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Curriculum in France: A National Frame in Transition

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INTRODUCTION AND HISTORICAL BACKGROUND

For the last two centuries, France has had a national curriculum elaborated by specialized committees and associated with textbooks as central for its implementation by teachers. For example, after the creation by Napoléon in 1802 of upper secondary schools (“Lycées” for students aged 11 to 18), a committee formed by three famous French mathematicians – Laplace, Monge, and Lacroix – published a sciences curriculum for these Lycées in 1803. Lacroix (1802) also wrote textbooks for the implementation of this curriculum. Some curricular reforms in France have had a strong impact on teaching beyond national borders, like the well-known reform of “Modern Maths ” in the 1970s, grounding mathematics teaching on very formal approaches (Trouche, 2016a). Figure 3.1 summarizes the history of the main curriculum changes beginning in 1802. Indeed, the official curriculum changes quite often as in the recent period in 2002, 2007, 2008, and 2015-2016 for primary school. It is often said that each new government wants its own curriculum.

Mathematics in the French Curriculum: A Tumultuous History

The history of French mathematics curriculum is a tumultuous one that is very sensitive to scientific, social, and political tensions. For a good understanding of this history, we need to compare and contrast the *official* and the *real* curriculum. Looking at the official curriculum, the view of mathematics in France's curricula grew stronger over time, particularly in three key moments: 1802, 1902, and 2002.

1802: Napoleon's ordinance of 19 *frimaire* of Year XI (December 10, 1802) stated: "The *Lycées* will essentially teach Latin and mathematics." Gispert notes, "In placing mathematics at the same level as Latin in the male secondary curriculum, [this ordinance] took into account the new situation following the French Revolution, in which mathematics had become a *core aspect* of an intellectual education *combining theory and practice*" (2014, p. 230).

1902: A new reform, following a survey launched by the French Parliament, reasserted this importance of mathematics education: "It was, for a time, the end of the monopoly on classical humanities by the *Lycées*, through the creation of a modern curriculum that was on par – at least in theory – with the classical curriculum. It also furthered the development of new disciplines such as the languages, sciences, and mathematics" (Gispert, 2014, p. 233).

2002: In 1999, the French education ministry appointed a commission, the CREM (National Commission for Reflection on the Teaching of Mathematics) headed by Jean-Pierre Kahane,¹ for rethinking the teaching of mathematics for the new century. In 2002, a report of the CREM stated "La mathématique est la plus ancienne des sciences et celle dont les valeurs sont les plus permanentes"² (Kahane, 2002). It situated mathematics among the other sciences and underlined the necessity of connecting their teaching in combining *rigor* and *imagination*.

Looking at the enacted curriculum (see e.g., Stein, Remillard, & Smith, 2007), mathematics education experienced less change than envisioned: after the ordinance of 1802, Gispert (ibid. p. 230) noticed that "actually the real teaching, after this ordinance, continues to favour Latin and the classical humanities until the end of the 19th century and to separate theory and practice." It appears that two kinds of mathematics teaching existed, according to social class and schooling structure (*Lycées* vs. primary schools): formation of the mind in *Lycées* for upper social classes, training for practice in primary schools for lower social classes.

Concerning the enacted curriculum in more recent years, differences with the official recommendations probably exist (e.g., concerning inquiry-based teaching, Grangeat (2011)), but we are not aware of research works proposing a comprehensive study of such differences.

Questions at the heart of mathematics teaching also appeared sensitive to social and political events. Gispert (ibid. p. 235) indicates, for example, "that the reform of the beginning of the 20th century – 1902 – was accused of being inspired by the German model of the *Realschule* to the detriment of the specificity of a 'French spirit' based on Latin and the classical humanities." If the place of mathematics increased during the two last centuries, it was always because of discussions full of passion, far from the view of an abstract discipline, independent of the human nature...

Figure 3.1. Mathematics in the French curriculum history

¹ Jean-Pierre Kahane is a famous mathematician, member of the Academy of Sciences, and former president of ICMI (1983-1990).

² Mathematics is the most ancient science and these whose values are the most stable" (our translation).

In this chapter, we first present an overall picture of the French mathematics curriculum, in particular since 2000; then we focus on the primary school curriculum and its recent evolutions. We follow with considerations of the controversies raised by the new curriculum (starting September 2016) for students aged 12 to 15. Next we focus on a particular content area, algorithmics across different grades. Textbooks are crucial resources for teaching, influencing the implemented curriculum (Pepin & Haggarty, 2001; Valverde et al., 2002). Deep changes in curriculum have resulted in the development of digital resources so we also focus on the development of a particular French e-textbook. In the conclusion, we synthesize the main aspects of curriculum content, design, and implementation in France.

THE NATIONAL CURRICULUM IN FRANCE: AN EVOLVING FRAMEWORK FOR ITS STRUCTURE AND CONCEPTION

In this section, we first introduce the general organization of the school system in France. We next examine the main changes in the curriculum content since 2000. Last, we discuss evolutions concerning the design mode of this curriculum.

