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H_-/L_∞ Fault Detection Observer for Discrete-time Takagi-Sugeno Fuzzy Systems

Meng Zhou^{1,2}, Mickael Rodrigues², Zhenhua Wang¹, Yi Shen¹, Didier Theilliol³

Abstract—In this paper, an H_-/L_∞ fault detection observer is proposed for discrete-time Takagi-Sugeno (T-S) fuzzy systems. The main contribution of this paper lies as, first, the finite frequency H_- index is used to measure the worst-case fault sensitivity. Second, unlike the common H_-/H_∞ fault detection observer, this paper uses L_∞ norm to measure the worst effect of disturbance on residual instead of H_∞ norm. Besides, a residual evaluation is synthesized based on the L_∞ norm to achieve an efficient fault detection despite of the disturbance occurrence. Besides, sufficient conditions of the observer design are derived based on piecewise Lyapunov functions. The effectiveness of the proposed method is demonstrated by numerical simulations.

I. INTRODUCTION

Faults are usually inevitable for complex nonlinear systems. In order to prevent severe events, fault detection has been extensively investigated over the last decades [1-3]. Takagi-Sugeno (T-S) fuzzy models are usually used to describe complex nonlinear systems, which are usually subject to modelling uncertainties, measurement noise and other disturbances. The disturbances may bring false alarms and missing alarms for fault detection, such that robustness against disturbances has been one of the most important requirements. H_∞ technique is one of the most effective methods to attenuate disturbances and has been widely used in fault detection for T-S fuzzy systems (see, e.g. [4-6], and the reference therein). Actually, H_∞ norm is a measurement of energy-to-energy gain, it requires that the exogenous disturbances are supposedly of energy-bounded signals. However, in practice, some signals are not energy-bounded but have bounded peak values. For such types of disturbances, it is not easy to calculate the energy on entire time-domain accurately. When peak bounded input signals are considered, the L_∞ norm appears to be more adequate as performance criterion for the observer design, which aims to minimize the peak-to-peak gain [7]. In [8], peak-to-peak gain minimization problem of linear continuous-time systems has been studied based on the bounding reachable set with inescapable ellipsoids. [9] proposes a peak-to-peak filtering for a class of Markov

jump systems with uncertain parameters. In [7], the robust L_∞ filter is proposed to stochastic systems. In [10], residual evaluation and threshold computation methods based on H_∞ norm and L_∞ norm are compared and it is shown that L_∞ norm method has better performance in fault detection threshold than H_∞ norm method.

In order to detect faults accurately and timely, only considering the disturbance robustness in observer design is not enough, high fault sensitivity to residual is important and necessary. A robust fault detection method should minimize the effect of the disturbance whilst maximize the effect of fault on residual, simultaneously [11]. H_-/H_∞ fault detection observer design method has drawn much researchers' attention, in which H_- index is considered as a measurement of the worst-case fault sensitivity and H_∞ norm is used to measure the disturbance attenuation performance. A great deal of literatures can be found in [12-14], just to name a few. However, to the best of authors' knowledge, very little work has been done on H_-/L_∞ fault detection observer design. In [15], an H_-/L_∞ fault detection observer is proposed for continuous-time LPV systems, in which the L_∞ norm is used to measure the effect of the disturbance on residual. The major merit of this method is that the H_-/L_∞ fault detection observer can provide a dynamic threshold for residual evaluation in fault detection. However, [15] is only suitable for continuous-time systems, besides, it is based on a common Lyapunov function approach, which brings some conservatism. Motivated by [15], in this paper we extend the H_-/L_∞ fault detection observer designed method to discrete-time T-S fuzzy systems. The main contribution of this paper lies in the following aspects. First, the fault sensitivity to residual in finite frequency domain is maximized by using H_- index. Then, considering the peak bounded disturbance, L_∞ norm is used to measure the effect of the disturbance to residual. Besides, a dynamic threshold is generated for residual evaluation and decision making based on L_∞ norm. In order to reduce some conservatism, the sufficient conditions of the proposed observer are derived based on piecewise Lyapunov function method and formulated as a set of LMIs. Furthermore, some extra design freedom of parameters are added to reduce some conservatism.

The rest of this paper is organized as follows. The preliminaries and system descriptions are given in Section II. An H_-/L_∞ fault detection observer is proposed for discrete-time T-S fuzzy systems in Section III, the sufficient conditions of the proposed observer are given and transformed as a set of LMIs. In Section IV, numerical simulations are used

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¹Meng Zhou, Zhenhua Wang and Yi Shen are with the Department of Control Science and Engineering, Harbin Institute of Technology, 150001 Harbin, China zhenhua.wang@hit.edu.cn

²Mickael Rodrigues is with the Laboratoire d'Automatique et Génie des Procédés, Université de Lyon, Lyon, F-69003, France mickael.rodrigues@univ-lyon1.fr

³Didier Theilliol is with the Centre de Recherche en Automatique de Nancy, Université de Lorraine, CNRS UMR 7039, F-54506 Vandoeuvre-lès-Nancy, France didier.theilliol@univ-lorraine.fr

to demonstrate the effectiveness of the proposed method. Section V gives the conclusions.

