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# Remaining Useful Life prediction method using an observer and statistical inference estimation methods

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## ABSTRACT

In this paper, we propose an approach for failure prognosis. The approach deals with a closed-loop control system in which the actuator stochastically degrades through time. The degradation of a system disturbs and affects its characteristic parameters. This is reflected by a change in one or more of them. The latter may remain partially or totally hidden given that the type of control. The aim of this work was to estimate online the duration before the system performance requirement is no longer met. This without adding sensors. The proposed approach is based on the system behavior model. The models describing the dynamics of the parameters have been assumed to be known a priori, but degradation is assumed to be unmeasurable. It was conducted in two phases: the first used the data available on this system to estimate unmeasured states and relevant parameters which are able to characterize system performance. To carry out this phase, we used an observer. In the second phase, to estimate when the desired performance will no longer be met over a specified mission, the historical states and parameters obtained in the first phase were exploited. Thus, in order to identify the models describing the parameter dynamics, statistical inference estimation methods such as the maximum likelihood method and the Bayesian estimation were used. To illustrate the performances of the approach, a simulated tank level control system was used.

## 1. INTRODUCTION

Currently, due to the strong interactions between systems

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and processes, monitoring and anticipating degradation have become very complex. In addition to the hybridity (continuous and /or discrete) of deterministic processes, the stochastic character imposed by component failures or uncertainties in the knowledge of the system must be taken into account. In this setting, many studies on the predictive assessment of systems operation have been conducted in recent years (Sikorska, Hodkiewicz and Ma, 2011). This predictive assessment is mainly based on prognosis (Dragomir, Gouriveau, Dragomir, Minca and Zerhouni, 2009). While diagnosis consists in detecting, isolating and identifying failures and their causes, prognosis aims to predict the future state before failures occur (Jardine, Lin and Banjevic, 2006), (Lee, Wu, Zhao, Ghaffari, Liao and Siegel, 2014). Its key element is estimation of the Remaining Useful Life (RUL) (Si, Wang, Hu and Zhou, 2011). Various approaches have been established in the literature to address issues related to prognosis. These approaches have been subject to several classifications (Jardine et al, 2006), (Byington and Roemer, 2002) and (Heng, Zhang, Tan and Mathew, 2009), the most common classification being whether the approach is based on: a physical model, or an experience-based model and or data-driven model. Prognosis can also be based on the fusion of more than one approach (Pecht and Jaai, 2010). Prognosis approaches differ in terms of data used, assumptions related to failure, operating modes and modeling strategies.

The proposed approach exploits the system behavior model. It deals with a closed-loop control system subject to degradations. The aim of this work was to estimate online the duration before the system performance requirement is no longer met. The estimate must be achieved without adding sensors.

The paper is organized as follows. Section 2 describes the closed-loop control system and presents the two phases of the proposed approach. Section 3 presents some evaluation metrics for prognosis approaches. Section 4 illustrates the approach on a tank level control system and Section 5 concludes the paper.

## 2. GENERAL SYSTEM AND PROGNOSIS METHODOLOGY

### 2.1. Closed-loop Control System

A control system is a set of interacting physical components. The aim is to adjust the system output so as to achieve the desired results from the control variables. In this work, we consider a closed-loop control system in which the actuator stochastically degrades through time (Figure 1). Thus, for this control structure, the effects of disturbances and / or degradation signals are compensated.

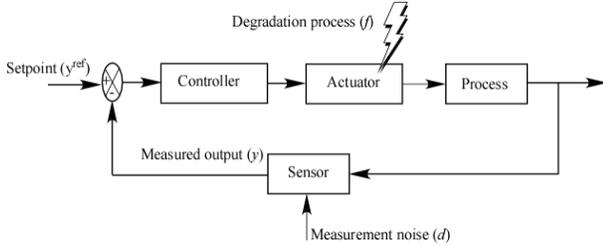


Figure 1. A closed-loop control system.

The command applied on the actuator is calculated in such a way that the difference between the inputs-outputs verifies the following constraint:

$$\lim_{t \rightarrow \infty} (y^{ref}(t) - y(t)) \approx 0 \quad \forall d(t), f(t) \quad (1)$$

where  $y^{ref}(t)$  is the set point (desired input),  $y(t)$  the controlled output,  $d(t)$  the noise and  $f(t)$  the degradation signals.

