Active Noise Control: Adaptive vs. Robust Approach

Ioan Doré Landau, Raúl Antonio Meléndez

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***Abstract*—Active noise control is often concerned with the strong attenuation of single or multiple tonal noise disturbances which may have unknown and time varying frequencies. Currently in applications, adaptive feed-forward compensation is used which requires the use of an additional transducer and introduces an instability risk due to a positive internal coupling. However for these types of noise a feedback approach can be efficiently used and this will be illustrated in this paper. One considers the case of two tonal disturbances located in two distinct frequency regions subject to frequency variations within a given range as well the case of interferences between tonal disturbances of very close frequencies. The objective is to minimize the measured residual noise in a predefined location. These problems occurs often in ventilation systems (active silencers). To solve these problems robust and adaptive solutions are considered. A robust controller design using sensitivity function shaping is considered. The maximum achievable attenuation is inverse proportional to the range of frequency variations of the tonal disturbances. To further improve the performance an add-on direct adaptive feedback approach using the Internal Model Principle and the Youla Kucera parametrization is considered. The adaptive approach allows both to improve the performance within the given frequency ranges as well as to extend the admissible domain of frequencies variations. Experimental results obtained on a relevant test bench will illustrates the performance of the two designs.**

***Index Terms*—Active noise control, System Identification, Internal model principle, Youla-Kučera parametrization, Adaptive control, Robust control.**

I. Introduction

Active disturbance rejection is a key issue in active vibration control and active noise control. The popular approach for active noise control is to use adaptive feed-forward compensation. This approach, inspired by Widrows technique for adaptive noise cancellation, see [1], ignores the possibilities offered by feedback control systems and have a number of disadvantages: 1) it requires the use of an additional transducer for obtaining an image of the disturbance, 2) difficult choice for positioning this additional transducer and, 3) in most cases, there exists a "positive" coupling between the compensator system and the disturbance image's measurement, which can cause instabilities. See for example [2]. To achieve the disturbance's rejection (asymptotically) without measuring it, a feedback solution can be considered. This approach is particularly pertinent for single or multiple time varying tonal or narrowband disturbance noise.

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The common framework is the assumption that a narrow band or a tonal disturbance noise is the result of a white noise or a Dirac impulse passed through the "disturbance's model." More specifically the model for a single narrow-band or tonal disturbance is a notch filter with poles on the unit circle and zeros inside the unit circle (for details see [3]).

In managing the vibration attenuation by feedback, the shape of the modulus of the "output sensitivity function" (the transfer function between the disturbance and the residual acceleration/force) is fundamental both from performance and robustness considerations. Three basic concepts are to be considered: the Bode Integral, the Modulus margin and the Internal Model Principle (IMP). The problem of robust control design in the context of active noise control has been considered in [4] and the shaping of the output sensitivity function has been achieved using the convex optimization procedure introduced in [5]. See also [6], [7] for Hinf and LMI approaches.

In this paper, one considers multiple unknown and time varying tonal disturbances located within two distinct relatively small frequency ranges. To be specific, two cases will be considered: (i) the case of two time varying tonal disturbances located in two distinct frequency regions and (ii) the case of four simultaneous tonal disturbances, two located in one limited frequency range and the other two in another frequency range. In this context, a very important problem is to be able to counteract the very low frequency oscillations which are generated when the two frequencies are very close (interference). Since these disturbances are located within two relatively small frequency ranges, it is possible to consider a robust linear control design which will shape the output sensitivity function in such a way that a sufficient attenuation is introduced in these two frequency regions but avoiding significant amplification at other frequencies (both for performance and robustness reason). It will be shown in this paper that an elementary procedure for shaping appropriately the modulus of the sensitivity functions can be implemented using stop band filters as shaping tools. For a basic reference on this approach see [3].

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To further improve the performance an add-on direct adaptive feedback approach using the Internal Model Principle and the Youla Kucera parametrization is considered. The adaptive approach allows both to improve the performance within the given frequency ranges as well as to extend the admissible domain of frequencies variations.

The performance of these approaches depend to a large extent on the quality of the dynamic model of the compensator system used for controller design. To obtain such reliable

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model, identification from data of a finite dimensional discrete time model has to be used since the physical modelling does not in general provide enough good models for design.