Main Principles of the Curriculum

France is a centralized country, sharing a national curriculum for all disciplines and all class levels from Kindergarten (starting at age 3) to upper secondary school (grade 12). An overview of the French system is presented in Table 3.1.

Table 3.1. Overview of the Educational System in France³ as of April 2016

Age (years)	School	School Time Each Week (hours)
3 to 5-6	Ecole Maternelle (Kindergarten)	24
6 to 10-11	Ecole Primaire (primary school)	24
11 to 14-15	Collège (lower secondary)	25 - 29
<i>National Assessment: Brevet des Collèges</i>		
15 to 17-18	Lycée Général (general upper secondary school)	28 - 30
	Lycée Technologique (technological upper secondary school)	
	Lycée Professionnel (vocational upper secondary school)	
<i>National Assessment: Baccalauréat</i>		

School is compulsory in France from age 6 to 16, but nearly all children start school at 3 years old. *Ecole maternelle* (students from ages 3 to 6) is already really a school – teachers have the same credentials as primary school teachers, and there is an official curriculum integrating mathematics through titles such as “discovering numbers and their use” or “exploring shapes and quantities.” School is the same for all students from 6 to 15 (in *Ecole Maternelle*, *Ecole Primaire*, and *Collège*). The *Lycée* is organized in three different streams: *Lycée general*, *Lycée technologique*, and *Lycée professionnel*. Within each stream, there are also different options: humanities, sciences, sciences and economics, for example, in the general stream.

³ For more details, see <http://eduscol.education.fr/cid66998/eduscol-the-portal-for-education-players.html>

In the humanities section, the main subject is philosophy (8 hours each week in Grade 12, last year of the *Lycée*); in the scientific section, the main subject is mathematics (6 hours, plus 2 hours for the students who choose the “mathematics specialty” in Grade 12). The sciences and economics section is more balanced: for grade 12 each week 5 hours of economics, 4 hours of philosophy and 4 hours of mathematics (for the main subjects).

At the end of the *Lycée*, students stand a written examination: *Baccalauréat*. The overall success rate (general, technological, and professional) was 90.6% in 2015; around 77% of each generation obtains the baccalaureate (an aim of 80% was set in the 1980s).

Mathematics holds an important place in the curriculum for all levels and for different streams. The scientific stream at general upper secondary, labelled “S,” is considered the best stream. Even at lower secondary school, mathematics is sometimes presented as a selection tool, whose usefulness resides mainly in sorting the “good” and “bad” students.

Concerning mathematics, the curriculum is presented in different official documents. The first is the “program,” presenting the content and associated skills that students should master with some comments. Before 2015, all these “programs” were organized according to each grade. The new program, starting in September 2016, is written according to three-year cycles. (For a detailed analysis of the curriculum planned for Grades 1 – 6 and grades 7 to 9, see subsequent sections.)

The program presents the content with many details. We give here a brief overview of the current grade 10 program (started in September 2009) to provide the reader with an insight into these documents and their possible contents. The grade 10 program comprises 10 pages, starting with an introduction stating general principles and objectives, and then giving details about three different domains (Functions, Geometry, Probability and Statistics) and two transversal domains

(Algorithmics⁴, and Reasoning and Logic). Table 3.2 presents the first line (our translation) of the program about Functions – the actual program table comprises eight such lines.

Table 3.2. Extract of the Grade 10 Curriculum Concerning Functions

Content	Expected Skills	Comments
Functions Image, pre-image, graph.	Interpret the link between two quantities with a formula. For a function defined by a graph, a table or a formula: <ul style="list-style-type: none"> • Identify the variable and sometimes the definition set; • Determine the image of a number; • Search for the pre-images of a number. 	The functions are generally functions of one real variable, their definition set is given. Some examples of functions defined on a finite set or on integers or even functions of two variables (area as function of lengths) should be presented.

Other texts, called “accompanying resources,” offer detailed propositions with mathematical explanations about specific subjects; for example, the accompanying resource for Algorithmics at grade 10 is 33 pages long. These resources are written by experts (e.g., inspectors, teacher educators; we give more details later in the chapter), officially to support teachers in the design of their courses. However, teachers do not seem to use them, probably because they consider these texts as too complex. In contrast, textbook authors seem to find these “accompanying resources” very useful: almost all the examples of mathematical situations proposed in these resources are transformed into “activities” in the textbooks, after didactical transposition work (Chevallard, 1992) that makes them accessible for students.

A Process of Deep Change

The national curriculum and the associated classroom practices have deeply evolved in France from the beginning of the 21st century, in general as well as in mathematics. As mentioned in

⁴ *Algorithmics* is mentioned throughout the chapter, as it is an important evolution in the recent French curriculum. The precise definition of this topic is given in the section focusing on it, and we invite the reader to refer to this section.