The difficulty in applying prognosis to a closed-loop control system is due to the fact that a small difference between the controlled output and the setpoint may be analyzed as no degradation whereas it is due to effective control. Furthermore, the system input is correlated with the system output by the feedback. This leads to "exciting" the system with a signal already affected by the disturbance and / or degradation signal.

### 2.2. Prognosis methodology

The proposed approach is based on the Model-based prognosis architecture for purposes of practical use (Figure 2) presented in (Sankararaman, Daigle and Goebel, 2014) and (Daigle and Goebel, 2011). The model must be able to represent the behavior of the system.

The approach consists of two phases:

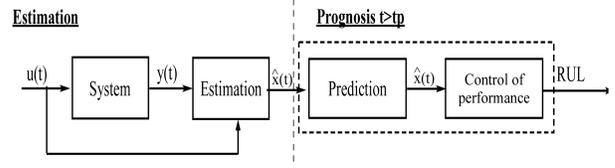


Figure 2. Model-based prognosis architecture (Sankararaman et al, 2014), (Daigle et al, 2011).

- The first phase involves simultaneous estimation of states and parameters. This phase consists in exploiting the various measures collected on the system to estimate unmeasured states and relevant parameters which are able to characterize system performance. Degradation of a system disturbs and affects its characteristic parameters. This is reflected by a change in one or more of them. In these conditions, monitoring the parameters enables any deviations from nominal behavior of the system to be detected. To carry out this phase we used an observer increased by the parameter vector.

- The second phase consists of determining when the desired performance will no longer be met over a specified mission time. Predicting the performance of the system begins with the identification of models describing the parameter dynamics. Then, the estimation of the RUL is obtained by comparing the performance estimated by simulation with the desired performance.

The following subsections describe in more detail the two phases and the tools used.

#### 2.2.1. Estimation phase

This phase exploits the different data collected on the system to estimate the states and unmeasured parameters that can characterize system performance.

In this phase, we consider an estimation model as follows:

$$\begin{cases} \dot{\hat{X}} = H(\hat{X}, u, t) \\ \hat{y} = G(\hat{X}, u, t) \end{cases} \quad (2)$$

with  $\hat{X}$  is the estimate of augmented state vector  $[\hat{x}, \hat{\theta}]^T$ , where  $\hat{\theta}$  is the estimated parameter vector;  $u$  is the system input and  $\hat{y}$  the estimated output.  $H$  and  $G$  are functions supposed known. It can be noted that the quality of the estimate, will depend on the system considered and the observer used. Hence, for this phase, it is important to select the most suitable methodology for observer synthesis.

#### 2.2.2. Prognosis phase

A realistic estimate of the RUL of a system at a time ( $t_p$ ) requires considering values of the states/parameters at this time  $\hat{X}(t_p)$ , future operating conditions ( $u_t, t > t_p$ ) (assumed known) and future values of the parameters ( $\hat{\theta}_t, t > t_p$ ) (Figure 3). For the estimation of the latter, we

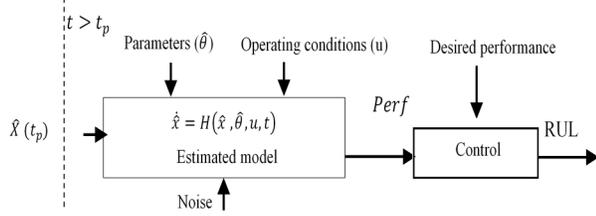


Figure 3. The variables considered in the prognosis phase.

propose to exploit the history of their estimates  $(\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \dots, \hat{\theta}_{t_p})$  by the using statistical inference estimation methods. Methods used will be described in the following subsection. If different modes of operation corresponding to different setpoint speeds are present during the degradation, for example, if there are two modes: normal mode and stressed mode, it is possible to integrate stress into the modelling by a coefficient.