Experimental results on a noise silencer for noise attenuation in ducts will illustrate comparatively the performance of the robust and adaptive approach.

II. The Test Bench

The detailed scheme of the noise silencer test bench used for the experiments is given in Fig. 1. Its actual photo is shown in Fig. 2. The speaker used as the source of disturbances is labeled as 1, the control speaker is 2 and finally, at pipe's open end, the microphone that measures the system's output (residual noise) is denoted as 3. The transfer function between the disturbance's speaker and the microphone (1→3) is denominated *Primary Path*, while the transfer function between the control speaker and the microphone (2→3) is denominated *Secondary Path*. Both speakers are connected to a PC Target computer with Simulink Real Time® environment through a pair of high definition power amplifiers and a data acquisition card. In Fig. 1, $y(t)$ is the system's output (residual noise measurement) and $u(t)$ is the control signal. Both primary and secondary paths have a double differentiator behaviour, since as input we have the voice coil displacement, and as output the air acoustic pressure. A second computer is used for development, design and operation with Matlab®.

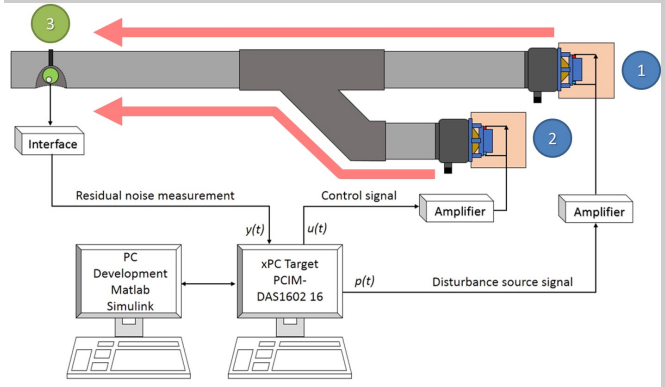

Fig. 1. Active noise control test bench diagram.

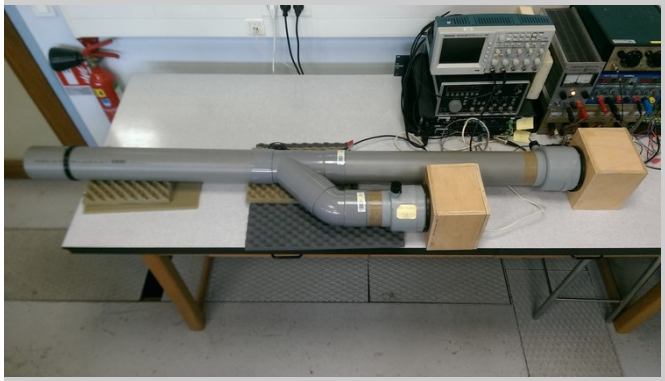

Fig. 2. Active noise control test bench (Photo).

PVC pipes of 0.10 m diameter are used in this test bench, with couplings of 135° for the control speaker. Distances between disturbance loudspeaker and microphone are 1.65 m, and to control input 0.80 m. Speakers are isolated inside wood boxes filled with special foam in order to create anechoic chambers and reduce the radiation noise produced.

III. System Description

The linear time invariant *(LTI)* discrete-time model of the secondary path, (the plant), used for controller design is

$$G(z^{-1}) = \frac{z^{-d}B(z^{-1})}{A(z^{-1})} = \frac{z^{-d}B'(z^{-1})D_F(z^{-1})}{A(z^{-1})}, \qquad (1)$$

where $D_F(z^{-1}) = (1-z^{-1})^2$ is a double differentiator filter and

$$A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_{n_A} z^{-n_A}, \qquad (2)$$

$$B'(z^{-1}) = b_1 z^{-1} + \cdots + b_{n_{B'}} z^{-n_{B'}}, \qquad (3)$$

with d as the plant pure time delay in number of sampling periods[1]. The system's order (without the double differentiator) is:

$$n = max(n_A, n_{B'} + d) \qquad (4)$$

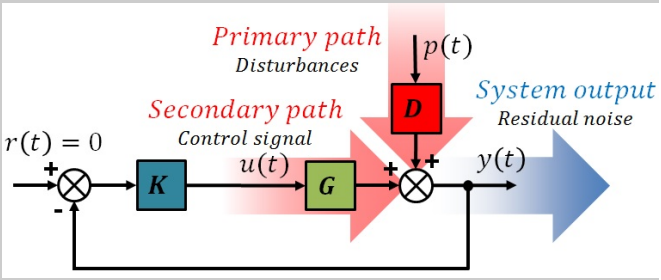

Fig. 3. Feedback regulation scheme.