Figure 3.1, an important step was the report of the CREM (Kahane, 2002). One of the central objectives grounding the commission's recommendations was to bring school mathematics and "living" mathematics closer. For example, the commission recommended opening "mathematics laboratories" (Kahane 2002, p.268) in secondary schools, in order to "create a new image of mathematics and of its experimental aspect" (Kahane 2002, p.269). It also proposed a comprehensive notion of *mathematical sciences*, encompassing the mathematical practices in physics, economy, or computer science (see the commission's recommendations concerning computer science and their consequences in a later section of the chapter). The CREM emphasized the importance of reasoning and proof, stressing that "we need the alliance between imagination and reasoning present in the mathematical approach, from the formulation of statements to the proof of their consequences" (Kahane, 2002, p. 265). The orientations proposed by this commission are still very influential regarding the aims and content of mathematics teaching.

Educational policies in France are also largely influenced by the PISA assessments and their results. The 2012 PISA tests demonstrated that, in France, students' achievements are highly correlated with their socio-economic backgrounds, and this was an important motivation for proposing a new curriculum in 2016. Concerning mathematics, the results in France are close to the OECD average; however, they decreased between 2003 and 2012. France was *above average* in 2003, and close to countries like Korea, Finland, and the Netherlands, whereas the 2012 results were just *on average* and on par with countries like the UK, Norway, and Denmark. In particular, the proportion of low achievers (below level 1 of PISA) increased from 5.6% in 2003 to 8.7 % in 2012. The PISA tests have been cited as one of the reasons for the introduction in 2005 in the French curriculum of the *common core* (Bodin, 2008): a set of knowledge, skills,

and attitudes that all students should acquire during compulsory education. The official curriculum (including this common core) has been formulated since 2005 according to knowledge, skills, and attitudes coming from the French version of the Key Competencies for Lifelong Learning (European Parliament, 2006).

The introduction of the common core and competencies was also associated with new modes of assessment; since 2009, each student has a personal competencies booklet covering all the disciplines. Two other recommended curricular evolutions linked with the common core can be considered consequences of the PISA tests: an evolution towards more individualized teaching practices taking into account each student and proposing “personal support,” and the development of inquiry-based mathematics learning (Dorier & Garcia, 2013). Inquiry-based learning also aims at motivating more students for scientific studies, following the recommendations of the European Commission (Rocard et al., 2007).

The main evolutions of the curriculum since 2000 can be summarized as:

- At a general level, the formulation of the official curriculum and the assessment of students in terms of competencies; a development of individualized practices;
- For mathematics, an effort to bring “living” mathematics in school, with the development of problem solving and of inquiry-based approaches (with, at the same time, an important place kept for rigorous proof at secondary school) and with an increasing importance of algorithmics.

Evolutions of the Design Mode of the Curriculum, Role of the Mathematics Education Community

France has a strong community involved in mathematics education. The teachers naturally belong to several associations, the most important being the Association of Mathematics

Teachers in Public Schools (APMEP), whose website⁵ and yearly conference are very popular. Most researchers in mathematics education are members of the Association for Research in Mathematics Didactics (ARDM⁶); teacher educators, a minority being also researchers, work in Schools for Teacher Education. Mathematicians working in universities can also belong to several societies, the most important being the Société Mathématique de France (SMF⁷) and the Société de Mathématiques Appliquées et Industrielles (SMAI⁸).

Within universities, “Institutes for Research on Mathematics Teaching” (IREM, Trouche 2016a) offer opportunities to gather these different actors in research groups to study professional questions and design teaching resources. This community usually expresses its opinion regarding ongoing curriculum reforms, even if this community is not in charge of the reforms. Indeed, in France several kinds of inspectors are responsible for the management of the educational system: assessing the teachers, organizing the national and regional examinations, etc. The “general inspectors” are the highest authority and are under the responsibility of the Ministry of Education; they are responsible for the text of the national curriculum. Nevertheless, groups writing this curriculum comprise different kinds of authors, most often general inspectors, local inspectors, and teachers. These groups sometimes integrate teacher educators and researchers in mathematics education, especially in the case of the primary school curriculum. Members of the IREMs (Trouche 2016a) have been regularly involved in such groups, in particular in the CREM. They have also been involved in the commission “Assessing the Implementation of the Grade 10 Curriculum” created in 2013. This idea of assessing the implementation and impact of a new curriculum is very recent in France.

⁵ <http://www.apmep.fr>

⁶ <http://www.ardm.eu>

⁷ French Mathematical Society, <http://smf.emath.fr/>

⁸ Society for Applied and Industrial Mathematics, <http://smai.emath.fr/>

Other significant evolutions took place in 2013 in the design process of the programs, with the creation of the “High Council of Programs” (CSP). This council, composed of members with different backgrounds (academics, members of the parliament, etc.) was created by the Ministry of Education to formulate propositions for new educational orientations and started their work in September 2014. It managed, in particular, the design of the new curriculum for primary and secondary school (from grade 1 to grade 9) with subgroups organized for each three-year cycle and each discipline. These groups comprised inspectors, academics, teacher educators, and teachers.

A first version of the program was published in May 2015 and a large national consultation, open to all, was organized on the ministry website for one month. Some suggestions were integrated into the final version of the program published at the end of November 2015. This means that, over the period of one year, a deep modification of the whole curriculum from grade 1 to 9 was prepared, and should have been implemented simultaneously at all levels in September 2016.