The system is considered to have failed when it is no longer able to meet the desired performance. The performance of a control system can be evaluated using its time responses. The main criteria are speed and amortization in the transitional regime, stability and accuracy in the steady state. Therefore, performance (*Perf*) can be expressed through a set of constraints  $C_i$  on  $(\hat{x}, \hat{\theta}, u, t)$ , with  $C_i(\hat{x}, \hat{\theta}, u, t) = 1$  if the constraint is satisfied and 0 otherwise. These combined constraints can be expressed as follows:

$$Perf(t) = \begin{cases} 1, & \forall i, C_i(\hat{x}, \hat{\theta}, u, t) = 1 \\ 0 & \end{cases} \quad (3)$$

Accordingly the RUL at time  $t_p$  is defined by:

$$Rul_{t_p} = \inf(t_i \in \mathbb{R}: t_i \geq t_p \wedge Perf(t_i) = 0) - t_p \quad (4)$$

### 2.2. 3. Identification of models describing the parameter dynamics

For identification of models describing the parameter dynamics, several approaches were used in the literature. For example, in (Chelidze and Chatterjee, 2002), the system was considered as a hierarchical dynamical system consisting of a “fast time” directly observable, subsystem coupled to a “slow time” subsystem. The tracking procedure was based on phase space reconstruction. In (Gucik-Derigny, Outbib and Ouladsine, 2016), the problem of determining the damage state and the parameters of its dynamics was expressed as a problem of unknown input reconstruction. In (Daigle et al, 2011), the methodology adopted was based on a particulate filter. It was applied to a model of a pneumatic valve with different damage mechanisms. In (Aggab, Kratz, Vrignat and Avila, 2017), assuming unknown the models describing the parameter dynamics, the approach exploited history of the estimates  $(\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \dots, \hat{\theta}_{t_p})$  by using learning-based methods. In particular, the Support Vector

Regression (SVR) and the Adaptive Neuro Fuzzy Inference System (ANFIS) were used. It was applied to a model of a Li-ion battery. In this paper, the models describing the dynamics of the parameters have been assumed to be known a priori. These models can be obtained by collecting data, provided by manufacturers or established by experts. Of course, these models can be tainted with uncertainties and errors. Therefore, we use statistical inference estimation methods. The problem to solve is to determine a density function  $f(\hat{\theta}|\phi)$  where the parameter vector  $\phi = \phi_1, \dots, \phi_n$  is unknown. Once the parametric model is constructed, the aim is to perform an inference on the unknown parameters  $\phi_1, \dots, \phi_n$ . In the literature, several methods have been described (Greenland, 2011). We examined the known methods: the maximum likelihood method (i.e. the history of the estimates obtained in the first phase  $(\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \dots, \hat{\theta}_{t_p})$  is the sole source of knowledge), and the Bayesian estimation (i.e. in addition to estimates of the history, knowledge of the model parameters is added).

#### 1. Estimation by Maximum Likelihood

In this method, only historical data  $(\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \dots, \hat{\theta}_{t_p})$  are used to estimate the unknown model parameters  $\phi = \phi_1, \dots, \phi_n$ . A likelihood function is used. It is written as follows:

$$l(\phi|\hat{\theta}) = f(\hat{\theta}|\phi) \quad (5)$$

where  $l(\hat{\theta}|\phi)$  characterizes the likelihood of the data knowing the parameters  $\phi$ .

The estimator that maximizes this function is given by:

$$\hat{\phi}_{MV} = \arg \max [l(\phi|\hat{\theta})] \quad (6)$$

#### 2. Bayesian estimation

The Bayesian estimation is based on Bayes' theorem, for two random events A and B. This theorem is written as follows:

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(B|A)\mathbb{P}(A)}{\mathbb{P}(B)} \quad (7)$$

where  $\mathbb{P}(A)$  represents the a priori probability and  $\mathbb{P}(A|B)$  is the posterior probability.

In contrast to the parametric estimation based on maximum likelihood, Bayesian estimation assumes that the parameters of interest  $\phi_1, \dots, \phi_n$  are considered as random variables characterized by probability densities  $\pi(\phi)$ . These densities are called a priori densities.

