

THREE DIMENSIONAL CONTOUR INTEGRAL GENERALIZATIONS: ANALYTICAL FORMULATION

S El Kabir, Rostand Moutou Pitti, F. Dubois, N Recho, Y. Lapusta

► To cite this version:

S El Kabir, Rostand Moutou Pitti, F. Dubois, N Recho, Y. Lapusta. THREE DIMENSIONAL CONTOUR INTEGRAL GENERALIZATIONS: ANALYTICAL FORMULATION. 14th International Conference on Fracture (ICF 14), Jun 2017, Rhodes, Greece. hal-01616970

HAL Id: hal-01616970 https://hal.science/hal-01616970

Submitted on 15 Oct 2017

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

THREE DIMENSIONAL CONTOUR INTEGRAL GENERALIZATIONS: ANALYTICAL FORMULATION

S. El Kabir¹, R. Moutou Pitti, F. Dubois, N. Recho and Y. Lapusta

GEMH Laboratory, Limoges University, Civil Enginering Center, 19300, Egletons, France Université Blaise Pascal, Institut Pascal, PB 10448, 63000, Clermont-Ferrand, France CNRS, UMR 6602, Institut Pascal, 63171, Aubière, France French Institute of Advanced Mechanics, Université Clermont Auvergne, Institut Pascal UMR 6602 /UBP /CNRS / IFMA, PB 265, 63175, Aubière, France

Abstract: This paper deals with the numerical development of the J-integral concept for three dimensional problems. A new integral parameter in real three-dimensional case, which computes the energy release rate combining an arbitrary crack front, is developed. The Independence of path integral is verified in quasi-static condition. The generalization of the J^{3D} -integral toward its G_{θ}^{3D} implementation form is realised.

1. Introduction

The study of the stress field close to the crack front in three-dimensional environment is more complicated than in two dimensional one. The aim of this paper is focused on the generalization of the J-integral formalism for a 3D problem and the adaptability of the G-theta method for a future finite element implementation. We are considering a 3D problem including any planar crack shape under any external loading orientations. In this paper we define a specific J-integral, Called J^{3D} -integral, adapted for 3D problems. It can be employed in the determination of the average energy release rate calculated along the crack front line.

2. Results

In two dimensional case, the well known method is the development of the J-integral proposed by Rice [1]. The generalization of the J-integral formalism for a three dimensional problem can be described as (Figure 1):



Figure 1. Description of a crack in a two dimensional case (a) and in a three dimensional case (b)

¹ Corresponding author

E-mail address: soliman.el-kabir@unilim.fr (S. El kabir)

By considering an infinitesimal variation dW, the Nother's theorem assumes a nonvariation of the Lagrangian. Following the same procedure described by Dubois et al [2], and by introducing a Gauss-Ostrogradski transformation, the Lagrangian's invariance becomes:

$$\int_{W} \left(W. n_1 - \left(\sigma_{ij}. n_j. u_{i,1} \right) \right) . dS - \int_{V} \left(W_{,1} - \sigma_{ij}. \left(\varepsilon_{ij} \right)_{,1} \right) . dV = 0$$
(1)

Thanks to the use of the generalized Noether theorem and after a set of mathematical developments the J^{3D} -integral can be defined as bellow:

$$J^{3D} = \int_{S_{\Gamma_{out}}} \left(W.n_{1} - (\sigma_{ij}.n_{j}.u_{i,1}) \right) . dS - \int_{S_{cr}} \sigma_{ij}.u_{i,1}.n_{j}. dS - \int_{V_{\Gamma_{out}}} \left(W_{,1} - \sigma_{ij}.(\varepsilon_{ij})_{,1} \right) . dV$$
(2)

So, the J^{3D} -integral can be used to determine of an average energy release rate \tilde{G} along the crack front line (cfl) such as:

$$\tilde{G} = \frac{J^{3D}}{\int_{cfl} dl} \tag{3}$$

Three-dimensional medium, presents difficulties to be implemented within finite element method [2]. That fore, a generalization of the J^{3D} -integral toward its G_{θ}^{3D} form is done on the basis of a Gauss- Ostrogradski transformation in order to allow this implementation. One can obtain:

$$G_{\theta}^{3D} = \int_{V} P_{j,k} \cdot \theta_{k,j} \cdot dS - \int_{S_{cr}} \sigma_{ij} \cdot u_{i,k} \cdot n_j \cdot \theta_k \cdot dS - \int_{V_{\Gamma_{out}}} \left(W_{,k} - \sigma_{ij} \cdot \left(\varepsilon_{ij} \right)_{,k} \right) \cdot \theta_k \cdot dV$$

$$(4)$$

The Gauss-Ostrogradski transformation assumes that the vector field $\vec{\theta}$ is derivable.

3. Conclusions

This paper deals with a new formulation of the J-integral to study the fracture behavior three dimensional medium. Applying the theta method, a G_{θ}^{3D} integral is developed. In future work, it will be necessary to extend the J integral to a mixed mode loading case for three dimensional problems allowing a mode separation process for mixed mode loadings by integrating virtual fields deduced by the singular form of the stress and strain tensors in the crack front line vicinity [3].

Acknowledgements

The authors wish to strongly acknowledge the National Agency of Research (ANR) for its financial support of this work through the project CLIMBOIS N° ANR-13-JS09-0003-01 labeled by ViaMeca.

References

- [1] J. R. Rice, A path independent integral and the approximate analysis of strain concentrations by notches and cracks, *Journal of Applied Mechanics* 1968;35:379-386.
- [2] F. Dubois, C. Petit, Modelling of the crack growth initiation in viscoelastic media by the $G\theta_{v}$ -integral, *Engineering Fracture Mechanics* 2005;72: 2821-2836.
- [3] R. Moutou Pitti, F. Dubois, C. Petit, N. Sauvat, and O. Pop, A new integral parameter for mixedmode crack growth in viscoelastic orthotropic media, *Engineering Fracture Mechanics* 2008;75:4450-4465.