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Catherine Goldstein

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Hermite and Lipschitz: A Correspondence and Its Echoes

Catherine Goldstein*

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Abstract

In the second half of the nineteenth century, the French mathematician Charles Hermite wrote thousands of letters to dozens of correspondents. Mixing personal, political, academic and mathematical matters, as well as views on mathematics and its development, these letters offer a vivid picture of the mathematical landscape of the time. Particularly interesting is the fact that many themes appear repetitively among several correspondents, while some others, on the contrary, are specific to one only. Such echoes and contradictions are of course evocative, but also constitute a challenge to a potential editor: neither strict chronology, nor restriction to one correspondent, allow us to take them into account. We discuss here these problems and some solutions while focussing on the exchanges between Hermite and the German mathematician Rudolf Lipschitz.

1 Charles Hermite, Rudolf Lipschitz and their correspondence

1.1 Parallel Lives

Charles Hermite (1822–1901) and Rudolf Lipschitz (1832–1903) were separated by a decade and a frontier, but their professional lives evolved in intertwined patterns. In 1842, Hermite succeeded in entering the École polytechnique, the main incubator of French mathematicians at the time. ***Because he had a lame foot, ***which barred his access to the standard careers of Polytechnique students/graduate*** and because he possessed a greater enthusiasm for pure mathematics than for engineering, however, Hermite left the school after a year, cutting short the royal road to French mathematical excellency. Passing examinations as a prerequisite to a career as a high-school teacher, which did not appeal

*I would like to thank Jim Ritter for his revision of my English, Alexandre Guilbaud for useful discussions on text encoding, and the personnel of the Universitäts- und Landesbibliothek Bonn for their help during and after my stay in the Archives. My trip to Bonn was funded by the program EMANMATH of the MSH Lorraine, directed by Scott Walter.

much to him either and which he never entered upon, giving lectures at the Collège de France, then holding positions as *répétiteur* and *examinateur* at the Polytechnique, but above all obtaining important results on quadratic forms and elliptic functions paved Hermite's unorthodox and slightly chaotic way to institutional recognition.*** When he was elected to the French Academy of Sciences in 1856, he still did not have any professorship.

At first glance, Lipschitz's path seems more straightforward, if not more prestigious: from 1847 on, he studied in Königsberg, then in Berlin. After his 1853 *Doktorarbeit*, he became a high-school teacher for a few years, during which time he published papers on quadratic forms and series, and prepared his *Habilitation*. From 1857 on, then, he was Privatdozent in Bonn, but left five years later for an extraordinary professorship at the University of Breslau. This very year, 1862, an equivalent position was at last created for Hermite, as *maître de conférences* at the École normale supérieure that the recently appointed director of studies, Louis Pasteur, wanted to reorganize. But two years later, Lipschitz returned to Bonn, this time as full professor; Hermite would not obtain such a position before the end of the decade, first at Polytechnique, then—and for a while simultaneously— at the Sorbonne.

In the 1870s, both mathematicians had reached the forefront of their professions in their respective countries. Lipschitz's choice to stay at Bonn, despite other attractive proposals, may seem from our perspective today to have put him in a backwater, compared to Hermite, who in Paris was at the center of all things mathematical. On the other hand, however, Lipschitz committed himself to important administrative duties, for example as *Rektor* (chancellor) of his university in 1874, and editor of the celebrated *Journal für die reine und angewandte Mathematik*, duties that Hermite sought to avoid at all costs for his entire life.

Hermite had come to Berlin in the early 1850s, in order to meet some of the most famous representatives of the German mathematical intelligentsia: Peter Gustav Lejeune Dirichlet, Gotthold Eisenstein and Ernst Eduard Kummer in particular,[1] but did not seem to have been acquainted with Lipschitz at this time. Dirichlet, however, was a key figure for both men: Lipschitz's *Doktorarbeit* was written under his supervision, and according to his necrologist, Hermann Kortum: "Lipschitz's mathematical thought was defined through Dirichlet, whose pupil he considered himself," [27, p. 57]. As for Hermite, he also described himself on several occasions as one of Dirichlet's disciples.[2] This is not the only point in common between the two men: both were offered a position in Göttingen (which they finally both declined), both had a large spectrum of mathematical interests, from number theory to forms to mechanics, as well as a deep committment to analysis. Their national and international recognition is well attested, by the number of mathematical journals to which they were both asked to contribute, as well as by their election to prestigious Academies of Sciences ; both, for instance, were correspondents of the Accademia dei Lincei in Rome and of the Berlin Akademie der Wissenschaften.

[1] See [16, p. 379, n. 8]. Mathematics in Berlin at the time are described in [4, 3]. On this particular circle, see also [34].

[2] He wrote for instance just before his death to Eugen Jahnke: "I have always been and will be until the end the disciple of your great mathematicians, Gauss, Jacobi, Dirichlet." In a 1853 letter to Dirichlet himself after his trip to Berlin, Hermite even evoked "a law of [his] destiny not to do anything in arithmetic other than unearth some of the discoveries you made a long time ago", [16, p. 399–400].

LettreLipRat.pdf	LettreHbl.pdf

Figure 1: Two extracts of the correspondence: left, a draft by Lipschitz (6*, 1878) ; right, a letter from Hermite (109, 1884). Repr. with the kind authorization of the Archives of Universitäts- und Landesbibliothek Bonn.

1.2 Correspondence

Another point in common is that both mathematicians were situated at the center of a vast correspondence network, with large points of overlap*. We know they both exchanges letters with Eugenio Beltrami, Georg Cantor, Jules Hoüel, Leopold Kronecker, Leo Königsberger, Gösta Mittag-Leffler, Henri Poincaré and James Joseph Sylvester, for instance. For both of them, a number of these letters were published as technical articles during their lifetime. However, important differences exist between their correspondence, then and now.

Hermite wrote exclusively in French, and most of his correspondents followed suit. The bulk of his passive correspondence, transmitted after his death to one of his sons-in-law, the mathematician Émile Picard, is said to have been destroyed in a fire. His correspondence with Thomas Stieltjes, edited in 2 volumes as early as 1905 by Benjamin Baillaud & Henry Bourget, is one of the rare cases to offer letters in both directions.[3] On the other hand, Hermite's letters to a variety of mathematicians, deposited in a matching variety of archives and libraries, have been published since the beginning of the twentieth century: for example, those to Paul Du Bois-Reymond by Emil Lampe in 1916; to Andrei Markoff, by Helen Ogigova in 1967; to Gösta Mittag-Leffler, by Pierre Dugac in 1984–1989; to Ernesto Cesàro (and partially to Eugène Catalan), by Paul Butzer, Luciano Carbone, François Jongmans and Franco Palladino in 2000; to Angelo Genocchi, by Giacomo Michelacci in 2003; to Georg Cantor, by Anne-Marie Decaillot in 2008. To this must be added the publication of many selected letters, e.g., with Italian mathematicians, by Umberto Bottazzini or with Sylvester, by Karen Parshall.[4]

Lipschitz's correspondence, on the other hand, include letters in German, French, English and Italian; Lipschitz kept also drafts of his own letters, which, in some

[3] This edition, [21], is unfortunately bowdlerized ... and was only partially completed by Pierre Dugac in 1983, [15].