To estimate the parameters a posteriori, the Bayes formula is used. It gives the following result:

$$\pi(\phi|\hat{\theta}) = \frac{f(\hat{\theta}|\phi)\pi(\phi)}{\int_{\phi} f(\hat{\theta}|\phi)\pi(\phi)d\phi} \quad (8)$$

The estimator that maximizes this function, called a maximum a posteriori (MAP) is given by:

$$\hat{\phi}_{MAP} = \arg \max [l(\phi|\hat{\theta}) \pi(\phi)] \quad (9)$$

### 3. PROGNOSIS PERFORMANCE METRICS

The proposed approach was assessed by implementing certain metrics proposed in the literature (Vachtsevanos, Lewis, Roemer, Hess and Wu, 2006), (Kurfess, Billington and Liang, 2006) and (Medjaher, Tobon-Mejia and Zerhouni, 2012). The metrics associated with the calculation of the RUL were chosen. They are described below:

Accuracy: a value close to zero means that the prognosis is poor while a value close to 1 corresponds to a good prognosis.

$$Accuracy = \frac{1}{T} \sum_{t=1}^T e^{-\frac{|RUL_{real}(t) - RUL(t)|}{RUL_{real}(t)}} \quad (10)$$

Precision: this measure quantifies the dispersion of the prognosis error around its mean.

$$Precision = \sqrt{\frac{\sum_{t=1}^T (\varepsilon(t) - \bar{\varepsilon})^2}{T}} \quad (11)$$

where:  $\varepsilon(t) = RUL_{real}(t) - RUL(t)$  and  $\bar{\varepsilon} = \frac{1}{T} \sum_{t=1}^T \varepsilon(t)$

Mean Absolute Percentage Error (MAPER): this measure quantifies the mean percentage error.

$$MAPER = \frac{1}{T} \sum_{t=1}^T \left| \frac{100 \cdot \varepsilon(t)}{RUL_{real}(t)} \right| \quad (12)$$

Prognostics Horizon (PH): it estimates the time at which the prognosis approach gives its first prediction within the confidence interval defined by  $a_c \in [0; 1]$ .

$$PH = T - t \quad \forall RUL(t) \in RUL_{real}(t) \cdot [(1 - a_c), (1 + a_c)] \quad (13)$$

Relative Accuracy measure (RA): it assesses the accuracy of the estimate at different times.

$$RA = 1 - \frac{|RUL_{real}(t) - RUL(t)|}{RUL_{real}(t)} \quad (14)$$

### 4. APPLICATION AND SIMULATION RESULTS

In order to illustrate the proposed approach, we consider a double-tank level control system with a stochastic degradation process for the pump motor. This case is presented in (Nguyen, Dieulle and Grall, 2014).

#### 4.1. Description of the system

The double-tank level control system is shown in Figure 4. The water is injected into the first tank with cross-sectional area  $S_1$  by a pump motor drive. Then, the outflow through valve 1 feeds the second tank with cross-sectional area  $S_2$ . The water level of the second tank is the system output (controlled measure). It is measured by a level measurement sensor and controlled by adjusting the pump motor control input which is calculated by a PID controller.

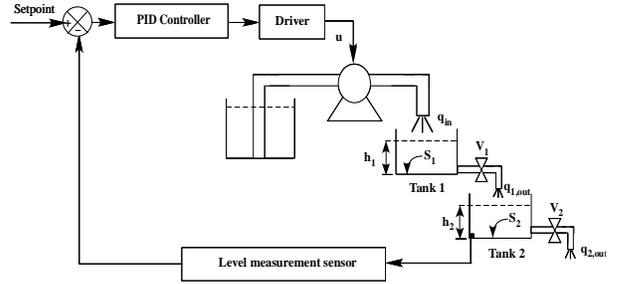


Figure 4. A double-tank level control system.

In order to consider the real response of the pump motor, the relation between the inlet flow rate  $q_{in}$  and the pump motor control input  $u$  is represented as a first order system.

$$\frac{dq_{in}}{dt} = -\frac{1}{\tau_a} q_{in} + \frac{K_a}{\tau_a} u \quad (15)$$

where  $\tau_a$  is the time constant of the pump motor,  $K_a(t)$  is the servo amplifier gain with the initial gain  $K_a(0) = K_{a\_init}$ .