Figure 3 shows the closed loop feedback regulation scheme, where the controller K is described by

$$K(z^{-1}) = \frac{R}{S} = \frac{r_0 + r_1 z^{-1} + \cdots + r_{n_R} z^{-n_R}}{1 + s_1 z^{-1} + \cdots + s_{n_S} z^{-n_S}}. \qquad (5)$$

The plant's output $y(t)$ and the input $u(t)$ may be written as (see Fig. 3):

$$y(t) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})} \cdot u(t) + p(t), \qquad (6)$$

$$S(q^{-1}) \cdot u(t) = -R(q^{-1}) \cdot y(t). \qquad (7)$$

In (6), $p(t)$ is the disturbances' effect on the measured output[2] and $R(z^{-1})$, $S(z^{-1})$ are polynomials in z^{-1} having the following expressions:

$$R = H_R \cdot R' = H_R \cdot (r'_0 + r'_1 z^{-1} + \ldots + r'_{n_{R'}} z^{-n_{R'}}), \qquad (8)$$

$$S = H_S \cdot S' = H_S \cdot (1 + s'_1 z^{-1} + \ldots + s'_{n_{S'}} z^{-n_{S'}}), \qquad (9)$$

[1]The complex variable z^{-1} is used to characterize the system's behavior in the frequency domain and the delay operator q^{-1} for the time domain analysis.

[2]The disturbance passes through the *primary path*, and $p(t)$ is its output.

where $H_S(z^{-1})$ and $H_R(z^{-1})$ represent prespecified parts of the controller (used for example to incorporate the internal model of a disturbance, or to open the loop at some frequencies) and $S'(z^{-1})$ and $R'(z^{-1})$ are solutions of the Bezout equation:

$$P = (A \cdot H_S) \cdot S' + \left(z^{-d} B \cdot H_R\right) \cdot R'. \tag{10}$$

In (10) $P(z^{-1})$ represents the characteristic polynomial, which specifies the desired closed loop poles of the system.

The transfer functions between the disturbance $p(t)$ and the system's output $y(t)$ and the control input $u(t)$, denoted respectively *output sensitivity* and *input sensitivity functions*, are given by

$$S_{yp}(z^{-1}) = \frac{A(z^{-1})S(z^{-1})}{P(z^{-1})} \tag{11}$$

and

$$S_{up}(z^{-1}) = -\frac{A(z^{-1})R(z^{-1})}{P(z^{-1})}, \tag{12}$$

IV. System Identification

System identification from experimental data (see [3], [8]) will be used for obtaining the dynamic model of the compensator system used for controller design .

For design and application reasons (the objective is to reject tonal disturbances up to 400 Hz), the sampling frequency was selected as $f_s = 2500\,\text{Hz}$ ($T_s = 0.0004\,\text{s}$) i.e. approximatively 6 times the maximum frequency to attenuated (see [3]).

A Pseudo Random Binary Sequence (PRBS) has been used as excitation signal. Its characteristics are: magnitude = 0.15 V, register length = 17, frequency divider of 1, sequence length of $2^{17} - 1 = 131{,}071$ samples, guaranteeing a uniform power spectrum from about 70 Hz to 1250 Hz. Since the transfer functions has a double differentiator behaviour (input: speaker's coil position, output: acoustic pressure), this is considered as a system's known part and the objective will be to identify the unknown part only. To do this, the input sequence will be filtered by a double discrete-time differentiator $D_F = (1-q^{-1})^2$ such that $u'(t) = D_F \cdot u(t)$. The double differentiator will be concatenated with the identified model of the unknown part in the final model used for controller design.