[4] See, respectively, [22, 23, 24, 7, 25, 11, 5, 32].

cases, allow a more complete view of the exchange. ***Lipschitz's *Nachlass*, at the Universitäts- und Landesbibliothek Bonn (Abteilung Handschriften und Rara) hosts most of the surviving letters, about 600 of them, including 455 letters or cards to Lipschitz from 61 correspondents. Winfried Scharlau has published a selection of them, [30], consisting of 140 letters or extract of letters, from a few lines to ten pages long, among which 26 are from Lipschitz. Scharlau's edition includes 13 letters from Hermite and 2 to him. A few isolated letters from or to Lipschitz have also been published in translation.[5]

1.3 The Correspondence between Hermite and Lipschitz

The surviving correspondence between Hermite and Lipschitz is kept in Lipschitz's *Nachlass*. It contains 148 letters and 9 postcards from Hermite to Lipschitz, the first dated August 19, 1877 and the last July 14, 1900, six months before Hermite's death. As expected, they are all in French. We also find in the *Nachlass* 70 drafts of letters from Lipschitz to Hermite written mostly between 1877 and 1886 (one only is dated 1892), again, all in French, with four exceptions (still) in German. ***Morevoer, Hermite's file in the Archives of the French Academy of Sciences contains two letters from Lipschitz, one of them corresponding to a draft kept in Bonn. *** Although it is clear from allusions in the surviving correspondence that several letters are missing, this is by far the most extensive correspondence known from or to Lipschitz (the second most numerous in this *Nachlass* ***is composed of*** 57 letters written by Carl Borchardt). *** It is, however, not easy to decipher/Their material appearance, however, is not very appealing***. As noted by W. Scharlau, Lipschitz's drafts are very badly written, with many corrections and deletions, often difficult to decipher, [30, p. xvi]. On some of Hermite's letters, the ink has almost disappeared. To this can be added that the microfilm scans are of extremely poor quality.

The correspondence starts in 1877 when Lipschitz sends to Hermite the first volume of his *Lehrbuch der Analysis* (textbook on analysis), [28]. Hermite writes back: "You offer me an opportunity which I eagerly seize to remind you of me while thanking you for the first volume of your treatise on analysis that you have bestowed on me the honor of sending to me".[6] The acquaintance between the two mathematicians dated back only a few months earlier when they had both attended the March 30, 1877 Göttingen ceremony for the centenary of Carl Friedrich Gauss' birth. This episode is recalled by Hermite on the occasion of New Year 1878, and again twenty years later on December 29, 1897: "You call to mind, Sir, our first encounter in Göttingen, during the centennial ceremony in honor of Gauss, which left me with unforgettable memories".[7] The exchange then accelerates rather quickly, culminating in the mid-eighties (see Fig. (2)), with one or more letter per month on each side. Several times, letters cross each other, a circumstance duly noted and a cause for another exchange; more than once, Hermite sends a postcard in the

[5]For instance, some letters exchanged with Richard Dedekind are translated in [14].

[6]Except otherwise indicated, all the quotes come from Lipschitz's *Nachlass*. ***In the *Nachlass*, the letters are organized by senders and, for each sender, are numbered independently. Here, we use these numbers, but, for the sake of clarity, add a star to those of Lipschitz's drafts***. Letter 1: *Vous m'offrez une occasion que je saisis avec empressement, de me rappeler à votre bon souvenir en venant vous remercier du premier volume de votre traité d'analyse que vous m'avez fait l'honneur de m'envoyer.*

[7]Letter 150: *Vous me rappelez, Monsieur, notre première rencontre à Gottingue, lors des fêtes du centenaire de Gauss qui m'a laissé d'inoubliables souvenirs.*

immediate aftermath of his regular letter, to signal a formula that needs correction or a reprint he has forgotten to request.

Besides dates, the heading of the letters keeps a trace of Hermite's wanderings. Every year, he spends holidays in his native Lorraine (which had in part been assigned to Germany after the 1870 war), or with one of his married daughters in the western part of France ; he also visits these places for family events. After an illness, he is also obliged to go to thermal spas. Last, but not least, in November 1886, a short letter announces his upcoming arrival in Bonn for a visit to Lipschitz. These trips do not deter him from writing, nor, as the letters themselves reveal, from working on mathematics, although Hermite often complains of his own laziness or fatigue. On the other hand, all the drafts we have from Lipschitz, except one, are written from Bonn itself, although we know for instance that in 1881 he spends several weeks in Switzerland after a serious health problem.

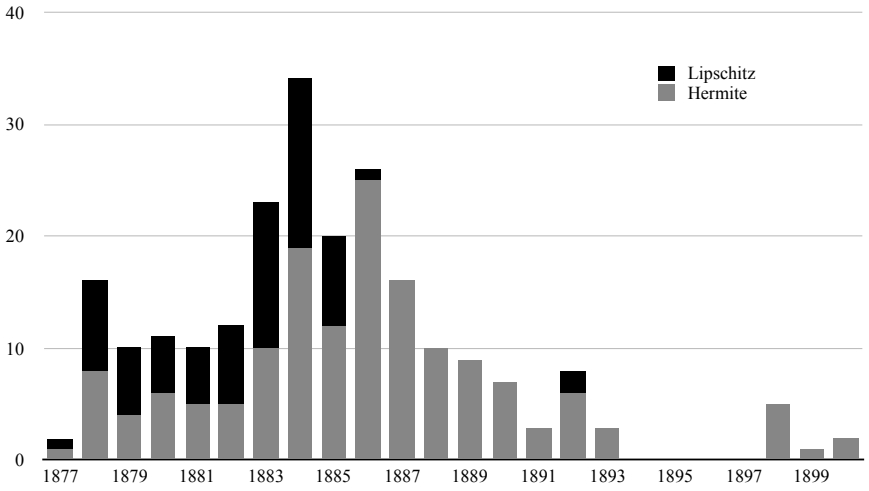

Figure 2: Distribution by years of the surviving letters between Hermite and Lipschitz: in grey, those authored by Hermite, in black, by Lipschitz.

Until 1888, Hermite's letters are simply adressed to "Monsieur Lipschitz, Professeur à l'université, Bonn (Prusse rhénane)" (after 1888, Koenigstrasse 34, Bonn (Prusse rhénane)), a good testimony to the efficiency of mail deliveries at the time and of the status of university professors. The salutations between both men never vary : "Monsieur" (Sir) is used all along in both directions. This contrasts with the "Mon cher Monsieur Schwartz" (my dear M. Schwarz) Hermite uses for Hermann Amandus Schwartz or with "Mon cher ami" (My dear friend) which appears in his letters to Carl Borchardt, Thomas Stieltjes or Gösta Mittag-Leffler. Stieltjes and Mittag-Leffler are much younger than Hermite, but the case of Borchardt, born in 1817, five years before Hermite, shows that it is not only a generational issue; but Hermite and Borchardt had met in Paris as young men, ***as early as 1847***. Borchardt himself uses "Verehrter Freund" (Esteemed friend) in his numerous letters to Lipschitz, while Helmholtz addresses him as "Bester Freund" or "Lieber

Freund" (dear friend), [30, pp. 13–24, 120–128, resp.]. The appellation Hermite and Lipschitz use to one another appears thus rather formal. Still, it does not hamper the raising of personal issues, nor some emphasis on the emotional importance of the exchange for both men. Lipschitz thanks Hermite for "[his] communications which are so dear to me",[8] while Hermite ends his letters with "his feelings of the most sincere friendship" (January 30, 1883), ***expresses regrets that they are not geographically closer and colleagues at the Sorbonne (January 5, 1884)*** and confides in 1892:[9]

> I consider as one of the greatest joys of my scientific career that for nearly 20 years I have always had your counsel near at hand.

