



HAL
open science

A fair and efficient payoff based modeling the coalition formation process for games with valuations

Stéphane Airiau

► **To cite this version:**

Stéphane Airiau. A fair and efficient payoff based modeling the coalition formation process for games with valuations. Septièmes journées francophones Modèles Formels de l'Interaction MFI'13, Jul 2013, Lille, France. <hal-01616377>

HAL Id: hal-01616377

<https://hal.science/hal-01616377v1>

Submitted on 13 Oct 2017

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



HAL Authorization

A fair and efficient payoff based modeling the coalition formation process for games with valuations

Stéphane Airiau

stephane.airiau@dauphine.fr

LAMSADE Université Paris-Dauphine
Place du Maréchal de Lattre de Tassigny
Paris – France

Abstract :

On s'intéresse à une situation où des agents rationnels peuvent former des coalitions pour améliorer leur bien-être. Nous nous plaçons dans des jeux coopératifs avec externalités et plus précisément des jeux avec valeurs ("games with valuations"): l'utilité de l'agent est définie pour chaque structure de coalitions (SC). Pour une SC donnée, un agent peut changer de coalition en rejoignant une coalition existante dans la SC. Si un changement lui est bénéfique, il a intérêt à changer de coalition. Une SC est stable lorsqu'aucun n'agent n'a intérêt de changer de coalition. La stabilité est une propriété désirable car les agents ne gachent pas de temps ou d'énergie à changer sans cesse de coalitions. Si aucune SC n'est stable, les agents risquent passer leur temps à changer de coalition, sauf si un agent décide d'accepter une solution sous-optimale. Lorsqu'une SC stable existe, elle n'est pas forcément unique, et son choix peut avantager certains agents plutôt que d'autres, ce qui pose un problème d'équité. De plus, il n'est pas forcément toujours possible d'atteindre une SC stable en utilisant une suite de changements qui profitent à l'agent qui change de coalition. L'objet de notre étude est de proposer une notion différente de la stabilité. Si les agents sont gloutons et myopes (i.e. un agent change de coalition si le changement améliore son bien être), chaque agent i peut calculer son utilité espérée \bar{u}_i (s'il connaît la SC de départ ou s'il possède une distribution de probabilité sur les états de départ). Nous proposons le choix d'une SC et une répartition des gains de cette SC de telle sorte que chaque agent i obtienne au moins \bar{u}_i . Tous les agents ont donc intérêt de choisir notre solution, et donc rester dans leur coalition. Malheureusement, notre solution est difficile à calculer.

Keywords: Jeux coopératifs, solutions équitables, chaînes de Markov.

1 Introduction

In the literature on coalition formation, valuation functions are typically defined only over a coalition, and the agents need to decide or negotiate a payoff distribution. We are interested in cases where the payoff distribution is defined for each partition of the population into coalitions, such partition is called a coalition structure (CS). In other words, each agent knows its payoff for any CS. This model, called *games with valuations*, can be viewed from two perspectives. The first perspective is that, even though agents cooperate within a coalition, the payoff of each member is clearly defined and may de-

pend on the other coalitions present in the environment. For example, when agents have different individual goals, members of a coalition help each other and compete with agents outside the coalition. The payoff of an agent depends on the extent of which her own goal was reached using the help of her coalition's members and the competing coalitions. Another example is that of firms forming coalitions in a supply chain domain: each firm in a coalition provides preferred rates or discounts for its services to other members of its coalition. The benefit of each member of the coalition depends on the behaviors of other firms. Each firm in the coalition is autonomous: each sells and buys goods, and makes its own profit or loss. Note that firms still benefit from being in a coalition but the benefit varies from firm to firm in any given CS. The second perspective is to consider that the payoff distribution has already been computed using a simple rule, for instance the marginal contribution, or using a more complex rule, for example a stability criterion, e.g., a payoff in the Kernel [4] or the Nucleolus [12]. Let us consider two different CSs with associated stable payoff distributions. In both cases, the payoff distributions are stable, but an agent may prefer to form the first CS when another agent would prefer the second: even if agents are using a stable payoff distribution, agents may still have incentive to change CS.

The question for games with valuations is which CS will form. Stability is a key property: the agents need to agree to stay on a chosen coalition and not change coalition. However, there may be some games such that there is no CS where no agent would like to change coalition. For such game, it is unclear what CS should form. There are also some games for which there exists more than one stable CS. In that case, choosing one CS over another may not be fair.