Notations: The notations in this paper are standard. \mathbb{R}^n denotes the n -dimensional Euclidean space and I represents the identity matrix with appropriate dimension. For a matrix A , A^T denotes its transpose, $\mathbf{He}(A)$ is expressed as $\mathbf{He}(A) := A + A^T$, and $A > 0$ ($A < 0$) means that A is positive definite (negative definite). The asterisk \star represents the transposed block induced by symmetry in a symmetric matrix. For a vector $x \in \mathbb{R}^n$, $\|x\|_2 = \sqrt{x^T x}$ is used to denote its \mathcal{L}_2 -norm. For a signal $x(k) \in \mathbb{R}^n$, we define the L_∞ norm (the peak norm) of $x(k)$ to be the sup over all time of the \mathcal{L}_2 -norm at each time instant, i.e. $\|x(k)\|_\infty := \sup_{k \geq 0} \|x(k)\|_2$.

II. SYSTEM DESCRIPTION AND PRELIMINARIES

A nonlinear dynamic system can be described by Takagi-Sugeno (T-S) fuzzy models which is represented by fuzzy IF-THEN rules [14]. The i -th rule of T-S fuzzy model is expressed as

Model Rule i : IF $\xi_1(k)$ is M_{i1} and \dots and $\xi_s(k)$ is M_{is} , THEN

$$\begin{cases} x(k+1) = A_i x(k) + B_i u(k) + F_i f(k) + D_{xi} w(k) \\ y(k) = C x(k) + D_y v(k), \quad i = 1, 2, \dots, N \end{cases} \quad (1)$$

where $x(k) \in \mathbb{R}^{n_x}$ denotes the state vector, $u(k) \in \mathbb{R}^{n_u}$ is the input vector, $y(k) \in \mathbb{R}^{n_y}$ is the measured output vector, $f(k) \in \mathbb{R}^{n_f}$ represents the actuator fault vector, $w(k) \in \mathbb{R}^{n_w}$ denotes the unknown disturbance, $v(k) \in \mathbb{R}^{n_v}$ is the measurement noise. $A_i \in \mathbb{R}^{n_x \times n_x}$, $B_i \in \mathbb{R}^{n_x \times n_u}$, $F_i \in \mathbb{R}^{n_x \times n_f}$, $C \in \mathbb{R}^{n_y \times n_x}$, $D_{xi} \in \mathbb{R}^{n_x \times n_w}$ and $D_y \in \mathbb{R}^{n_y \times n_v}$ are known system matrices. M_{ij} , ($i = 1, 2, \dots, N, j = 1, 2, \dots, s$) are the fuzzy sets and $\xi_i(k)$, $i = 1, 2, \dots, s$ are known premise variables. s is the number of premise variables and N is the number of model rules.

Then, the global fuzzy dynamics is inferred as follows

$$\begin{cases} x(k+1) = \sum_{i=1}^N \mu_i(\xi(k)) [A_i x(k) + B_i u(k) \\ \quad + F_i f(k) + D_{xi} w(k)] \\ y(k) = C x(k) + D_y v(k) \end{cases} \quad (2)$$

where

$$\xi(k) = [\xi_1(k), \xi_2(k), \dots, \xi_s(k)]^T \quad (3)$$

$$\mu_i(\xi(k)) = \frac{\omega_i(\xi(k))}{\sum_{i=1}^N \omega_i(\xi(k))}, \quad \omega_i(\xi(k)) = \prod_{j=1}^s M_{ij}(\xi_j(k)) \quad (4)$$

For all k , the term $M_{ij}(\xi_j(k))$ is the grade of membership of $\xi_j(k)$, $\omega_i(\xi(k))$ and $\mu_i(\xi(k))$ satisfy that

$$\begin{aligned} \omega_i(\xi(k)) &\geq 0, \quad i = 1, 2, \dots, N, \quad \sum_{i=1}^N \omega_i(\xi(k)) > 0 \\ \mu_i(\xi(k)) &\geq 0, \quad i = 1, 2, \dots, N, \quad \sum_{i=1}^N \mu_i(\xi(k)) = 1 \end{aligned} \quad (5)$$

For brevity of notation, the following notations are defined as follows

$$\begin{aligned} A(\mu) &= \sum_{i=1}^N \mu_i(\xi(k)) A_i, \quad B(\mu) = \sum_{i=1}^N \mu_i(\xi(k)) B_i, \\ F(\mu) &= \sum_{i=1}^N \mu_i(\xi(k)) F_i, \quad D_x(\mu) = \sum_{i=1}^N \mu_i(\xi(k)) D_{xi} \end{aligned} \quad (6)$$

Then, the T-S fuzzy model (2) is rewritten such that

$$\begin{cases} x(k+1) = A(\mu)x(k) + B(\mu)u(k) + F(\mu)f(k) \\ \quad + D_x(\mu)w(k) \\ y(k) = Cx(k) + D_yv(k) \end{cases} \quad (7)$$

The main objective of this paper is to design an observer for system (7) such that the residual is sensitive to faults and robust against disturbances, simultaneously.