The pump saturates at a maximum input  $u_{max}$ . Therefore, regardless of the time  $t$  considered  $u(t) \in [0; u_{max}]$ .

The water flows out at the bottom of each tank through valves at flow rates according to Torricelli's law:

$$q_{j,out} = K_{Vj} \sqrt{2gh_j} \quad j=1, 2 \quad (16)$$

where  $h_j$  is the level of tank  $j$ ,  $g$  is the acceleration of gravity, and  $K_{Vj}$  is the specified parameter of the valve  $j$ .

Using the mass balance equation, the process can be described by the following equations

$$\begin{cases} \frac{dh_1(t)}{dt} = \frac{1}{S_1} q_{in}(t) - \frac{K_{V1}}{S_1} \sqrt{2gh_1(t)} \\ \frac{dh_2(t)}{dt} = \frac{K_{V1}}{S_2} \sqrt{2gh_1(t)} - \frac{K_{V2}}{S_2} \sqrt{2gh_2(t)} \end{cases} \quad (17)$$

The aim of control is to maintain the water level of the second tank (only measured) at the desired setpoint  $y^{ref}$ . The measurement of the water level in the second tank  $h_2^m$  given by a level measurement sensor is affected by measurement noise  $d$ .

$$h_2^m(t) = h_2(t) + d(t) \quad (18)$$

#### 4.1.1. System Performance

To evaluate system performance, we consider its time response. The system is considered failed if one of the two constraints below is no longer satisfied.

- $C_1$ : constraint related to the transitional regime: the system is no longer able during a set-point change to satisfy the time necessary to reach the area  $\pm 5\%$  of change in less than  $T_1$  (in our example equal to 1.5 response time obtained under nominal conditions).
- $C_2$ : constraint in steady state: the system is no longer able to satisfy the condition to find the area  $\pm 5\%$  in a time less than  $T_2$  (in our example equal to half the time response).

It must be noted that other constraints can be selected.

#### 4.1.2. Degradation

Control systems may be viewed as subject to deterioration via actuators (Nguyen et al, 2014). Indeed, the loss of partial or total capacity of an actuator may cause a system performance loss (in that it changes its behavior from the desired behavior). For the degradation process, it is assumed that the pump (actuator) capacity decreases from its nominal capacity according to a non-stationary Gamma Process  $Ga(m(t), \beta)$  with  $m(t)$  of the form  $pt^q$ . It should be noted that other models can be used for describing the degradation process. Hence, the capacity at time  $t$  before its failure can be expressed as:

$$K_a(t) = K_a(0) - Z(t) \quad (19)$$

where  $K_a(0)$  is the initial capacity of the pump, and  $Z(t)$  describes the accumulated degradation of the pump at time  $t$ .

For all  $t > 0$  and  $\Delta t > 0$ , the law of growth  $Z(t + \Delta t) - Z(t)$  is a gamma distribution  $Ga(m(t + \Delta t) - m(t), \beta)$  with a density:

$$f(z) = \frac{\beta^{-(m(t+\Delta t)-m(t))}}{\Gamma(m(t+\Delta t)-m(t))} z^{(m(t+\Delta t)-m(t))-1} e^{-\frac{z}{\beta}}, z \geq 0 \quad (20)$$

with  $\Gamma$  gamma function.

#### 4.1.3. System Operation

In this subsection, an example (Figure 5) of the system behavior until failure with 2 set-points whose evolution is described by a Markov chain is presented. The pump capacity decreases according to a non-stationary Gamma Process. The numerical values summed up in (Table 1) are associated with the behavior of the system (Nguyen et al, 2014). For the tests performed, we accelerated the speed of degradation

compared to what seems realistic. Indeed, the pump degradation time scale is in years.