The contribution of this paper is to propose a different view on stability. We assume that agents are rational and myopic. In this situation,

if they have the opportunity to change coalition to improve their utility, they will do so. If the valuation function $v_i : \mathcal{S} \rightarrow \mathbb{R}$ of each agent i is common knowledge, every agent i could compute her expected utility \bar{u}_i given an initial state (or a probability distribution over the possible initial states). We propose to the agents to form a specific coalition s^* (one that maximises utilitarian social welfare) and we propose that agents make side payments p_i so that each agent gets a utility $w_i = v_i(s^*) + p_i \geq \bar{u}_i$. The agent are better off following our proposal and stay in their respective coalitions: on expectation, each agent is better off. Hence, we can ensure stability. From the societal point of view, this solution is also good as we use an optimal CS. To obtain these good properties, we need to make a non-standard assumption: we need to allow side payments between members of different coalitions. Traditionally, side payments are only allowed between members of the same coalition. In our work, some coalitions will send payment to other coalitions to support stability.

The paper is organized as follows. In Section 2.1, we present the coalition framework and the existing stability concepts for coalition formation when in the non transferable utility case. In Section 4, we show how to build a Markov chain that models the coalition formation process, how to use it to compute the expected utility, and finally, we present the payoff obtained by the agents when they follow our proposed solution.

2 Coalition Framework

2.1 Problem Description

We consider a set N of n agents; N is also known as the grand coalition. A coalition structure (CS) $s = \{\mathcal{S}_1, \dots, \mathcal{S}_m\}$ is a partition of N , where \mathcal{S}_i is the i^{th} coalition of agents with $\cup_{i \in [1..m]} \mathcal{S}_i = N$ and $i \neq j \Rightarrow \mathcal{S}_i \cap \mathcal{S}_j = \emptyset$. \mathcal{S} is the set of all CSs. The coalition of agent i in s is noted as $s(i)$. We consider that an agent i has a preference order \succ_i over \mathcal{S} and for a CS s , an agent i has a utility $u_i(s)$. These assumptions have two consequences.

The first consequence is that each agent has a private utility which depends on the other agents present in the coalition, as is done in hedonic coalition formation [1, 3, 2, 6]. Coalitions do not always receive a reward as a whole: each

agent has a private cost and benefit which depends on the organization of the agents. Members of a coalition help each other, which can globally reduce the cost or increase the private benefit of each member. For example, soccer players have a private utility, or satisfaction, which depends on the other members of the team.

The second consequence is that, unlike in the hedonic coalition formation case, we are working in the more general case where the valuation of a coalition depends on the other coalitions present in the population. For an agent i such that $i \in \mathcal{C}$ and two CSs s_1 and s_2 such that $\mathcal{C} \in s_1$ and $\mathcal{C} \in s_2$, it is possible that $u_i(s_1) \neq u_i(s_2)$. In our soccer example, the satisfaction of a player in a team playing a league may also depend on how the remaining players are dispatched in the other teams, for example, he may prefer that the best players are put in different team than put all together in a dream team. A more generic example involves agents competing for an environmental niche. The payoff of a coalition may be higher when the competitors work alone than when the competitors also decide to team together to form a more competitive group. Ray and Vohra [11] consider this problem and propose a protocol where agents propose a coalition and a distribution of the coalitions' worth. Other agents can accept or reject the proposition. One issue is that, when proposing a coalition, an agent does not know which CS will ultimately form. Hence, the payoff distribution proposed by an agent is conditioned on the CS that is finally selected. Ray and Vohra consider that the agents' offer contains a payoff distribution for each possible CS, which is not realistic for large populations. But such elaborate offers allow them to show the existence of an equilibrium.

We further assume that there is no coordinated change of coalitions; one agent at a time can change coalition. This assumption prevents uncertainties about the state of the CS. For example, let agents i , j and k form singleton coalitions. At this point, agent i would like to join agent k , and agent j would like to join agent i , but neither i or j would like to form the grand coalition. If we allowed simultaneous moves, the resulting state would be unclear. The grand coalition may be formed though it was not the intent of agent i or j . The resultant CS could also be $\{\{i, k\}, \{j\}\}$ where agent i joined agent k , and agent j tried to join the coalition of agent i , but ended up joining an empty coal-

tion. It could also be $\{\{i, j\}, \{k\}\}$ where agent j joined agent i , and agent k refused that both agent i and j joined it at the same time. We avoid such ambiguities with this assumption.

Finally, we assume that agents are myopic and rational, and members of a coalition accepts a new member only when all members agree. After a change of CS, it is possible, if not likely, that another agent changes its coalition, leading to a different CS. As it is computationally expensive to perform multi-steps look ahead because of the large state space, we consider myopic agents that change coalition to maximize their immediate reward. We believe it is reasonable to assume that current members can control when other agents can join a coalition. Moreover, it would not be myopic rational for a member i to accept a new agent if this meant a payoff loss for i . Hence, we also assume that all members of a coalition must agree to accept a new member and, if some member i refuses, we will say that agent i vetoes the transition. We also make the implicit assumption that members of a coalition cannot prevent a member to leave, even if some of the remaining members lose utility.