In the sequel, the following Assumptions and Lemma are given and used.

Assumption 1. The actuator fault $f(k)$ is assumed to belong to a low frequency range as

$$\Theta_f = \{\theta \in \mathbb{R} : |\theta| \leq \vartheta_l\} \quad (8)$$

where θ denotes the frequency of fault $f(k)$ and $\vartheta_l > 0$ is a known scalar.

Assumption 2. The unknown disturbance and noise are assumed to be peak bounded, i.e.

$$\|w(k)\|_\infty \leq \epsilon_w, \quad \|v(k)\|_\infty \leq \epsilon_v \quad (9)$$

where $\epsilon_w > 0$ and $\epsilon_v > 0$ are known scalars.

Lemma 1. [13] Considering a T-S fuzzy system as

$$\begin{cases} x(k+1) = A(\mu)x(k) + B(\mu)u(k) \\ y(k) = C(\mu)x(k) + D(\mu)u(k) \end{cases} \quad (10)$$

Assuming that system (10) is asymptotically stable and controllable, letting $\gamma > 0$ be a given constant, then under zero initial conditions, system (10) has a finite frequency performance with γ , i.e.

$$\sum_{k=0}^{\infty} y^T(k)y(k) \leq \gamma^2 \sum_{k=0}^{\infty} u^T(k)u(k) \quad (11)$$

if there exists a fuzzy-basis-dependent matrix $P(\mu) = P^T(\mu)$ and a matrix $Q = Q^T > 0$, $\mu^+ := (\mu_1(\xi(k+1)), \dots, \mu_r(\xi(k+1)))$ such that

$$\begin{aligned} \begin{bmatrix} A(\mu) & B(\mu) \\ I & 0 \end{bmatrix}^T \Xi \begin{bmatrix} A(\mu) & B(\mu) \\ I & 0 \end{bmatrix} + \\ \begin{bmatrix} C(\mu) & D(\mu) \\ 0 & I \end{bmatrix}^T \Pi \begin{bmatrix} C(\mu) & D(\mu) \\ 0 & I \end{bmatrix} \leq 0 \end{aligned} \quad (12)$$

holds for any μ , where

$$\Pi = \begin{bmatrix} I & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \quad (13)$$

and for a low-frequency domain $\|\theta\| \leq \vartheta_l$, the matrix Ξ is given as

$$\Xi = \begin{bmatrix} -P(\mu^+) & Q \\ Q & P(\mu) - 2 \cos \vartheta_l Q \end{bmatrix} \quad (14)$$

III. H_-/L_∞ FAULT DETECTION OBSERVER DESIGN

In order to detect the actuator fault for system (7), an observer is proposed with the following structure

$$\begin{cases} \hat{x}(k+1) = A(\mu)\hat{x}(k) + B(\mu)u(k) \\ \quad + L(\mu)(y(k) - C\hat{x}(k)) \\ r(k) = y(k) - C\hat{x}(k) \end{cases} \quad (15)$$

where $\hat{x}(k) \in \mathbb{R}^{n_x}$ denotes the estimation of state vector $x(k)$ and $r(k) \in \mathbb{R}^{n_y}$ stands for the residual. $L(\mu) \in \mathbb{R}^{n_x \times n_y}$ is the observer gain matrix to be determined with the following form

$$L(\mu) = \sum_{i=1}^N \mu_i(\xi(k)) L_i \quad (16)$$

The state estimation error vector can be defined such that

$$e(k) = x(k) - \hat{x}(k) \quad (17)$$

By subtracting (15) from (7), the estimation error dynamics is obtained as

$$\begin{cases} e(k+1) = [A(\mu) - L(\mu)C]e(k) + F(\mu)f(k) \\ \quad + [\bar{\mu}_x(h) - L(\mu)\bar{D}_y]d(k) \\ r(k) = Ce(k) + \bar{D}_y d(k) \end{cases} \quad (18)$$

where

$$d(k) = \begin{bmatrix} w(k) \\ v(k) \end{bmatrix}, \quad \bar{D}_x(\mu) = [D_x(\mu) \quad 0], \quad \bar{D}_y = [0 \quad D_y]$$

According to Assumption 2, it has

$$\|d(k)\|_\infty \leq \bar{d} \quad (19)$$

where \bar{d} is the maximum scalar of ϵ_w and ϵ_v .

According to [15], the observer (15) is an H_-/L_∞ observer if the following conditions are satisfied:

- (1) The estimation error dynamics (18) is asymptotically stable;
- (2) The state estimation error system (18) has the following peak-to-peak performance, i.e.

$$\|r(k)\|_2 \leq \sqrt{\gamma^2 \|d(k)\|_\infty^2 + \mathcal{N}(e(0), k)} \quad (20)$$

where $\mathcal{N}(e(0), k)$ is a quadratic function of initial estimation error $e(0)$ and will converge to zero as $k \rightarrow \infty$;

- (3) The residual $r(k)$ in (18) is sensitive to actuator faults $f(k)$ in a finite frequency range in (8), i.e.

$$\sum_{k=0}^{\infty} r^T(k)r(k) \geq \beta^2 \sum_{k=0}^{\infty} f^T(k)f(k), \quad \|\theta\| \leq \vartheta_l \quad (21)$$

To facilitate the H_-/L_∞ fault detection observer design, the following Theorem is proposed.

