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► **To cite this version:**

Tian Ma, Shouhong Wang. RADIATIONS AND POTENTIALS OF FOUR FUNDAMENTAL INTERACTIONS. 2017. hal-01616874

**HAL Id: hal-01616874**

**<https://hal.science/hal-01616874>**

Preprint submitted on 15 Oct 2017

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# RADIATIONS AND POTENTIALS OF FOUR FUNDAMENTAL INTERACTIONS

TIAN MA AND SHOUHONG WANG

ABSTRACT. First we demonstrate that the principle of general relativity, the Lorentz and gauge invariance are basic symmetries, dictating the Lagrangian actions of the four fundamental interactions, and the field equations are then derived with the principle of interaction dynamics (PID) and the principle of representation invariance (PRI). Second, we identify that each individual interaction possesses two basic attributes: 1) field particles and their radiation, and 2) the interaction potential and force formulas. In particular, due to the presence of dark matter and dark energy, the Einstein general theory of relativity can be uniquely modified using PID to take into account the effect of dark energy and dark matter phenomena, and to preserve the Einstein's two fundamental principles: the principle of equivalence and the principle of general relativity. The gravitational field particle is described by the dual field  $\{\Phi_\mu\}$ , which is a spin-1 massless particle. This field particle can be regarded as the dark matter, and the energy it carries is the dark energy.

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*Date:* October 10, 2017.

*1991 Mathematics Subject Classification.* 83C, 70S15.

*Key words and phrases.* fundamental interactions, principle of general relativity, principle of equivalence, principle of gauge invariance, principle of representation invariance, principle of interaction dynamics, principle of symmetry-breaking, interaction potential, gravitational field equations, gravitational field particles, gravitational radiation, weak radiation, strong radiation, electromagnetic radiation.

The work was supported in part by the US National Science Foundation (NSF), the Office of Naval Research (ONR) and by the Chinese National Science Foundation.

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## 1. INTRODUCTION

There are four fundamental interactions in nature: the gravity, the electromagnetism, the weak and the strong interactions. It was to the credit of Albert Einstein, Herman Weyl and Paul Dirac that physical laws of these four interactions should be built based on fundamental first principles. There have been extensive studies in the last 100 years or so; see among many others [9, 22, 15, 19, 8, 4, 18, 21, 16, 17, 3, 6, 1, 5, 7, 20, 2, 23] and the references therein.

The main objectives of this paper are to examine these first principles, and to study radiations and potentials for the four interactions. Hereafter we address the main ingredients of this study.

*First*, the Einstein theory of general relativity is the most profound scientific theory in the recorded human history. The Einstein theory is built on two first principles: the principle of equivalence (PE) and the principle of general relativity (PGR). In essence, PE amounts to saying that space time is a four-dimensional Riemannian manifold  $(\mathcal{M}, \{g_{\mu\nu}\})$  with the metric being the gravitational potential.

The PGR is a symmetry principle, and says that the law of gravity is the same (covariant) under all coordinate systems. In other words, the Lagrangian action of gravity, called the Einstein-Hilbert action, is invariant under all coordinate transformations.

The Einstein–Hilbert functional (2.1) is *uniquely dictated* by this profound and simple looking symmetry principle, together with simplicity of laws of Nature. Indeed, in Riemannian geometry, the invariant quantities satisfying the principle of general relativity and containing the second order derivative terms of  $\{g_{\mu\nu}\}$  is just the scalar curvature  $R$ , which is unique.

The presence of the dark matter and dark energy phenomena requires the inevitable need for modifying the Einstein general theory of relativity. Such modification needs to preserve the following basic physical requirements:

- conservation of energy-momentum,
- inclusion of dark matter and dark energy effect, and
- preservation of Einstein's two principles: PE and PGR.

We have shown that, under these basic requirements, the unique route for altering the Einstein general theory of relativity is through the principle of interaction dynamics (PID), which takes variation of the Einstein-Hilbert action subject of energy-momentum conservation constraint. This leads to the new gravitational field equations (2.7), stated here as well for clarity [12]:

$$(1.1) \quad R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\frac{8\pi G}{c^4}T_{\mu\nu} - \nabla_\mu\Phi_\nu.$$

Also, we have shown [13] that PID is the direct consequence of the presence of dark energy and dark matter, is the requirement of the presence of the Higgs field for the weak interaction, and is the consequence of the quark confinement phenomena for the strong interaction.

*Second*, it has been shown that the basic symmetry for the electromagnetic, the weak and the strong interactions is the gauge invariance and the Lorentz invariance. We discovered the principle of representation invariance (PRI) [10], which is a logic requirement for the gauge theory. Basically, PRI requires that the gauge theory be independent of the choices of the representation generators. These representation generators play the same role as coordinates, and in this sense, PRI is a coordinate-free invariance/covariance, reminiscent of the Einstein principle of general relativity.

PRI shows that the unification of four interactions through large symmetric group is not mathematically feasible. Consequently, PRI leads us to postulate the principle of symmetry-breaking (PSB); see [11, 13].

The three principles PID, PRI and PSB offer us to establish a unique route of unification for the four fundamental interactions:

- the PGR dictates uniquely the Einstein-Hilbert action, and the Lorentz and gauge invariances dictate uniquely the actions for the electromagnetic, the weak and the strong interactions;
- the PID and PRI determine the field equations.

Our unification is the unique unification, which provides the unified field equations and is based on first principles. In fact, all other known

attempts of unification are not able to provide proper field equations. It has successfully provided solutions to a number of challenge problems associated with the four fundamental interactions that modern physics has faced for many decades, including for example the following:

- (1) what are dark energy and dark matter?
- (2) what is the mechanism of active galactic nucleus jets?
- (3) What are the strong and weak interaction potential formulas?
- (4) Why the Yukawa potential of nucleus is in disagreement with experiments in short range?
- (5) What is the mechanism of quark confinement and asymptotic freedom?
- (6) Why do leptons not participate in strong interactions?
- (7) Is there a first principle approach to the Higgs mechanism?
- (8) What is the mechanism of subatomic decays and reactions?
- (9) What is the mechanism of bremsstrahlung?
- (10) What are the field equations of the strong and weak interactions?

*Third*, our PID and PRI based models of the four interactions demonstrate the following two characteristics:

- (1) Each individual interaction possesses its own field particles and its own radiations, and the field particles are massless, electric neutral bosons;
- (2) each interaction possesses its own potentials and force.