The goal of our work is to choose a CS and a new, efficient payoff distribution that is also fair, i.e., given the existing valuation function, we compute a different valuation function that is fair, and agents will adjust their payoff by making side-payments.

2.2 Stability Concepts

We first start to give the definition of stability concepts in the non-transferable utility case when the value function depends only on the members of the coalition [3]

Definition 2.1. A coalition structure s is *core stable* iff $\nexists C \subset N \mid \forall i \in C, C \succ_i s(i)$.

Definition 2.2. A coalition structure s is *Nash stable* $(\forall i \in N) (\forall C \in s \cup \{\emptyset\}) s(i) \succeq_i C \cup \{i\}$

Definition 2.3. A coalition structure s is *individually stable* iff $(\nexists i \in N) (\nexists C \in 2^N \cup \{\emptyset\}) \mid (C \cup \{i\} \succ_i s(i))$ and $(\forall j \in C, C \cup \{i\} \succeq_j C)$

Definition 2.4. A coalition structure s is *contractually individually stable* iff $(\nexists i \in N) (\nexists C \in 2^N \cup \{\emptyset\}) \mid (C \cup \{i\} \succ_i s(i))$ and $(\forall j \in C, C \cup \{i\} \succeq_j C)$ and $(\forall j \in s(i) \setminus \{i\}, s(i) \setminus \{i\} \succeq_j s(i))$

If a CS is core stable, no subset of agents has incentive to leave their respective coalition to form a new one. In a Nash stable CS s , no single agent i has an incentive to leave its coalition $s(i)$ to join an existing coalition in s or create the singleton coalition $\{i\}$. The two other criteria add a constraint on the members of the coalition joined or left by the agent. For an individually stable CS, there is no agent that can change coalition from $s(i)$ to \mathcal{S} yielding better payoff for itself, and the members of \mathcal{S} should not lose utility. The contractually individual stability requires that in addition, the members of $s(i)$, the coalition left by the agent, should not lose utility.

The definition of Nash, individually and contractually individually stability can be extended to the case where the value of a coalition depends on the CS. Another criterion for a rational agent to be a member of a coalition is individual rationality: an agent i would consider joining a coalition only when it is beneficial for itself. The agent compares the situation when it is on its own and when it is a member of a coalition. However, the payoff the agent gets when it is by itself depends on the CS. The minimum payoff that agent i can guarantee on its own is $r_i = \min_{s \in \mathcal{S}, \{i\} \in s} v(s, i)$ [5] (the minimum is over all the CSs where agent i forms a coalition on its own). An agent is individual rational when its payoff in a coalition with other agents is greater than the minimum payoff it can get on its own.

For some coalition formation problem, it is possible that no CS satisfies any of these stability criteria. Satisfying the individually or contractually individually stability criteria may depend on the protocol used by the agents to form coalition. We can consider that agents in a coalition have the power to veto the entrance of a newcomer, but cannot prevent a member from leaving a coalition. For example, an academic can freely leaves its department to join a new one, provided that no member of the new department will suffer from its presence. In some cases, the coalition left is allowed to demand compensation. For example, as pointed out in [6], a player of a soccer team can join another club, but its former club can receive a compensation for the transfer. In the following, we will only assume that members of a coalition can veto the entrance of new agent in their coalition.

We can construct the **preference graph of the coalition formation process** by building a

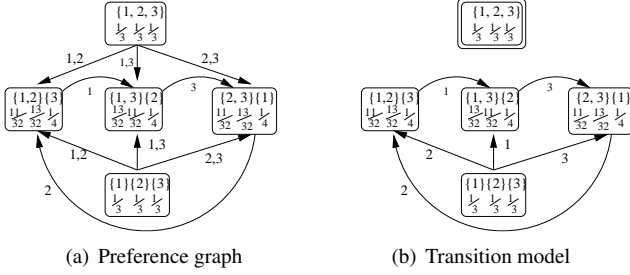


Figure 1: Preference graph and transition model for the same valuation function. The valuation function has one absorbing state and also contains a cycle (double boxed CS are absorbing CS).

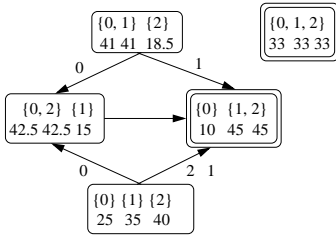


Figure 2: Transition model with two absorbing CSs (double boxed CS are absorbing CS)

graph where each node is a CS, and there is an arrow from node S to node T when $\exists i \in N \mid T \succ_i S$. From the preferences graph, we can construct the **transition graph of the coalition formation process** by first removing arrows that are vetoed, and by selecting only the change of coalition that maximizes immediate payoff.