Theorem 1. Given positive scalars $0 < \alpha < 1$, γ and β , observer (15) is asymptotically stable and satisfies the H_- performance in (21) and L_∞ performance such that

$$\|r(k)\|_2 \leq \sqrt{\gamma^2 \|C\|_2^2 \alpha^k V(0) + (\gamma^2 \|C\|_2^2 + \|\bar{D}_y\|_2^2) \bar{d}^2} \quad (22)$$

if there exist fuzzy-basis-dependent matrices $P_1(\mu) = P_1^T(\mu) \in \mathbb{R}^{n_x \times n_x}$ and $P_2(\mu) = P_2^T(\mu) > 0 \in \mathbb{R}^{n_x \times n_x}$ and matrix $Q = Q^T > 0 \in \mathbb{R}^{n_x \times n_x}$ such that

$$\begin{bmatrix} \Phi_1 & \Phi_2 \\ \star & -F(\mu)^T P_1(\mu^+) F(\mu) + \beta^2 I \end{bmatrix} < 0 \quad (23)$$

$$\begin{bmatrix} \Pi_1 & \Pi_2 \\ \star & \Pi_3 \end{bmatrix} < 0 \quad (24)$$

$$I - \gamma^2 P_2(\mu) < 0 \quad (25)$$

where

$$\begin{cases} \Phi_1 = -[A(\mu) - L(\mu)C]^T P_1(\mu^+) [A(\mu) - L(\mu)C] \\ \quad + Q[A(\mu) - L(\mu)C] + [A(\mu) - L(\mu)C]^T Q \\ \quad + P_1(\mu) - 2 \cos \vartheta_l Q - C^T C \\ \Phi_2 = -[A(\mu) - L(\mu)C]^T P_1(\mu^+) F(\mu) + QF(\mu) \\ \Pi_1 = [A(\mu) - L(\mu)C]^T P_2(\mu^+) [A(\mu) - L(\mu)C] \\ \quad - \alpha P_1(\mu) \\ \Pi_2 = [A(\mu) - L(\mu)C]^T P_2(\mu^+) [\bar{D}_x(\mu) - L(\mu)\bar{D}_y] \\ \Pi_3 = [\bar{D}_x(\mu) - L(\mu)\bar{D}_y]^T P_2(\mu^+) [\bar{D}_x(\mu) - L(\mu)\bar{D}_y] \\ \quad - (1 - \alpha)I \end{cases}$$

Proof: The proof is divided into the following two parts.

A. H_- fault sensitivity condition

When only consider the effect of actuator fault, by setting $d(k) = 0$ and (18) becomes

$$\begin{cases} e(k+1) = [A(\mu) - L(\mu)C]e(k) + F(\mu)f(k) \\ r(k) = Ce(k) \end{cases} \quad (26)$$

In this paper, the actuator fault is assumed to belong to a given low finite frequency domain (8), and according to Lemma 1, for a given positive scalar $\beta > 0$, letting

$$\Pi = \begin{bmatrix} -I & 0 \\ 0 & \beta^2 I \end{bmatrix} \quad (27)$$

and substituting $[A(\mu) - L(\mu)C] \rightarrow A(\mu)$, $F(\mu) \rightarrow B(\mu)$, $C \rightarrow C(\mu)$, $0 \rightarrow D(\mu)$ and $P_1(\mu) \rightarrow P(\mu)$ in (12), then (12) is transformed in (23), and system (26) has a finite frequency H_- index performance as (21).

B. L_∞ disturbance attenuation condition

By setting $f(k) = 0$, the estimation error is only affected by the disturbance and can be re-written as

$$\begin{cases} e(k+1) = [A(\mu) - L(\mu)C]e(k) + [\bar{D}_x(\mu) \\ \quad - L(\mu)\bar{D}_y]d(k) \\ r(k) = Ce(k) + \bar{D}_y d(k) \end{cases} \quad (28)$$

A Lyapunov candidate is chosen such that

$$V(k) = e^T(k)P_2(\mu)e(k) \quad (29)$$

Given a positive scalar $\alpha > 0$, under zero initial condition, it is assumed that the following inequality is satisfied.

$$V(k+1) < \alpha V(k) + (1 - \alpha)d^T(k)d(k) \quad (30)$$

When $k = 0$, we have

$$V(1) < \alpha V(0) + (1 - \alpha)d^T(0)d(0) \quad (31)$$

By iterating, we can obtain that

$$V(k) < \alpha^k V(0) + (1 - \alpha) \sum_{i=0}^{k-1} \alpha^i d^T(k-1-i)d(k-1-i) \quad (32)$$