*Fourth*, the gravitational field particle is described by the dual field  $\{\Phi_\mu\}$  in (1.1), and the governing radiation equations are (3.3), recalled here for convenience:

$$(1.2) \quad \nabla^\mu \nabla_\mu \Phi_\nu = -\frac{8\pi G}{c^4} \nabla^\mu T_{\mu\nu},$$

where  $T_{\mu\nu}$  stands for the energy-momentum tensor of the visible matter, and  $\{\Phi_\mu\}$  is a massless, spin-1 and electric neutral boson.

In fact, the gravitational field particle  $\{\Phi_\mu\}$  represents the dark matter that we have been searching for, and the energy that  $\{\Phi_\mu\}$  carries is the dark energy. Equation (1.2) is the field equations for dark matter and dark energy. The gravitational effect of the field particle  $\{\Phi_\mu\}$  is manifested through the mutual coupling and interaction with the gravitational potential  $\{g_{\mu\nu}\}$ , through the field equations (1.1), leading to both attractive and repulsive behavior of gravity, which is exactly the dark energy and dark matter phenomena.

*Fifth*, the basic symmetry for the field particles and radiations of the electromagnetic and the strong interactions are respectively the

U(1) and SU(3) gauge invariances. The corresponding field particles for electromagnetism are the scalar and vector photons, and the field particles for strong interactions are the vector and scalar gluons.

The *massive*  $W^\pm$  and Z particles do not qualify for the field particles for the weak interaction as they are massive. We conjecture that the symmetry for the field particles and radiation of the weak interaction should be SU(2) gauge symmetry, and the corresponding field particles should be the  $\nu$ -particles:  $\nu_1, \nu_2, \nu_3$ , whose weakton constituents are  $\nu_1 = \nu_e \bar{\nu}_e$ ,  $\nu_2 = \nu_\mu \bar{\nu}_\mu$  and  $\nu_3 = \nu_\tau \bar{\nu}_\tau$ .

The field equations for the field particles and radiations are then given by

$$\begin{aligned}
\text{scalar photon:} & \quad \partial^\mu \partial_\mu \phi = 0, \\
\text{vector photon:} & \quad \partial^\mu (\partial_\mu A_\nu - \partial_\nu A_\mu) = e J_\nu, \\
\text{vector } \nu\text{-particles:} & \quad \partial^\mu A_{\mu\nu}^k - \frac{g_w}{\hbar c} \varepsilon_{ij}^k g^{\alpha\beta} A_{\alpha\nu}^i A_\beta^j = g_w Q_\nu^{k?} \\
\text{scalar } \nu\text{-particles:} & \quad \partial^\mu \partial_\mu \phi_w^k = -g_w \partial^\nu Q_\nu^{k?} \\
\text{vector gluons:} & \quad \partial^\mu A_{\mu\nu}^a - \frac{g_s}{\hbar c} \lambda_{bc}^a g^{\alpha\beta} A_{\alpha\nu}^b A_\beta^c = g_s Q_\nu^a, \\
\text{scalar gluons:} & \quad \partial^\mu \partial_\mu \phi_s^a = -g_s \partial^\nu Q_\nu^a,
\end{aligned}$$

*Sixth*, by the modified gravitational field equations (1.1), we derive the following approximate gravitational force formula:

$$F(r) = mMG \left[ -\frac{1}{r^2} - \frac{k_0}{r} + k_1 r \right], \quad k_0 = 4 \times 10^{-18} \text{km}^{-1}, \quad k_1 = 10^{-57} \text{km}^{-3}.$$

*Seventh*, the potentials for electromagnetic, the weak and the strong interactions depend on the number  $N$  of particles carrying the same charges. For an  $N$  particle system with each particle carries the same charge  $g$ , the system obeys the SU(N) gauge invariance, and the governing field equations are

$$\begin{aligned}
(1.3) \quad \partial^\mu A_{\mu\nu}^a - \frac{g}{\hbar c} \lambda_{bc}^a g^{\alpha\beta} A_{\alpha\nu}^b A_\beta^c - g \bar{\Psi} \gamma_\nu \tau^a \Psi \\
= \left( \partial_\nu - \frac{1}{4} k^2 x_\nu + \frac{g}{\hbar c} \gamma A_\nu \right) \phi^a,
\end{aligned}$$

$$(1.4) \quad \left( i \gamma^\mu D_\mu - \frac{mc}{\hbar} \right) \Psi = 0.$$

By PRI, using the contraction with an SU(N) vector, we deduce the total potential

$$(1.5) \quad A_\mu = (A_0, \vec{A}) = \alpha_a A_\mu^a, \quad \phi = \alpha_a \phi^a,$$

where  $A_0$  is the interaction charge potential, and  $\vec{A}$  is the corresponding interaction magnetic potential. Namely, the interaction force  $F$  and the interaction magnetic force  $F_m$  are given by

$$F = -g\nabla A_0, \quad F_m = \frac{g}{c}v \times \text{curl}\vec{A},$$

where  $v$  is the velocity of the particle with charge  $g$ . We can then derive the unified layered interaction potential  $A_0$  for the electromagnetic, the weak and the strong interactions:

$$A_0 = qe^{-k_1t} \left[ \frac{1}{r} - \frac{A}{\rho}(1 - kr)e^{-(k-k_1)t} \right], \quad q = nN \left( \frac{\rho_0}{\rho} \right)^3 g,$$

where  $n$  is the number of charges  $g$ ,  $\rho_0$  is the radius of the particle, and  $\rho$  is the radius of the system of  $N$  particles.

The paper is organized as follows. In Section 2, we recall the fundamental first principles for the four interactions. Section 3 is devoted to field particles and radiations, and Section 4 derives interaction potential and force formulas.

## 2. FIRST PRINCIPLES OF FUNDAMENTAL INTERACTIONS

**2.1. Two intrinsic properties of interactions.** There are four fundamental interactions in Nature:

gravitational, electromagnetic, strong, weak.

There are two basic attributes for each fundamental interaction:

- (a) the field particles and their radiations, and
- (b) the potentials and forces.

In this paper, we demonstrate that the above two attributes of each interaction are governed by their respective physical laws which are determined by the following first principles:

- (1) the principle of general relativity, which dictates the Lagrangian action of the gravity,
- (2) the principle of gauge invariance and the principle of Lorentz invariance, which dictate the Lagrangian actions of the electromagnetic, weak and strong interactions, and
- (3) the principle of interaction dynamics and the principle of representation invariance, which dictate the field equations through their Lagrangian actions.