In spite of the presence of educational researchers within the working groups, research results have not always been a central source for writing the official curriculum. Different opinions have been expressed during the discussions, and the suggestions formulated by educational researchers were not considered as more valuable than others. Nevertheless, the increasing involvement of researchers in such groups, as required by the Ministry of Education, is an important trend in the present evolutions.

FOCUS ON MATHEMATICS CURRICULUM IN PRIMARY SCHOOL

In France, primary school receives students from 6 to 10-11 years old. Until 2015, it was organized into two cycles: “cycle 2” (grade 1 to grade 2) and “cycle 3” (grade 3 to grade 5). In

September 2016, it will still be organized into two cycles, but “cycle 2” will cover grade 1 to grade 3, and “cycle 3” will cover grade 4 to grade 6, the first grade of lower secondary school. In this section, we present the new mathematics curriculum of primary school, which will be implemented in September 2016. Compared with the former curriculum in place since September 2008, there are many changes. We have chosen three points to illustrate the main lines of change. These points embody the aim of the CSP of improving mathematics teaching and learning. Will their implementation meet this major challenge? Further research is needed to answer this question. We present evidence of the difficulties likely to arise.

A Curriculum Reorganized Around Three-Year Cycles

The new curriculum is no longer organized by discipline, but by the common core. Taking into account the learning process of each student, even of students with specific needs, is much more manageable with a curriculum built according to three-year cycles, without giving details of what has to be done during each level of the cycle. However, such a yearly division of the curriculum is very likely to be soon available on the Internet, as teachers' main question is knowing what they have to teach in mathematics in their class level. How will the institution support teachers to take into account this new way of thinking about students' learning of mathematics?

Although textbooks can support this change, most of the available updated versions as of March 2016 have continued to propose a yearly division of the mathematics curriculum. We have analyzed the new 2016 offering of seven textbook publishers. Each publisher distributes several collections of mathematics textbooks. Eleven existing collections have been updated following the new curriculum requirements, but all propose a yearly-division of the curriculum! No link seems to be made in these collections of textbooks with the last year of primary school (grade 5, which is the second year of cycle 3) and the first year of lower secondary school (grade

6, which is the third and last year of cycle 3). Furthermore, publishers' catalogs highlight interdisciplinary activities for only four collections, and activities using new technology for only three collections. Only one completely new textbook is organized at the level of the cycle: M.A.T.H. cycle 3 (Peltier et al., 2016). It is structured around ten mathematics themes and a progression at the cycle level is proposed for each theme: years during which a notion of the theme is introduced, years during which a notion is studied, and years during which a notion has already been seen (consolidation). For example, for the theme "computation with decimal numbers, problems":

- The notion "multiply a decimal number by a decimal number" is introduced in grade 4, studied in grade 5, and considered as already been seen in grade 6;
- The notion "column addition" is studied in grade 4 and considered as already seen in grades 5 and 6.

The publisher states that this textbook is perfect as a complement to a specific grade textbook to facilitate student differentiation. It does not seem to be presented to teachers as a self-sufficient textbook.

The cycles' reform has joined the two last years of primary school with the first year of lower secondary school in the same cycle, trying to ensure continuity for students between primary and secondary school. But questions arise, such as how to develop a dialogue between primary and secondary teachers in a reasonable time frame, when primary school teachers are in charge of all the disciplines and secondary teachers are specialists of one discipline? In grade 6, each class has 9 or 10 different teachers, one for each discipline. Will primary teachers have to find time to discuss with each of them? This question of time will be of high importance during the coming years. For example, the cycle 2 program has 85 pages (among which only 14 concern

mathematics) and the cycle 3 program has 127 pages (among which only 17 concern mathematics). A teacher with a double level class of CE2 (Cours Élémentaire 2^d year, grade 3, last year of cycle 2, 8-9 years old) and CM1 (Cours Moyen first year, grade 4, first year of cycle 3, 9-10 years old) has to read 212 pages!

A Curriculum that Takes into Account Research Findings in Mathematical Education

New programs of cycles 2 and 3 have been elaborated by working groups comprising researchers. Some of their suggestions have been taken into account. The influence of didactical research can explain most of the changes that occurred in this new curriculum.

Concerning “space and geometry,” spatial activities used to be restricted to cycle 1, but this is no longer the case. Curriculum designers recognized the importance of spatial knowledge for the teaching of geometry according to the work of Berthelot and Salin (1999). For example, being able to use spatial references to complete a move on a map is now part of the cycle 3 curriculum.

The topic of “Quantities and Measurement” has been greatly modified relying on the research of Chambris (2010, 2015). Relations between “quantities and measurement” and “numbers and computation” are emphasized through:

- Links between teaching of the metric system and teaching of the system of place value for whole numbers instead of using mostly a conversion table (e.g., 1 m = 100 cm); in fact, the word “conversion,” used before, has disappeared from the new program;
- Introduction of the “number line”: a graduated straight line with numbers (Vilette, Mawart, & Rusinek, 2010) in both topics.