1.4 A closer look: a year and a letter

To grasp more concretely the nature of the correspondence, let us consider as an example the year 1881: at the very end of the preceding December, Hermite had announced the wedding of his daughter Marie with his young colleague and protégé Émile Picard. A day before the wedding, on January 3, Lipschitz thanks Hermite for the various bits of news and expresses his best wishes for the young couple. Hermite answers on January 31 with some news of the family, a summary of the way one of his students, Jules Tannery, had constructed following Karl Weierstrass a new example of discontinuities of functions expressed as series, and his own comments on the issue; the end of the letter alludes to the upcoming election at the Academy of Sciences in which Gaston Darboux and Camille Jordan are competing. Because of a long illness, he explains, Lipschitz delays his answer until April 1st, in which he comments informally on a mathematical remark by Hermite on the theory of transformations of quadratic forms; a few weeks later, on April 25, he completes his views on both topics, series and forms, to which Hermite quickly replies, on May 3, urging Lipschitz to publish his new theory. The French mathematician adds some explanations on his current course on analysis at the Sorbonne, concluding with surprise and regret about the silence surrounding Borchardt's death (on June 27, 1880), in particular in the *Journal für die reine und angewandte Mathematik* of which Borchardt had been chief editor since 1856. The following letter, addressed from Hermite's vacation resort, explains in a self-mockingly desperate tone his failed attempts to extract some results on the Γ function from a formula given by Cauchy

$$\int_0^{\frac{\pi}{2}} (2\cos x)^{a+b} \cos(a-b)x dx = \frac{\pi}{2} \frac{\Gamma(a+b+1)}{\Gamma(a+1)\Gamma(b+1)} : \tag{1}$$

"But how, I ask you, how to disentangle what then becomes of the definite integral? The only thing I have seen clearly is that analysis was created for the chastisement of pride and that it inflicts frequent and salutary humiliation."[10] He also proposes to ask Darboux, the editor of the *Bulletin des sciences mathématiques*, to publish a

[8] Draft 23, April 25, 1881: *vos communications qui me sont si chères*.

[9] Letter 148, December 30, 1892: *je regarde comme un des plus grands bonheurs de ma carrière scientifique que depuis près de 20 ans j'ai toujours eu près de moi vos conseils*.

[10] Letter 22, August 4, 1881: *Comment je vous le demande, comment débrouiller ce que devient alors l'intégrale définie ? La seule chose que j'ai vue clairement c'est que l'analyse a été créée pour le châtiment de l'orgueil et qu'elle inflige de fréquentes et salutaires humiliations*.

translation of Lipschitz's self presentation of his textbook. At the time in Switzerland for his health, Lipschitz writes as soon as he returns home, on October 30, and gives a direct proof of Cauchy's formula (1); he also joins his photograph to the letter—a current practice of the time. Due to several deaths in his family, Hermite only reciprocates on December 13th, with his own photograph, as well as a follow-up on the issue of the *Bulletin*; he also transmits some laudatory commentaries on Lipschitz's treatise from various French mathematicians. He compares in particular the rigid programs of French courses with the flexible ones he thinks are possible in Germany, which allow professors to introduce innovations more easily. Hermite also regretfully evokes Eduard Heine, who has died on October 21. The last two letters of the year, one from Hermite on December 27 and the grateful answer of Lipschitz on the 30th, are devoted to the ongoing publication of Cauchy's complete works on behalf of the French Academy of Sciences: Hermite, indeed, has obtained one of the coveted books for Lipschitz and another for his Bonn colleague, Rudolf Clausius.

This mixture is quite typical of the whole correspondence. Discussing the editorial problems attached to it, Winfried Scharlau comments:[11]

> Then, the letters most prominently discuss a variety of mathematical questions and it would be necessary for a proper commentary to reconstruct these questions jointly from both the letters and the original papers.

The second half of this comment raises a fundamental issue to which I shall return in section 3 below. But before this, I would like to look more closely at the first assertion through the observation of a concrete example, Hermite's letter of December 5th, 1883; it is composed of seven and a half pages, with about 16 to 19 lines of writing on each page. It begins by the announcement of the death of Hermite's sister-in-law (c. 5 lines), before turning to mathematics, more specifically to a comparison between the class numbers of properly and improperly primitive binary quadratic forms of determinant $-D$, for $D \equiv +3 \bmod 8$.[12] Hermite recalls the relation stated by Gauss, as well as a relevant article published by himself in 1862 (13 lines). He then proves the required relation via the series expansion of θ-functions (98 lines). This, Hermite says, will be a part of a future article for the *Bulletin of the Saint Petersburg Academy* that he sketches and which will involve the use of a formula communicated to him by Lipschitz (9 lines).[13] Finally a post-scriptum (9 lines 1/2) adds:[14]

[11] [30, p. xvi]: *zweitens geht es in den Briefen ganz überwiegend um mathematische Fragen verschiedenster Art und für einen sachgerechten Kommentar wäre es erforderlich, diese Fragen aus den Briefen und Originalarbeiten zusammenhängend zu rekonstruieren.*

[12] In Gauss's normalization, a (binary quadratic) form is an expression of the type $Ax^2 + 2Bxy + Cy^2$, here with integral coefficients A, B, C; the determinant D is $B^2 - AC$. The form is properly (resp. improperly) primitive when $\gcd(A,B,C) = \gcd(A,2B,C) = 1$, resp. when $\gcd(A,B,C) = 1, \gcd(A,2B,C) = 2$. Two forms are equivalent when they can be transformed into each other by an invertible linear change of variables of determinant ± 1; forms equivalent to a properly (resp. improperly) primitive form are properly (resp. improperly) primitive form. For a given D, the number of classes of equivalent forms with integral coefficients is finite, thus also the number of classes of properly primitive (or improperly primitive) forms. The computation of these numbers was one of the difficult problems nineteenth-century number theorists inherited from Gauss.

[13] This article was published in 1884, in the volume 29 of the *Bulletin*, and reproduced the same year in the 5th volume of *Acta Mathematica*.

[14] Letter 39 and 40 (this letter is separated into two parts in the Archives, each numbered separately)***

> Mr Bischoffsheim, the member of Parliament who spoke to the House about the courses at the Sorbonne in the manner you know [i.e., very critically] is a candidate for a seat as a free member of the Academy of Sciences; you may well imagine that I shall not give him my vote. But his generosity in favor of astronomy and even of the Sorbonne, to which he offered a magnificent portrait of the great physiologist Claude Bernard, will give him many votes and he may well succeed.

Mathematics per se, results and proofs, represent here about three quarters of the letter, in line with W. Scharlau's statement. However, the remaining quarter draws a rich tapestry of nineteenth-century scientific life, from family to political intelligence. In order to understand the uses of this correspondence, including the communication of mathematical results, a more systematic overview of the matters at hand in the letters is in fact worthwhile. The classification is mine and mostly proposed for the sake of clarity—the quotes will make clear how intertwined the topics are.

2 A Variety of Topics

2.1 Personal life

Family deaths, births, weddings are evoked very frequently, either explicitly, or, for the former, through the black frame of official mourning letters. Consolations and congratulations are exchanged on such occasions, as we have already seen. "We have lost Madame Duhamel after a long illness which had deprived her almost completely of the use of reason, and we had ceased a long time ago to have any hope of saving her.[15] explains Hermite on June 16, 1878. On Lipschitz's side, in 1882: "On March 11, I received the telegraphic announcement that my mother, who had been attacked by a serious illness since Christmas, who had shared all my interests from my childhood until her final days, and in whom clarity of mind and warmth of feeling could vanish only with life itself, had died on the preceding day."[16] Besides glimpses of family relations, personal services are sometimes required, for instance when Hermite asks Lipschitz's help in favor of his nephew

: *Mr Bischoffsheim, le député qui a parlé à la Chambre des cours de la Sorbonne comme vous savez, se présente à une place de membre libre de l'Académie des sciences ; vous pensez que je ne lui donnerai point ma voix. Mais ses générosités en faveur de l'Astronomie et même de la Sorbonne, à qui il a fait don d'un magnifique portrait du grand physiologiste Claude Bernard lui vaudront beaucoup de suffrages, et peut-être va-t-il réussir.* The banker Raphaël Bischoffsheim (1923–1906) had been elected at the Parliament in 1881 and, ***unlike Hermite***, supported Republicanism. He had launched attacks against the Sorbonne professors as old-fashioned and ignorant of the most innovative approaches***. He was finally elected at the Academy of Sciences, but only in 1890…

[15]Letter 6: *Nous avons perdu Madame Duhamel après une longue maladie qui lui avait enlevé à peu près complètement l'usage de la raison, et depuis longtemps nous ne pouvions plus avoir l'espérance de la conserver*. Virginie Duhamel, the wife of the mathematician Jean-Marie Duhamel, to whose positions Hermite succeeded both at the Sorbonne and at the Polytechnique, was the sister of Joseph and Alexandre Bertrand's father, and thus the aunt of Hermite's wife.