**2.2. Symmetry principles.** Symmetry plays a fundamental role in understanding Nature. The Einstein principle of general relativity (PGR), the principle of gauge invariance, the principle of Lorentz invariance and the principle of representation invariance are all symmetry principles. Together with the simplicity of laws of Nature, these symmetry principles dictate the Lagrangian actions for the four fundamental interactions.

1) *Gravity.* Among the four interactions, the gravity is unique and different from the other three interactions. The Lagrange action of gravity is the Einstein–Hilbert functional:

$$(2.1) \quad L_{\text{EH}} = \int_M \left[ R + \frac{8\pi G}{c^4} S \right] \sqrt{-g} dx,$$

where  $M$  is the 4-dimensional Riemann space, representing the three dimensional space and the one dimensional time,  $R$  is the scalar curvature of the space-time,  $S$  is the energy-momentum density,  $G$  is the gravitational constant,  $c$  is the speed of light, and

$$g = \det(g_{\mu\nu}).$$

Here  $\{g_{\mu\nu}\}$  is the Riemannian metric of  $M$ , representing the gravitational potential.

The basic symmetry principle for gravity is the principle of general relativity (PGR), which is recalled here:

**Principle of General Relativity 2.1** (Einstein, 1915). *The law of gravity is the same (co-variant) under all coordinate systems. In other words, the Lagrange action of gravity is invariant under any coordinate transformations.*

The Einstein–Hilbert functional (2.1) is dictated by this profound and simple looking symmetry principle, together with simplicity of laws of Nature. Indeed, in Riemannian geometry, the invariant quantities satisfying the principle of general relativity and containing the second order derivative terms of  $\{g_{\mu\nu}\}$  is just the scalar curvature  $R$ , which is unique.

In fact, we believe it is this very reason that Albert Einstein stated in different occasions that his gravitational theory is unique, and we believe that the uniqueness refers to the uniqueness of the action (2.1). In short, any modification of the action (2.1) is not warranted.

2). *Electromagnetic, weak, and strong interactions.* Modern theory and experimental evidence have suggested that the electromagnetic, the weak and the strong interactions obey the gauge symmetry, to be stated below as the principle of gauge invariance. The Lagrangian



actions of the electromagnetic, weak and strong interactions are the functionals of gauge fields:

$$(2.2) \quad \begin{aligned} \Psi &= \{\psi^1, \dots, \psi^N\}, \\ A_\mu^a &= \{A_\mu^1, \dots, A_\mu^{N^2-1}\}, \end{aligned}$$

where  $N \geq 1$  represents the number of particles in the associated interaction system,  $\psi^i$  is the wave function corresponding to the  $i$ -th particle,  $A_\mu^a (1 \leq a \leq N^2 - 1)$  are the functions of  $SU(N)$  gauge fields, and for each  $a$ ,

$$A_\mu^a = (A_0^a, A_1^a, A_2^a, A_3^a)$$

is a 4-dimensional vector field defined on the space-time manifold  $M$ .

The Lagrangian action of the  $SU(N)$  gauge field (2.2) is expressed as

$$(2.3) \quad L_{SU(N)} = \int_M \left[ -\frac{1}{4} \mathcal{G}_{ab} g^{\mu\alpha} g^{\nu\beta} A_{\mu\nu}^a A_{\alpha\beta}^b + \bar{\Psi} \left( i\gamma^\mu D_\mu - \frac{mc}{\hbar} \right) \Psi \right] dx$$

where  $\{g^{\mu\nu}\}$  is the Minkowski metric,  $\bar{\Psi} = \Psi^\dagger \gamma_0$ ,  $\gamma_\nu$  is the Dirac matrix,  $\gamma^\mu = g^{\mu\nu} \gamma_\nu$ ,

$$(2.4) \quad \begin{aligned} A_{\mu\nu}^a &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g \lambda_{bc}^a A_\mu^b A_\nu^c, \\ D_\mu &= \partial_\mu + ig A_\mu^a \tau_a, \end{aligned}$$

where  $g$  is interactional charge,  $A_\mu^a$  as in (2.2),  $\lambda_{bc}^a$  are the structure constants of  $SU(N)$ , and  $\{\tau_a \mid 1 \leq a \leq N^2 - 1\}$  are the representation generators of  $SU(N)$ ,  $\mathcal{G}_{ab} = \frac{1}{2} Tr(\tau_a \tau_b^+)$ .

The expression of  $L_{SU(N)}$  in (2.3) is essentially determined by the principle of Lorentz invariance and the principle of gauge invariance, which are stated as follows.

**Principle of Lorentz Invariance 2.2.** *Physical laws are invariant under the Lorentz transformation, i.e. the Lagrangian action of the electromagnetic, weak and strong interactions is invariant under the Lorentz transformation.*

**Principle of Gauge Invariance 2.3.** *The electromagnetic, weak and strong interactions obey gauge invariance. Namely, for an  $N$ -particle system with the interactions mentioned above, its Lagrangian action is invariant under the following  $SU(N)$  gauge transformation*

$$(2.5) \quad (\tilde{\Psi}, \tilde{A}_\mu^a \tau_a) = \left( \Omega \Psi, A_\mu^a \Omega \tau_a \Omega^{-1} + \frac{i}{g} (\partial_\mu \Omega) \Omega^{-1} \right),$$

for any  $\Omega = e^{i\theta^a \tau_a} \in SU(N)$ , where  $\theta^a(x) (1 \leq a \leq N^2 - 1)$  are functions,  $\{\tau_a\}$  are the representation generators of  $SU(N)$ ,  $g$  is the charge,  $(\Psi, A_\mu^a)$  are the gauge fields as in (2.2).

**2.3. Principle of Interaction Dynamics (PID).** The three symmetry principles stated above dictate the expressions (2.1) and (2.3) of Lagrangian actions for the four interactions. Then PID determines the field equations from the Lagrangian actions (2.1) and (2.3). PID is stated as follows.

**Principle of Interaction Dynamics 2.4.** *Let  $L(u)$  be a Lagrangian action of an interaction system, and  $u$  be the field functions with  $u = g_{\mu\nu}$  for the gravity, and  $u = (\Psi, A_\mu^a)$  for the electromagnetic, weak, strong interactions. Then the state of  $u$  is the extremum point of  $L(u)$  with the constraint of energy-momentum conservation, i.e.*

$$(2.6) \quad \frac{d}{d\lambda} L(u + \lambda X) = \int_M \delta L(X) \cdot X dx = 0, \quad \forall \text{div} X = 0.$$

Based on the PID, the field equations of the four fundamental interactions are derived as follows:

1). *Gravitational field equations.* For the Einstein–Hilbert functional (2.1), the equation (2.6) can be written in the form

$$(2.7) \quad R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{\mu\nu} - \nabla_\mu \Phi_\nu,$$

where  $\Phi_\nu$  is a 4-dimensional vector field, called the dual gravitational potential.