Concerning “numbers and calculation,” the significant importance of deconstruction and reconstruction on small numbers is directly influenced by the research results of ACE⁹ (Arithmetic and Comprehension in primary school), whose aim is to design a progression for numbers and calculus topics in cycle 2. Butlen’s (2007) research about mental arithmetic has also been taken into account in structuring the different types of computation in cycle 3. Besides, following the writing of the new mathematical program concerning numbers and computation, a national conference named “consensus conference¹⁰” on whole numbers was organized by the National Council of Evaluation of the Scholar System (CNESCO¹¹) to set up a dialogue between “experts” and members of the teaching community to establish research-based recommendations for the teaching of whole numbers.

As we can see, the new mathematics curriculum for primary school places great importance on didactical research findings and presents ambitious content related to recent research developments. Can this text be fully understood by teachers who are not specialists of mathematics and of the didactics of mathematics? The answer to this question is not clearly positive. There is a critical need for training; unfortunately, inservice teacher education has been quite reduced in France (for financial reasons in a difficult national economical context).

Using Technology in Mathematics in Primary School

Another important change in the new curriculum is the place given to the use of new technologies in mathematics classrooms. Integrating new technologies is expected starting from grade 1 (age 6). In the former curriculum, the only mention of technological tools concerned the

⁹ <http://python.espe-bretagne.fr/ace/>. ACE is a research project supported by the Ministry of Education.

¹⁰ <http://www.cnESCO.fr/fr/conference-de-consensus-numeration/>

¹¹ <http://www.cnESCO.fr/fr/accueil/>

use of calculators in grade 3 (age 8). Now, the use of technological tools is required in two topics: “numbers and calculation” and “space and geometry.”

Concerning “numbers and calculation,” calculators are introduced from grade 1 to “calculate, estimate or verify a result.” In cycle 3, “instrumented calculation” is seen as one of the three types of calculation to practice with students; the two others are “mental arithmetic” and column method calculation. In France, the institutional demand to use calculators in primary school is not new; until now, this integration of calculators in mathematics classrooms remained low. Indeed, there is resistance to the integration of calculators (Assude, 2007; Trouche, 2016b). A number of teachers still think that using calculators will lead to poor development of students' calculation knowledge.

Concerning “space and geometry,” two types of technological tools are introduced: software allowing programming of the movement of a robot or a character on a screen and dynamic geometry software (DGS). During cycle 2, activities concerning programming the moves of a robot or a character on the screen (like with the Logo turtle) lead children to formulate simple algorithms. Such activities are related to the spatial competency “finding one's way and moving using references.” In cycle 3, an introduction to program writing is seen in “geometry” as well as the use of DGS. Using DGS in lower secondary school is quite common, but its use remains limited in primary school (Soury-Lavergne & Maschietto, 2015). Besides, many primary schools in France are still poorly equipped with computers, interactive white boards, and video projectors. Nevertheless, several researchers (Gueudet, Bueno-Ravel, & Poisard, 2014; Ruthven, 2012) have demonstrated that material aspects cannot be the only factor for low integration of technologies in classrooms. For example, concerning algorithmics, teachers' conceptions of mathematics (in particular the importance devoted to “rigorous proof”) explain most of the

failure of the first attempt to introduce algorithmics in number theory teaching in grade 12 (age 17) in the 2000 curriculum change in secondary school (Ravel, 2003). Once again, the questions of teachers' training and teachers' resources on this subject are central. In the new textbooks, only three collections among 12 propose content in relation to the institutional requirements to use new technologies in mathematics classrooms.

FOCUS ON SOME CONTROVERSIES: “CYCLE 4” NEW CURRICULUM

The design of the new curriculum lasted for more than one year. In this section, we study three points that appeared as critical during this design process, in particular in the discussions between the CSP and the various actors of mathematics teaching: the position of mathematics among the other disciplines; the place of proving among mathematics activities; and the place of interdisciplinary activities for learning mathematics. We focus on the case of cycle 4 (grades 7 to 9, ages 12 to 15), which is the last cycle of the French lower secondary school.

The Position of Mathematics Among the Other Disciplines

In the global structure of the whole cycle 4 curriculum, mathematics is mentioned at three places: in the presentation of the general features of this cycle; in the description of the contribution of each subject (mathematics, history, etc.) to the common core; and naturally in the presentation of the mathematical content to be taught. In the first component, mathematics is not explicitly mentioned. Actually, the text mentions overall general features of cycle 4 [as “a cycle for deepening the learning”] that apply to all disciplines. Regarding the second component, the discussion between CSP and the mathematics education community helped the program to evolve. At the beginning of this discussion, the domain “languages” only related to the learning of French and other languages and, actually, mathematics appeared only for its contribution to

the second domain (“methods and tools for learning”). The discussion led to a more balanced view, giving to mathematics a responsibility in each domain of learning, for example:

- In the third domain (Educating the person and the citizen), “Mathematics as well as technical and scientific culture support the development of critical thinking and the taste for truth;” (BOEN, 2015, p.223)
- In the fourth domain (Natural and technical systems), “this domain helps to initiate students to the scientific modelling, and to understand the power of mathematics.” (BOEN, 2015, p.223)

