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High-order time-domain simulation of acoustic impedance models using diffusive representation

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Context: impedance boundary condition

- Euler equation for isentropic acoustical perturbations p, u

$$\forall x \in \Omega, \begin{cases} \partial_t p + \nabla \cdot u + \nabla \cdot pu = 0 \\ \partial_t u + \partial_t pu + \nabla \cdot u \otimes u + \nabla \cdot pu \otimes u = -\nabla p \end{cases} \quad (1)$$

- Non-linear terms neglected in the spatial discretisation



- Time-domain impedance boundary condition (TDIBC)

$$\forall x \in \Gamma_z, p(t, x) = [z(\cdot, x) * u \cdot n(\cdot, x)](t) \quad (2)$$

- Causal and tempered distribution $z \in \mathcal{D}'_+(\mathbb{R}) \cap \mathcal{S}'(\mathbb{R})$

- Integro-differential equation between p and $u \cdot n$

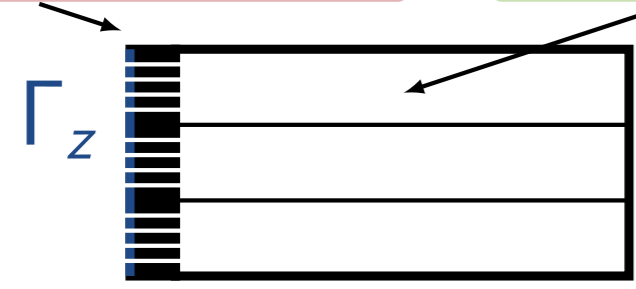
Intuition $|\hat{z}| \gg 1$ (high impedance) $\Rightarrow u \cdot n = 0 \quad \partial_n p = 0$ (rigid wall)

Application linear modelling of a locally-reactive propagation medium Ω_z

Acoustical impedance models

- Ω_z is an acoustic liner

perforated plate (p) cavity (c)



- Impedance models¹

$$\hat{z}(s) = \hat{z}_p(s) + \hat{z}_c(s) \quad (3)$$

$$\hat{z}_p(s) = a_0 + a_{1/2} \sqrt{s} + a_1 s \quad (4)$$

$$\hat{z}_c(s) = \coth(b_0 + b_{1/2} \sqrt{s} + b_1 s) \quad (5)$$

Objectives

- Design a numerical TDIBC for time-domain simulation from acoustical models (\neq state of the art)
- Investigate well-posedness and stability

Challenges of (2):

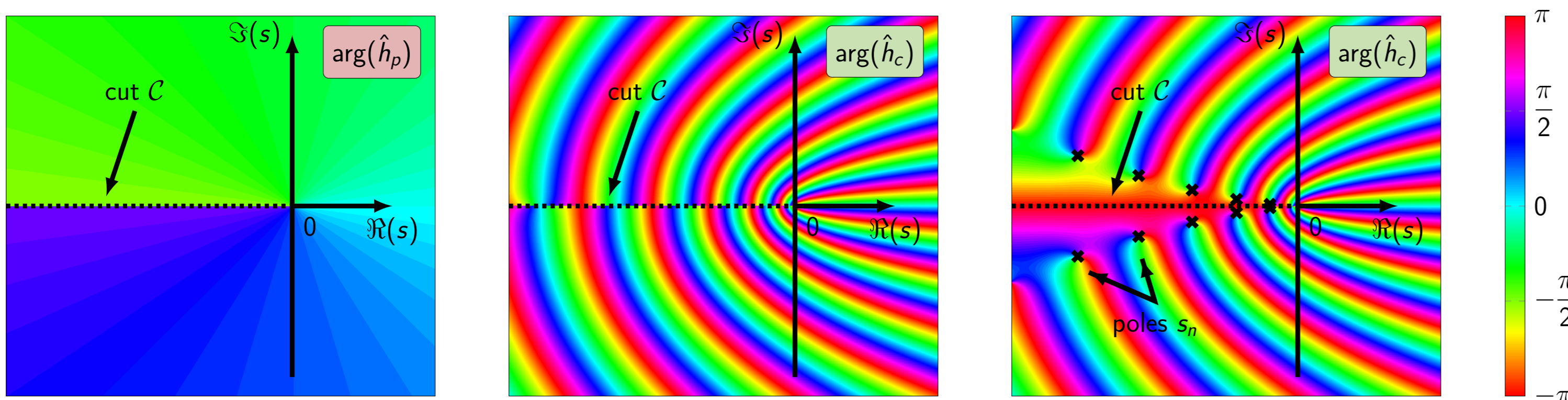
- Generally does not reduce to Dirichlet, Neumann or Robin
- Acoustical models for z only known in the frequency domain (i.e. $\hat{z}(s)$ only)