Property 1. A CS s is individually stable iff there is no outgoing edges in the transition graph of the coalition formation.

The proof is obvious given the definition of the transition graph. In Figure 1, we present an example with three agents where the payoff of an agent is shown below its label in a coalition. In this example, no CS is core or Nash stable and $\{\{1, 2, 3\}\}$ is individually stable. However, if the agents start from the bottom of the lattice (where each agent forms a singleton coalition) or any other CS in the mid level, the agents will be trapped in a cycle: for each CS in the mid-level, one agent benefits from leaving its coalition in that CS to join the singleton agent. We present a different scenario in Figure 2: the CS $\{\{0\}\{1, 2\}\}$ is Nash stable, core stable (and hence individually stable), and the grand coalition is individually stable. From any CS, it is possible to reach an individually stable CS.

3 A Markov Chain model

A myopic rational agent will change coalitions if it can immediately gain utility by doing so. In this paper, we assume that the valuation is common knowledge. It is therefore possible to build and analyze the transition model. The agents will build a directed graph where the nodes represent the CSs, and edges are valid transitions between two CSs. A transition from node S_s to node S_d exists if there is an agent i such that:

- $S_d \succ S_s$, i.e., i is better off in S_d than in S_s .
- and this transition is not vetoed by the members of the coalition joined by i (of course, i is always allowed to form a singleton coalition).

Incidentally, another agent j may also prefer S_d over S_s (for example, when i moves to an existing coalition C , all agents in C may benefit). Hence, a transition may be beneficial for more than one agent. However, only the agent that changes the coalition can induce the transition. Even if it is beneficial for members of some coalition C that agent i joins, C 's members cannot force i to leave its current coalition to join them (this action would be considered to be a group action whereas in our model, we consider only individual actions). In the case where two agents i and j that were previously forming singleton coalitions now form a coalition of two agents in the new CS, it may be difficult to interpret which agent induced the transition: as it is beneficial for both agents, an interpretation of the transition can be that agent i joins the coalition $\{j\}$ or vice versa. Our interpretation is that both agents are responsible for this transition. Hence we make an exception for this case.

Since we assume that agents are myopically rational, for a given CS, each agent will only choose the transition that yields the maximum immediate payoff gain over all its possible legal moves. For each state, there can then be at most n outgoing edges, one for each of the n agents (this happens when every agent prefer another CS over the current one). This prunes the number of transitions from the preference graph to the transition graph.

We now have a transition graph that models the best action that each myopically rational agent can take. If a CS is absorbing, there is no outgoing edge in the graph from that CS. Otherwise,

there will be one outgoing edge for each agent that wants to change from the current CS. Given the assumption that only one agent at a time can change coalition, we are now in position to estimate the probability of transition between any two CSs. For each outgoing edge e from CS s , the probability of making this transition is

- $\frac{1}{o(s)}$, where $o(s)$ is the out degree of a node, i.e., the number of agents that want to change from s .
- $\frac{2}{o(s)}$ when two agents i and j that are each forming a singleton coalition merge to form the two-agent coalition $\{i, j\}$ and it is the best choice for both i and j .

As the probability of a transition does not depend upon the prior states of the population, the Markov assumption is verified. We have now completely defined a Markov chain. From the above specified transition model, we can construct the transition matrix P of the Markov chain. The size of the matrix is $\mathcal{B}(n) \times \mathcal{B}(n)$, where n is the number of agents and $\mathcal{B}(n)$ is the Bell function. The dimension of the matrix can be quite large, however, the matrix is sparse: for each row of the matrix, there can be only up to n positive entries¹.

To compute the expected utility, we study its long term behavior. A Markov chain has transient and ergodic states: ergodic states are states that the chain will keep coming back to, whereas transient states are states that the chain will eventually leave to never visit again. In the long term, the chain will be in one of the ergodic states. The ergodic states can form multiple strongly connected components. If the size of such a strongly connected component is one, it means that the corresponding CS is individually stable (it may also be core or Nash stable, but not necessarily). The study of the Markov chain will tell us, given a probability distribution over the initial state, the probability to reach each strongly connected component, and, once reached, what is the proportion of time spent in each ergodic states. Hence, the value of the expected utility is an average over the possible stable CSs, and the CSs that are parts of some cycle.