The Place of Proving in Mathematics Learning

The third component of the cycle 4 curriculum emphasizes six main competencies for mathematics learning: searching, modelling, representing, reasoning, computing, and communicating. These competencies are clearly connected with the Key competencies defined in 2006 by the European Parliament (Table 3.3). A discussion took place about the status to be given to *proving*, seen as a critical aspect of mathematics learning. At the beginning of the discussion, proving was presented only in a restrictive way, suggesting to avoid requests for formal proof. This” evidenced a kind of fear of the CSP to give too much importance to proving (perhaps due to the memory of the “Modern Math” episode), and proving was seen as restricted to geometry. The discussion led to a richer view, underlining that:

The education to reasoning and the initiation to proving are essential objectives of the cycle 4. The reasoning, in the heart of mathematical activity, must be based on various situations [...]. Investigative practices (trial and error, conjecture, validation, etc.) are essential and can rely both on manipulation or research on paper / pencil, on the use of

digital tools (spreadsheets, DGS, etc.). It is important to provide a progression in learning to proving and not to have too many requirements on formal proof. (BOEN, 2015, p.366)

Far from being restricted to mathematics, the activity of reasoning concerns all the disciplines as stated in the program: “All the disciplines are intended to underpin and broaden modes of reasoning and proving.” (BOEN, 2015, p.223). Discussions during the curriculum design process started with the intention of better situating proving within mathematics, and ended with situating reasoning and proving at the heart of learning processes.

What Should be the Place of Interdisciplinary Activities?

The new curriculum proposes a new frame for crossing disciplines: “interdisciplinary practical teaching” (Enseignements pratiques interdisciplinaires in French, EPI in the following), with six possible themes (see the third component in Table 3.3) appearing far from the usual perimeter of mathematics teaching. At first glance, the 7th theme, “science, technology and society” can appear as the only one that could be related to mathematics. The designers of the curriculum appear aware of the difficulty for mathematics teachers to contribute to the different themes grounding EPI. The curriculum underlines:

Mathematics occupies an essential place in EPI. It provides tools for calculation and representation (using tables, diagrams, graphs), methods (based on different types of reasoning) that organize, prioritize and interpret information for various origins [...]. The variety of professions in which mathematics plays an important or essential role can be explored in the EPI. The use of media in foreign or regional language, in addition to greater exposure to other language, provides an opening to another approach to mathematics and allows students to enrol in the EPI Foreign languages and cultures. (BOEN 2015, p. 379)

In spite of these general statements, the curriculum does not give very convincing examples of EPI giving a relevant place to mathematics. The reason is probably that the programs for each discipline have been conceived by teams of experts who are specialists in each discipline at stake. The result is that the teachers have to conceive themselves the way to engage with their colleagues in fruitful interdisciplinary projects.

Finally, these controversies demonstrated the need for teaching resources in a time of strong evolution: evolution of the mathematical content, evolution of the teaching environment, but also evolution of the frontiers of mathematics. Kahane (2002) evokes “the mathematical sciences,” and suggests envisioning mathematics as part of a network incorporating signal processing, theoretical physics, and also naturally computer science.

Table 3.3 General Structure of Curriculum Cycle 4, Based on Three Components, from a General One to a Discipline Specific One

The Cycle 4 Curriculum, Cycle for Deepening Learning Three Components (1st and 2nd common to all disciplines, 3rd specific for each discipline)	
First Component: Special Features of Cycle 4	
<i>Presentation of the features at stake</i>	<i>Disciplines mentioned for reaching this feature</i>
Building a new relationship with oneself and with the complexity of the world	All the disciplinary and interdisciplinary activities
Switching from one language to another	Physical, artistic, and scientific languages
Managing an abundance of information and understanding the challenges of the world	Historical dimension of knowledge
Abstracting and modelling	All the disciplines
Appropriating the great human works and developing personal creativity	Artistic and cultural activities
Being responsible and collaborating with others	Moral and civic education
Acknowledging the cultural common norms and developing a personal thinking	All the disciplines
Second Component: Contribution of Each Discipline to the Common Core	
<i>Presentation of the 5 domains</i>	<i>Place allocated to mathematics</i>
Languages for thinking and communicating	Languages of mathematics, of sciences and computer science
Methods and tools for learning	Mathematics, due to the necessity of memorization and to solve complex tasks
Personal and citizen education	Mathematics and scientific culture develop critical thinking and the “taste of the truth”
Natural and technical systems	Approach of the scientific modelling and first understanding of the power of mathematics
Representing the world and human activity	Scientific and technological culture; history of sciences and techniques
Third Component: Mathematics Programme for Cycle 4	
<i>6 competencies</i>	<i>Searching, Modelling, Representing, Reasoning, Computing, Communicating</i>
<i>5 mathematical topics to be taught</i> (for each topic are given: knowledge, associated competencies, and situations for constructing them)	Numbers and computation; Organizing data and function; Quantities and measurement; Space and geometry; Algorithmics and programming.
<i>8 interdisciplinary practical teaching</i> (EPI) to be chosen by volunteer teachers	Body, health, well being and security; Culture and artistic creation; Ecological transition and sustainable development; Information, communication and citizenship; Languages and culture of the Antiquity; Foreign languages and cultures; Economical and professional word; Sciences, technology and society.

