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**PUBLIC INPUTS, ALLOCATABLE FIXED FACTORS
AND THE THEORY OF THE MULTIPRODUCT FIRM :
A CLARIFICATION ¹**

by

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SUMMARY

1. Introduction
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3. Public inputs and cost complementarities
4. Allocatable fixed inputs and netput relationships
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1. INTRODUCTION

As noted by Lynne (1988), "some of the most challenging production economic concepts are those of "technical interdependence" and "jointness" as pertaining to the products in a multiproduct firm". Extending the works of Samuelson (1966), Lau (1972, 1978) and Hall (1973), Kohli (1981, 1983) has defined four forms of non-jointness : in input quantities, in output quantities, in input prices and in output prices. But only non-jointness in input quantities, or, more simply, non-jointness since the qualification in input quantities is frequently omitted, has received a great deal of attention by agricultural economists.

Two sources of jointness, at least, are particularly relevant to agricultural production : the "public" nature of some inputs whose use in one process does not diminish their use in another (Baumol et al., 1982), and the "quasi-fixity" of some allocatable inputs whose supply to the firm is fixed, but whose allocation among production processes is variable (Hugues, 1981 ; Shumway et al., 1984).

On the other hand, Sakai (1974) has defined four propositions which govern the market conduct of the multiproduct-multifactor firm in the "normal" case : the total marginal cost of an output decreases with increases in other output quantities (cost complementarities) and does not decrease with increases in input prices (input normality), and the total marginal revenue of an input does not decrease with increases in other input quantities ("cooperant" inputs) and output prices (output normality). Since cost complementarities have no equivalent in the single-output case, they can be viewed as the most important proposition of Sakai. Baumol et al. have shown that the total cost function is joint in input quantities and exhibits cost complementarities when the firm uses one normal public input. By contrast, Moschini (1988, 1989) has shown that cost complementarities are not verified when jointness in input quantities is due to normal allocatable inputs.

The first purpose of this note is to reconcile Baumol et al. and Moschini's results and to show that there is no contradiction between their propositions. More specifically, we will show first that the proposition of Baumol et al. cannot be generalized to the multiinput case without additional assumptions on netput relationships. We will show then that the results of Moschini may be explained by the fact that the model he considers is non-joint in input quantities in the long run, i.e. when the firm optimizes on all inputs, whereas the model of Baumol et al. is non-joint in input quantities in the short run, i.e. when the public input is fixed. In the model of Moschini, the firm moves from unconstrained to constrained regime with a tendency towards more substitutability

between outputs. In the model of Baumol et al., there is a tendency towards more complementarity between outputs because the firm moves from constrained to unconstrained regime.

2. DEFINING THE "NORMAL CASE"

According to Sakai, a multiproduct - multifactor technology is normal if the following propositions, stated in terms of weak inequalities on the total or long-run cost and revenue functions, are satisfied

$$\delta^2 C / \delta y_i \delta y_j \leq 0 \quad v_i, v_j, i \neq j \quad [1]$$

$$\delta^2 C / \delta y_i \delta w_k \geq 0 \quad v_i, v_k \quad [2]$$

$$\delta^2 R / \delta x_h \delta x_k \geq 0 \quad v_h, v_k, h \neq k \quad [3]$$

$$\delta^2 R / \delta x_h \delta p_j \geq 0 \quad v_h, v_j \quad [4]$$

where $C(y, w) = \min_x [w'x ; x \in T(y)]$; and

$$R(p, x) = \max_y [p'y ; y \in T(x)]$$

$C(y, w)$ is the unconstrained cost function, $R(p, x)$ is the unconstrained revenue function, y is the vector of M output quantities with corresponding vector price p , and x is the vector of N input quantities with corresponding vector price w . Price and quantity vectors are positive vectors of real numbers. T is the production possibility set; it is assumed nonempty, closed, bounded from above and convex; it permits also free disposal. Finally, $T(y)$ and $T(x)$ are the input and output possibility sets, respectively.

Hertel (1984, 1987) and Moschini (1988, 1989) have analyzed at some length these four propositions, particularly in order to evaluate their validity in the context of agriculture at the firm and sector levels.

Propositions [2] and [4] rule out inferior inputs and outputs, respectively, while proposition [3] is a generalization to the multiproduct case of Rader's (1968) concept of cooperant factors of production defined in the single-output case. As noted by Moschini (1989), proposition [1] has no equivalent in the single-output case. It implies that the total marginal cost of an output tends to decrease when the quantities of other outputs increase.