[16]Letter 26*: *Le 11 mars, j'ai reçu l'avertissement télégraphique que ma mère qui était attaquée d'un mal grave depuis la fête de Noël, qui a partagé tous mes intérêts dès mon enfance jusqu'à ses derniers jours, chez laquelle la clarté de l'esprit et la chaleur des sentiments ne se sont évanouis qu'avec la vie même a été décédée le jour précédent*. Lipschitz's French, although good, is not always correct. I have left it untouched in the original but have given a grammatically correct English translation.

Georges Bertrand (the son of Alexandre Bertrand) who wants to spend five months in Bonn in order to learn German (letter 70). The intricacy of family and professional links among the members of the nineteenth-century European intelligentsia is well-known, [38], and confirmed here by hints at professional discussions over family dinners. We also learn about travel plans and details of the personal organization of work. But some letters also provide information concerning personal relations *between third parties*. For example, on January 20, 1882, Hermite tells Lipschitz that "M. Chasles was M. de Jonquières's friend for a long time, but this bond that nothing should have broken was destroyed over a priority dispute."[17]

2.2 Circulation of mathematics

The correspondence also documents the usually tacit character of a variety of mathematical collaborations. On the occasion of Eduard Heine's death, in October 1881, Hermite reveals for instance: "I cannot tell you how afflicted I am by the loss of M. Heine, who was an excellent man as well as a first-rate mathematician, whose works will remain forever a part of science. I wrote to him frequently and he did important favors for me by teaching me things from Riemann that though very well-known in Germany were not so to me."[18] Or on March 12, 1878: "May I venture to ask you to tell me if you know of a paper 'On Rotation' in the *Mathematisches Wörterbuch* of MM. Hoffmann and Natani, written, so M. Borchardt tells me, on the basis of lectures that M. Weierstrass gave at the Berlin University and which present a close analogy to what I myself have just done."[19]

Thanks to the proximity of their mathematical interests, more specialized issues are also tackled. For instance, on February 20, 1878, Hermite asks: "Allow me to call your attention to a question concerning elliptic functions which worries me and on which I would like to have your opinion. You know that M. Rosenhain represents the four fundamental Jacobi functions $\Theta(x)$, $H(x)$, $H_1(x)$ and $\Theta_1(x)$ by $\theta_0(x)$, $\theta_1(x)$, $\theta_2(x)$, $\theta_3(x)$; this notation seems to me of real importance, as it allows us to encompass within a single equation a group of four relations, save for, as you will see, some difficulties that I cannot succeed in overcoming."[20]

[17]Letter 26: *Mr Chasles avait été longtemps l'ami de Mr de Jonquières, mais cette liaison que rien n'aurait dû rompre, a été détruite par une question de priorité.*

[18]Letter 24: *Je ne puis vous dire combien j'ai été affecté de la perte de M. Heine, qui était un excellent homme en même temps qu'un géomètre de premier ordre dont les travaux resteront à jamais dans la science. Je lui écrivais fréquemment et il m'a rendu les plus signalés services en m'apprenant des choses de Riemann, extrêmement connues en Allemagne, et que j'ignorais.*

[19]Letter 4: *Oserais-je aussi vous prier de me faire savoir si vous avez connaissance d'un article Sur la rotation du Mathematisches Wörterbuch de MM Hoffmann et Natani rédigé m'a dit Mr Borchardt d'après les leçons de M. Weierstrass données à l'Université de Berlin et qui offriraient une grande analogie avec ce que je viens de faire moi-même.*

[20]Letter 3: *Permettez aussi d'appeler votre attention sur une question relative aux fonctions elliptiques qui me préoccupe et sur laquelle j'aimerais avoir votre avis. Vous savez que M. Rosenhain représente les quatre fonctions fondamentales de Jacobi,* $\Theta(x)$, $H(x)$, $H_1(x)$ *et* $\Theta_1(x)$ *par* $\theta_0(x)$, $\theta_1(x)$, $\theta_2(x)$, $\theta_3(x)$ *; c'est cette notation qui me paraît avoir une importance réelle, en permettant de comprendre dans une seule équation, un groupe de quatre relations, sauf toutefois les difficultés que vous allez voir et que je ne puis réussir à lever.*

2.3 Reflections on science

Technical mathematics is thus sometimes an incentive to display more general points of view be it, as here, concerning notation or, more generally, concerning the development of mathematics. When Hermite explains to Lipschitz his ideas on cuts—Hermite's analytical version of Riemann's more geometrical ideas on complex functions—he adds: "How greatly have ideas in analysis been modified because of all these facts, since the time when infinity first seemed to be the only possible discontinuity and the more recent period when the study of Fourier series revealed sudden jumps from one continuous series to another, completely different one."[21] Hermite often expresses his deeply-felt epistemological convictions, for instance on April 12, 1882:[22]

> I believe that in science and especially in mathematics, we are less masters than servants of our work. I do not deny free will, but I think that it coexists with the action of a force which, while it arises from us, acts outside us, and unwittingly directs us where we would not like to go ... I recognize that the conception of Riemann spaces frightens me, but that it has its utility.

2.4 Publication

On a more material note, we also learn about the concrete functioning of publications at the time. On a 1880 postcard, Hermite summarizes: " I have just finished correcting the proofs of the first part of your article that was presented to the session of the Academy, and I informed M. Gauthier-Villars that you wished to have reprints, but it would be necessary to tell him how many copies you want. Please write him a note to inform him without delay."[23] Or, concerning one of his own texts: [24]

> My lessons at the Sorbonne are being published this year in lithography. They have been written down by a student of the Ecole normale, and with the authorization of the Faculty. But the publisher did not think it relevant to give me a single copy, so that the one I possess I had to buy with my own money. One of my friends told me on this

[21] Letter 19: *Combien les idées en Analyse se sont modifiées en présence de tous ces faits, depuis le temps ou l'infini avait paru d'abord la seule discontinuité, et l'époque plus récente ou l'étude de la série de Fourier a révélé des sauts brusques d'une série continue, à une autre toute différente.*

[22] Letter 27: *Je crois que nous sommes dans les sciences et tout particulièrement dans les mathématiques, moins les maîtres que les serviteurs de notre œuvre. Je ne nie point le libre arbitre, mais que je pense qu'il coexiste avec l'action d'une force qui naissant par notre fait, agit en dehors de nous, et à notre insu, nous dirige là même où nous ne voudrions pas aller ... je reconnais à la fois que la conception des espaces de Riemann m'effraye, et qu'elle a sa raison d'être.*

[23] Card 18: *Je viens de corriger les épreuves de la première partie de votre article qui a été présenté à la séance de l'Académie, et j'ai prévenu Mr Gauthier-Villars que vous désiriez en avoir un tirage à part, mais il serait nécessaire de lui faire connaître combien vous voulez d'exemplaires. Permettez-moi de vous prier de lui écrire un mot pour l'en informer sans retard.*

[24] Letter 27: *Les leçons à la Sorbonne se publient cette année sous forme de feuilles lithographiées, qui ont été rédigées par un élève de l'Ecole Normale, et avec l'autorisation de la Faculté. Mais l'éditeur n'a point jugé à propos de m'en donner un seul exemplaire, de sorte que celui que je possède je l'ai acheté de mes deniers. Un de mes amis m'a dit à cette occasion, que j'avais été pour cet éditeur, une poule à plumer, et j'en ai bien ri. Mais l'essentiel c'est que les élèves puissent avec la rédaction sommaire des leçons suivrent (sic) facilement un cours dont très peu profitaient m'a-t-on dit les années précédentes.*

> occasion that I for this publisher I was a goose to be plucked and I had a good laugh. But the main point is that the students can, with the short draft of the lessons, easily follow a course from which, I have been told, very few were able to profit in preceding years.