¹ \mathcal{S} can be represented by a lattice where each CS at a given level of the lattice contains the same number of coalitions. For each level i in the lattice, an agent has at most i actions: joining one of the existing $i - 1$ coalitions and forming a singleton coalition if it is not already forming one. As there are n levels, the maximum number of transitions from a CS is n .

	$\{1, 2, 3\}$	$\{1, 2\}\{3\}$	$\{1, 3\}\{2\}$	$\{2, 3\}\{1\}$	$\{1\}\{2\}\{3\}$
$\{1, 2, 3\}$	1	0	0	0	0
$\{1, 2\}\{3\}$	0	0	1	0	0
$\{1, 3\}\{2\}$	0	0	0	1	0
$\{2, 3\}\{1\}$	0	1	0	0	0
$\{1\}\{2\}\{3\}$	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0

Table 1: Transition Matrices for Figure 1

In Table 1, we present the transition matrix for the example of Figure 1. As agents change from one CS to another, the chain moves from one state to another. We are interested in the long-term characteristics of the chain. Depending on the property of the Markov chain, the agents may be trapped in one of the strongly connected components of the chain. Let \mathcal{E} be the set of ergodic states of the Markov chain. For each component $X \subset \mathcal{E}$, we compute the probability p_X to reach X , and then for each state s of X , we compute the fraction of time p_s spent in s of X in the limit. If a CS s is at least individually stable, then the size of the corresponding component is one, and $p_s = 1$. For each ergodic state $s \in \mathcal{E}$, let $X(s)$ be the component of s . The expected utility $E(v_i)$ is then

$$E(v_i) = \sum_{s \in \mathcal{E}} p_{X(s)} \cdot p_s \cdot v(i, s).$$

In Figure 3, we present an example issue from the Agent Reputation and Trust testbed [7]. In the testbed, agents provide appraisals about artifacts and compete for a pool of clients. To improve their appraisals, agents can ask other agents for appraisals for artifacts and reputation of other agents. We consider collusion of agents: agents can form a coalition where members provide their truthful appraisals, which benefits all members. In a domain with 8 agents, we computed the valuation function and the associated Markov chain for a particular instance, and the outcome is presented in Figure 3. In that instance, the Markov chain contains 4,140 CSs, 26,641 transitions, 62 stable CSs and 5 additional ergodic states which correspond to some strongly connected components.

4 A Fair Payoff Distribution for Myopic Rational Agent

It is possible that some coalition formation problem do not have any stable CS. To operate efficiently, we require that the agents remain in

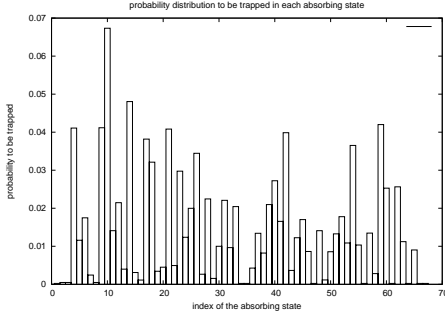


Figure 3: Example of the ART domain: probability to be in an ergodic state

a CS. We propose that the agent forms a CS s^* that maximizes social welfare. However, s^* may not be stable, hence we want to share the utility u^* of s^* that provides the agent an incentive to stay in that CS.

The utility function, as a whole, tells how good the agent is. A first candidate is to share u^* proportionally to the average utility over all the CSs. This assumes that each CS is equally important and we believe it is not so. Another candidate is to consider an average over the stable CSs. However, such stable CSs may not always exist, and even if they do, there may not be a path allowing to reach a stable CS (as in the example of Figure 1). If we assume any CS is likely to be the initial CS, we can compute an expected utility when the agents are myopic, rational, and when members of a coalition can veto the entrance of new members. The expected utility is a great metric to determine, and compare the strength of each agent in the coalition formation process. We will show that the payoff obtained is at least its expected utility, which is a sufficient incentive for using our proposed payoff distribution.

4.1 Choice of Final Payoff Distribution and Corresponding CS

The expected value $E(v_i)$ we computed using the Markov chain assumes that the initial CS is chosen uniformly over \mathcal{S} , in other word, it is no biased by the initial CS². $E(v_i)$ reflects the utility that agent i receives on average when all agents are myopically rational. We consider that this value represents the strength of an agent

²Of course, if an application specifies a probability distribution over the initial CSs (or requires to start from a specific CS such as the one where each agent is in a singleton), we can still use our analysis to compute the expected utility.

given the valuation function. Agents with high $E(v_i)$ should obtain a larger payoff than agents with lower $E(v_i)$.