2). *SU(N) gauge field equations.* For the Lagrangian action (2.3) of SU(N) gauge field, the field equation (2.6) can be expressed as

$$(2.8) \quad \mathcal{G}_{ab} \left[ \partial^\mu A_{\mu\nu}^b - \frac{g}{\hbar c} \lambda_{cd}^b g^{\alpha\beta} A_{\alpha\nu}^c A_\beta^d \right] - g \bar{\Psi} \gamma_\nu \tau_a \Psi = \left[ \partial_\nu - \frac{1}{4} k^2 x_\nu + \frac{g}{\hbar c} \gamma A_\nu \right] \phi_a,$$

$$(2.9) \quad \left( i\gamma^\mu D_\mu - \frac{mc}{\hbar} \right) \Psi = 0,$$

where  $(\Psi, A_\mu^a)$  are as in (2.2),  $g$  is the interaction charge,  $A_{\mu\nu}^a$  and  $D_\mu$  are as in (2.4),  $k, \gamma$  are constants,  $A_\mu = \partial_a A_\mu^a$  is the SU(N) tensor construction, representing the total interacting potential,  $\mathcal{G}_{ab} = \frac{1}{2} \text{Tr}(\tau_a \tau_b^+)$ .

**2.4. Principle of representation invariance (PRI).** We note that in (2.5), the matrices  $\Omega$  of gauge transformation are expressed as

$$(2.10) \quad \Omega = e^{i\theta^a \tau_a} \in SU(N),$$

where  $\tau_a$  ( $1 \leq a \leq N^2 - 1$ ) are the representation generators of SU(N), which constitute a coordinate basis of the tangent space of SU(N) at

the identity matrix  $I$ . In fact, if we choose another basis  $\{\tilde{\tau}_a\}$ , i.e. take a transformation

$$(2.11) \quad \tilde{\tau}_a = x_a^b \tau_b,$$

where  $A = (x_a^b)$  is an  $(N^2 - 1) \times (N^2 - 1)$  complex matrix, then the gauge fields  $\{A_\mu^a\}$  transform as

$$(2.12) \quad \tilde{A}_\mu^a = y_b^a A_\mu^b,$$

where  $(y_b^a) = A^{-1}$  is the inverse matrix of  $A$ . In addition, the quantities  $\lambda_{bc}^a$  and  $\mathcal{G}_{ab}$  transform also as  $(N^2 - 1)$ -dimensional tensors:

$$(2.13) \quad \tilde{\lambda}_{bc}^a = y_k^a x_b^i x_c^j \lambda_{ij}^k, \quad \tilde{\mathcal{G}}_{ab} = x_a^i x_b^j \mathcal{G}_{ij}.$$

However, basic logic indicates that the gauge theory should be invariant under the transformation (2.11), i.e. the Lagrangian action of gauge field should be invariant. Consequently, we have the following principle of representation invariance (PRI):

**Principle of Representation Invariance 2.5** (Ma & Wang [13]).  
*For the  $SU(N)$  gauge theory, under the representation transformation (2.11), the Lagrangian action of gauge field (2.2) is invariant, and the field equations are covariant.*

As we have indicated, PRI is a basic logic requirement for an  $SU(N)$  gauge theory, and has profound physical implications. The quantities satisfying (2.12) and (2.13) are called the  $SU(N)$  tensors.

It is easy to see that the field equations (2.8)–(2.9) are covariant under the transformation (2.11), and

$$(2.14) \quad A_\mu = \alpha_b A_\mu^b$$

is invariant, where  $\{\alpha_b\}$  and  $\{A_\mu^b\}$  are  $SU(N)$  tensors.

Furthermore, PRI leads to the following physical conclusions:

- (1) the coupling constant  $g$  in (2.4) represents the interaction charge, playing the same role as the electric charge  $e$  in the  $U(1)$  gauge theory for quantum electrodynamics;
- (2) the potential  $A_\mu = \alpha_b A_\mu^b$  represents the total interacting potential, and each  $\alpha_b$  represents the portion of the component  $A_\mu^a$  contributed to the total interaction potential; and
- (3) for  $A_\mu = (A_0, \vec{A})$ , the temporal component  $A_0$  and the spatial components  $\vec{A} = (A_1, A_2, A_3)$  represent respectively the interaction potential and the interaction magnetic potential. The force and the magnetic force generated by the interaction are

given by

$$F = -g\nabla A_0, \quad F_m = \frac{g}{c}v \times \text{curl}\vec{A},$$

where  $v$  is the velocity of the particle with charge  $g$ .

### 3. FIELD PARTICLES AND RADIATIONS

**3.1. Gravitational field particles and radiations.** First we recall the PID gravitational field equations (2.7) for convenience:

$$(3.1) \quad R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\frac{8\pi G}{c^4}T_{\mu\nu} - \nabla_\mu\Phi_\nu.$$

We deduce then from (3.1) the following conclusions for the gravitational interaction, which will be explored in detail below:

- (a) gravity possesses a field particle  $\Phi_\nu$ , called the graviton, which is massless with no electric charge and spin  $J = 1$ ;
- (b) there exists gravitational radiations, emitting gravitons at the speed of light.

1). *Gravitational field particle.* In the field equations (3.1), there are three groups of state functions, whose physical meaning are given as follows:

- (1)  $\{g_{\mu\nu}\}$  represents the gravitational potential, depicting the curved space-time;
- (2)  $\{\Phi_\mu\}$  is the dual gravitational potential, representing the gravitational field particle and carrying the field energy; and
- (3)  $\{T_{\mu\nu}\}$  is the energy-momentum tensor, representing the visible matter field.

Physically, the dual gravitational potential  $\{\Phi_\mu\}$  plays the role of the dark matter and dark energy. In fact, as a state function describing the gravitational field particle,

$$(3.2) \quad \{\Phi_\mu\} \text{ represents the graviton.}$$

In this sense,  $\{\Phi_\mu\}$  is the dark matter, and the energy carried by  $\{\Phi_\mu\}$  is the dark energy.

