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## PROBABILISTIC ANALYSIS OF MIXED MODE CRACK GROWTH IN WOOD USING DIMENSION REDUCTION METHOD

H. Riahi<sup>1</sup> and R. Moutou-Pitti

*LARIS-ISTIA, Université d'Angers, 62 avenue notre Dame du Lac, 49000 Angers, France*

**Abstract:** An efficient probabilistic approach is developed to assess the reliability of wood structures subjected to crack growth. The crack driving forces, such as the energy release rate and the stress intensity factors are computed through a new formulation of independent path integral, namely M-integral, which allows taking into account mechanical loads on the crack edges in mixed mode-mode ratio. The statistical moments of the crack driving forces are obtained by dimension reduction integration technique. In addition, the probability density function is constructed by performing Monte-Carlo simulations on a probabilistic surface responses obtained by projection on Lagrange polynomial basis. Numerical results show that the effect of uncertainty associated to the mechanical properties of wood on the variability of the fracture parameters is significant since the coefficient of variation exceeds 10%.

### 1. Crack driving forces computation

Independent path integral techniques, such as M-integral adapted to anisotropic material [1], seem to be a good tool to compute the fracture parameters. Let us consider a cracked orthotropic body  $\Omega$  subjected to mechanical loads, and  $\vec{\theta}$  a continuous derivable vector field defined around the crack tip, the M-integral reads:

$$M = \int_{\Omega} \frac{1}{2} [\sigma_{ij,k}^v u_i - \sigma_{ij}^u v_{i,k}] \theta_{k,j} ds + \int_{\Gamma} \frac{1}{2} F_i v_{i,j} \theta_j dx_1 \quad (1)$$

The first term is the generalized definition of the M-integral which represents the effect of mechanical loads applied far away from the crack tip, where  $\sigma_{ij}^v$  and  $\sigma_{ij}^u$  are stress components associated to the auxiliary displacement field  $v$  and the real displacement field  $u$ , respectively. The second term represents the effect of mechanical load  $F_i$  applied on the crack edges  $\Gamma$ . The real stress intensity factors  $K_I^u$  and  $K_{II}^u$ , associated to the opening and in plan shear fracture modes respectively, can be easily derived by performing two distinct computations for particular values of the auxiliary stress intensity factors  $K_I^v$  and  $K_{II}^v$ , such as:

$$K_I^u = \frac{8M(K_I^v = 1, K_{II}^v = 0)}{C_1}, \quad K_{II}^u = \frac{8M(K_I^v = 0, K_{II}^v = 1)}{C_2} \quad (2)$$

where  $C_1$  and  $C_2$  are the elastic compliances associated to the opening and in plan shear fracture modes, respectively.

### 2. Uncertainty propagation through an implicit mechanical model

Let us consider a mechanical model  $f$  having  $N$  uncertain parameters gathered in the  $N$ -dimensional random variable  $\mathbf{x} = \{x_1, \dots, x_N\}^T$  which is fully described by its joint probability density function  $p_{\mathbf{x}}$ . By definition, the  $l^{\text{th}}$  statistical moment of the scalar random variable  $y = f(\mathbf{x})$  associated to the mechanical model response, can be written as:

$$m_y^l = \mathbb{E}[y^l(\mathbf{x})] = \int_{\mathbb{R}^N} y^l p_{\mathbf{x}}(\mathbf{x}) d\mathbf{x} \quad (3)$$

For engineering applications, the computation of the above  $N$ -dimensional is not a trivial task, since the number of uncertain parameters can be very large and the mechanical models are often time consuming. To overcome this problem, the idea is to use the dimension decomposition method (DDM) [2] to reduce the integration dimension. In other words, the  $N$ -