Due to $d^T(k)d(k) \leq \|d(k)\|_\infty^2 \leq \bar{d}^2$, then we have

$$V(k) \leq \alpha^k V(0) + (1 - \alpha^{k-1})\bar{d}^2 \quad (33)$$

As $0 < \alpha < 1$, such that $0 < \alpha^{k-1} < 1$, we can get

$$V(k) < \alpha^k V(0) + \bar{d}^2 \quad (34)$$

Furthermore, if it is satisfied that

$$I < \gamma^2 P_2(\mu) \quad (35)$$

then it obtains

$$e^T(k)e(k) < \gamma^2 e^T(k)P_2(\mu)e(k) \quad (36)$$

According to the definition of residual $r(k)$ in (28), it comes that

$$\|r(k)\|_2^2 \leq \|C\|_2^2 \|e(k)\|_2^2 + \|\bar{D}_y\|_2^2 \|d(k)\|_2^2 \quad (37)$$

By using (32), (36) and $\|d(k)\|_2^2 \leq \|d(k)\|_\infty^2 \leq \bar{d}^2$, we obtain

$$\begin{aligned} \|r(k)\|_2 &\leq \sqrt{\|C\|_2^2 \gamma^2 P_2(\mu) \|e(k)\|_2^2 + \|\bar{D}_y\|_2^2 \bar{d}^2} \\ &\leq \sqrt{\gamma^2 \|C\|_2^2 \alpha^k V(0) + (\gamma^2 \|C\|_2^2 + \|\bar{D}_y\|_2^2) \bar{d}^2} \end{aligned} \quad (38)$$

By substituting (29) into (30), then (24) can be derived, which completes the proof. ■

Remark 1. Inequality in (23) and (24) of Theorem 1 are not jointly convex due to the coupling of the terms $P_1(\mu)L(\mu)$ and $P_2(\mu)L(\mu)$, which makes the observer difficult to be solved. In order to facilitate the observer design, the following Theorem is given.

Theorem 2. Given scalars $0 < \alpha < 1$, $\gamma > 0$, η_1 , η_2 and matrix $K \in \mathbb{R}^{n_x \times n_u}$, the observer (15) is an H_-/L_∞ observer, if there exist matrices $Q = Q^T > 0 \in \mathbb{R}^{n_x \times n_x}$, $P_{1i} = P_{1i}^T \in \mathbb{R}^{n_x \times n_x}$, $P_{2i} = P_{2i}^T > 0 \in \mathbb{R}^{n_x \times n_x}$, $G \in \mathbb{R}^{n_x \times n_x}$ and $W_i \in \mathbb{R}^{n_x \times n_y}$ such that

$$\begin{aligned} \Psi_{ii} &< 0, \quad i = 1, \dots, N \\ \Psi_{ij} + \Psi_{ji} &< 0, \quad 1 \leq i < j \leq N \end{aligned} \quad (39)$$

$$\begin{aligned} \Omega_{ii} &< 0, \quad i = 1, \dots, N \\ \Omega_{ij} + \Omega_{ji} &< 0, \quad 1 \leq i < j \leq N \end{aligned} \quad (40)$$

$$I - \gamma^2 P_{2i} < 0 \quad (41)$$

where

$$\Psi_{ij} = \begin{bmatrix} \psi_1 & \psi_2 & \psi_3 \\ \star & \psi_4 & -K^T G + F_i^T G^T \\ \star & \star & -P_{1j} - G - G^T \end{bmatrix}$$

$$\Omega_{ij} = \begin{bmatrix} \omega_1 & \omega_2 & \omega_3 \\ \star & -(1 - \alpha)I & \bar{D}_{xi}^T G^T - \bar{D}_y^T W_i^T \\ \star & \star & P_{2j} - G - G^T \end{bmatrix}$$

$$\begin{cases} \psi_1 = P_{1i} - 2 \cos \vartheta_l Q - C^T C + \mathbf{He}\{\eta_1(GA_i - W_i C)\} \\ \psi_2 = \eta_1 G F_i + A_i^T G^T K - C^T W_i^T K \\ \psi_3 = -\eta_1 G + Q + A_i^T G^T - C^T W_i^T \\ \psi_4 = \beta^2 I + \mathbf{He}\{K^T G F_i\} \\ \omega_1 = -\alpha P_{2i} + \mathbf{He}\{\eta_2(GA_i - W_i C)\} \\ \omega_2 = \eta_2(GD_{xi} - W_i D_y) \\ \omega_3 = -\eta_2 G + A_i^T G^T - C^T W_i^T \end{cases}$$

The observer can be determined by maximizing the H_- index β and the optimization problem is expressed as

$$\begin{aligned} \max \quad & \beta \\ \text{s.t.} \quad & (39) - (41) \end{aligned} \quad (42)$$

