *The Journal of Business* 38 (1): 34‑ 105.

———. 1965b. « Random Walks in Stock Market Prices ». *Selected Papers of the Graduate School of Business , University of Chicago , reprinted in the Financial Analysts Journal (September - October 1965), The Analysts Journal, London (1966), The Institutional Investor , 1968.*, octobre, 55‑ 59.

———. 1970. « Efficient Capital Markets: A Review of Theory and Empirical Work* ». *The Journal of Finance* 25 (2): 383‑ 417. doi:10.1111/j.1540-6261.1970.tb00518.x.

———. 1976a. *Foundations of finance: portfolio decisions and securities prices*. New York: Basic Books.

———. 1976b. « Efficient Capital Markets: Reply ». *The Journal of Finance* 31 (1): 143. doi:10.2307/2326404.

———. 1991. « Efficient Capital Markets: II ». *The Journal of Finance* 46 (5): 1575‑ 1617. doi:10.1111/j.1540-6261.1991.tb04636.x.

———. 2011. « My Life in Finance ». *Annual Review of Financial Economics* 3 (1): 1‑15. doi:10.1146/annurev-financial-102710-144858.
Fox, Justin. 2011. *The Myth of the Rational Market: A History of Risk, Reward, and Delusion on Wall Street*. Reprint edition. New York, NY: HarperBusiness.
Jovanovic, Franck. 2009. « Le modèle de marche aléatoire dans l'économie financière de 1863 à 1976 ». *Revue d'Histoire des Sciences Humaines* 20 (1): 51. doi:10.3917/rhsh.020.0051.
Kendall, Maurice. 1953. « The analys of Economic Time-Series-Part I: Prices ». *Journal of the Royal Society. Serie A (General)* 116 (1): 11‑34.
LeRoy, Stephen F. 1989. « Efficient Capital Markets and Martingales ». *Journal of Economic Literature* 27 (4): 1583‑1621.
Lo, Andrew W. 2004. « The Adaptive Markets Hypothesis: Market Efficiency from an Evolutionary Perspective ». SSRN Scholarly Paper ID 602222. Rochester, NY: Social Science Research Network. http://papers.ssrn.com/abstract=602222.
MacKenzie, Donald. 2008. *An Engine, Not a Camera: How Financial Models Shape Markets*. Inside Technology. Cambridge, Mass.: MIT.
Mandelbrot, Benoit. 1965. « Forecasts of Future Prices, Unbiased Markets, and "Martingale" Models ». *The Journal of Business* 39 (1): 242‑242. doi:10.1086/294850.
Merton, Robert C. 2006. « Paul Samuelson and Financial Economics ». *The American Economist* 50 (2): 9‑31.
Mignon, Valérie. 2008. « Les ambiguïtés de la théorie de l'efficience informationnelle des marchés financiers ». *Regards croisés sur l'économie*, n° 3 (mars): 104‑17.
Osborne, MF Maury. 1959. « Brownian motion in the stock market ». *Operations research* 7 (2): 145‑73.
Rhea, Robert. 1932. *The Dow Theory*. Burlington, VT: Fraser Publishing Co.
Roberts, Harry V. 1959. « Stock-Market Patterns and Financial Analysis: Methodological Suggestions ». *The Journal of Finance* 14 (1): 1‑10. doi:10.1111/j.1540-6261.1959.tb00481.x.
Samuelson, Paul A. 1965. « Rational Theory of Warrant Pricing ». *Industrial Management Review* 6 (2): 13‑39.
———. 1973a. « Mathematics of speculative price ». *Society for INdustrial and Applied Mathematics* 15 (1): 1‑42.
———. 1973b. « Proof That Properly Discounted Present Values of Assets Vibrate Randomly ». *Bell Journal of Economics* 4 (2): 369‑74.
———. 2002. « Modern Finance Theory Within One Lifetime ». In *Mathematical Finance — Bachelier Congress 2000*, édité par Hélyette Geman, Dilip Madan, Stanley R. Pliska, et Ton Vorst, 41‑45. Springer Finance. Springer Berlin Heidelberg. doi:10.1007/978-3-662-12429-1_2.
Shiller, Robert J. 1990. « Market Volatility and Investor Behavior ». *The American Economic Review* 80 (2): 58‑62.
Shiller, Robert J. 2003. « From efficient markets theory to behavioral finance ». *The Journal of Economic Perspectives* 17 (1): 83‑104.
Shleifer, Andrei, et Robert W. Vishny. 1997. « The Limits of Arbitrage ». *The Journal of Finance* 52 (1): 35‑55. doi:10.1111/j.1540-6261.1997.tb03807.x.
Summers, Lawrence H. 1985. « On Economics and Finance ». *The Journal of Finance* 40 (3): 633‑35. doi:10.2307/2327785.

———. 1986. « Does the Stock Market Rationally Reflect Fundamental Values? » *The Journal of Finance* 41 (3): 591‑601. doi:10.1111/j.1540-6261.1986.tb04519.x.
Thaler, Richard H. 1999. « The End of Behavioral Finance ». *Financial Analysts Journal* 55 (6): 12‑17.
Walter, Christian. 1996. « Une histoire du concept d'efficience sur les marchés financiers ». *Annales. Histoire, Sciences Sociales* 51 (4): 873‑905. doi:10.3406/ahess.1996.410892.
———. 2013. *Le modèle de marché au hasard en finance*. Paris: Économica.
Working, Holbrook. 1934. « A Random-Difference Series for Use in the Analysis of Time Series ». *Journal of the American Statistical Association* 29 (185): 11‑24. doi:10.2307/2278456.
———. 1949. « The investigation of economic expectations ». *The American Economic Review* 39 (3): 150‑66.