To be used in a real world application, it is not desirable to have agents continuously change coalitions: agents should form a stable CS and have no incentive to further change coalition. To maximize the agents' payoff, we choose as the final CS s^* one of the CSs that maximizes social welfare. This CS may not be a stable, but it guarantees maximal total payoff to the agents. As we view the expected utility value as a measure of the strength of each agent, we propose a distribution of $v(s^*)$ to all agents proportional to the expected payoff of the agents, i.e., we prescribe the payoff to agent i to be

$$u_i = \frac{E(v_i)}{\sum_{j \in N} E(v_j)} v(s^*).$$

Note that this value is guaranteed to be at least as good as $E(v_i)$, as shown by Property 2. So, when agents share the payoff we propose, they are guaranteed to have at least the expected value when they were changing coalitions to maximize their immediate reward, and in general, they may get more. In addition, the payoff distribution is Pareto Optimal as we share the value of a social welfare maximizing CS (if an agent gets more utility, at least another agent must lose some). We believe that these incentives are sufficient for the agents to accept our proposed value. Not only is the payoff distribution fair, as the share of utility the agents receive is proportional to their expected utility over the chain, but the outcome is also efficient as it maximizes social welfare.

Property 2. $u_i = \frac{E(v_i)}{\sum_{j \in N} E(v_j)} v(s^*) \geq E(v_i)$, i.e., the payoff of an agent is at least as good as the expected utility that an agent would get on average if the agents are myopically rational.

Proof. Let \mathcal{E} denote the set of the ergodic states of a Markov chain. For player i , the expected payoff is a weighted average over the ergodic states: $E(v_i) = \sum_{s \in \mathcal{E}} \alpha_s v(i, s)$, where α_s is the weight of the ergodic state s and we have $\sum_{s \in \mathcal{E}} \alpha_s = 1$. The transient states are only used to determine the probability of leading to one of the ergodic sets: the α_s 's are determined by the transient and the ergodic states (when there is a cycle or a regular sub-chain).

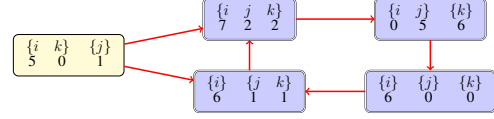
$$\begin{aligned}
\forall s, v(s) &\leq v(s^*) \\
\forall s, \alpha_s \cdot v(s) &\leq \alpha_s \cdot v(s^*) \text{ as } \alpha_s \geq 0 \\
\sum_{s \in \mathcal{E}} \alpha_s \cdot v(s) &\leq \sum_{s \in \mathcal{E}} \alpha_s \cdot v(s^*) \\
\sum_{s \in \mathcal{E}} \alpha_s \cdot v(s) &\leq v(s^*) \cdot \sum_{s \in \mathcal{E}} \alpha_s \\
\sum_{s \in \mathcal{E}} \alpha_s \cdot v(s) &\leq v(s^*), \text{ as } \sum_{s \in \mathcal{E}} \alpha_s = 1 \\
\sum_{s \in \mathcal{E}} \alpha_s \sum_{j \in N} v(j, s) &\leq v(s^*) \\
\sum_{j \in N} \sum_{s \in \mathcal{E}} \alpha_s v(j, s) &\leq v(s^*) \\
\sum_{j \in N} E(v_j) &\leq v(s^*) \\
E(v_i) &\leq \frac{E(v_i)}{\sum_{j \in N} E(v_j)} v(s^*) \\
&\text{as } E(v_i) \geq 0.
\end{aligned}$$

□

Another important question is to determine whether the payoff distribution v_i is individually rational: is an agent guaranteed to get as much as when an agent is forming a singleton coalition? The minimum payoff an agent can guarantee for itself is $r_i = \min_{s \in \mathcal{S}, \{i\} \in s} v(i, s)$. For example, consider the three-agent example in Figure 4. The value obtained by i is $\frac{209}{36} = 5.806$, which is lower than 6, the minimum payoff that agent i receives when it forms a singleton coalition. This means that the payoff obtained by an agent in a coalition from our protocol is less than the worst payoff obtained by the agent when it forms a singleton coalition. Although possible in the general case, this may not be likely in practice: the worst case scenario for an agent should be when it forms a singleton coalition and when all other agents in the population try to minimize its payoff. As shown by Property 3, if the worst payoff for an agent occurs when it is forming a singleton coalition, our protocol is individual rational.

