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## THREE DIMENSIONAL CONTOUR INTEGRAL GENERALIZATIONS: ANALYTICAL FORMULATION

S. El Kabir<sup>1</sup>, R. Moutou Pitti, F. Dubois, N. Recho and Y. Lapusta

*GEMH Laboratory, Limoges University, Civil Engineering Center, 19300, Egletons, France*

*Université Blaise Pascal, Institut Pascal, PB 10448, 63000, Clermont-Ferrand, France*

*CNRS, UMR 6602, Institut Pascal, 63171, Aubière, France*

*French Institute of Advanced Mechanics, Université Clermont Auvergne, Institut Pascal UMR 6602 /UBP /CNRS /*

*IFMA, PB 265, 63175, Aubière, France*

**Abstract:** This paper deals with the numerical development of the J-integral concept for three dimensional problems. A new integral parameter in real three-dimensional case, which computes the energy release rate combining an arbitrary crack front, is developed. The Independence of path integral is verified in quasi-static condition. The generalization of the  $J^{3D}$ -integral toward its  $G_{\theta}^{3D}$  implementation form is realised.

### 1. Introduction

The study of the stress field close to the crack front in three-dimensional environment is more complicated than in two dimensional one. The aim of this paper is focused on the generalization of the J-integral formalism for a 3D problem and the adaptability of the G-theta method for a future finite element implementation. We are considering a 3D problem including any planar crack shape under any external loading orientations. In this paper we define a specific J-integral, Called  $J^{3D}$ -integral, adapted for 3D problems. It can be employed in the determination of the average energy release rate calculated along the crack front line.

### 2. Results

In two dimensional case, the well known method is the development of the J-integral proposed by Rice [1]. The generalization of the J-integral formalism for a three dimensional problem can be described as (Figure 1):

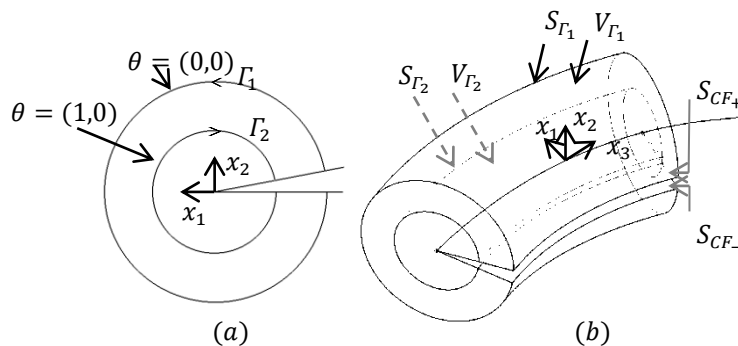


Figure 1. Description of a crack in a two dimensional case (a) and in a three dimensional case (b)

<sup>1</sup> Corresponding author

E-mail address: [soliman.el-kabir@unilim.fr](mailto:soliman.el-kabir@unilim.fr) (S. El kabir)

By considering an infinitesimal variation  $dW$ , the Nother's theorem assumes a non-variation of the Lagrangian. Following the same procedure described by Dubois et al [2], and by introducing a Gauss-Ostrogradski transformation, the Lagrangian's invariance becomes:

$$\int_{\partial V} \left( W \cdot n_1 - (\sigma_{ij} \cdot n_j \cdot u_{i,1}) \right) \cdot dS - \int_V \left( W_{,1} - \sigma_{ij} \cdot (\varepsilon_{ij})_{,1} \right) \cdot dV = 0 \quad (1)$$

Thanks to the use of the generalized Noether theorem and after a set of mathematical developments the  $J^{3D}$ -integral can be defined as bellow:

$$J^{3D} = \int_{S_{\Gamma_{out}}} \left( W \cdot n_1 - (\sigma_{ij} \cdot n_j \cdot u_{i,1}) \right) \cdot dS - \int_{S_{cr}} \sigma_{ij} \cdot u_{i,1} \cdot n_j \cdot dS - \int_{V_{\Gamma_{out}}} \left( W_{,1} - \sigma_{ij} \cdot (\varepsilon_{ij})_{,1} \right) \cdot dV \quad (2)$$

So, the  $J^{3D}$ -integral can be used to determine of an average energy release rate  $\tilde{G}$  along the crack front line ( $cfl$ ) such as:

$$\tilde{G} = \frac{J^{3D}}{\int_{cfl} dl} \quad (3)$$

Three-dimensional medium, presents difficulties to be implemented within finite element method [2]. That fore, a generalization of the  $J^{3D}$ -integral toward its  $G_{\theta}^{3D}$  form is done on the basis of a Gauss- Ostrogradski transformation in order to allow this implementation. One can obtain:

$$G_{\theta}^{3D} = \int_V P_{j,k} \cdot \theta_{k,j} \cdot dS - \int_{S_{cr}} \sigma_{ij} \cdot u_{i,k} \cdot n_j \cdot \theta_k \cdot dS - \int_{V_{\Gamma_{out}}} \left( W_{,k} - \sigma_{ij} \cdot (\varepsilon_{ij})_{,k} \right) \cdot \theta_k \cdot dV \quad (4)$$

The Gauss-Ostrogradski transformation assumes that the vector field  $\vec{\theta}$  is derivable.

### 3. Conclusions

This paper deals with a new formulation of the J-integral to study the fracture behavior three dimensional medium. Applying the theta method, a  $G_{\theta}^{3D}$  integral is developed. In future work, it will be necessary to extend the J integral to a mixed mode loading case for three dimensional problems allowing a mode separation process for mixed mode loadings by integrating virtual fields deduced by the singular form of the stress and strain tensors in the crack front line vicinity [3].

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