The criterion used for order estimation has the form:

$$J_{IV}(\hat{n}, N) = V_{IV}(\hat{n}, N) + 2\hat{n}\frac{\log N}{N}, \tag{13}$$

where $\hat{n}$ is the estimated order of the system and N is the number of data and the optimal estimated order is the one which minimize the criterion J_{IV}. The first term of the criterion $V_{IV}(\hat{n}, N)$ is a prediction error criterion to which a term penalizing the model's complexity is added. The effective order estimation was done using the algorithms given in [3], [9] which uses *instrumental variables*for obtaining an unbiased value for the error criterion V_{IV}. Once an estimated order $\hat{n}$ is selected, one can apply a similar procedure to estimate $\hat{n}_A$, $\hat{n} - \hat{d}$, and $\hat{n}_{B'} + \hat{d}$, from which $\hat{n}_A$, $\hat{n}_{B'}$ and $\hat{d}$ are obtained.

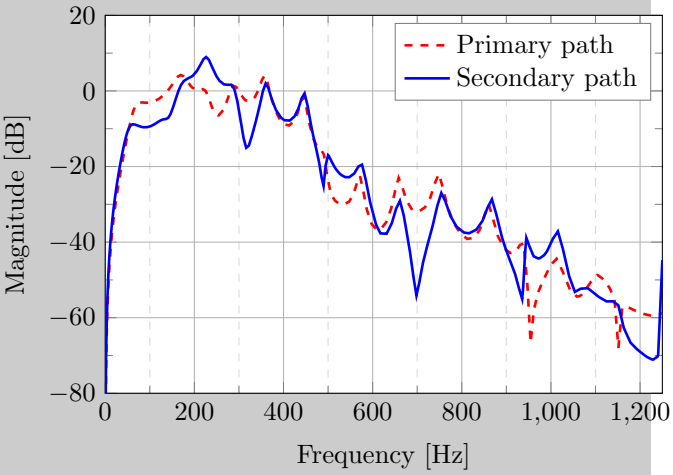

Fig. 4. Frequency characteristics of the identified primary and secondary paths models.

A model with the orders n=40, nA=38, nB'=30 and d=8 has been chosen.

Comparison of several models obtained with various parameter estimation algorithms in terms of statistical validation led to the conclusion that an ARMAX model representation is the most appropriate for this system. Among the various methods which can be used for this structure[3], *XOLOE* algorithm gives the best results for a given order model, in terms of whiteness test validation (see [3]).

Therefore the *XOLOE* model with $n = 40$ has been chosen. It has 18 oscillatory modes with damping comprised between 0.0097 and 0.3129; also 13 pairs of oscillatory zeros with damping comprised between −0.0159 and 0.5438. Fig. 4 gives the frequency characteristics of the identified complete models for the primary and secondary paths[4].

V. Robust Control Design

Control specifications

The controller will be designed to attenuate in regions of ±5 Hz around the two nominal frequencies 170 Hz and 285 Hz. The attenuation must be al least of −17 dB and any undesired amplification should be less that 7 dB. Also since our model may be not fully representative of the system's behaviour at high frequencies, magnitudes at the input sensitivity function should be below −20 dB at frequencies over 600 Hz (improving robustness versus additive plant model uncertainties in high frequencies).

In addition the gain of the controller should be zero at 0 Hz since the plant does not have gain at zero frequency and the gain of the controller should be zero also at the Nyquist frequency $(0.5 f_s)$ for robustness reasons. These control specifications will be achieved through the sensitivity functions' shaping.

[3]Recursive Extended Least Squares *(RELS)*, Output Error with Extended Prediction Model *(OEEPM)* or *(XOLOE)*, Recursive Maximum Likelihood *(RML)*

[4]Primary path model has been identified using the same procedure. This model is used for simulations only

Design procedure

To achieve the constraints at 0Hz and at $0.5f_s$, a fixed part (H_R) [5] will be introduced in the controller:

$$H_R(q^{-1}) = (1-q^{-1})(1+q^{-1}) = 1-q^{-2}, \tag{14}$$

Three major tools will be used for design

- Choice of the dominant poles
- Use of the band stop filters for shaping the sensitivity functions
- Choice of the auxiliary poles for further improving performance and robustness

The use of auxiliary poles will be done such that the characteristic polynomial take the form

$$P(z^{-1}) = P_D(z^{-1}) \cdot P_F(z^{-1}), \tag{15}$$

where P_D are the dominant poles obtained from the identified dynamic model, and P_F will be the auxiliary poles determined by the controller's requirements.

