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# High-order time-domain simulation of acoustic impedance models using diffusive representation

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## Context: impedance boundary condition

- Euler equation for isentropic acoustical perturbations  $p, u$

$$\forall x \in \Omega, \begin{cases} \partial_t p + \nabla \cdot u + \nabla \cdot pu = 0 \\ \partial_t u + \partial_t pu + \nabla \cdot u \otimes u + \nabla \cdot pu \otimes u = -\nabla p \end{cases} \quad (1)$$

- Non-linear terms neglected in the spatial discretisation



- Time-domain impedance boundary condition (TDIBC)

$$\forall x \in \Gamma_z, p(t, x) = [z(\cdot, x) * u \cdot n(\cdot, x)](t) \quad (2)$$

- Causal and tempered distribution  $z \in \mathcal{D}'_+(\mathbb{R}) \cap \mathcal{S}'(\mathbb{R})$

- Integro-differential equation between  $p$  and  $u \cdot n$

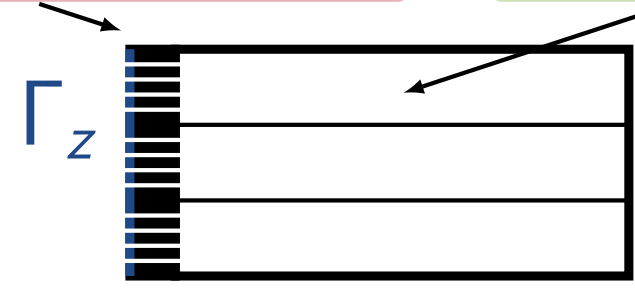
Intuition  $|\hat{z}| \gg 1$  (high impedance)  $\Rightarrow u \cdot n = 0 \quad \partial_n p = 0$  (rigid wall)

Application linear modelling of a locally-reactive propagation medium  $\Omega_z$

## Acoustical impedance models

- $\Omega_z$  is an acoustic liner

perforated plate (p) cavity (c)



- Impedance models<sup>1</sup>

$$\hat{z}(s) = \hat{z}_p(s) + \hat{z}_c(s) \quad (3)$$

$$\hat{z}_p(s) = a_0 + a_{1/2} \sqrt{s} + a_1 s \quad (4)$$

$$\hat{z}_c(s) = \coth(b_0 + b_{1/2} \sqrt{s} + b_1 s) \quad (5)$$

## Objectives

- Design a numerical TDIBC for time-domain simulation from acoustical models ( $\neq$  state of the art)
- Investigate well-posedness and stability

Challenges of (2):

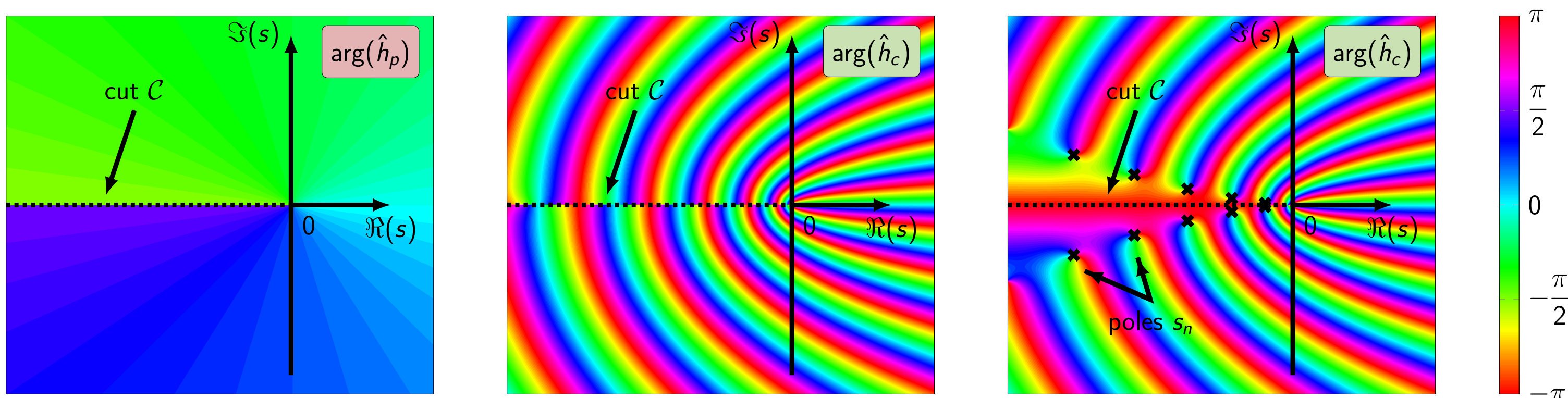
- Generally does not reduce to Dirichlet, Neumann or Robin
- Acoustical models for  $z$  only known in the frequency domain (i.e.  $\hat{z}(s)$  only)