French scientific journals published only in French and translations had to be negociated. "I had the occasion of discussing you with M. Darboux who expressed the desire to have a reprint of the note you gave to the Göttingen *Nachrichten*, on your treatise on analysis, in order to give it to one of his coworkers who will do a translation to appear in the *Bulletin*."[25]

Despite the continuity of the title, the change of editors of the *Journal de mathématiques pures et appliquées* gives rise on July 23, 1884 to Hermite's comment: "I shall make it the subject of an article that M. Camille Jordan asked me to publish in the first issue of a *Journal de mathématiques*, of which he will become the main editor. This future *Journal* is that of M. Resal, which continued with mediocre success that of M. Liouville, and which the publisher M. Gauthier-Villars wants to revive and transform."[26]

2.5 Teaching

The publication of textbooks is not the only mention of teaching in this correspondence. The various reforms of the curricula, the difference between the situation in France and in Germany, and even specific pedagogical issues are discussed in detail. For instance Lipschitz explains, on November 13, 1877: "I have come to believe that the understanding of the fundamental theorem of algebraic equations requires from the beginner a particular effort and that a longer way, that leads to the proof while teaching how to find a root of an equation by computation, is to be preferred to a shorter, but less illuminating, way."[27]

Or Hermite, on December 5, 1882: "None of our legislators could imagine that M. Bouquet and myself agreed, after the war, to combine our efforts to raise the level of the teaching of analysis at the faculty, and that, with this objective, I dropped my course on advanced algebra in order to be the assistant in the basic course, on differential and integral calculus."[28]

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[25] Letter 25: *J'ai eu l'occasion de m'entretenir de vous avec Mr Darboux qui m'a exprimé le désir d'avoir un exemplaire séparé de la notice que vous avez donnée dans les Nachrichten de Gottingue, sur votre traité d'Analyse, afin de le donner à celui de ses collaborateurs qui en fera la traduction destinée à paraître dans le Bulletin*. On the translations into French of foreign articles during the nineteenth century, see [6].

[26] Letter 52: *J'en ferais le sujet d'un article que Mr Camille Jordan m'a demandé pour paraître dans le premier n^o d'un Journal de Mathématiques dont il sera le rédacteur en chef. Ce futur Journal est celui de Mr Résal, qui a succédé avec un succès médiocre à celui de Mr Liouville, et que l'éditeur Mr Gauthier-Villars veut relever et transformer*.

[27] Letter 1*: *Je suis parvenu à croire que l'entendement du théorème fondamental des équations algébriques exige des commençants un effort tout particulier et qu'un chemin plus long qui mène à la démonstration en apprenant comme on puisse trouver une racine d'une équation par le calcul soit préférable à un chemin plus court, mais moins lumineux*.

[28] Letter 29: *Aucun de nos législateurs ne s'est douté que Mr Bouquet et moi nous sommes convenus, après la guerre, de réunir nos efforts pour relever l'enseignement de l'analyse à la faculté, et que dans ce but j'ai renoncé à mon cours d'algèbre supérieure afin de me faire l'auxiliaire du cours fondamental, de calcul différentiel et de calcul intégral*.

2.6 Scientific policy

As shown by the letter of December 5th, 1883, summarized earlier, battles for a position or recruitment to a scientific society occupy a rather prominent place in the letters. On December 28, 1880, Hermite explains that "[t]he geometry section [of the Academy] will have some difficulty in deciding between the two main candidates, M. Camille Jordan and M. Darboux who both have good credentials, but have unequal chances of success. The first is more favored by more of our colleagues, but I must confess that the second seems to me to have done more and better work, entirely free of the obscurity for which one can only too easily reproach M. Jordan's work on the theory of equations."[29] Or on February 24, 1885: "A great pitched battle was just fought at the Faculty around the choice of a substitute for the chair of analysis; it was a fight between the students of the Polytechnique and those of the École normale."[30]

Administrative duties are also commented on in general, at least from Hermite's side, who regularly complains or jokes about them. On October 5, 1889, he describes for instance the ceremony of the inauguration of the new Sorbonne which has taken place in August, "in pomp and circumstance, in front of the President of the Republic, several ministers, representatives of the main bodies of the State, eminent characters such as M. Pasteur, M. Duruy, M. Jules Simon, etc. and a thousand students from every part of the world, with their national costumes and the banners of their countries."[31]

2.7 Politics

There is thus just one step from scientific administration to general politics. Hermite, who despised the Third Republic, loses no occasion to express his proximity with Germany and the values he thinks incarnated by the Prussian state. In December 14, 1882, he declares for instance : "My intimate feeling, which is more an impression than a deduction, is that radicals like M. Paul Bert, M. Laisant, etc., are leading us to imminent and horrible catastrophes. I would go further, I believe that M. Wirchow and his party, who in your country want ministerial responsibility, are thus moving to the revolutionary side. There is no need, it seems to me, of this responsibility in order to resist M. von Bismarck as fully as necessary.[32]" Or

[29] Letter 19: *la section de géométrie aura fort à faire pour se prononcer entre deux candidats principaux Mr Camille Jordan et Mr Darboux qui tout deux ont bien des titres, mais avec des chances inégales de succès. C'est le premier qui est le plus en faveur auprès du plus grand nombre de nos confrères, mais je vous avoue que le second me semble avoir fait plus de travaux et des travaux meilleurs, entièrement exempts de l'obscurité qu'on n'a que trop à reprocher à ceux de Mr Jordan, sur la théorie des équations*. Jordan will nonetheless be elected in 1881, to replace Michel Chasles, who had died on December 18, 1880.

[30] Letter 63: *Une grande bataille rangée vient de se livrer à la Faculté pour le choix d'un suppléant à la chaire d'analyse ; c'était la lutte entre les Polytechniciens et les Normaliens*.

[31] Letter 23: *C'est le 8 Aout qu'a eu lieu la cérémonie de l'inauguration de la Sorbonne, en grand apparat, par devant le Président de la République, plusieurs ministres, des représentants des grands corps de l'Etat, d'éminents personnages comme Mr Pasteur, Mr Duruy, Mr Jules Simon, etc. etc., et un millier d'étudiants de toutes les parties du monde avec leurs costumes nationaux, et les bannières de leurs pays*.

[32] Letter 30: *Mon sentiment intime qui est plutôt une impression qu'une déduction, c'est que les radicaux tels que Mr Paul Bert, Mr Laisant, etc. nous conduisent à de prochaines et d'affreuses catastrophes. J'irai plus loin, je crois que Mr Wirchow et son parti qui veulent chez vous la responsabilité ministérielle vont ainsi du côté de la révolution. Point n'est besoin, ce me semble, de cette responsabilité, pour résister autant qu'il est nécessaire à Mr de Bismarck*. The physiologist Paul Bert was Minister of Education in 1881–1882 and an advocate of a secular and free school system (he was also in favor of colonization and of a republican racism).

again, on December 28, 1899, a year before his death: "At least M. Doumer reassured us with respect to Germany, with whom our relations are better, it is said, so that we should expect an alliance between the two nations, against England, rather than a new war. What a marvellous thing it would be to march into battle with our previous adversaries!"[33] ***On this aspect, the correspondence is not symmetrical: as far as we can see from the drafts, Lipschitz does not reciprocate and remains very circumspect on political issues***.

3 Echoes: The Correspondence as a non-closed corpus

3.1 The intricasies of privacy

Letters are both social and textual links. Scientific letters have sometimes been described as defining open networks of scientific communication, as opposed to those of closed institutions, academies or journals, which require membership or referee processes or entrance examinations, [29]. Sophie Germain, for instance, could not enter the Polytechnique, nor be a member of the Academy of Sciences, but she ***was at liberty to/could*** write to Gauss. As we have seen, the exchanges between Hermite and Lipschitz were born in, and supported by, a professional setting. ***But they were not strictly professional, they were initiated and reinforced by personal encounters, in Göttingen or Bonn***, and as such, their correspondence is far from open:[34] it relies on strong ties between the two participants, which at first sight appear to be both personal and scientific, or at least to develop in such a way.