It is shown in [3] that very accurate shaping of the output or the input sensitivity functions can be obtained by the use of band-stop filters (BSF). These are IIR filters obtained from the discretization of continuous-time filters of the form

$$F(s) = \frac{s^2 + 2\zeta_{num}\omega_0 s + \omega_0^2}{s^2 + 2\zeta_{den}\omega_0 s + \omega_0^2} \tag{16}$$

using the bilinear transform $s = \frac{2}{Ts}\frac{1-z^{-1}}{1+z^{-1}}$. The use of BSFs introduces an attenuation $M = 20\log\left(\frac{\zeta_{num}}{\zeta_{den}}\right)$ at the normalized discretized frequency $\omega_d = 2 \cdot \arctan\left(\frac{\omega_0 T_S}{2}\right)$. Depending on whether the filter is designed for shaping the output or the input sensitivity function, the numerator of the discretized filter is included in the fixed part of the controller denominator H_{S_0} or numerator H_{R_0}, respectively. The filter denominator is always included in the desired closed loop characteristic polynomial. As such, the filter denominator influences the design of the controller indirectly since S'_0 and R'_0 are solutions of the Bezout equation (10). These filters will be used for a fine shaping of both the output and input sensitivity functions.

The steps for the design of the linear controller are:

1) include all (stable) secondary path poles in the closed loop characteristic polynomial.
2) open the loop at 0 Hz and at 1250 Hz by setting the fixed part of the controller numerator as in Eq. (14).
3) 7 BSFs on S_{yp} have been used around each of the frequencies where attenuation is desired in order to assure the desired attenuation within ± 5 Hz .
4) 11 BSF has been used on S_{up} to reduce its magnitude above 600 Hz.
5) to improve robustness 17 auxiliary real poles located at 0.17 have been added to the characteristic polynomial.

Figure 5 shows the characteristics of the output sensitivity function through the various steps of the design. It is clear that the performance and robustness specifications are achieved.

[5] $H_i, H_{R_i}, H_{R_i}, P_{F_i}$ will denote any given controller's fixed part.

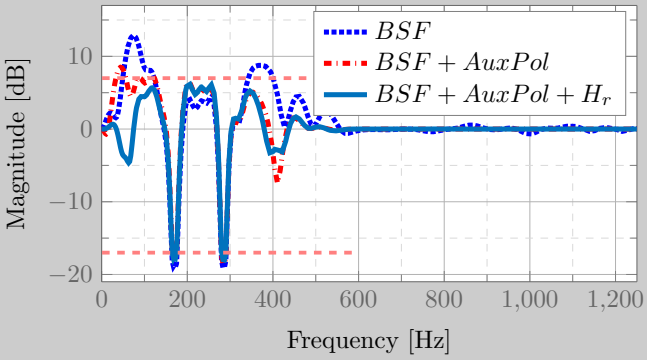

Fig. 5. Robust controller design: Output sensitivity function.

VI. Adaptive Control Design

The adaptive approach uses the Youla-Kucera parametrization of the controller combined with the Internal Model Principle. The basic reference for this approach used in active vibration control is [3] A key aspect of this methodology is the use of the Internal Model Principle (IMP). It is supposed that $p(t)$ is a deterministic disturbance given by

$$p(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t), \tag{17}$$

where $\delta(t)$ is a Dirac impulse and N_p, D_p are coprime polynomials of degrees n_{N_p} and n_{D_p}, respectively[6]. In the case of stationary narrow-band disturbances, the roots of $D_p(z^{-1})$ are on the unit circle.