2). *Gravitational radiation.* By the Bianchi identity,

$$\nabla^\mu(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R) = 0.$$

Hence we infer from (3.1) that

$$(3.3) \quad \nabla^\mu\nabla_\mu\Phi_\nu = -\frac{8\pi G}{c^4}\nabla^\mu T_{\mu\nu}.$$

For the case where the gravity is weak,  $\nabla^\mu \nabla_\mu$  is approximated by the wave operator:

$$(3.4) \quad \nabla^\mu \nabla_\mu \simeq \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2.$$

Consequently, (3.2) is the graviton and (3.3) is the gravitational radiation equation, representing the field equation of gravitons.

3). *Gravitational radiation wave.* In vacuum,  $T_{\mu\nu} = 0$ . Then equations (3.3) becomes

$$(3.5) \quad \left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) \Phi_\nu = 0,$$

which is the wave equation of gravitational radiation.

**Remark 3.1.** The gravitational radiation wave is different from the gravitational wave. The radiation wave is about the field particle  $\{\Phi_\mu\}$  in (3.2), representing the propagating of gravitational field energy. The gravitational wave is about space metric  $\{g_{\mu\nu}\}$ , representing the propagation of space deformation. In fact, because the gravitational wave is very weak,  $\{g_{\mu\nu}\}$  can be written as

$$(3.6) \quad g_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu},$$

where  $\delta_{\mu\nu}$  is the Kronecker sign, and  $|h_{\mu\nu}| \ll 1$ . Then  $h_{\mu\nu}$  satisfy the wave equation

$$(3.7) \quad \left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) h_{\mu\nu} = 0.$$

Therefore, (3.7) describes the wave of space-time metric (3.6), reflecting the propagation of space-time deformation.

**3.2. Electromagnetic field particles and radiation.** The symmetry dictating electromagnetic field particle is the U(1) gauge invariance. The particle described by  $\psi$  is

$$(3.8) \quad \psi \text{ particle in the U(1) gauge theory} = \text{electron.}$$

By (2.8), the U(1) gauge field equation is written as

$$(3.9) \quad \partial^\mu A_{\mu\nu} - e J_\nu = \partial_\nu \phi,$$

where  $J_\nu = \bar{\psi} \gamma_\nu \psi$  is the 4-dimensional current density, and

$$A_{\mu\nu} = \frac{\partial A_\nu}{\partial x_\mu} - \frac{\partial A_\mu}{\partial x_\nu}.$$

Based on (3.9), we can deduce the following conclusions:

1). *Electromagnetic field particles.* According to (3.9), the electromagnetic field particles are a pair of dual photons:

$$(3.10) \quad \begin{aligned} A_\mu &= \text{vector (spin-1) photon } \gamma, \\ \phi &= \text{scalar (spin-0) photon } \gamma_0. \end{aligned}$$

The weakton constituents of  $\gamma$  and  $\gamma_0$  are

$$(3.11) \quad \begin{aligned} \gamma &= \cos \theta w_1 \bar{w}_1 - \sin \theta w_2 \bar{w}_2 \quad (\uparrow\uparrow, \downarrow\downarrow), \\ \gamma_0 &= \cos \theta w_1 \bar{w}_1 - \sin \theta w_2 \bar{w}_2 \quad (\uparrow\downarrow, \downarrow\uparrow). \end{aligned}$$

where  $w_1, w_2$  are the weaktons, and  $\bar{w}_1, \bar{w}_2$  are their antiparticles; see Ma and Wang [14, 13].

2). *Electromagnetic radiation.* By the identity and the electric charge conservation:

$$\partial^\nu \partial^\mu A_{\mu\nu} = 0 \quad \text{and} \quad \partial^\nu J_\nu = 0,$$

we derive from (3.9) the radiation equation for the scalar photon  $\gamma_0$ :

$$(3.12) \quad \left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) \phi = 0.$$

Taking  $\phi = 0$  in (3.9), we get the radiation equation for the vector photon  $\gamma$  as follows

$$(3.13) \quad \frac{\partial}{\partial x_\mu} \left( \frac{\partial A_\nu}{\partial x_\mu} - \frac{\partial A_\mu}{\partial x_\nu} \right) = e J_\nu.$$

The equation (3.13) is the Maxwell equation. Equations (3.12) and (3.13) are also regarded as the field equations for the scalar photon  $\gamma_0$  and the vector photon  $\gamma$ .

**3.3. Strong interaction field particles and radiations.** Based on QCD, the symmetry dictating strong interaction field particles is the SU(3) gauge invariance. The particles described by  $\psi_1, \psi_2, \psi_3$  are the quarks with three different colors: red, blue, and green:

$$(3.14) \quad \text{three particles } \psi_1, \psi_2, \psi_3 \text{ of } \text{SU}(3) = (q_1, q_2, q_3),$$

where  $q_1 = q_{\text{red}}, q_2 = q_{\text{blue}}, q_3 = q_{\text{green}}$ .

The gauge field equation (2.8) for SU(3) is written as

$$(3.15) \quad \partial^\mu A_{\mu\nu}^a - \frac{g_s}{\hbar c} \lambda_{bc}^a g^{\alpha\beta} A_{\alpha\nu}^b A_\beta^c - g_s Q_\nu^a = \partial_\nu \phi^a,$$

where  $g_s$  is the strong charge, and we have used the Gell-Mann matrices  $\{\tau_a\}$  as the generators of SU(3) so that

$$A_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + \frac{g_s}{\hbar c} \lambda_{bc}^a A_\mu^b A_\nu^c,$$

$$\begin{aligned} Q_\nu^a &= \bar{\Psi} \gamma_\nu \tau^a \Psi, \\ \mathcal{G}_{ab} &= \delta_{ab}. \end{aligned}$$

Here  $\tau^a = \tau_a$ , and  $\Psi = (\psi_1, \psi_2, \psi_3)$  satisfies the Dirac equation (2.9).

Based on (3.15), we have the following conclusions.

1). *Strong interacting field particles.* For SU(3) gauge theory of the strong interaction, the field particles are two groups of gluons, each of which consists of eight gluons with either spin  $J = 1$  or spin  $J = 0$ :

$$(3.16) \quad \begin{aligned} \{A_\mu^a\} &= \text{spin-1 gluons } \{g_a \mid 1 \leq a \leq 8\}, \\ \{\phi^a\} &= \text{spin-0 gluons } \{g_a^0 \mid 1 \leq a \leq 8\}. \end{aligned}$$

The weakton constituents for  $g_a$  and  $g_a^0$  are

$$(3.17) \quad g_a = w^* \bar{w}^* (\uparrow\uparrow, \downarrow\downarrow), \quad g_a^0 = w^* \bar{w}^* (\uparrow\downarrow, \downarrow\uparrow),$$

where the index  $a$  corresponds to the different combinations of colors of  $w^*$  and  $\bar{w}^*$ .