Property 3. *If $(\forall s \in \mathcal{S}) v(i, s) \geq r_i = \min_{s \in \mathcal{S} | \{i\} \in s} v(i, s)$, then $u_i \geq r_i$, i.e., the payoff distribution u_i is individually rational.*

Proof. The hypothesis $\forall s \in \mathcal{S}, v(i, s) \geq r_i$ means that for any CS, the valuation of agent i is at least equal to i 's minimum valuation when it forms a singleton coalition, i.e., the payoff of an agent in a coalition with at least another agent should be at least the minimum payoff the



There is a cycle with 4 states, hence, the proportion spent in each state is $\frac{1}{4}$. The value of the optimal CS is 11. The minimum value of agent i when it is in a singleton coalition is 6. $E(v_i) = \frac{1}{4}(7+0+6+6) = 4.75$, $v_i = \frac{4.75}{1.75+2+2.25} \cdot 11 = 5.8056 < 6 = \frac{216}{36}$
 $E(v_j) = \frac{1}{4}(2+5+0+1) = 2$, $v_j = \frac{2}{1.75+2+2.25} \cdot 11 = 2.4444 > 0$
 $E(v_k) = \frac{1}{4}(2+6+0+1) = 2.25$, $v_k = \frac{2.25}{1.75+2+2.25} \cdot 11 = 2.75 > 0$

Figure 4: Case where the protocol is not individually rational: i 's payoff is lower than r_i , i 's minimum payoff when it forms a singleton.

agent receives when it is on its own in a singleton coalition. Hence, we have $\sum_{s \in \mathcal{E}} \alpha_s v(i, s) \geq \sum_{s \in \mathcal{E}} \alpha_s r_i$, and then $E(v_i) \geq r_i$ as $\sum_{s \in \mathcal{E}} \alpha_s = 1$. From Proposition 2, we have $u_i \geq E(v_i) \geq r_i$. □

4.2 Computational Complexity of the centralized algorithm

We now consider the complexity of computing the payoff distribution if a centralized entity was used. To compute the canonical form of a stochastic matrix, we first need to compute the communication classes of the matrix and this operation is polynomial in the size of the matrix ($O(\mathcal{B}(n)^2)$). Then, to determine the canonical form of the matrix, we need to find the permutation matrix, which can also be done in quadratic time, hence in $O(\mathcal{B}(n)^2)$. To compute the limit behavior of the Markov chain, either a matrix has to be inverted (which can be done in $O(\mathcal{B}(n)^3)$, or a linear system needs to be solved (iterative methods can also be used here). The complexity is then $O(\mathcal{B}(n)^3)$. The fact that the matrix is sparse should allow for faster computation. The search of the optimal CS is $O(\mathcal{B}(n))$ if the brute force method is applied. As we consider valuation function that depends on CS, we cannot use the faster algorithm in [10]. The computation of the side-payments and the execution of the payments has linear complexity. Hence, the complexity of the protocol is $O(\mathcal{B}(n)^3)$.

4.3 Discussion on the payoff distribution

Our protocol uses global properties of the valuation function and shares the utility of the optimal CS, s^* , in a fair manner. The distribution of the valuation of s^* , however, is not according to the actual coalitions present in s^* .

In other words, given the payoff function v_i , it is possible that, for each coalition $\mathcal{C} \in s^*$, $\sum_{i \in \mathcal{C}} u_i \neq \sum_{i \in \mathcal{C}} v(i, s^*)$.

This is different from the traditional assumption in game theory where agents share the value of their coalition. For some agents i , $v(i, c^*) > u_i$, and these agents may not consider the payoff distribution fair. The alternative for these agents, however, is to reject the proposed final CS and payoff distribution and continue to change coalitions to maximize immediate reward. From Proposition 2, we see that the expected utility from such a process may be smaller than the value proposed by the protocol and hence the agents have an incentive to accept the guaranteed value while saving on the “cost” of continual change. Hence, on one hand, we want the entire population of agents to cooperate and work together, which has a flavor of using the grand coalition. On the other hand, we want to use the synergy between the agents, and thus form a CS that maximizes social welfare. The reward the agent obtain is designed to be fair for all agents and reflects the performance of the agents over all CSs.

To compute the expected utility of an agent, we have assumed that the coalition formation process starts in a CS picked randomly from a uniform distribution. Of course, some probability distribution for the initial CSs will benefit some agents in detriment of others. We believe that the probability distribution of the initial CS is part of the definition of the coalition formation problem, and agents do not have any control over it. It is from the entire definition of the coalition formation problem that we compute the expected utility, which we use as a measure of the strength of an agent. If it the distribution is not uniform, the probability to reach the strongly connected components will be different (some components may not be reachable). In addition, the search of the CS that maximizes social welfare should be performed on the subset of CSs that are reachable from the set of possible initial CSs. Minor modification of our computations are needed to address these changes.