Then, the gain matrices L_i is calculated by

$$L_i = G^{-1} W_i, \quad i = 1, 2, \dots, N \quad (43)$$

Proof: First, (23) is equivalent to

$$\Upsilon_1 + \mathcal{A}_1^T \mathcal{Q}^T + \mathcal{Q} \mathcal{A}_1 - \mathcal{A}_1^T P_1(\mu^+) \mathcal{A}_1 < 0 \quad (44)$$

where

$$\Upsilon_1 = \begin{bmatrix} P_1(\mu) - 2 \cos \vartheta_l Q - C^T C & 0 \\ \star & \beta^2 I \end{bmatrix}, \quad (45)$$

$$\mathcal{A}_1 = [(A(\mu) - L(\mu)C) \quad F(\mu)], \quad \mathcal{Q} = \begin{bmatrix} Q \\ 0 \end{bmatrix} \quad (46)$$

A sufficient condition of (44) is

$$\begin{bmatrix} \Upsilon_1 + \mathcal{Y}_1 \mathcal{A}_1 + \mathcal{A}_1^T \mathcal{Y}_1 & -\mathcal{Y}_1 + \mathcal{Q} + \mathcal{A}_1^T G^T \\ \star & -P_1(\mu^+) - G - G^T \end{bmatrix} < 0 \quad (47)$$

It can be verified by pre- and post- multiplying (47) with $[I \quad \mathcal{A}_1^T]$ and its transpose, respectively.

In order to remove the coupling between G and \mathcal{Y}_1 , by denoting

$$\mathcal{Y}_1 = \begin{bmatrix} \eta_1 G \\ K^T G \end{bmatrix} \quad (48)$$

By substituting (48) and (56) into (47), we have

$$\begin{bmatrix} \tilde{\psi}_1 & \tilde{\psi}_2 & \tilde{\psi}_3 \\ \star & \beta^2 I + \mathbf{He}\{K^T G F(\mu)\} & -K^T G + F(\mu)^T G^T \\ \star & \star & -P_1(\mu^+) - G - G^T \end{bmatrix} < 0 \quad (49)$$

where

$$\begin{cases} \tilde{\psi}_1 = P_1(\mu) - 2 \cos \vartheta_l Q - C^T C + \mathbf{He}\{\eta_1 G[A(\mu) - L(\mu)C]\} \\ \tilde{\psi}_2 = \eta_1 G F(\mu) + A(\mu)^T G^T K - C^T L(\mu)^T G^T K \\ \tilde{\psi}_3 = -\eta_1 G + Q + A(\mu)^T G^T - C^T L(\mu)^T G^T \end{cases}$$

Similarly, (24) can be rewritten as follows

$$\Upsilon_2 + \mathcal{A}_2^T P_2(\mu^+) \mathcal{A}_2 < 0 \quad (50)$$

where

$$\Upsilon_2 = \begin{bmatrix} -\alpha P_2(\mu) & 0 \\ \star & -(1 - \alpha)I \end{bmatrix}, \quad (51)$$

$$\mathcal{A}_2 = [(A(\mu) - L(\mu)C) \quad (\bar{D}_x(\mu) - L(\mu)\bar{D}_y)] \quad (52)$$

A sufficient condition of (50) is

$$\begin{bmatrix} \Upsilon_2 + \mathcal{Y}_2 \mathcal{A}_2 + \mathcal{A}_2^T \mathcal{Y}_2^T & -\mathcal{Y}_2 + \mathcal{A}_2^T G^T \\ \star & P_2(\mu^+) - G - G^T \end{bmatrix} < 0 \quad (53)$$

It can also be easily shown by pre- and post- multiplying (50) with $\begin{bmatrix} I & \mathcal{A}_2^T \end{bmatrix}$ and its transpose, respectively.

By denoting that

$$\mathcal{Y}_2 = \begin{bmatrix} \eta_2 G \\ 0 \end{bmatrix} \quad (54)$$

Substituting (54) into (53), we have

$$\begin{bmatrix} \tilde{\omega}_1 & \tilde{\omega}_2 & \tilde{\omega}_3 \\ \star & -(1-\alpha)I & (\bar{D}_x(\mu) - L(h)\bar{D}_y)^T G^T \\ \star & \star & P_2(\mu^+) - G - G^T \end{bmatrix} < 0 \quad (55)$$

where

$$\begin{cases} \tilde{\omega}_1 = -\alpha P_2(\mu) + \mathbf{He}\{\eta_2 G(A(\mu) - L(\mu)C)\} \\ \tilde{\omega}_2 = \eta_2 G(\bar{D}_x(\mu) - L(\mu)\bar{D}_y) \\ \tilde{\omega}_3 = -\eta_2 G + (A(\mu) - L(\mu)C)^T G^T \end{cases}$$

and by noting that

$$GL(\mu) = W(\mu) \quad (56)$$

and combining the definitions in (6) and (16), according to [4], the conditions (49) and (55) are equivalent to (39) and (40), respectively, which completes the proof. ■

Remark 2. The tuned parameters η_1 and η_2 , matrices K and G are added in Theorem 2, which can provide extra design degrees of freedom and further reduce some conservatism. In the implementation, the values of η_1 , η_2 and matrix K can be determined beforehand by trial and error method.