Furthermore, Sakai has shown that propositions [1] to [4] imply some restrictions on the technology at the unconstrained Marshallian equilibrium: gross substitution among inputs and outputs and regressive (or inferiority) relationships between inputs and outputs are ruled out, i.e.

$$\delta^2 \pi / \delta p_i \delta p_j \geq 0 \quad \forall i, \forall j \quad [5]$$

$$\delta^2 \pi / \delta w_h \delta w_k \geq 0 \quad \forall h, \forall k \quad [6]$$

$$\delta^2 \pi / \delta p_i \delta w_h = \delta^2 \pi / \delta w_h \delta p_i \leq 0 \quad \forall i, \forall h \quad [7]$$

where $\pi(p, w)$ is the total or long-run profit function defined by

$$\pi(p, w) = \max_{y, x} [p'y - w'x ; (-x, y) \in T] \quad [8a]$$

$$= \max_y [p'y - C(w, y)] \quad [8b]$$

$$= \max_x [R(p, x) - w'x] \quad [8c]$$

3. PUBLIC INPUTS AND COST COMPLEMENTARITIES

Baumol et al. have shown that, if the source of jointness arises uniquely from an input whose use in one production process does not diminish its use in another (in this sense, it is a public input), then the joint unconstrained cost function exhibits cost complementarities, i.e. proposition [1] is satisfied, under normality assumption of the public input. More specifically, the unconstrained cost function defined by Baumol et al. may be written as

$$C(y, w) = \min_{x^0} \left[\sum_{i=1}^M CR^i(y_i, w^1, x^0) + w^0 x^0 \right] \quad [9]$$

where $CR^i(y_i, w^1, x^0)$ is the constrained or short-run cost function ($i = 1, \dots, M$), x^0 is the public input (price w^0), and w^1 is the price vector of variable inputs x^1 . If the public input is normal, i.e. if $\delta^2 CR^i / \delta y_i \delta x^0 \leq 0$, we have (for any $i \neq j$)

$$\delta^2 C / \delta y_i \delta y_j = -(\delta^2 CR^i / \delta y_i \delta x^0) (\delta^2 CR^j / \delta y_j \delta x^0) / \sum_{i=1}^M (\delta^2 CR^i / \delta x^0 \delta x^0) \leq 0 \quad [10]$$

In the single-output case, the normality of an input with respect to the output is traditionally defined in terms of output-compensated input demands, i.e. $\delta x_h(y, w) / \delta y \geq 0$ (see, for example, Chambers, 1988, p. 69). Proposition [2] is a straightforward generalization to the multiproduct case of this definition. Nevertheless, Baumol et al. use a different definition of input normality stated in terms of partial second derivatives of the constrained cost function $CR^i(y_i, w^1, x^0)$ with respect to levels x^0 and y_i . In the single-public input case, it is easy to show that $\delta^2 C / \delta w^0 \delta y_i \geq 0$ implies $\delta^2 CR^i / \delta x^0 \delta y_i \leq 0$. Unfortunately, in the multiple-public input case, this implication does not hold in general.

To show this, it is convenient to rewrite [9] as

$$C(y, w) = \min_{x^0} \left[\sum_{i=1}^M CR^i(y_i, w^1, x^0) + w^{0'} x^0 \right] \quad [11a]$$

$$= \min_{x^0} [CR(y, w^1, x^0) + w^{0'} x^0] \quad [11b]$$

where x^0 is now a vector of public inputs with corresponding vector price w^0 . In matrix notation, we have then¹

$$\begin{aligned} [\delta^2 C / \delta y_i \delta y_j] &= [\delta^2 CR^i / \delta y_i \delta y_j] \\ &- [\delta^2 CR^i / \delta x^0 \delta y_i]' \left[\sum_{i=1}^M \delta^2 CR^i / \delta x^0 \delta x^0 \right]^{-1} [\delta^2 CR^j / \delta x^0 \delta y_j] \end{aligned} \quad [12]$$

or, equivalently, by using [11b]

$$\begin{aligned} [\delta^2 C / \delta y_i \delta y_j] &= [\delta^2 CR / \delta y_i \delta y_j] \\ &- [\delta^2 CR / \delta x^0 \delta y_i]' [\delta^2 CR / \delta x^0 \delta x^0]^{-1} [\delta^2 CR / \delta x^0 \delta y_j] \end{aligned} \quad [13]$$