FOCUS ON A SPECIFIC CONTENT: ALGORITHMICS IN THE FRENCH SECONDARY SCHOOL CURRICULUM

In this section, we focus on the introduction of *algorithmics* in the mathematical curricula. By algorithmics, we refer to the branch of mathematics and computer science that is interested in the design of algorithms to solve problems and the analysis of algorithms as objects of study. (For more details, see Knuth, 2000 or Lagrange, 2014.) Such an introduction of algorithmics in connection with mathematics is specific to France and deserves to be examined here.

Computer Science and Its Relation with Mathematics in the French Curriculum: A Historical Overview

In the 1980s, after a few experiments, the teaching of computer science at upper secondary school was introduced in France for the first time with an optional teaching called “informatique des Lycées.” This teaching was done by specially trained teachers, essentially mathematics teachers, and was centered on *algorithmics and programming*; the ministry invested a lot in teacher education for this option. Then, computer science as a teaching subject disappeared in the 1990s, replaced by teaching how to use computers as tools in every discipline. (See Baron & Bruillard (2011) for details about the history of teaching computer science in France.)

In the 2000s, the CREM (Kahane, 2002) recommended in its report to “introduce some computer science in the teaching of mathematics and in teachers’ education” (Kahane, 2002, p.44) and defended the importance of interactions between mathematics and computer science. The report addressed many arguments, summarized as follows:

- Algorithmic thinking, implicit in the teaching of mathematics, could be developed and enlightened with the instruments of algorithmics;
- Programming promotes formalized reasoning;

- Questions about effectiveness of algorithms involve mathematics;
- Data processing and digital computations are common in other disciplines;
- And finally, Computer Science transformed mathematics, bringing new points of view on objects, bringing new questions, creating new fields in mathematics that are expanding rapidly, and changing the mathematician's activity with new tools.

Just after this report, algorithmics was introduced in mathematics at grades 11 and 12, but only for Literature and Art majors, and in optional mathematics courses (called mathematics specialty) in the last year of the economics option (in an introduction to graph theory) and the sciences option (in an introduction to number theory). Then, between 2009 and 2012 in new official programs, algorithmics was generalized in mathematics for all options of the general stream (literature, economics, sciences).

Finally, in the 2010s, computer science as a discipline came back in the upper secondary school. In 2012, a new optional teaching of computer science (called ISN, “Informatique et Sciences du Numérique”) was proposed in grade 12 for students of the scientific stream; a similar option in grade 10 has been tried since 2015. Computer science will also be taught in cycle 4, divided between mathematics and technology classes.

After this brief historical perspective, we examine the content of “algorithmics” in the current programs for grades 10 to 12 in the general stream. This is followed by a discussion of “Algorithmics and Programming” in the new programs for cycle 4.

Algorithmics in Grade 10 to 12 (Lycée, age 15 to 18)

It is only since 2009 for grade 10 and then since 2011 and 2012 for grades 11 and 12 that algorithmics has been introduced in the teaching of mathematics for every option of the general stream. Exactly as in the experimentation done for Literature and Art majors, algorithmics

content (just like logic content that returned to the curriculum at the same moment) has a special status in the program: it must not be taught as a course chapter (like functions, trigonometry, etc.) but integrated with the other chapters and content. The program guide (BOEN, 2009) asserts that “algorithms have a natural place in all the fields of mathematics.” (p. 9) Another specificity is that at all levels and in all options, the objectives are the same and given as “objective for the end of Lycée.” (p.9). This can be interpreted as a prefiguration of the organization in cycles now proposed for grades 1 to 9. Those objectives include general competencies about “describing algorithms in natural language and symbolic languages,” “realizing some algorithms” with different tools, and “interpreting more complex algorithms.” (BOEN, 2009, p.10). Some standard content in programming and algorithmics is specified as “elementary instructions (assignment, computation, input/output)” and the three classic control structures *if-then-else*, *for*, and *while* loops. Some specific algorithms or algorithmic activities are then mentioned at each level, for example, the bisection method for finding the root of a function in grade 10 or algorithms for computing the n^{th} element of a sequence defined by a recursive relation in grade 11.

An accompanying resource for grade 10, published in 2009, gives more information about the goals of this teaching of algorithmics and its place in modern mathematics. This resource has very deeply influenced the content of textbooks, and these textbooks have influenced teachers’ practices. Many mathematics teachers admit that they have not been trained enough in algorithmics and have difficulties to link algorithmics with the mathematics content (Ministère de l’Éducation Nationale, 2014a).