Based on the L_∞ performance in (22), in this paper, at time k , the \mathcal{L}_2 norm of the residual $\|r(k)\|_2$ is considered to evaluate the residual. And a dynamic threshold can be determined such as

$$J_{th}(k) = \sqrt{\gamma^2 \|C\|_2^2 \alpha^k V(0) + (\gamma^2 \|C\|_2^2 + \|\bar{D}_y\|_2^2) \bar{d}^2} \quad (57)$$

As (22), the fault detection logic is given as

$$\begin{cases} \|r(k)\|_2 > J_{th}(k) \Rightarrow \text{Faulty} \\ \|r(k)\|_2 \leq J_{th}(k) \Rightarrow \text{Fault-free} \end{cases} \quad (58)$$

IV. SIMULATION RESULTS

In this section, numerical simulations are given to illustrate the effectiveness of the proposed method. Borrowed from [16], a nonlinear system is expressed as

$$\begin{cases} x(k+1) = A(\xi(k))x(k) + B(\xi(k))u(k) \\ \quad + F(\xi(k))f(k) + D_x(\xi(k))d(k) \\ y(k) = Cx(k) + D_y d(k) \end{cases} \quad (59)$$

where the system matrices are

$$A(\xi(k)) = \begin{bmatrix} 0.3 & 0.2 & \xi_2(k) \\ 0.6 & \xi_1(k) & 0.1 \\ 2\xi_2(k) & 0.3 & 0.5 \end{bmatrix}, \quad (60)$$

$$B(\xi(k)) = \begin{bmatrix} 0.8 + \xi_1(k) & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, F(\xi(k)) = B(\xi(k)), \quad (61)$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, D_x = \begin{bmatrix} 0 \\ 0.02 \\ 0.01 \end{bmatrix}, D_y = \begin{bmatrix} 0.01 \\ 0 \end{bmatrix}. \quad (62)$$

In this paper, the nonlinear system is represented by the following subsystems.

Plant rule 1: IF $\xi_1(k)$ is 0.1 and $\xi_2(k)$ is 0.1, THEN

$$x(k+1) = A_1 x(k) + B_1 u(k) + F_1 f(k) + D_x d(k)$$

Plant rule 2: IF $\xi_1(k)$ is 0.1 and $\xi_2(k)$ is 0.3, THEN

$$x(k+1) = A_2 x(k) + B_2 u(k) + F_2 f(k) + D_x d(k)$$

Plant rule 3: IF $\xi_1(k)$ is 0.3 and $\xi_2(k)$ is 0.1, THEN

$$x(k+1) = A_3 x(k) + B_3 u(k) + F_3 f(k) + D_x d(k)$$

Plant rule 4: IF $\xi_1(k)$ is 0.3 and $\xi_2(k)$ is 0.3, THEN

$$x(k+1) = A_4 x(k) + B_4 u(k) + F_4 f(k) + D_x d(k)$$

The subsystem matrices A_i , B_i and F_i , $i = 1, \dots, 4$ can be easily obtained based on (60) and (61).

In the simulations, the premise variables $\xi_1(k)$ and $\xi_2(k)$ are assumed as Fig. 1. The membership functions for rule i , $i = 1, \dots, 4$ are given as

$$\begin{aligned} \mu_1(\xi(k)) &= \frac{(0.3 - \xi_1(k))(0.3 - \xi_2(k))}{0.04}, \\ \mu_2(\xi(k)) &= \frac{(0.3 - \xi_1(k))(\xi_2(k) - 0.1)}{0.04}, \\ \mu_3(\xi(k)) &= \frac{(\xi_1(k) - 0.1)(0.3 - \xi_2(k))}{0.04}, \\ \mu_4(\xi(k)) &= \frac{(\xi_1(k) - 0.1)(\xi_2(k) - 0.1)}{0.04}. \end{aligned}$$

In this paper, the actuator fault is assumed to belong to a given frequency domain in $\theta \in [-0.1, 0.1]$. In the simulations, the parameters are chosen as $\alpha = 0.9$, $\eta_1 = 0.2$, $\eta_2 = 0.2$, $K = -9F_1$ and $\gamma = 0.3$. By solving the optimization problem in (42), the maximum H_∞ index is calculated as $\beta = 5.1918$ and the gain matrices of the proposed observer are obtained as

$$\begin{aligned} L_1 &= \begin{bmatrix} -0.6185 & 0.2028 \\ 0.5991 & -0.7712 \\ 0.2009 & 0.2912 \end{bmatrix}, L_2 = \begin{bmatrix} -0.6189 & 0.2040 \\ 0.5996 & -0.7634 \\ 0.6004 & 0.2944 \end{bmatrix}, \\ L_3 &= \begin{bmatrix} -0.5969 & 0.2023 \\ 0.5991 & -0.5695 \\ 0.2011 & 0.2949 \end{bmatrix}, L_4 = \begin{bmatrix} -0.5970 & 0.2028 \\ 0.5993 & -0.5695 \\ 0.6009 & 0.2949 \end{bmatrix}. \end{aligned}$$