2). *Gluon radiation.* The equations (3.15) provide the model describing gluon radiations. As  $\{g_a^0\}$  and  $\{g_a\}$  are independent to each other in strong interaction radiation, we deduce from (3.15) the field equations for  $\{g_a^0\}$  as:

$$(3.18) \quad \left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) \phi^a = -g_s \partial^\nu Q_\nu^a \quad (1 \leq a \leq 8),$$

and the field equations for  $\{g_a\}$  as:

$$(3.19) \quad \partial^\mu A_{\mu\nu}^a - \frac{g_s}{\hbar c} \lambda_{bc}^a g^{\alpha\beta} A_{\alpha\nu}^b A_\beta^c = g_s Q_\nu^a \quad (1 \leq a \leq 8).$$

**3.4. Weak interaction field particles and radiation.** In the classical standard model, the weak interaction field particles are dictated by SU(2) gauge invariance, and are given as the three massive bosons:

$$(3.20) \quad W^\pm, \quad Z \quad \text{and Higgs } H^0.$$

However, in the weakton model by Ma and Wang [14, 13], the massive bosons (3.20) are just the transition particles in decaying and scattering processes. Also, for an interaction radiation, the following basic rule should be satisfied:

radiation particles must be massless and electric-neutral.

Hence the  $W$  and  $Z$  particles (3.20) are not the weak interaction field particles.

Due to the limitation of physical experiments, the massless and electric neutral particles are hard to be directly detected. Therefore, the

theory for the weak interaction radiation is completely open. The key point is that we don't know what the weak interaction field particles are. It is likely that the field particles are confined in the interiors of electrons and quarks.

We conjecture that the symmetry for the field particles and radiation of the weak interaction should be  $SU(2)$  gauge symmetry, and the corresponding field particles should be the  $\nu$ -particles:  $\nu_1, \nu_2, \nu_3$ , whose weakon constituents are  $\nu_1 = \nu_e \bar{\nu}_e$ ,  $\nu_2 = \nu_\mu \bar{\nu}_\mu$  and  $\nu_3 = \nu_\tau \bar{\nu}_\tau$ . In this case, the field equations for the field particles and radiations are then given by

$$\begin{aligned} \partial^\mu A_{\mu\nu}^k - \frac{g_w}{\hbar c} \varepsilon_{ij}^k g^{\alpha\beta} A_{\alpha\nu}^i A_\beta^j &= g_w Q_\nu^k, \\ \partial^\mu \partial_\mu \phi_w^k &= -g_w \partial^\nu Q_\nu^k, \\ Q_\nu^k &= \bar{\Psi} \gamma_\nu \tau^k \Psi, \quad \text{for } k = 1, 2, 3. \end{aligned}$$

#### 4. INTERACTION POTENTIALS AND FORCES

**4.1. Gravitational potentials.** Gravity is different from the other three fundamental interactions, and the difference is reflected in four aspects as follows:

- (1) the symmetry that gravity obeys is the principle of general relativity, rather than the gauge symmetry;
- (2) the field equations of the gravitational potential are the same as these of gravitational radiation;
- (3) the gravitational force is the curved effect of the space-time, and the electromagnetic, weak, strong interactions are the twisted effects of the underlying complex vector bundles  $M \otimes_p \mathbb{C}^N$ ;
- (4) the gravitational mass charge is continuous, and the electric, weak, strong charges are discrete.

**Remark 4.1.** We will see that the electromagnetic, weak and strong interaction potentials are layered. In fact, it is the difference (4) between gravity and the other three interactions that makes the gravitational potential non-layered.  $\square$

Recall that for the Newtonian gravitational potential and force are given by

$$(4.1) \quad \varphi = -\frac{MG}{r},$$

$$(4.2) \quad F = -m\nabla\varphi = -\frac{mMG}{r^2}.$$



In the general theory of relativity, the gravitational potentials are

$$(4.3) \quad \{g_{\mu\nu}\} \text{ and the dual potential } \{\Phi_\mu\}.$$

In the Newtonian law, there is a single potential function  $\varphi$  in (4.1), which yields the gravitational force  $F$  in (4.2). However, in the general theory of relativity, there are many components in the gravitational potentials in (4.3). A natural question is that which component of (4.3) represents the gravitational force  $F$ , and how the potentials  $\{g_{\mu\nu}\}$  and  $\{\Phi_\mu\}$  are related? We address this question as follows.

1). *Gravitational potentials in spherically symmetric fields.* Let  $M$  be the total mass of a centrally symmetric ball. The gravitational potentials of (4.3) are the functions of  $r$ , i.e.  $g_{\mu\nu} = g_{\mu\nu}(r)$ ,  $\Phi_\mu = \Phi_\mu(r)$ . In this case, the time-component  $g_{00}$  of  $g_{\mu\nu}$  and the Newton potential  $\varphi$  have the following relation

$$g_{00} = -\left(1 + \frac{2}{c^2}\varphi\right).$$

Also the gravitational force  $F$  exerted on an object of mass  $m$  is written as

$$(4.4) \quad F = -m\nabla\varphi = -\frac{c^2m}{2}\frac{dg_{00}}{dr}.$$

In addition, mathematically the Riemann metric is

$$(4.5) \quad ds^2 = g_{00}c^2dt^2 + g_{11}dr^2 + g_{22}d\theta^2 + g_{33}d\varphi^2,$$

where  $(r, \theta, \varphi)$  is the spherical coordinate, and

$$g_{00} = -e^{u(r)}, \quad g_{11} = e^{v(r)}, \quad g_{22} = r^2, \quad g_{33} = r^2 \sin^2 \theta,$$

and the dual potential  $\Phi_\mu$  is given by

$$\Phi_\mu = \nabla_\mu\phi(r).$$

Thus, the gravitational potential and dual potential become

$$(4.6) \quad u = u(r), \quad v = v(r) \quad \text{and} \quad \phi = \phi(r),$$

and the force (4.4) is rewritten as

$$(4.7) \quad F = -\frac{c^2m}{2}e^u\frac{du}{dr}.$$

Let  $r_0$  be the radius of the ball with mass  $M$ . Then, for  $r > r_0$  the energy-momentum tensors is zero, i.e.

$$T_{\mu\nu} = 0.$$

Thus, the gravitational field equations (3.1) are the ordinary differential equations, written as

$$(4.8) \quad \begin{aligned} v' + \frac{1}{r}(e^v - 1) &= -\frac{r}{2}u'\phi', \\ u' - \frac{1}{r}(e^v - 1) &= r(\phi'' - \frac{1}{2}v'\phi'), \\ u'' + (\frac{1}{2}u' + \frac{1}{r})(u' - v') &= -\frac{2}{r}\phi', \end{aligned}$$

where  $u, v, \phi$  are as in (4.6), and the prime stands for derivative in  $r$ .