Internal Model Principle[10]: The effect of the disturbance (17) upon the output

$$y(t) = \frac{A(q^{-1})S(q^{-1})}{P(q^{-1})} \cdot \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t), \tag{18}$$

where $D_p(z^{-1})$ is a polynomial with roots on the unit circle and $P(z^{-1})$ is an asymptotically stable polynomial, converges asymptotically towards zero *iff* the polynomial $S(z^{-1})$ in the RS controller has the form (based on eq. (9))

$$S(z^{-1}) = D_p(z^{-1})H_{S_0}(z^{-1})S'(z^{-1}). \tag{19}$$

Thus, the pre-specified part of $S(z^{-1})$ should be chosen as $H_S(z^{-1}) = D_p(z^{-1})H_{S_0}(z^{-1})$ and the controller is computed solving

$$P = AD_pH_{S_0}S' + z^{-d}BH_{R_0}R', \tag{20}$$

where P, D_p, A, B, H_{R_0}, H_{S_0} and d are given[7].
In the context of this paper for the Youla-Kučera parametrization, one considers a finite impulse response (FIR) filter of the form:

$$Q(z^{-1}) = q_0 + q_1 z^{-1} + \cdots + q_{n_Q} z^{-n_Q}, \tag{21}$$

to which one associate the vector of parameters:

$$\theta = [q_0 \; q_1 \ldots q_{n_Q}]^T. \tag{22}$$

[6] Throughout the paper, n_X denotes the degree of the polynomial X.
[7] Of course, it is assumed that D_p and B do not have common factors.

Under Youla-Kučera parametrization or *Q-parametrization*, the equivalent polynomials $R(z^{-1})$ and $S(z^{-1}$ of the controller $K(q^{-1})$ take the form

$$R(q^{-1}) = R_0 + A \cdot Q \cdot H_{S_0} \cdot H_{R_0} \quad (23)$$

$$S(q^{-1}) = S_0 - q^{-d} B \cdot Q \cdot H_{S_0} \cdot H_{R_0}, \quad (24)$$

with

$$R_0(z^{-1}) = r_0^0 + r_1^0 z^{-1} + \ldots + r_{nR}^0 z^{-n_{R_0}} = R'_0 \cdot H_{R_0} \quad (25)$$

$$S_0(z^{-1}) = 1 + s_1^0 z^{-1} + \ldots + s_{nS}^0 z^{-n_{S_0}} = S'_0 \cdot H_{S_0}, \quad (26)$$

where A, B and d correspond to the identified model of the secondary path, $R_0(z^{-1})$, $S_0(z^{-1})$ are the central controller's polynomials, and H_{S_0}, H_{R_0} are the controller fixed parts.

Using the output sensitivity function, the expression of the output can be written as:

$$y(t) = \frac{S_0}{P} \cdot w(t) - Q \cdot \frac{q^{-d} B H_{S_0} H_{R_0}}{P} \cdot w(t), \quad (27)$$

with

$$w(t) = A \cdot y(t) - q^{-d} B \cdot u(t) = A \cdot p(t) \quad (28)$$

as a disturbance's observer. The objective is to find a value of Q such that $y(t)$ is driven to zero.

A block diagram of the adpative scheme is given in Figure 6.

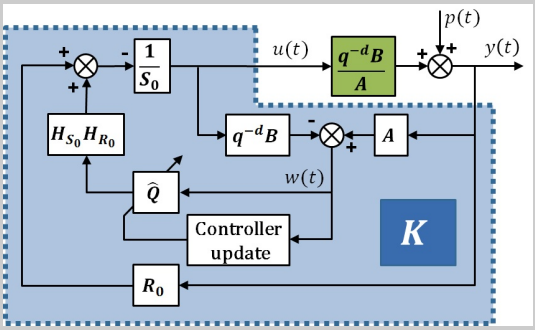

Fig. 6. Adaptive Youla-Kučera parametrization scheme.