The traditional classificatory dichotomies: private/public and personal /professional, are particularly called into question.[35] The exchanges, as we have seen, are situated at the margins of the professional world, crossing regularly but partially the frontier. The letters display a large spectrum of subjects, from strictly confidential matters, both professional and personal (for instance when comparative opinions on candidates for a position are requested or details on a family member are provided), to public matters in the most obvious sense, such as those letters published verbatim in mathematical journals. It can become domestic, or

The Polytechnician Charles-Ange Laisant was a mathematician who supported *Boulangisme* in the 1880s; he was later a cofounder of the journal *L'Enseignement mathématique*, see [2]. Both men sat on the extreme-left in Parliament. The pathologist Rudolf Virchow cofounded the radical Deutsche Fortschrittspartei, he defended the idea that ministers should be held responsible for state expenditures engaged without authorization of Parliament.

[33] Letter 157: *Au moins Monsieur Doumer nous a rassurés à l'égard de l'Allemagne, avec qui nos rapports sont meilleurs, dit-on, de sorte qu'on devrait plutôt croire à une alliance entre les deux nations, contre l'Angleterre, qu'à une nouvelle guerre. Quelle chose merveilleuse ce serait de marcher au combat avec nos anciens adversaires !*. Paul Doumer, a future president of the French Republic, was at the time Governor-General of French Indochina.

[34] The difference between a one-to-one exchange and a correspondence network involving several persons, such as that relative to the editing committee of a journal, is of course decisive. On this point, for another period, see [17]. This is true even if one focusses on the exchanges between two persons inside a more collective setting; a good exemple here are is exchange between Lipschitz and Darboux for the *Bulletin des sciences mathématiques*, [30, pp. 44–46].

[35] This classification has been extensively discussed for the nineteenth century, see for example [8, 35, 12, 10].

even intimate, as when Hermite vents his regrets or frustration on his own work. But this intimacy is clearly delimited: it does not involve sharing thoughts on their marriages, nor even comments on novels one of them may have read, or concerts they may have attended.[36] From the social point of view, then, this correspondence appears to be both closed and restricted in its content.

From a textual point of view, on the other hand, it is not a closed corpus. This may appear to be a trivial remark: many parts of the letters are of course not understandable to the modern reader without a recourse to external information (as illustrated in several footnotes of the preceding sections), information which would have been obvious to any nineteenth-century cultured person, that pertaining for instance to politics, or to any nineteenth-century mathematician, for instance on the current abbreviations for the titles of journals. Allusions to people, political events, recent mathematical results, are of this kind. They delineate tacit knowledge, operating at different scales, from what is shared by all contemporaries to what is shared by the two correspondents; and, in this sense, the corpus is not closed textually, all the more *because* it is closed socially.

But what I mean to say is different: contextualization through external sources is decisive for a proper understanding of the place of the correspondence in the work and life of Hermite and Lipschitz themselves, and of its role in the more general scientific communication network. To take an example, the election in the Academy of Sciences to replace Michel Chasles in 1881–1882 diffuses through a variety of writings: the opinion required from Lipschitz—on Darboux's and Jordan's respective merits—appears to be in fact a simple sidetrack of the main issue, that is to avoid at all costs the election of the engineer Amédée Mannheim, [24, I, pp. 99–100, 117–118]. Hermite is pushed by some of his colleagues to put Jordan (who for a variety of reasons is more likely than Darboux to be elected) alone in the first line in order to secure the votes against Mannheim. ***Although they do not mention Mannheim,*** the letters to Lipschitz express Hermite's resistance to this strategy *** and his attempts to circumvent it through international support***: this*** required external documents to be understood.

3.2 Publication echoes

The role of their correspondence in the work and life of each mathematician is indeed impossible to evaluate from within the correspondence itself. We have already seen how both expressed their appreciation in the letters. But external information—a comparison with their other correspondence— has to be used to state that this appreciation did not imply the same familiarity the two men may have with others. To decide further if the warm description of their relation inside their correspondence has meaning beyond the basic politeness of the time also requires confirmation from outside. ***In 1881, in an attempt to have Lipschitz elected at the Berlin Academy, Hermite writes to Kronecker that "among so many distinguished mathematicians [...], I value and love above all M. Lipschitz".[37]*** To Mittag-Leffler, he mentions how he "greatly treasures" Lipschitz's opinion, [24, I, p. 229], while Lipschitz, after his meeting with Hermite in Göttingen, confides to

[36] In 1898, however, Lipschitz sends folk and military songs to Hermite after an exchange on the memories of past wars, and Hermite evokes music listened to at Lipschitz's home during his visit, letter 155.

[37] Letter from September 30, 1881. A copy of these letters is kept in Hermite's file in the Archives of the French Academy of Sciences.

Richard Dedekind how "Hermite's personality especially inspires trust," and later that he has "developed an affection towards him,"[30, p. 87, p. 90]. In the 1890s, Mittag-Leffler refers to Lipschitz as one of the two German geometers (with Fuchs) who had the closest relations with Hermite, [30, I, p. 193].

To appreciate the scientific role of the correspondence, again we need external help. Hermite had published in German journals since the beginning of his career but did not mention Lipschitz before their meeting in Göttingen; among the 92 articles Hermite published after 1877—many of them letters to a variety of people—only 9 articles refer to Lipschitz (with 14 mentions of his name[38]), all published between 1877 and 1887: we find in particular two articles cosigned with him in *Acta* and a comment to a note by Lipschitz in the *Comptes rendus*. ****Hermite often discusses his own results in letters to Lipschitz, but personal communication with Lipschitz (supported by the evidence of the correspondence) is explicitely hinted at in 3 published papers only, while 3 refer to specific results of Lipschitz.****

The situation is not symmetrical. Let us look for instance at the list of Lipschitz's publications, provided in [30, pp. 235–244]. Fifty items are listed before 1877, the year the first volume of Lipschitz's treatise appears and the correspondence between Hermite and Lipschitz is launched. Among them, only four appear in French journals—one is a note in the *Comptes rendus* of the French Academy,[39] the three others appear in Darboux's *Bulletin des sciences mathématiques*. The *Bulletin* had published short reviews of Lipschitz's articles, as they did for all papers published in *Crelle's Journal* and in 1872, Darboux explicitly requests from Lipschitz a longer analysis of a series of his papers, [30, p.44-45]. Besides this, Lipschitz addressed a letter to the journal to complete and correct bibliographical references. The third paper in the *Bulletin* and the note to the Academy only summarize longer contributions published elsewhere.

After 1877, Lipschitz's mode of publication obviously changed. Among the 47 articles published after this date, 26 are published in French journals. Among them, 17 articles are notes in the *Comptes rendus*, including 14 extracts from letters to Hermite! Athough not explicitly indicated as letters, two others are communicated by Hermite (and discussed in the correspondence). Only 1 note on probability, and written quite late, in 1898, is communicated by Bertrand. Moreover, among the 9 papers in other French journals, 4 are explicit extracts from letters to Hermite, 2 can be traced to them, and 2 are only summaries or translations of work published in German elsewhere. To this can be added 2 letters to Hermite published outside France (one in *Acta mathematica*, 1 in *Crelle's Journal*) and the 2 articles in *Acta mathematica* cosigned with Hermite. To summarize, after his direct acquaintance with Hermite, Lipschitz published 27 articles (out of 47) which are directly connected with the correspondence*** and his recognition on the French scene has significantly improved***.

[38]Kronecker, on the other hand, is cited in 24 papers, Jacobi in more than 50. Lipschitz is one of 20 authors born between 1830 and 1850 and cited by Hermite after 1880. These data are established and discussed in [19].

[39]There is no indication of the member of the Academy who communicated it, which suggests that Lipschitz sent it directly to the Academy and its *secrétaire perpétuel* Bertrand. This was the course indicated to Lipschitz as being the "most natural" by Borchardt in December 1875, [30, p. 21].

3.3 ***Views on creation in proper perspective***

This need to contextualize items of the correspondence from outside sources is not restricted to mathematics per se. It also touches upon epistemic issues. We have quoted for instance Hermite's letter from April 12, 1882:

> I believe that in science and especially in mathematics, we are less masters than servants of our work. I do not deny free will, but I think that it coexists with the action of a force which, while it arises from us, acts outside us, and unwittingly directs us where we would not like to go.