<sup>1</sup> Corresponding author

E-mail address: hassan.riahi@univ-angers.fr (H. Riahi)

dimensional integral (3) can be substituted by a set of elementary integrals of lower order. According to the DDM, the  $s$ -dimensional approximation  $\tilde{y}_s$  of the mechanical response  $y$  is:

$$\tilde{y}_s = \sum_{i=0}^s (-1)^i C_{N-s+i-1}^i \sum_{k_1=1}^{N-s+i+1} \cdots \sum_{k_{s-i}=k_{s-i-1}+1}^N f_{k_1, \dots, k_{s-i}}(\tilde{\mathbf{x}}) \quad (4)$$

where  $f_{k_1, \dots, k_{s-i}}$  are  $(s-i)$ -dimensional functions representing the interaction between the uncertain parameters  $x_{k_1}, \dots, x_{k_{s-i}}$  gathered in the vector  $\tilde{\mathbf{x}}$ . Now, by replacing  $y$  in (3) by its approximation (4), the  $l^{\text{th}}$  statistical moment  $m_y^l$  is rewritten as:

$$m_y^l \cong \sum_{i=0}^s (-1)^i C_{N-s+i-1}^i \sum_{k_1=1}^{N-s+i+1} \cdots \sum_{k_{s-i}=k_{s-i-1}+1}^N \int_{\mathbb{R}^{s-i}} f_{k_1, \dots, k_{s-i}}^l(\tilde{\mathbf{x}}) p_{\tilde{\mathbf{x}}}(\tilde{\mathbf{x}}) d\tilde{\mathbf{x}} \quad (5)$$

It is clear from (5), that the estimation of  $m_y^l$ , requires only evaluation of a set of elementary integrals having at most a dimension  $(s-i) \ll N$ , which can be computed efficiently using classical numerical integration techniques, such as cubature rules.

### 3. Results and discussions

Numerical analysis is conducted on a modified notched three-point bending specimen ( $54 \times 5 \times 1 \text{ cm}$ ), made on pin spruce having the following mechanical properties: longitudinal modulus  $E_1 = 1500 \text{ MPa}$ , transverse modulus  $E_2 = 600 \text{ MPa}$ , shear modulus  $G_{12} = 700 \text{ MPa}$ , and Poisson's coefficient  $\nu_{12} = 0.5$ . Due to the shift between the crack plane and the loading direction, mixed mode crack growth will occur. The stress intensity factors  $K_I$  and  $K_{II}$  associated to the opening and shear fracture modes respectively, are obtained from a finite element model based on the M-integral formulation presented above. Then, probabilistic analysis, based on dimension reduction method ( $s = 2$ ), is performed to assess the effect of uncertainty of mechanical properties ( $E_1, E_2, G_{12}$ ), load amplitude  $P$  and specimen thickness  $t$ , on the fracture parameters. As can be seen from figures (a) and (b), the effect of the uncertainty of the input parameters is significant, since the coefficient of variation of  $K_I$  and  $K_{II}$ , are about 9.8% and 10.2%, respectively. The Probability Density Function (PDF) gives an idea about the most probable events associated to the fracture parameters. This information is very useful to designers to avoid the structural failure.

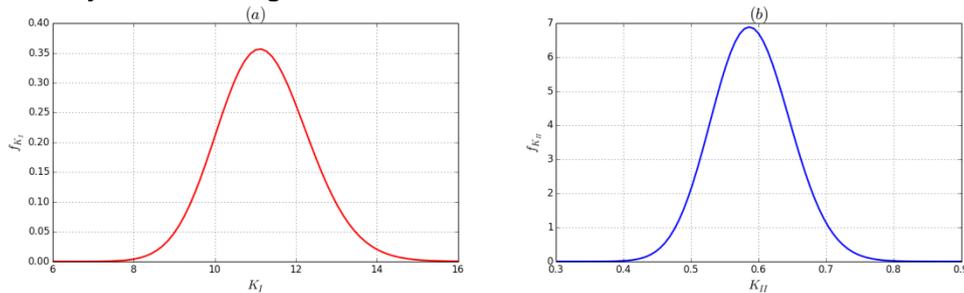


Figure 1. Probability density function (a) opening mode  $K_I$  (b) shear mode  $K_{II}$

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