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To cite this version:
S El Kabir, Rostand Moutou Pitti, F. Dubois, N Recho, Y. Lapusta. THREE DIMENSIONAL CONTOUR INTEGRAL GENERALIZATIONS: ANALYTICAL FORMULATION. 14th International Conference on Fracture (ICF 14), Jun 2017, Rhodes, Greece. hal-01616970

HAL Id: hal-01616970
https://hal.science/hal-01616970
Submitted on 15 Oct 2017
THREE DIMENSIONAL CONTOUR INTEGRAL GENERALIZATIONS: 
ANALYTICAL FORMULATION

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Abstract: This paper deals with the numerical development of the J-integral concept for three dimensional problems. A new integral parameter in real three-dimensional case, which computes the energy release rate combining an arbitrary crack front, is developed. The Independence of path integral is verified in quasi-static condition. The generalization of the J-integral toward its $G_{th}$ implementation form is realised.

1. Introduction
The study of the stress field close to the crack front in three-dimensional environment is more complicated than in two dimensional one. The aim of this paper is focused on the generalization of the J-integral formalism for a 3D problem and the adaptability of the G-theta method for a future finite element implementation. We are considering a 3D problem including any planar crack shape under any external loading orientations. In this paper we define a specific J-integral, Called $J^{3D}$-integral, adapted for 3D problems. It can be employed in the determination of the average energy release rate calculated along the crack front line.

2. Results
In two dimensional case, the well known method is the development of the J-integral proposed by Rice [1]. The generalization of the J-integral formalism for a three dimensional problem can be described as (Figure 1):

![Figure 1. Description of a crack in a two dimensional case (a) and in a three dimensional case (b)](image)

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By considering an infinitesimal variation \( dW \), the Noether's theorem assumes a non-variation of the Lagrangian. Following the same procedure described by Dubois et al [2], and by introducing a Gauss-Ostrogradski transformation, the Lagrangian's invariance becomes:

\[
\int \left( \frac{\partial}{\partial y} \left( W, n_1 - (\sigma_{ij}, n_j, u_{i,1}) \right) \right) \cdot dS - \int \left( \frac{\partial}{\partial y} \left( W, \sigma_{ij}, (\varepsilon_{ij})_A \right) \right) \cdot dV = 0 \tag{1}
\]

Thanks to the use of the generalized Noether theorem and after a set of mathematical developments the \( J^{3D} \)-integral can be defined as bellow:

\[
J^{3D} = \int_{S_{\text{out}}} \left( W, n_1 - (\sigma_{ij}, n_j, u_{i,1}) \right) \cdot dS - \int_{S_{\text{cr}}} \sigma_{ij} \cdot u_{i,1} \cdot n_j \cdot dS - \int_{V_{\text{out}}} \left( W, \sigma_{ij}, (\varepsilon_{ij})_A \right) \cdot dV \tag{2}
\]

So, the \( J^{3D} \)-integral can be used to determine of an average energy release rate \( \bar{G} \) along the crack front line (cfl) such as:

\[
\bar{G} = \frac{J^{3D}}{\int_{\text{cfl}} dl} \tag{3}
\]

Three-dimensional medium, presents difficulties to be implemented within finite element method [2]. That fore, a generalization of the \( J^{3D} \)-integral toward its \( G_{\theta}^{3D} \) form is done on the basis of a Gauss-Ostrogradski transformation in order to allow this implementation. One can obtain:

\[
G_{\theta}^{3D} = \int_{V} P_{j,k} \cdot \theta_{k,j} \cdot dS - \int_{S_{\text{cr}}} \sigma_{ij} \cdot u_{i,\theta_{k}} \cdot n_j \cdot \theta_k \cdot dS - \int_{V_{\text{out}}} \left( W, \theta_k - \sigma_{ij}, (\varepsilon_{ij})_k \right) \cdot \theta \cdot dV \tag{4}
\]

The Gauss-Ostrogradski transformation assumes that the vector field \( \theta \) is derivable.

### 3. Conclusions

This paper deals with a new formulation of the J-integral to study the fracture behavior three dimensional medium. Applying the theta method, a \( G_{\theta}^{3D} \) integral is developed. In future work, it will be necessary to extend the J integral to a mixed mode loading case for three dimensional problems allowing a mode separation process for mixed mode loadings by integrating virtual fields deduced by the singular form of the stress and strain tensors in the crack front line vicinity [3].

### Acknowledgements

The authors wish to strongly acknowledge the National Agency of Research (ANR) for its financial support of this work through the project CLIMBOIS N° ANR-13-JS09-0003-01 labeled by ViaMeca.

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