The local context, in the letter itself, is that of an opposition between German ways of doing mathematics, presented as abstract, and French ones, presented as more concrete; specifically, it concerns Lipschitz's research on the movement of a body in a Riemannian space.[40]

> If I were your colleague and neighbor, I would hold you as hard as I could by the hem of your garment, so that you would not start down such a prodigiously abstract path, while so many questions present themselves, which are of immediate interest and, if I may say so, more concrete. Between us it would be the great combat of Germans and Latins; however, the fight would imply on my side a restriction which I will reveal to you. I believe that in science and especially in mathematics, we are less masters than servants of our work. I do not deny free will, but I think that it coexists with the action of a force which, while it arises from us, acts outside us, and unwittingly directs us where we would not like to go. [...] Something pushes you forward perhaps, something above you and me; I recognize that the conception of Riemann spaces frightens me, but that it has its utility.

The main issue here seems to be the status of Riemannian geometry. ***Although Hermite was a main actor in the importation of Riemannian ideas into France (organizing in particular the publication in French of Riemann's works), he was also a defender of a down-to-earth analysis against any ad-hoc ontologization, in particular geometrical one, and thus expressed several times doubts with respect to certain interpretations or uses of Riemann's results, [18]. Here, as was the case for Cantor's ideas, his reluctance is framed in a national setting. The opposition, thus the main theme, is just tempered by the possible intervention of an external force which may lead mathematics, and almost inwillingly mathematicians, in this new, apparently abstract, way***. The occurrence of the "masters vs servant" theme, on the other hand, remains isolated in Hermite's correspondence with Lipschitz.

[40] Letter 27: *Si j'étais votre collègue et votre voisin, je vous retiendrais autant que je pourrais, par un pan de votre habit, pour ne pas vous engager dans une telle voie si prodigieusement abstraite, lorsque tant de questions s'offrent qui sont d'un intérêt immédiat et plus tangible, si je puis dire. Entre nous ce serait le grand combat des germains et des latins ; cependant la lutte impliquerait de ma part une réserve que je vais vous dire. Je crois que nous sommes dans les sciences et tout particulièrement dans les mathématiques, moins les maîtres que les serviteurs de notre œuvre. Je ne nie point le libre arbitre, mais que je pense qu'il coexiste avec l'action d'une force qui naissant par notre fait, agit en dehors de nous, et à notre insu, nous dirige là même où nous ne voudrions pas aller ... Quelque chose vous pousse peut-être qui est au dessus de vous et de moi ; je reconnais à la fois que la conception des espaces de Riemann m'effraye, et qu'elle a sa raison d'être.*

However, taking into account other known correspondence involving Hermite offers a different picture. In March 1876, Hermite writes to Leo Königsberger :"I reject as totally wrong the idea that mathematicians are the creators of their science," [18, pp. 156–157]. On February 19, 1880, to Genocchi, this time, he adds: " I reject as totally wrong the idea that mathematicians are the creators of their science. Mathematicians seem to me as much servants as masters of their science," [25, p. 25]. The expression serves once more in a well-known letter to Du Bois-Reymond, on March 24, 1882, a few weeks before that to Lipschitz: "in mathematics which seems the fruit of the most complete intellectual freedom, we are nonetheless more servants than masters," [22]. Or to Mittag-Leffler in 1885: "in the development of mathematics, we are servants, much more than masters," [24, II, p. 100]. Each time, the direct, mathematical, context of the respective letters is different: a discussion on the role of computations in mathematics, some thoughts about good topics for academic prizes, Cantor's set theory, uniform functions arising from the study of second-order differential equations. The repetition of the theme and its variants, however, and of the words themselves, shows that it is much more than a passing remark; it points to a central conviction in Hermite's view of mathematical creation and development.[41] Echoes, from outside the Hermite-Lipschitz correspondence, are here the warrant of the meaningfulness—and finally of the meaning—of the sequence.

3.4 Mathematical links

The correspondence between Hermite and Lipschitz was the main locus of mathematical collaboration between the two mathematicians, and, as such, offers glimpses of the genesis of several articles of each author. To take a simple example, the second volume of *Acta mathematica*, published in 1883, contains two direct extracts of the correspondence (see letter 35 of Mai 12, 1883 and draft 37* of June 6, 1883) combined as a single contribution, "Sur quelques points dans la théorie des nombres, par Ch. Hermite et R. Lipschitz." ***Their point of departure is Dirichlet's memoir on mean values of artihmetical functions, [13], that they "both know and admire," in Hermite's terms. In this memoir, Dirichlet evaluates in particular an asymptotic approximation of $F(n) = \sum_{i=1}^{n} \phi(i)$, where $\phi(i)$ designates the number of divisors of the integer i. In his contribution, Hermite uses in order to prove the approximation a new expression of $F(n)$,

$$F(n) = 2\sum_{i=1}^{E}(\sqrt{n})E(\frac{n}{i}) - [E(\sqrt{n})]^2,$$

$E(x)$ being here the integral part of x (the largest integer less than or equal to x). Lipschitz's answer generalizes Hermite's formula to the arithmetical functions $F_s(n) = \sum_{i=1}^{n} \phi_s(i)$, where $\phi_s(i)$ is the number of divisors of the integer i which are also s-th powers (thus $s = 1$ is Hermite's case).

The exchange, thus, stimulates research and the link between the correspondence and the outside world, here the publications, is obvious.***

But another kind of external links is illustrated in ***the letter of January 31, 1881 (letter 20) already mentioned***. "It may interest you," writes Hermite, " to know that one of my students, Mr Tannery, has discovered a series which is much

[41] On this viewpoint and further contextualization with respect to his contemporaries' positions, see [18].

FormuleWeierstrass2.pdf

FormuleWeierstrass1.pdf

simpler than that expressed by Mr Weierstrass as :

$$\chi(x) = \frac{2x}{\pi}\Psi(1,1,xi) + \frac{2}{\pi x}\Psi(1,1,\frac{i}{x})$$
$$= \frac{2}{\pi}(x+x^{-1}) + \frac{2}{\pi}\sum[\frac{x}{(1-2\nu-2\nu' ix)(2\nu+2\nu' x)^2}$$
$$+ \frac{x^{-1}}{(1-2\nu-2\nu' ix^{-1})(2\nu+2\nu' x^{-1})^2}]$$

and has the same type of discontinuity. This is the following:

$$\frac{1+x^2}{1-x^2} + \frac{2x^2}{x^4-1} + \frac{2x^4}{x^8-1} + \frac{2x^8}{x^{16}-1} + \ldots$$

It has the value +1 or -1 according as the modulus of the variable is smaller or greater than unity."

The issue here is well-known [26, pp. 265–266]: Weierstrass had constructed by means of elliptic functions a series which converges to different analytic functions on different domains. ***If we only take into account the publications,*** we see that the series appears in a communication by Weierstrass to the Berlin Academy of Sciences, on August 12, 1880, [37, p. 735]. Its form and its notation are exactly those given by Hermite***. On February 21, 1881, a second communication reproduces a letter from Jules Tannery to Weierstrass, announcing the possibility of simplifying Weierstrass's example. The two communications, one following the other, are translated by Tannery himself into French for the April 1881 issue of the *Bulletin des sciences mathématiques* (pp. 157–181, and 181–183).