The estimation of the polynomial Q at time t is denoted:

$$\hat{Q}(t, q^{-1}) = \hat{q}_0(t) + \hat{q}_1(t) q^{-1} + \cdots + \hat{q}_{n_Q}(t) q^{-n_Q} \quad (29)$$

and is caracterized by the parameter vector:

$$\hat{\theta}(t) = [\hat{q}_0(t)\ \hat{q}_1(t) \ldots \hat{q}_{n_Q}(t)]^T, \quad (30)$$

Since this is a regulation problem $y(t)$ is expected to go to zero and as such it is an a priori adaptation error denoted $\varepsilon^0(t+1)$ for a given estimated polynomial $\hat{Q}(t, q^{-1})$:

$$\varepsilon^0(t+1) = \frac{S_0}{P} \cdot w(t+1) - \hat{Q}(t) \frac{q^{-d} B^* H_{S_0} H_{R_0}}{P} \cdot w(t), \quad (31)$$

with $B(q^{-1}) = q^{-1} \cdot B^*(q^{-1})$. In a similar way, we can define an *a posteriori* error like

$$\varepsilon(t+1) = \frac{S_0}{P} \cdot w(t+1) - \hat{Q}(t+1) \frac{q^{-d} B^* H_{S_0} H_{R_0}}{P} \cdot w(t), \quad (32)$$

which can be further expressed as

$$\varepsilon(t+1) = [Q - \hat{Q}(t+1)] \cdot \frac{q^{-d} B^* H_{S_0} H_{R_0}}{P} \cdot w(t) + \eta(t+1) \quad (33)$$

where $\eta(t)$ tends asymptotically towards zero (see [3]for details).

Denoting filtered versions of observer output $w(t)$ as

$$w_1(t) = \frac{S_0(q^{-1})}{P(q^{-1})} \cdot w(t) \quad (34)$$

$$w_2(t) = \frac{q^{-d} B^* H_{R_0} H_{S_0}}{P} \cdot w(t) \quad (35)$$

and

$$\varphi^T(t) = [w_2(t)\ w_2(t-1) \ldots w_2(t-n_Q)], \quad (36)$$

Eq. (33) can be rewritten as:

$$\varepsilon(t+1) = [\theta^T - \hat{\theta}^T(t+1)] \cdot \varphi(t) + \eta^*(t+1). \quad (37)$$

This type of equation allows immediately to develop an adaptation algorithm (see [11]):

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F(t) \varphi(t) \varepsilon(t+1) \quad (38)$$

$$\varepsilon(t+1) = \frac{\varepsilon^0(t+1)}{1 + \varphi^T(t) F(T) \varphi(t)} \quad (39)$$

$$\varepsilon^0(t+1) = w_1(t+1) - \hat{\theta}^T(t) \varphi(t). \quad (40)$$

$$F(t+1) = \frac{1}{\lambda_1(t)} \left[F(t) - \frac{F(t) \varphi(t) \varphi^T(t) F(t)}{\frac{\lambda_1(t)}{\lambda_2(t)} + \varphi^T(t) F(t) \varphi(t)} \right] \quad (41)$$

where λ_1, λ_2 allows to obtain different profiles for the evolution of the adaptation gain $F(t)$. Finally the control to be applied is given by

$$S_0 \cdot u(t+1) = -R_0 \cdot y(t+1) - H_{R_0} H_{S_0} \hat{Q}(t+1) \cdot w(t+1). \quad (42)$$

For the stability analysis of this algorithm see[11].

VII. Experimental Results

The robust and adaptive design have been comparatively evaluated on the duct silencer described in Section II. For all the adaptive experiments $n_Q = 3$ (4 parameters)

A. Interference test

Figure 7 shows the performance of the robust controller in the presence of two pairs of sinusoidal noise signals acting simultaneously, and located first at170 Hz and 170.5 Hz, 285 Hz from 10s to 20s and then with modified central frequencies located at 285.5 Hz, 180 Hz and 180.5 Hz, and respectively at 295 Hz and 295.5 Hz. One can see that the controller gives good performance from 10s to 20s(global attenuation of 36.56 dB) but the performance is degraded after 20s and this is understandable since one operates outside the the region considered for the design. Figure 8shows the performance of the adaptive controller for the same configuration. The performance are very good (global attenuation of 71.45 dB).