## Analysis of acoustical impedance models: oscillatory-diffusive representation

- Recast of models (4,5) leads to the definition of  $\hat{h}_p$  and  $\hat{h}_c$

$$\begin{aligned} \hat{z}_p(s) &= a_0 + a_{1/2} \hat{h}_p(s) s + a_1 s & \hat{z}_c(s) &= 1 + e^{-\Delta t s} \hat{h}_c(s) \\ z_p * u_n &= a_0 u_n + a_{1/2} \hat{h}_p * \dot{u}_n + a_1 \dot{u}_n & z_c * u_n &= u_n + \hat{h}_c * u_n(\cdot - \Delta t) \end{aligned} \quad (6)$$

- Analysis in the complex plane reveals both poles  $s_n$  and cut  $C$



- Oscillatory-diffusive representation<sup>2,3,4</sup> in the time domain

$$h(t) = \sum_{n \in \mathbb{Z}} r_n e^{s_n t} + \int_C e^{-\xi t} d\mu(\xi) \quad (7)$$

- Convolution is then recast into an observer of an infinite-dimensional state-space representation

$$h * u_n = \sum_{n \in \mathbb{Z}} r_n \psi_n + \int_C \varphi_\xi d\mu(\xi) \quad (8)$$

where variables  $\varphi_\xi$  and  $\psi_n$  follow a first order dynamic

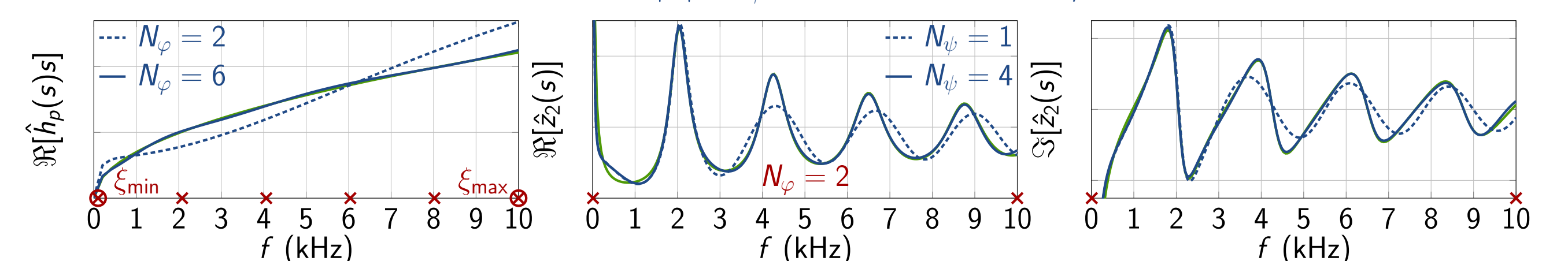
$$\dot{\varphi}_\xi = -\xi \varphi_\xi + u_n \quad \dot{\psi}_n = s_n \psi_n + u_n \quad (9)$$

- Time-local realisation of a hereditary operator
- Analysis applicable to a wide class of acoustical models<sup>4</sup>

## Numerical impedance boundary condition

- Numerical TDIBC stems from the discretisation of (8)
- DoF:  $N_\psi$  poles  $s_n$  and weights  $\tilde{r}_n$  /  $N_\varphi$  poles  $\xi_k$  and weight  $\tilde{\mu}_k$
- Optimisation of  $\tilde{r}_n$  and  $\tilde{\mu}_k$  in the frequency domain<sup>3</sup> (10)
- Broadband approximation through elementary linear least squares
- No optimisation against experimental data

$$h(j\omega) \simeq \tilde{h}(j\omega) = \sum_{|n| \leq N_\psi} \frac{\tilde{r}_n}{j\omega - s_n} + \sum_{k \leq N_\varphi} \frac{\tilde{\mu}_k}{j\omega + \xi_k} \quad (10)$$



## Spatial discretisation

- Discontinuous Galerkin method (high spatial order)
  - Numerical flux for the TDIBC<sup>5</sup>: centred, with fictive state  $q_z$
- $$f = A_i n_i \frac{q + q_z(q)}{2} \quad \Omega \quad \Gamma_z \quad \frac{n}{q} \quad \Omega_z \quad q = [p \ u]$$
- Additional variables  $(\psi_n, \varphi_k)_{n,k}$  at each node of  $\Gamma_z$

- Spatial discretisation leads to the following global formulation

$$\dot{X}(t) = AX(t) + CX(t - \Delta t) \quad (11)$$

on extended state  $X = [p_i \ u_i \ \psi_n \ \varphi_k]$

- TDIBC integrated at the semi-discrete level
- If  $C \neq 0$ , (11) is a delay differential equation (DDE)

## Time discretisation

- High-order time integration with Runge-Kutta method  $(a_{ij}, b_j)$
- If  $C \neq 0$ , use of a continuous extension<sup>6</sup>  $(a_{ij}, b_i(\theta))$

## Validation

- Asymptotic stability study of (11) on a monodimensional case
- TDIBC does not impact stability limit as long as  $\xi_{\max} \leq f_{\max}^{DG}$

## Well-posedness

- Admissibility conditions for  $z$ : reality, passivity, causality and stability<sup>7</sup>

- Semigroup theory on extended state<sup>8</sup>  $[p(\cdot, t) \ u(\cdot, t) \ \psi_n \ \varphi_\xi]$

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