Furthermore, using the Hessian identities approach of Lau (1976), we have also

$$[\delta^2 CR^i / \delta x^0 \delta y_i] = [\delta^2 CR / \delta x^0 \delta y_i] = [\delta^2 C / \delta w^0 \delta w^0]^{-1} [\delta^2 C / \delta w^0 \delta y_i] \quad [14]$$

$$\left[\sum_{i=1}^M \delta^2 CR^i / \delta x^0 \delta x^0 \right] = [\delta^2 CR / \delta x^0 \delta x^0] = -[\delta^2 C / \delta w^0 \delta w^0]^{-1} \quad [15]$$

The assumptions of concavity of the unconstrained cost function $C(y, w)$ in input prices w , and more particularly in public input prices w^0 , and of normality of public inputs x^0 with respect to outputs y are not sufficient to guarantee that the elements of the vector $[\delta^2 CR^i / \delta x^0 \delta y_i] = [\delta^2 CR / \delta x^0 \delta y_i]$ are all nonpositive. Therefore, in the

¹ Obviously, $\delta^2 CR^i / \delta y_i \delta y_j = \delta^2 CR / \delta y_i \delta y_j = 0$ when $i \neq j$.

multiple-public input case, $\delta^2 C / \delta w_h^0 \delta y_i \geq 0$ does not imply $\delta^2 CR^i / \delta x^0 \delta y_i = \delta^2 CR / \delta x^0 \delta y_i \leq 0$, and the sign of expressions [12] and [13] cannot be defined unambiguously. In other words, even if the public inputs are normal, i.e. satisfy inequality [2], the joint cost function $C(y, w)$ does not necessarily exhibit cost complementarities for all output pairs. Only in some cases (one public input, all public inputs are substitutable at the Hicksian equilibrium defined with respect to $C(w, y)^2$, ..., for example), expressions [12] and [13] are nonpositive without ambiguity.

4. ALLOCATABLE FIXED INPUTS AND NETPUT RELATIONSHIPS

Furthermore, Moschini (1989) has shown that propositions [5] to [7] "are affected under allocatable fixed inputs. In particular, [5] is always violated (indeed reversed) for any $i \neq j$, and [7] need not hold". In other words, "cost complementarities between outputs do not exist when outputs are joint because of fixed allocatable factors which are normal inputs" (Nelson, 1991), and "an output price increase may decrease the demand of some inputs" (Moschini). Below, we will show that Moschini's results may be explained by the fact that the model he considers is a constrained one, i.e. with some allocatable factors fixed at given levels, in the case where the unconstrained technology is normal but non-joint in input quantities (which is allowed in the normal case). Furthermore, we will show that Moschini's results do not depend on the nature of the inputs which are fixed (i.e. public or allocatable) and that [7] necessarily holds.

The joint short-run cost function analyzed by Moschini is the following

$$CR(y, w, z) = \min_{x^1, z^1} \left[\sum_{i=1}^M w^i x^i; y_i = f^i(x^i, z^i) \text{ and } \sum_{i=1}^M z^i \leq z \right] \quad [16]$$

where x^i and z^i are vectors of inputs used exclusively in the production of y^i ; $i = 1, \dots, M$; and f^i is a strictly concave production function. Variable inputs x are purchased at competitive prices w and allocatable fixed inputs z satisfy the availability constraint, i.e.

$$\sum_{i=1}^M z^i \leq z.$$

[16] may also be written as

$$CR(y, w, z) = \min_{z^1} \left[\sum_{i=1}^M C^i(y_i, w, z^i); \sum_{i=1}^M z^i \leq z \right] \quad [17]$$

² In that case, $\delta^2 C / \delta w_0 \delta w_0$ is an M matrix and its inverse possesses nonpositive elements only.

where $C^i(y_i, w, z^i)$ are "partially constrained" cost functions which are non-joint in input quantities, i.e. $\delta^2 C^i / \delta y_i \delta y_j = 0$, for any $i \neq j$. The long-run cost function can therefore be defined by

$$C(y, w, r) = \min_{x^i, z^i} \left[\sum_{i=1}^M w'x^i + r'z^i ; y_i = f^i(x^i, z^i) \right] \quad [18a]$$

$$= \min_{z^i} \left[\sum_{i=1}^M C^i(y_i, w, z^i) + r'z^i \right] \quad [18b]$$

$$= \min_z [CR(y, w, z) + r'z] \quad [18c]$$

where r is the price vector of allocatable factors z . Finally, we define the long-run profit function $\pi(p, w, r)$, the long-run revenue function $R(p, x, z)$ and the short-run profit function $\pi R(p, w, z)$ by the following equations

$$\pi(p, w, r) = \max_y [p'y - C(y, w, r)] \quad [19a]$$

$$= \max_z [\pi R(p, w, z) - r'z] \quad [19b]$$

$$= \max_{x, z} [R(p, x, z) - w'x - r'z] \quad [19c]$$

At this stage, it is interesting to note that definitions [18c], [19a], [19b] and [19c] do not depend on the nature of factors z and are satisfied in the cases where z are allocatable inputs and where they are public inputs.