Table 1. Double-tank model

Parameters		
$S_1=25 \text{ m}^2$	$K_{V_1}=8$	$\tau_a=1$
$S_2=20 \text{ m}^2$	$K_{V_2}=6$	$g=9.82 \text{ ms}^{-2}$
$u_{\text{max}}=100$	$d=N(0, 2.5 \cdot 10^{-4})$	
Initial condition		
$h_1(0)=0$	$h_2(0)=0$	$K_a(0)=5$

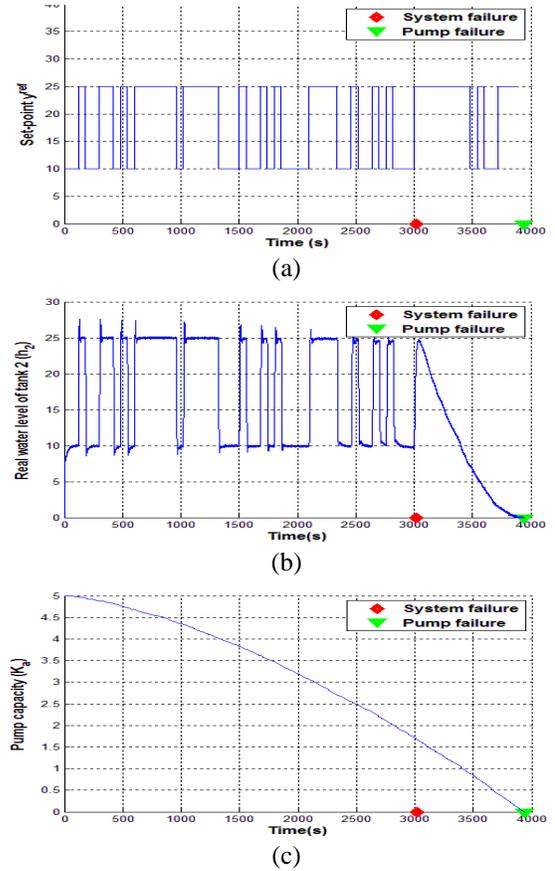


Figure 5. A trajectory of the water tank level control system until pump failure: (a) setpoint, (b) water level of tank 2 (c) pump capacity (not measured).

From Figure 5, it's clear that when the pump capacity decreases, the system compensates for the degradation until it cannot follow the evolution of the desired setpoint (does not meet the desired performance). It should be noted that the pump capacity (Figure 5. c) is not measurable.

## 4.2. Application and prognosis performance metrics

Now that the water tank level control system has been described, we start the implementation of the two phases of the proposed approach.

#### 4.2.1. Estimation phase

To carry out this phase, we used an observer extended to the parameter  $K_a$ . The observer is designed to exploit the available data provided by the sensors (the water level of tank 2 and control input applied on the pump ( $u$ )). The observer designed is the full order high-gain observer (Ayadi, Hajji, Smaoui, Chaari, and Farza, 2016).

The observer has the form.

$$\begin{cases} \frac{d\hat{q}_{in}(t)}{dt} = -\frac{1}{\tau_a}\hat{q}_{in}(t) + \frac{\hat{K}_a(t)}{\tau_a}u \\ \frac{d\hat{h}_1(t)}{dt} = \frac{1}{S_1}\hat{q}_{in}(t) - \frac{K_{V_1}}{S_1}\sqrt{2g\hat{h}_1(t)} + L_1\tilde{h}_2 \\ \frac{d\hat{h}_2(t)}{dt} = \frac{K_{V_1}}{S_1}\sqrt{2g\hat{h}_1(t)} - \frac{K_{V_2}}{S_2}\sqrt{2g\hat{h}_2(t)} + L_2\tilde{h}_2 \\ \frac{d\hat{K}_a(t)}{dt} = L_3\tilde{h}_2 \end{cases} \quad (21)$$

where:  $\tilde{h}_2 = h_2^m - \hat{h}_2$  is the estimation error on the water level of the second tank.  $L_1$ ,  $L_2$  and  $L_3$  the observer gains.

The performance of the synthesized observer is evaluated through the error between the measured and estimated output. It is found that the error is very low and the observer converges quite well for the observer gains chosen (Figs. 6 and 7).