Analysis of acoustical impedance models: oscillatory-diffusive representation

- Recast of models (4,5) leads to the definition of \hat{h}_p and \hat{h}_c

$$\begin{aligned} \hat{z}_p(s) &= a_0 + a_{1/2} \hat{h}_p(s) s + a_1 s & \hat{z}_c(s) &= 1 + e^{-\Delta t s} \hat{h}_c(s) \\ z_p * u_n &= a_0 u_n + a_{1/2} \hat{h}_p * \dot{u}_n + a_1 \dot{u}_n & z_c * u_n &= u_n + \hat{h}_c * u_n(\cdot - \Delta t) \end{aligned} \quad (6)$$

- Analysis in the complex plane reveals both poles s_n and cut C



- Oscillatory-diffusive representation^{2,3,4} in the time domain

$$h(t) = \sum_{n \in \mathbb{Z}} r_n e^{s_n t} + \int_C e^{-\xi t} d\mu(\xi) \quad (7)$$

- Convolution is then recast into an observer of an infinite-dimensional state-space representation

$$h * u_n = \sum_{n \in \mathbb{Z}} r_n \psi_n + \int_C \varphi_\xi d\mu(\xi) \quad (8)$$

where variables φ_ξ and ψ_n follow a first order dynamic

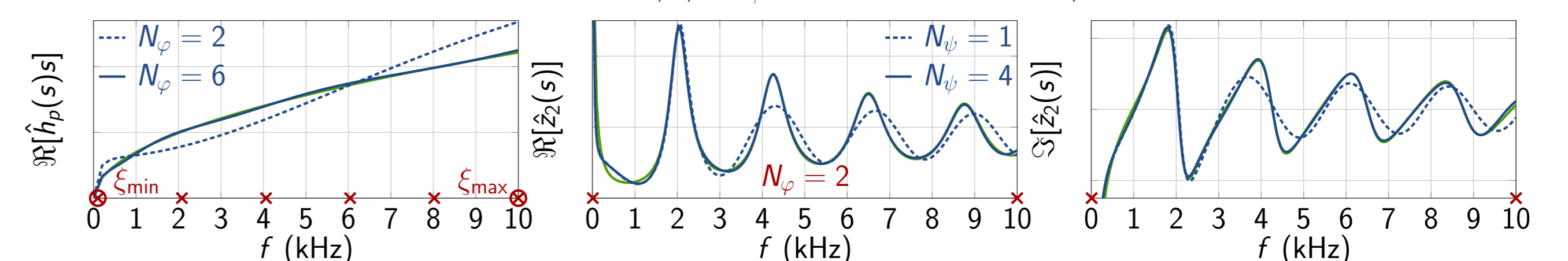
$$\dot{\varphi}_\xi = -\xi \varphi_\xi + u_n \quad \dot{\psi}_n = s_n \psi_n + u_n \quad (9)$$

- Time-local realisation of a hereditary operator
- Analysis applicable to a wide class of acoustical models⁴

Numerical impedance boundary condition

- Numerical TDIBC stems from the discretisation of (8)
- DoF: N_ψ poles s_n and weights \tilde{r}_n / N_φ poles ξ_k and weight $\tilde{\mu}_k$
- Optimisation of \tilde{r}_n and $\tilde{\mu}_k$ in the frequency domain³ (10)
- Broadband approximation through elementary linear least squares
- No optimisation against experimental data

$$h(j\omega) \simeq \tilde{h}(j\omega) = \sum_{|n| \leq N_\psi} \frac{\tilde{r}_n}{j\omega - s_n} + \sum_{k \leq N_\varphi} \frac{\tilde{\mu}_k}{j\omega + \xi_k} \quad (10)$$



Spatial discretisation

- Discontinuous Galerkin method (high spatial order)
- Numerical flux for the TDIBC⁵: centred, with fictive state q_z
- Additional variables $(\psi_n, \varphi_k)_{n,k}$ at each node of Γ_z

$$f = A_i n_i \frac{q + q_z(q)}{2} \quad \Omega \quad \Gamma_z \quad n \quad \Omega_z \quad q = [p \ u]$$

- Spatial discretisation leads to the following global formulation

$$\dot{X}(t) = A X(t) + C X(t - \Delta t) \quad (11)$$

on extended state $X = [p_i \ u_i \ \psi_n \ \varphi_k]$

- TDIBC integrated at the semi-discrete level
- If $C \neq 0$, (11) is a delay differential equation (DDE)

Time discretisation

- High-order time integration with Runge-Kutta method (a_{ij}, b_j)
- If $C \neq 0$, use of a continuous extension⁶ $(a_{ij}, b_i(\theta))$

Validation

- Asymptotic stability study of (11) on a monodimensional case
- TDIBC does not impact stability limit as long as $\xi_{\max} \leq f_{\max}^{DG}$

Well-posedness

- Admissibility conditions for z : reality, passivity, causality and stability⁷

- Semigroup theory on extended state⁸ $[p(\cdot, t) \ u(\cdot, t) \ \psi_n \ \varphi_\xi]$

Acknowledgments & Bibliography

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