By solving  $u, v, \phi$  from (4.8), we deduce the Riemannian metric (4.5) and the gravitational force (4.7) in  $r > r_0$ .

2). *Gravitational potential in astrophysical fluids.* In astronomy, many physical motions are governed by the astrophysical fluid dynamics equations, which are expressed in the following general form; see Ma and Wang [11, 13]

$$(4.9) \quad \begin{aligned} \frac{\partial u^i}{\partial t} + u^k \frac{\partial u^i}{\partial x^k} + \Gamma_{kj}^i u^k u^j &= \nu[\text{div}(\nabla u^i) + g^{ij} R_{jk} u^k] \\ &\quad - \frac{1}{\rho} g^{ij} \frac{\partial p}{\partial x^j} + \frac{mc^2}{2} g^{ij} \frac{\partial g_{00}}{\partial x_j}, \end{aligned}$$

$$(4.10) \quad \frac{\partial u^k}{\partial x^k} + \Gamma_{kj}^k u^j = 0,$$

where

$$g = \begin{pmatrix} g_{00} & 0 \\ 0 & g_{ij} \end{pmatrix}$$

is the gravitational potential,  $(g_{ij})$  represents the space metric and  $g_{00}$  is the time component; these potentials satisfy the gravitational field equations

$$(4.11) \quad R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\frac{8\pi G}{c^4}T_{\mu\nu} - \nabla_{\mu\nu}\phi,$$

where  $T^{\mu\nu} = (c^2\rho + p)u^\mu u^\nu + pg^{\mu\nu}$ ,  $R_{\mu\nu}$  is the Ricci curvature tensor,  $R = g^{\mu\nu}R_{\mu\nu}$  is the scalar curvature, and

$$\begin{aligned} \Gamma_{\mu\nu}^\alpha &= \frac{1}{2}g^{\alpha\beta} \left[ \frac{\partial g_{\mu\beta}}{\partial x^\nu} + \frac{\partial g_{\nu\beta}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^\beta} \right], \\ \text{div}(\Delta u^i) &= g^{kl} \left[ \frac{\partial}{\partial x^l} \left( \frac{\partial u^i}{\partial x^k} + \Gamma_{kj}^i u^j \right) \right. \\ &\quad \left. + \Gamma_{lj}^i \left( \frac{\partial u^j}{\partial x^k} + \Gamma_{ks}^j u^s \right) - \Gamma_{kl}^j \left( \frac{\partial u^i}{\partial x^j} + \Gamma_{js}^i u^s \right) \right]. \end{aligned}$$

We see from the model (4.9)–(4.11) that the fluid dynamical equations (4.9) and (4.10) are coupled with the gravitational field equations (4.11), to form astrophysical fluid dynamics equations. In this manner, the gravitational potentials  $\{g_{\mu\nu}\}$  play an important role in astrophysical fluid dynamics.

**4.2. Gravitational forces of central matter fields.** For a central matter field with radius  $r_0$ , the gravitational force in  $r > r_0$  is given by

$$(4.12) \quad F = -\frac{c^2 m}{2} \frac{d}{dr} e^{u(r)},$$

where  $g_{00} = e^{-u}$  is the gravitational potential generated by the central matter field with mass  $M$ , and  $m$  is the mass of a body at distance  $r$ , and  $u = u(r)$  is determined by (4.8).

Based on (4.8) and (4.12), for a central gravitational field we can deduce the following conclusions; also see Ma and Wang [12, 13].

- (1) The gravitational force  $F$  given by (4.12) is asymptotically zero at large distance:

$$F(r) \rightarrow 0 \quad \text{as} \quad r \rightarrow \infty.$$

- (2) There exists a sufficiently large  $r_1$  such that the gravitational force  $F$  is repulsive for  $r > r_1$ , i.e.

$$F(r) > 0 \quad \text{if} \quad r > r_1.$$

- (3) The force  $F$  has an approximate expression:

$$F(r) = mMG \left[ -\frac{1}{r^2} - \frac{k_0}{r} + k_1 r \right] \quad \text{for } r_0 < r < r_1,$$

where  $k_0$  and  $k_1$  are given by

$$k_0 = 4 \times 10^{-18} \text{km}^{-1}, \quad k_1 = 10^{-57} \text{km}^{-3}.$$

These conclusions (1)–(3) provide an explanation of the dark matter and dark energy phenomena. In other words, dark matter and dark energy are intrinsic properties of gravity, through the nonlinear interactions of the gravitational potentials  $\{g_{\mu\nu}\}$  and  $\{\Phi_\nu\}$  through the gravitational field equations (3.1).

**4.3. Electromagnetic, weak and strong potentials.** The electromagnetic, weak and strong interaction potentials are dictated by the  $SU(N)$  gauge invariance, where  $N$  is the number of particles in the interacting multi-particle system with the same interaction charge. If we

take the standard representation basis  $\{\tau_a \mid 1 \leq a \leq N^2 - 1\}$  of  $SU(N)$  such that

$$\frac{1}{2}\text{Tr}(\tau_a \tau_b^+) = \delta_{ab},$$

then the  $SU(N)$  gauge field equations (2.8) and (2.9) for the  $N$ -particle system are in the form

$$(4.13) \quad \partial^\mu A_{\mu\nu}^a - \frac{g}{\hbar c} \lambda_{bc}^a g^{\alpha\beta} A_{\alpha\nu}^b A_\beta^c - g \bar{\Psi} \gamma_\nu \tau^a \Psi = \left( \partial_\nu - \frac{1}{4} k^2 x_\nu + \frac{g}{\hbar c} \gamma A_\nu \right) \phi^a,$$

$$(4.14) \quad \left( i \gamma^\mu D_\mu - \frac{mc}{\hbar} \right) \Psi = 0,$$

where  $\Psi = (\psi_1, \dots, \psi^N)^T$  represents the wave functions of the  $N$  particles,  $k^{-1}$  is the interaction range, and

$$\gamma \begin{cases} \neq 0 & \text{for the weak interaction,} \\ = 0 & \text{for the electromagnetic and strong interactions.} \end{cases}$$

Also the  $SU(N)$  gauge potentials  $\{A_\mu^a \mid 1 \leq a \leq N^2 - 1\}$  represent the interaction potentials between the  $N$  particles; see Ma and Wang [13].