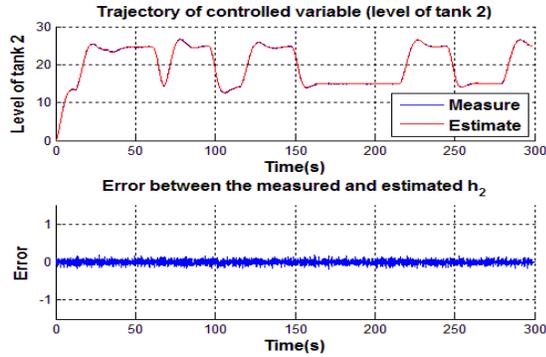


Figure 6. The water level of tank 2 and error estimation.

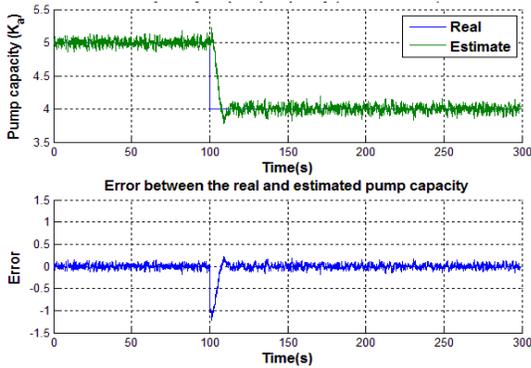


Figure 7. The pump capacity and error estimation.

#### 4.2.2. Prognosis phase

At an instant  $t_p$ , assessing the system performance and comparing it with admissible performance requires knowledge of the future operating conditions (assumed known over about 4000 s) and the future value of pump capacity. The latter are obtained through the capacity loss model. The estimation of the model parameters is carried out following smoothing by the Loess method (Cleveland and Devlin, 1988). The interest is to obtain a series of data which follows a trend, because we used a Gamma process. The RUL( $t$ ) being a distribution. In the results below, the RULs presented are those obtained by using the mean values of  $K_a$  estimated at the instants ( $t > t_p$ ). The results obtained by the implementation of the prognosis phase are presented below.

##### Maximum likelihood estimation

Results for the estimation of RUL are shown in Figure 8. The estimated RUL is shown in red (\*), the evolution of the real RUL in blue, and the evolutions characterizing the confidence interval by  $\alpha_c=0.3$  in green. This value enables a deviation of 30% from the real RUL to be tolerated. The results obtained from the implementation of the prognosis performance metrics are given in Table 2.

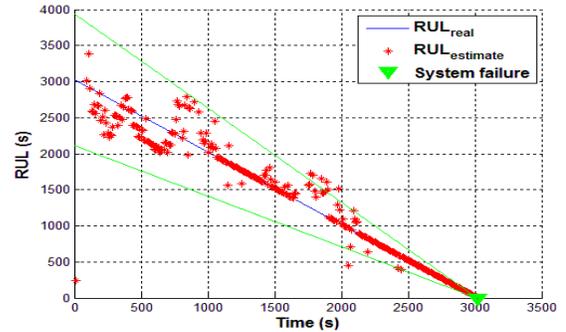


Figure 8. RUL estimation for the example.

Table 2. Performance measures

Performance	Value
Accuracy	0.85
PH ( $\alpha_c=0.3$ )	2930
RA [ $T/4$ $T/2$ $3T/4$ ]	[0.90 0.99 0.99]

In this example, we observe that the performance metrics of the approach are overall satisfactory: the accuracy is 0.85, the value of the HP measure is 2930 s and RA assessed for three times corresponding to the [ $T/4$ ,  $T/2$ ,  $3T/4$ ] are good. The precision and MAPER were not calculated due to the absence of certain values of the RUL (hypothesis on knowledge of the duration of future operating conditions).

##### Bayesian estimation

For this estimate, prior distributions for unknown parameters  $p$ ,  $q$  and  $\beta$  play a key role. They may be obtained by using past experiences, insights and experience of the expert. The knowledge is uncertain, so it is natural to model through laws

$\pi(\emptyset)$ . In practice,  $\pi(\emptyset)$  can be a normal distribution, Beta, Gamma, etc. The evolution of the estimated capacity of the pump until failure for 50 tests was used to formalize priori knowledge. The parameters  $p$ ,  $q$  and  $\beta$  of each test were calculated by the maximum likelihood method and modeled with a normal distribution. For the priori knowledge, the same weight was first attributed to the possible values of the parameters (uninformative a priori law). In a second step, the a priori probabilities of the unknown parameters were taken into account.