5 Conclusion, current and future work

Myopic rational agents who receive a private payoff that depends on the CS may never reach an agreement on the CS to be formed. It may be

possible that for each CS, at least one agent has an incentive to change coalition. We designed a protocol that computes a payoff distribution so that agents are guaranteed to have at least the expected utility from a process where each agent would change coalition to maximize its immediate reward. The protocol assumes that 1) the valuation function provides a payoff for each individual agent given a CS and 2) the agents are myopically rational. The payoff function we propose is based on the value of a social welfare maximizing CS and on the expected utility of the agents if they try to change coalitions to maximize their immediate reward. Following our protocol, the agents form the optimal CS, which makes the multiagent system efficient from the viewpoint of a system designer. The valuation of the optimal CS is shared proportionally to the expected utility of the agents. We argue that this is a fair distribution as the payoff obtained by an agent reflects the behavior of the agents over the entire space of CSs, i.e., it is a global property of the valuation function. When the agents follow our protocol, they are guaranteed to have a payoff which is at least their expected value if all agents try to maximize their immediate reward.

The drawback of our approach is its computational cost: the agents needs to build a Markov chain where the number of states is equal to the number of the CSs, which grows even faster than exponentially in the number of agents. Although the corresponding transition matrix is sparse, this method may not be suitable for large number of agents (10 and more). The agents can approximate the expected value by simulating the Markov chain. In that case, they only need to be able to evaluate the best coalitional move from a given CS.

Because of the computational cost, we are studying algorithms to approximate the computation of the Markov chain. By sampling the chain, we can obtain rapidly good estimate of the expected utility of the agents.

Another line of research is to look for a representation that can compactly represent much more compactly the transition graph. For example, if we represent the game using the idea of a marginal contribution network [8], it is unclear whether we can represent the Markov game much more compactly.

When the payoff of an agent is its marginal contribution, the expected payoff has some flavor of the Shapley value. A first study would be to

compare our value with existing variants of the Shapley value (see some references in [9]). Another interesting line of research is to come back to the traditional coalitional games where the value of a coalition only depends on the members of that coalition. In the coalition formation process, we can consider that an agent gets its marginal contribution. Then, an agent has an incentive to change coalition when its marginal contribution improves. This would be a more realistic assumption on the formation of coalitions. We will need to check whether we can always solve the Markov chain and if so, we will need to study the properties of the payoffs.

References

- [1] J. Alcalde and A. Romero-Medina. Coalition formation and stability. *Soc Choice Welfare*, 27(2):365–375, 2006.
- [2] S. Banerjee, H. Konishi, and T. Sönmez. Core in a simple coalition formation game. *Soc Choice Welfare*, 18(1):135–153, 2001.
- [3] A. Bogomolnaia and M. O. Jackson. The stability of hedonic coalition structures. *Games and Economic Behavior*, 38(2):201–230, 2002.
- [4] M. Davis and M. Maschler. The kernel of a cooperative game. *Naval Research Logistics Quarterly*, 12, 1965.
- [5] T. Dieckmann and U. Schwalbe. Dynamic coalition formation and the core. *Journal of Economic Behavior & Organization*, 49(3):363–380, November 2002.
- [6] J. H. Drèze and J. Greenberg. Hedonic coalitions: Optimality and stability. *Econometrica*, 48(4):987–1003, 1980.
- [7] K. K. Fullam, T. B. Klos, G. Muller, J. Sabater, A. Schlosser, Z. Topol, K. S. Barber, J. Rosenschein, L. Vercouter, , and M. Voss. A specification of the agent reputation and trust (ART) testbed: Experimentation and competition for trust in agent societies. In *Proceedings of the Fourth International Joint Conference on Autonomous Agents and Multiagent Systems*, pages 512–518. ACM Press, 2005.
- [8] S. Jeong and Y. Shoham. Marginal contribution nets: a compact representation scheme for coalitional games. In *EC '05: Proceedings of the 6th ACM conference on Electronic commerce*, pages 193–202, New York, NY, USA, 2005. ACM Press.
- [9] T. Michalak, T. Rahwan, D. Marciniak, M. Szamotulski, and N. R. Jennings. Computational aspects of extending the shapley value to coalitional games with externalities. In *Proceeding of the 2010 conference on ECAI 2010: 19th European Conference on Artificial Intelligence*, pages 197–202, Amsterdam, The Netherlands, The Netherlands, 2010. IOS Press.
- [10] T. Rahwan, S. D. Ramchurn, V. D. Dang, and N. R. Jennings. Near-optimal anytime coalition structure generation. In *Proceedings of the Twentieth International Joint Conference on Artificial Intelligence (IJCAI'07)*, pages 2365–2371, January 2007.
- [11] D. Ray and R. Vohra. A theory of endogenous coalition structures. *Games and Economic Behavior*, 26:286–336, 1999.
- [12] D. Schmeidler. The nucleolus of a characteristic function game. *SIAM Journal of applied mathematics*, 17, 1969.