**Remark 4.2.** The parameter  $k$  in (4.13) represents the attracting range of the electromagnetic, weak and strong interactions, and  $\gamma$  represents the weak interaction range.  $\square$

Taking the contraction (2.14) we obtain the total interaction potentials

$$(4.15) \quad A_\mu = \alpha_a A_\mu^a = (A_0, \vec{A}) \quad \text{and} \quad \phi = \alpha_a \phi^a.$$

The system (4.13) contains  $4(N^2 - 1)$  equations, and has  $5(N^2 - 1)$  unknown functions

$$A_\mu^a, \quad \phi^a, \quad 1 \leq a \leq N^2 - 1, \quad \mu = 0, 1, 2, 3.$$

Therefore we need to supply  $N^2 - 1$  equations for (4.13), which are called the gauge fixing equations.

In the following, we establish the field equations for the dual potentials  $A_0$  and  $\phi$  in (4.15) of the associated interaction from the fields (4.13)–(4.14).

Taking inner product of (4.13) with  $\alpha_a$ , we get

$$(4.16) \quad \partial^\mu A_{\mu\nu} - \frac{g}{\hbar c} \kappa_{bc} g^{\alpha\beta} A_{\alpha\nu}^b A_\beta^c - g Q_\nu = \left( \partial_\nu - \frac{1}{4} k^2 x_\nu + \frac{g}{\hbar c} \gamma A_\nu \right) \phi,$$

where  $\kappa_{bc} = \alpha_a \lambda_{bc}^a$ , and

$$\begin{aligned} Q_\nu &= \alpha_a \bar{\Psi} \gamma_\nu \tau^a \Psi, \\ A_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu + \frac{g_s}{\hbar c} \kappa_{bc} A_\mu^b A_\nu^c. \end{aligned}$$

As  $A_{\mu\nu} = -A_{\nu\mu}$ , we have

$$\partial^\nu \partial^\mu A_{\mu\nu} = 0.$$

Then taking divergence on both side of (4.16), we get the field equation of  $\phi$  as follows

$$(4.17) \quad \left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) \phi + k^2 \phi - g \partial^\nu Q_\nu = \frac{g}{\hbar c} \partial^\nu [\kappa_{ab} g^{\alpha\beta} A_{\alpha\nu}^a A_\beta^b + \gamma A_\nu \phi].$$

For the weak interaction,  $\gamma \neq 0$ . By the short-range property of the weak interaction, take the following transformation in (4.16)

$$\phi \rightarrow \phi + \phi_0, \quad \phi_0 = \text{constant}.$$

Then (4.16) become

$$(4.18) \quad \begin{aligned} &\partial^\mu (\partial_\mu A_\nu - \partial_\nu A_\mu) - k_1^2 A_\nu + \frac{1}{4} k^2 x_\nu \phi - g Q_\nu \\ &= \left( \partial_\nu + \frac{g\gamma}{\hbar c} A_\nu \right) \phi + \frac{1}{4} k^2 a_0 x_\nu - \frac{g}{\hbar c} \kappa_{ab} (\partial^\mu A_\mu^a A_\nu^b - g^{\alpha\beta} A_{\alpha\nu}^a A_\beta^b), \end{aligned}$$

where

$$(4.19) \quad k_1 = \begin{cases} \sqrt{\frac{g\gamma\phi_0}{\hbar c}} & \text{for weak interaction,} \\ 0 & \text{for electromagnetic and strong interactions.} \end{cases}$$

Based on the superposition property of the interaction charge forces, the dual potentials  $A_0$  and  $\phi$  should satisfy linear equations, i.e. the equation (4.17) and the time-component  $\nu = 0$  equation of (4.18) are linear. Therefore, the gauge fixing equations for the interactional potentials must contain the following three equations

$$\begin{aligned} &\partial^\nu (\kappa_{ab} g^{\alpha\beta} A_{\alpha\nu}^a A_\beta^b + \gamma A_\nu \phi) = 0, \\ &\left( \partial_\nu + \frac{g\gamma}{\hbar c} A_\nu \right) \phi + \frac{1}{4} k^2 a_0 x_\nu - \frac{g}{\hbar c} \kappa_{ab} (\partial^\mu A_\mu^a A_\nu^b - g^{\alpha\beta} A_{\alpha\nu}^a A_\beta^b) = 0, \\ &\frac{\partial}{\partial t} (\partial^\mu A_\mu) = 0. \end{aligned}$$

In addition, under the static conditions

$$\frac{\partial A_0}{\partial t} = 0, \quad \frac{\partial \phi}{\partial t} = 0,$$

the equation (4.17) and the time-component  $\nu = 0$  equation (4.18) become

$$(4.20) \quad \begin{aligned} -\Delta\phi + k^2\phi &= g\partial^\mu Q_\mu, \\ -\Delta A_0 + k_1^2 A_0 &= -gQ_0 - \frac{1}{4}k^2 c\tau\phi, \end{aligned}$$

where  $c\tau$  is the interaction wave length of  $\phi$ ,  $k_1$  is as in (4.19).

The equation of (4.20) determine the potential  $A_0$  in the form

$$(4.21) \quad A_0 = qe^{-k_1 t} \left[ \frac{1}{r} - \frac{A}{\rho} (1 - kr) e^{-(k-k_1)t} \right],$$

where  $A$  is a parameter,  $q$  is the effective charge written as

$$(4.22) \quad q = N \left( \frac{\rho_0}{\rho} \right)^3 g,$$

and  $\rho_0$  is the particle radius and  $\rho$  is the radius of the particle system. Then the interaction forces read as

$$\begin{aligned} \text{Coulomb force :} \quad f &= -e \frac{d}{dr} A_0 && \text{for electric charge } g = e, \\ \text{weak force :} \quad f &= -g_w \frac{d}{dr} A_0 && \text{for weak charge } g = g_w, \\ \text{strong force :} \quad f &= -g_s \frac{d}{dr} A_0 && \text{for strong charge } g = g_s. \end{aligned}$$

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