Using the Hessian identities approach of Lau, it follows that

$$[\delta^2 \pi R / \delta p \delta w] = [\delta^2 R / \delta p \delta x] [\delta^2 R / \delta x \delta x]^{-1} \quad [20]$$

We assume then that the unconstrained revenue function $R(p, x, z)$ satisfies properties [3] and [4]. We have therefore $\delta^2 R / \delta p_i \delta x_k \geq 0$, for any i and k , and $\delta^2 R / \delta x_h \delta x_k \geq 0$, for any $h \neq k$. Furthermore, R being concave in x and satisfying property [4], $[\delta^2 R / \delta x \delta x]^{-1}$ has nonpositive elements only. It is then easy to show that $\delta^2 \pi R / \delta p_i \delta w_k \leq 0$, for any i and k , which shows that [7] necessarily holds at the short-run equilibrium contrary to the third proposition of Moschini³.

Using the same approach in terms of Hessian identities, we have also

³ Since $\delta^2 \pi^i / \delta p_i \delta w_k$ (π^i is the unconstrained non-joint profit function) may be equal to zero in the normal case, Moschini claims to prove that $\delta^2 \pi R / \delta p_i \delta w_k$ may be nonnegative. Nevertheless, we have also the following relationship: $\delta^2 \pi^i / \delta p_i \delta w_k = -(\delta^2 C^i / \delta w_k \delta y_i) (\delta^2 \pi^i / \delta p_i \delta p_i)$. It is then easy to show that both right-hand side terms of Moschini's equation [12] are equal to zero (and not only the second right-hand side term) and that $\delta^2 \pi R / \delta p_i \delta w_k$ is equal to zero. Therefore, proposition [7] is not violated.

$$[\delta^2 \pi R / \delta p \delta z] = [\delta^2 R / \delta p \delta z] - [\delta^2 R / \delta p \delta x] [\delta^2 R / \delta x \delta x]^{-1} [\delta^2 R / \delta x \delta z] \quad [21]$$

and

$$[\delta^2 \pi R / \delta p \delta p] = [\delta^2 \pi / \delta p \delta p] + [\delta^2 \pi / \delta p \delta r] [\delta^2 \pi R / \delta z \delta p] \quad [22]$$

Using the same reasoning as previously, i.e. properties [3] and [4], and the concavity of R in x and z , we can show that the elements of $[\delta^2 \pi R / \delta p \delta z]$ are nonnegative. Property [7] implies that the elements of $[\delta^2 \pi / \delta p \delta r]$ are nonpositive. The long-run technology being non joint in input quantities, the matrix $[\delta^2 \pi / \delta p \delta p]$ is diagonal. It follows therefore that the off-diagonal elements of $[\delta^2 \pi R / \delta p \delta p]$ are nonpositive. Finally, it is easy to show that the off-diagonal elements of $[\delta^2 CR / \delta y \delta y]$ are nonnegative using the Hessian identity $[\delta^2 CR / \delta y \delta y] = [\delta^2 \pi R / \delta p \delta p]^{-1}$ and the properties of M matrices.

We have shown that a technology, which satisfies properties [3] and [4] and is non-joint in input quantities at the unconstrained equilibrium, verifies the property of cost substitutabilities in the short run, i.e. when some inputs are fixed. This implication does not depend on the nature, allocatable or public, of the inputs which are fixed in the short run ⁴. Therefore, our results ($\delta^2 \pi R / \delta p_i \delta p_j \leq 0$, for any $i \neq j$, and $\delta^2 CR / \delta y_i \delta y_j \geq 0$ for any $i \neq j$) are more general than propositions 1 and 2 of Moschini. Finally, we have shown that the third proposition of Moschini is not satisfied, i.e. we have $\delta^2 \pi R / \delta p_i \delta w_k \leq 0$, for any i and k . Finally, it is interesting to note that we do not use the property of input normality but only properties associated to the unconstrained revenue function.