In the approach, strategies without or with update of the mean of unknown parameters were tested. Either the a priori parameters obtained do not change from one moment to the next or the a posteriori parameters obtained are used to simulate the likelihood of the subsequent moment.

For the uninformative a priori law, results for the estimation of RUL are shown in Figure 9. The estimated RUL without update is shown in red (\*), while the estimated RUL with update is shown in blue (\*). The results of the prognosis performance metrics are given in Table 3.

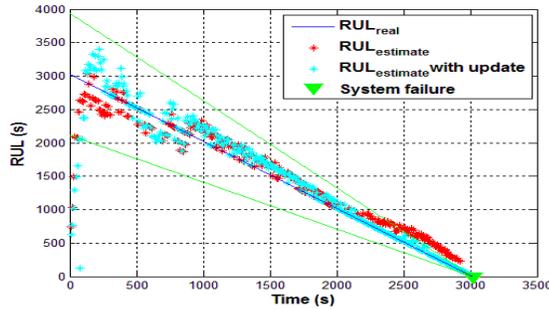


Figure 9. RUL estimation for the example.

Table 3. Performance measures

Performance	Without update	With update
Accuracy	0.86	<b>0.92</b>
Precision	<b>263.61</b>	356.14
MAPER	17.46	<b>8.38</b>
PH ( $\alpha_c=0.3$ )	<b>2960</b>	2920
RA [T/4 T/2 3T/4]	[0.92 0.88 0.85]	[ <b>0.95 0.92 0.99</b> ]

We observe that when the a priori knowledge of the degradation is updated, we obtained better results for the accuracy and MAPER and a slight decrease in precision and the PH measure.

The results for the case where probabilities are taken into account are shown in Figure 10 and performance measures are presented in Table 4. We observe that results obtained when the a priori knowledge is updated are better than those obtained without update. The accuracy and MAPER values improve substantially with the update.

From the various results presented (Table 2-4.), we conclude that the addition of knowledge on model parameters (loss of pump capacity) affects the results of a prognosis approach.

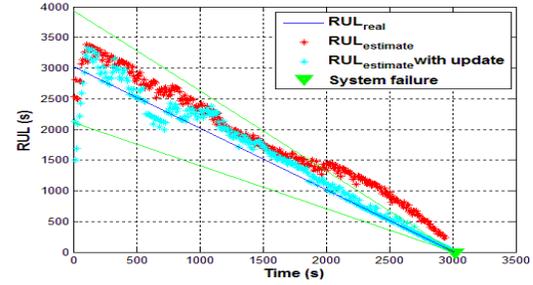


Figure 10. RUL estimation for the example.

Table 4. Performance measures

Performance	Without update	With update
Accuracy	0.72	<b>0.92</b>
Precision	<b>143.68</b>	206.65
MAPER	37.45	<b>8.77</b>
PH ( $\alpha_c=0.3$ )	<b>3010</b>	<b>3010</b>
RA [T/4 T/2 3T/4]	[0.83 0.83 0.42]	[ <b>0.96 0.87 0.99</b> ]

For the different strategies presented, the best results were obtained when taking into account the probability in the prior knowledge with updates of the mean of unknown parameters (Table 4). The results obtained with update of unknown parameters are better, which reflects that the model's update strategy increases the influence of likelihood and makes the a priori distribution less informative.

## 5. CONCLUSION

We have presented, in this paper, an approach for failure prognosis by online estimation of RUL on a closed-loop control system. The approach is based on the behavioral model. Its implementation requires knowledge of future operating conditions and of the dynamics of the system parameters (degradation causing a deviation of the system parameters). Initially, an observer was used to estimate the states and the unmeasured parameters capable of characterizing the system performance. The history of these estimates was then used to determine the parameter dynamics. Two strategies were presented, parameter estimation by the maximum likelihood method and the Bayesian estimation. The approach for failure prognosis was applied to a tank level control system subject to pump degradation (stochastic degradation process); its feasibility and also its performance were illustrated. A highly interesting perspective of the present study is to consider uncertainties in the knowledge of future operating conditions and thus provide robust decisions.

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