But ***instructive data on the sequence of events, in particular a closer datation, are offered by correspondence***: on November 27, 1880, Darboux asks Weierstrass for permission to publish a French translation of his 1880 communications to the Berlin Academy, [9, II, p.51] ; Tannery is in charge of it, [24, I, p. 100]. On December 24, in a sequence of letters which involves Weierstrass's, Mittag-Leffler's and Hermite's intertwined results on analytic functions, and their various publications and translations, Hermite comments to Mittag-Leffler : "What marvelous things are these discontinuous series of M. Weierstrass, which represent absolutely different functions in separate domains," [24, I, p. 87]. ***As explained above, Hermite's youngest daughter, Marie, marries Picard at the very beginning of January 1881 and on January 22, Hermite explains to Genocchi that he did not work much lately because of that event, but is now studying several articles by Weierstrass, in particular that of August 1880, in order to include them in his Sorbonne's lectures, [**?**, p. 27]. There is still no mention of Tannery's simpler example***. A letter[42] from Hermite to Tannery, while congratulating him on his result, suggests that he inform Weierstrass directly about it, and indeed, on February 8, 1881, Tannery thanks Weierstrass for the latter's interest in his "little remark," and his permission to translate the relevant articles, [9, II, p. 54]. As we have seen, Hermite writes to Lipschitz about Tannery's example as early as December 31. But it is not until February 13 that he communicates it to Mittag-Leffler, a result which, he "cannot doubt, will please [him] too," [24, I, p. 102]. *** Schwarz's

[42]***Dossier Hermite, Archives of the French Academy of Science, Paris. The letter is not dated, but the chronology we have reconstructed here suggests that it was probably written in late January 1831.

Nachlass also contains a letter from Weierstrass, dated March 6 (thus after the communication of Tannery's letter to the Berlin Academy) stating that "the editor of the Darboux Bulletin, J. Tannery, has communicated to [him] recently a very interesting remark" that his series can be replaced by a much more elementary one, [9, III, p. 76].

This episode displays the intricate role of correspondence in the communication of mathematics. If the main figure, Weierstrass, receives Tannery's result with priority/first and foremost***, it is noticeable that Hermite immediately dispatches the news to his own favorite correspondents: Lipschitz receives it even before Mittag-Leffler, while the latter was directly involved in the matter, both mathematically and as one of the mediators in the translation process ***. Even before publication, the business of French translations favors the transfer of knowledge through exchanges organizing them***, their corrections, their development. Mittag-Leffler makes no mistake; on February 19, 1881, he writes to Hermite just after receiving the announcement of Tannery's example : "M. Tannery's series interested me a lot. It is admirably simple and one sees the proof immediately. The translation of M. Weierstrass's memoirs is apparently not unfruitful, it seems, for the French mathematicians."[43]We come full circle with Tannery's own presentation of his result, [44].

> While translating for the *Bulletin* Weierstrass's communications to the Berlin Academy of Sciences "Zur Functionenlehre" (August 1880) [...] I noticed that one could replace by a simpler series the series Weierstrass had constructed [...]. The origin of the series I allowed me to communicate to Weierstrass is to be found in a problem [...] which was probably suggested to me while I studied the proof of the fundamental theorem of algebra that M. Lipschitz gave in his *Lehrbuch der Analysis*.

Published articles are thus only the tip of the iceberg, integrating parts of letters or integrated into them. Reciprocally, reconstructing this web of texts highlights the value of the correspondence as a place of fast diffusion between French and German mathematics.

4 Editorial Issues

The correspondence between Lipschitz and Hermite is an important platform for the observation of mathematical milieux in the second half of the nineteenth century, in particular for the renewal of the French-German relations after the 1870

[43]Letters from Mittag-Leffler to Hermite, File 53J, Archives of the French Academy of Sciences , Paris: *La série de M. Tannery m?a extrêmement intéressé. Elle est admirablement simple et on voit tout de suite la démonstration La traduction des mémoires de M. Weierstrass n?est pas sans fruit, il parait pour les géomètres français.*

[44][36, p. 19]: *En traduisant pour le* Bulletin *les communications "Zur Functionenlehre" de Weierstrass à l?Académie des sciences de Berlin (août 1880), [...] je remarquai qu?on pouvait remplacer, par une série plus simple, une série construite par Weierstrass [...]. L?origine de la série que je crus devoir communiquer à Weierstrass se trouve dans un problème [...] qui m?a été probablement suggéré en étudiant la démonstration du théorème fondamental de l?algèbre que M. Lipschitz a donnée dans son* Lehrbuch der Analysis

ThamousHermite1883.pdf	NavigationHermite1883.pdf

Figure 4: ***Thamous database: card of an article by Hermite (extracted from a letter to Lipschitz), and card of links from and to this article***.

war.[45] As explained above, it plays a decisive role in Lipschitz's manner of publishing. It is thus only natural to raise the issue of a complete edition, and more specifically to discuss*** how to properly take into account the echoes of which some instances have been explored above.

Letters have traditionally been edited as texts, organized chronologically (or by sender and addressee). Footnotes or endnotes are then generally*** used to explain the various allusions made in the letters.[46]This access to the whole text of the letters, without too many disturbing interventions of the editor, is of course essential.

However, as shown above, each letter can and should be envisioned as composed of a number of units of meaning: in order to take into account the richness of the correspondence, both as an informational device and as linking device, we should be able to connect each of these units to others, internal or external. Examples include parts of letters within parts of articles in certain journals as well as identical formulas appearing in different correspondence. Recently, electronic editions have envisioned the presentation of correspondence as a communication network, where each letter is a link between the sender and the addressee. Appropriate search functions would thus allow one to easily locate places where the letter was written or received, dates, sometimes persons or matters discussed in each letter; graphical representations could visualize such information.[47]

[45]On this issue, see [1].

[46]The difficulties already involved in this simple display*** are well-known for ancient texts and literary manuscripts, but are no less important in contemporary scientific texts. For instance, ***TEI-encoding offers interesting features for Lipschitz's drafts, as it permits a presentation of erasures, loose additions between lines and even certain types of links***, but its complete compatibility with LaTeX is still a delicate issue, see `http://www.tei-c.org/index.xml`***.

[47]Illuminating examples are the Van Gogh online correspondence, `http://vangoghletters.org/vg/`, the edition of D'Alembert's letters, `http://dalembert.academie-sciences.fr/Correspondance/` and Early Modern Letters On Line, `emlo.bodleian.ox.ac.uk/`. See also `http://www.newtonproject.sussex.ac.uk/view/texts/normalized/NATP00225:theproblemofmathematics`.

But relevant links are not only those connecting a letter to its writer or to the institution it is sent to. Even when they are materialized by chains of characters associated to such entities, the association can be variegated: a proper noun can appear as an addressee, a mathematical author quoted in the letter, a candidate for some position, an enemy. Or let us consider, for instance, the chain of characters "French Academy of Sciences." It is first a multiple link between Hermite and Lipschitz inside the correspondence: Hermite reports about his efforts to have Lipschitz elected as a corresponding member, he also presents notes of Lipschitz (often extracts of letters) to be inserted in the proceedings of the Academy. But it also constitutes a link between Hermite and Bischoffsheim (Hermite trying to prevent the election of Bischoffsheim), with Lipschitz as a depository and witness of this link. And of course, internal references are not sufficient: as explained forTannery's example, the echoes of important issues should be traced in the correspondence of others or in the publications of the Academy or in the minutes of its meetings. Mathematical journals, political events, but also mathematical formulas, for instance (see 3) constitute other possible links to be taken into account. Such links operate at a variety of levels and thus should themselves be capable of indexation, commentaries and labeling, in order to catch the multiplicity of operational aspects they encapsulate. An inspiring example of such commented and labeled links is provided by the collective database Thamous, initially constructed by Alain Herreman, see the illustration (4).

To summarize, we need a platform* which (i) is *open*, in order to be able to add in real time new documents, for instance newly discovered letters, or new links ; (ii) provides *selective displays*, to be able at will to only read the transcription of the letters or to have access exactly to those linked to a specific concept or reference ; (iii) is *homoiconic*, in the sense that, as is the case in Thamous for instance, links should be treated as data as well as the texts of the letters, with commentaries and links possibly added to them.

The identification of mathematical activities, through a variety of documents, and the efficient sharing of their multiple echoes to understand effective transfer procedures and concrete knowledge dynamics is perhaps the next challenge for the history of mathematics. A key step is the development of appropriate tools which would allow us to take into account the current reflection on sources and their uses.

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