In order to demonstrate the effectiveness of the proposed method, it is compared with the L_∞ observer without considering the H_∞ fault sensitivity. With the same disturbance attenuation performance as $\gamma = 0.3$, the gain matrices of L_∞ observer are calculated as

$$\begin{aligned} \hat{L}_1 &= \begin{bmatrix} -0.0180 & 0.1984 \\ 0.5919 & -0.1516 \\ 0.2351 & 0.3017 \end{bmatrix}, \hat{L}_2 = \begin{bmatrix} 0.0278 & 0.2047 \\ 0.5977 & -0.1342 \\ 0.6496 & 0.3031 \end{bmatrix}, \\ \hat{L}_3 &= \begin{bmatrix} -0.0023 & 0.1986 \\ 0.5914 & 0.0580 \\ 0.2392 & 0.3029 \end{bmatrix}, \hat{L}_4 = \begin{bmatrix} 0.0278 & 0.2049 \\ 0.5980 & 0.0582 \\ 0.6496 & 0.3034 \end{bmatrix}. \end{aligned}$$

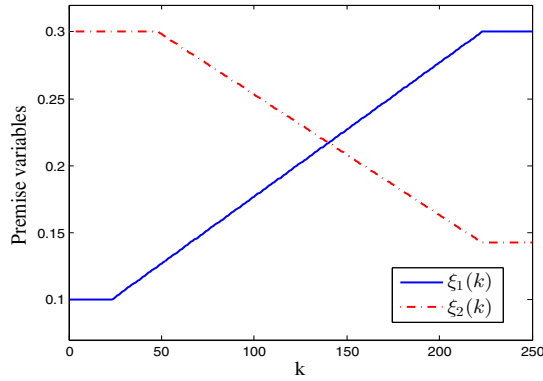


Fig. 1. Premise variables

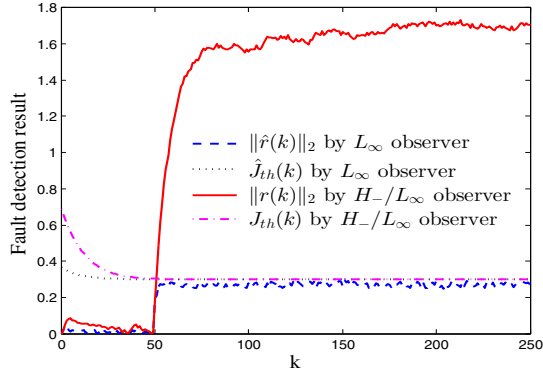


Fig. 2. Fault detection result for abrupt fault

In this paper, the initial condition of the nonlinear system is chosen as $x(0) = [0.02 \ 0.05 \ 0.1]^T$, the disturbance is simulated by a random vector bounded by $[-1, 1]$, such that $\bar{d} = 1$. To demonstrate the effectiveness of the proposed method, two fault scenarios are considered. First, an abrupt fault with the amplitude of 0.2 is assumed to happen in the second actuator after time $k = 50$, such that $f_2(k) = 0.2$. In the second fault scenario, a time-varying fault is considered to occur in the first actuator after time $k = 50$, the fault is formulated as $f_1(k) = 0.2 - 0.05 \sin(0.1\pi k - 5)$. Simulation results are depicted in Fig. 2–3. Herein, the solid lines and dash-dot lines represent the residual evaluation and threshold generated by the proposed method, respectively. The dashed lines and dotted lines are the residual evaluation and threshold generated by L_∞ observer, respectively. From Fig. 2 and Fig. 3, we can see that the proposed H_-/L_∞ fault detection observer can detect the abrupt fault and time-varying fault accurately and timely, while the L_∞ observer method is failed to detect the faults. The proposed method has better fault sensitivity than the L_∞ observer method. Besides, the generated dynamic threshold is able to detect the actuator faults in a timely manner.

V. CONCLUSIONS

This paper proposes an H_-/L_∞ fault detection observer design method for discrete-time T-S fuzzy systems. In this paper, the fault sensitivity to residual is maximized by using the finite frequency H_- index and the disturbance robustness to residual is measured by L_∞ norm. The proposed method exhibits a peak-to-peak disturbance attenuation performance and a dynamic threshold is provided for residual evaluation

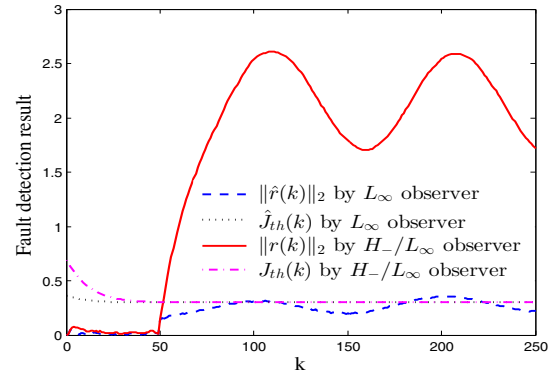


Fig. 3. Fault detection result for time-varying fault

based on L_∞ norm, it is suitable for disturbances with peak bounded. Simulations results demonstrate the effectiveness of the proposed method.

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