5. CONCLUDING COMMENTS

If production is joint only because some allocatable inputs are constraining, the restricted technology, defined with respect to constrained profit and cost functions, exhibits cost substitutabilities (i.e. $\delta^2 CR / \delta y_i \delta y_j \geq 0$ for any $i \neq j$) and gross substitutability among outputs (i.e. $\delta^2 \pi R / \delta p_i \delta p_j \leq 0$ for any $i \neq j$) if the unconstrained technology is normal and, more precisely, if the unconstrained revenue function satisfies properties [3] of [4] of Sakai. Therefore, in constrained regime, constraints on the availability of allocatable inputs imply that the technology is

⁴ Because we use only definitions of objective functions stated in terms of π , R and C and not π^i and C^i , the previous demonstration corresponds to the case where fixed inputs in the short run may be allocatable or public.

characterized by increasing marginal costs when the quantities of other outputs increase and decreasing marginal profits when the prices of other outputs increase.

These results differ from those obtained when joint production is due to public inputs. In that case, the marginal cost of an output tends to increase when the quantities of other outputs decrease (under normality assumption of public inputs, this proposition is strictly correct only in the case of one public input) and gross substitution among output is ruled out.

These apparent opposite results are not contradictory. In the allocatable-factor case, netput relationships are examined in constrained regime whereas they are analyzed in unconstrained situation in the public factor case. In the allocatable factor case, the unconstrained technology is non-joint in input quantities (i.e. verifies $\delta^2 \pi / \delta p_i \delta p_j = 0$ and $\delta^2 C / \delta y_i \delta y_j = 0$, for any $i \neq j$) and therefore does not violate the "normal" restrictions on C and π . In fact, cost substitutabilities in constrained regime do not depend on the nature of constrained inputs, but are due to non-jointness in input quantities (which is allowed in the normal case) in unconstrained regime.

REFERENCES

- Baumol, W.J., J.C. Panzar, and R.D. Willig. *Contestable Markets and the Theory of Industry Structure*. New-York: Harcourt Brace Jovanovich, 1982.
- Chambers, R.G. *Applied Production Analysis*. Cambridge: Cambridge University Press, 1988.
- Hall, R.E. "The Specification of Technology with Several Kinds of Outputs". *J. Polit. Econ.* 81(1973) : 878-92.
- Hertel, T.W. "Applications of Duality and Flexible Functional Forms : The Case of the Multiproduct Firm". Research Bulletin 980, Purdue University, Agricultural Experiment Station, West Lafayette, Indiana, 1984.
- Hertel, T.W. "Estimating Substitution and Expansion Effects: Comment." *Amer. J. Agr. Econ.* 69(1987):188-92.
- Hughes, J.P. "Giffen Inputs and the Theory of Multiple Production." *J. Econ. Theory* 25(1981): 287-301.
- Kohli, U.R. "Nonjointness and Factor Intensity in U.S. Production." *Int. Econ. Rev.* 22(1981):3-18.
- Kohli, U.R. "Non-joint Technologies." *Review Econ. Studies* 50(1983):209-19.
- Lau, L.J. "Profit Functions of Technologies with Multiple Inputs and Outputs." *Rev. Econ. and Statist.* 54(1972): 281-89.
- Lau, L.J. "A Characterization of the Normalized Restricted Profit Function." *J. Econ. Theory* 12(1976):131-63.
- Lau, L.J. "Applications of Profit Functions." *Production Economics: A Dual Approach to Theory and Applications*, ed. M. Fuss and D. McFadden, vol.1, pp. 133-216. Amsterdam: North-Holland Publishing Co., 1978.
- Lynne, G.D. "Allocatable Fixed Inputs and Jointness in Agricultural Production: Implications for Economic Modeling: Comment." *Amer. J. Agr. Econ.* 70(1988):947-49.
- Moschini, G. "A Model of Production with Supply Management for the Canadian Agricultural Sector." *Amer. J. Agr. Econ.* 70(1988):318-29.
- Moschini, G. "Normal Inputs and Joint Production with Allocatable Fixed Factors." *Amer. J. Agr. Econ.* 71 (1989): 1021-1024.
- Nelson, C.H. "A model of Production with Supply Management for the Canadian Agricultural Sector: Comment." *Amer. J. Agr. Econ.* 73(1991):867-70.
- Rader, T. "Normally, Factor Inputs Are Never Gross Substitutes." *J. Polit. Econ.* 76(1968):38-43.
- Sakai, Y. "Substitution and Expansion Effects in Production Theory: The Case of Joint Production." *J. Econ. Theory* 9(1974):255-74.

Samuelson, P.A. "The Fundamental Singularity Theorem for Non-Joint Production." *Int. Econ. Rev.* 7(1966):34-41.

Shumway, C.R., R.D. Pope, and E.K. Nash. "Allocatable Fixed Inputs and Jointness in Agricultural Production: Implications for Economic Modeling." *Amer. J. Agr. Econ.* 66(1984):72-78

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