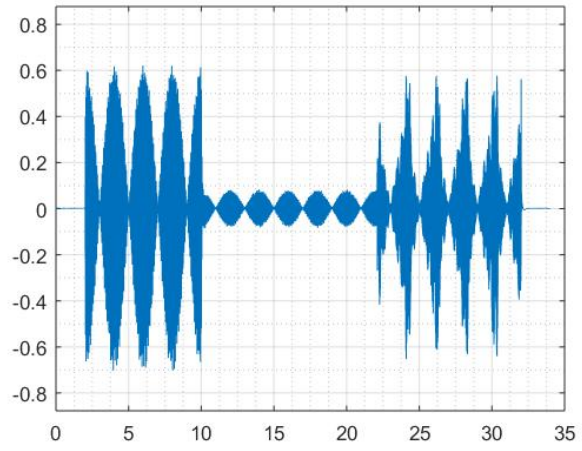

Fig. 7. Acoustic interference attenuation using a robust controller. Noise frequencies:170,170.5,285,285.5 Hz then 180,180.5,295, 295.5 Hz.

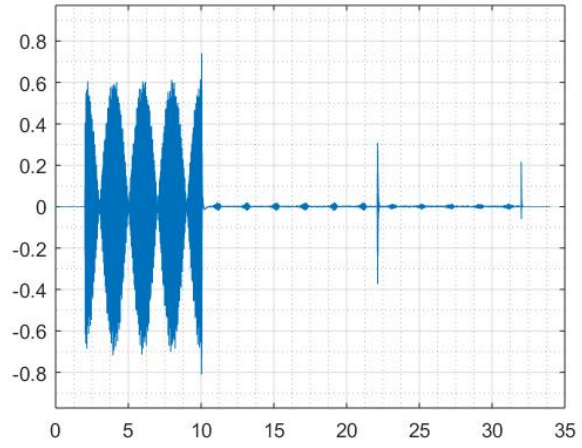

Fig. 8. Acoustic interference attenuation using an adaptive controller. Noise frequencies:170,170.5,285,285.5 Hz then 180,180.5,295, 295.5 Hz.

B. Step Changes in Frequency

In this test two simultaneous signals of constant frequency act as disturbances. After a given amount of time a step change in the frequencies of both signals is done. Both frequencies are decreased or increased with a constant value and remain at those new constant frequencies for 4s . Figures 9 and 10 show the performance of the robust and adaptive controller. The red curves gives the magnitude of the residual noise in open loop and the blue curves give the magnitude of the residual noise in closed loop. The frequencies of the disturbances are indicated in the plots. One can clearly see that the adaptive controller has better performance than the robust controller even within the frequency domain of variations used for robust controller design.

VIII. Conclusion

The paper has shown that robust and adaptive approaches can be considered for active attenuation of multiple narrow band noise disturbances by feedback. However the adaptive approach offer better performance.

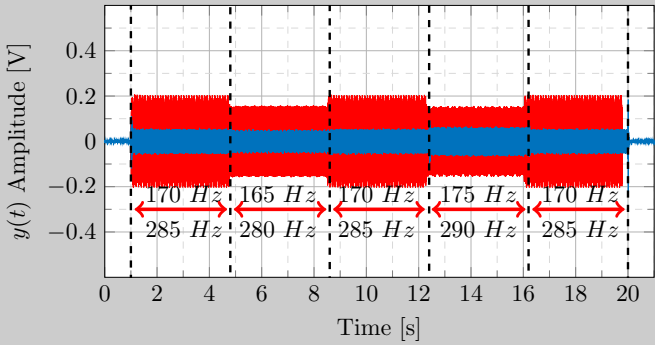

Fig. 9. Step changes in frequencies using the robust controller.

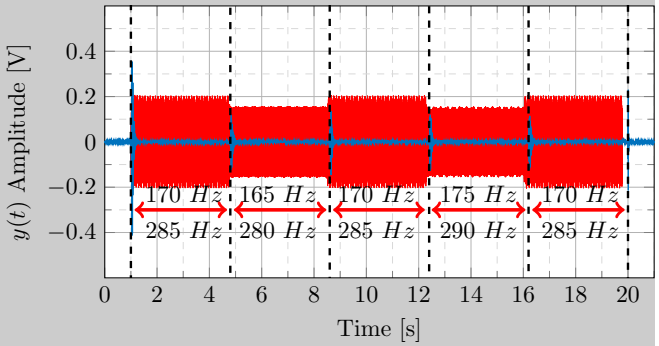

Fig. 10. Step changes in frequencies using the adaptive controller.

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