A specific didactical transposition (Chevallard, 1992) of the concept of algorithm happened in this curriculum (Modeste, 2012). Activities in algorithmics are directed toward

language activity, such as reading an algorithm, translating from natural language to a programming language, and understanding an algorithm written in a programming language. Actually, in the accompanying resource, even the concept of algorithm is defined through language: an algorithm is composed of three steps – preparation of treatment, treatment, and output of the results. The treatment is composed of instructions that are: assignation of data in variables, reading (or input) of data, sequence of instructions, and control instructions (alternative structure and repetitive structures). This construction is very close to the description of the grammar of a programming language. The resource provides a specific algorithmic language, a convention, to describe algorithms as in Figures 3.2 and 3.3.

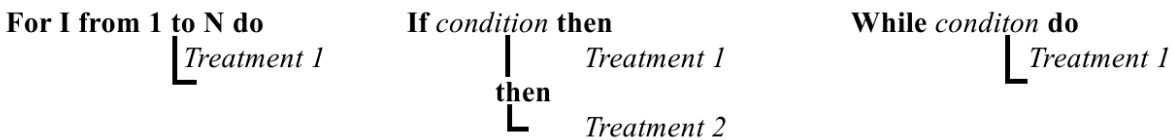


Figure 3.2. Description of the control structures in the accompanying resource (our translation)

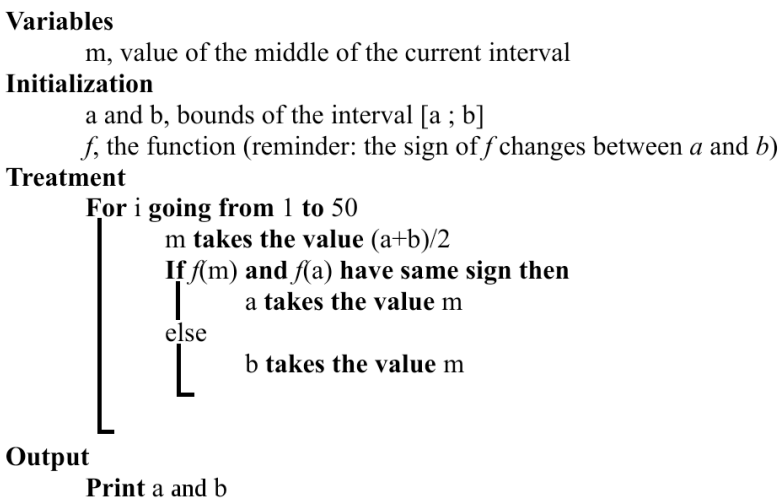


Figure 3.3. An example of algorithm described in the accompanying resource, the bisection method (our translation)

In Figure 3.3, we notice that the “natural” language used to describe algorithms is very close to the programming language, and already includes instructions that refer to the computer (declaration of variables, printing values); the programming of the algorithm is then very close to the algorithm produced. It is a good illustration of the representation of algorithms in *Lycée*, where the notions of algorithm and program seem not very well distinguished.

Another important point is the place given to algorithmics in the experimental activity. Indeed, the role given to algorithms is often to be programmed and used to generate conjectures. For example, algorithms simulating repetitions of random experiences (rolling die, flipping coins, random draws, ...) are present at every level for illustrating properties or generating conjectures in the probability and statistics chapters. We also notice that no mathematical content or activities are proposed to deal with algorithms as objects (such as discussion about particular types of algorithms, or proof and analysis of algorithms, for example).

In conclusion, the orientation of these programs is to deal with algorithms as tools for mathematics, in particular for experimental activity. This can explain the focus on language activities and the programming of most of the algorithms proposed. The final step of the didactical transposition process is curriculum implementation. Although no specific programming language is required, it seems that two types of tools for programming are currently used: the languages of calculators (mostly Casio and Texas Instruments) and a language designed by some teachers specifically for algorithmics of the *Lycée* called “Algobox.”¹² We think that these languages strongly influence the activities developed by teachers.

¹² <http://www.xmlmath.net/algobox/>

This didactical transposition should also be compared to the curriculum of option ISN for grade 12, where algorithmics is one of the four branches of computer science (with representation of the information, languages and programming, and material architectures). In this curriculum, algorithm is considered as a concept and defined, activities and content involve algorithms as tools and as objects, and some algorithms are introduced without being programmed.

A New Theme in Mathematics for Cycle 4: Algorithmics and Programming

In the new common core for compulsory education and the new curriculum for cycles 3 and 4, computer science appears as a science and a technology that contribute to the development of knowledge and competencies. This is a big change for teachers who have never been trained to teach computer science; this was discussed in the process of designing the new curriculum as previously described. In cycle 3, it will be complex to guarantee a continuity in the teaching of this new content, not only at the transition between primary school and lower secondary school (*collège*), but also at the transition with cycle 4, which we are going to discuss below.

Concerning computer science, the most important changes take place at cycle 4. In *collège*, as there is no independent teaching of computer science or teachers of computer science, the content has been separated between two disciplines: mathematics and technology. In mathematics, this content appears as one separated theme (of five) called “Algorithmics and Programming.” It has been developed through consultation with mathematics and computer science education communities in the process previously described. The curriculum includes general content in algorithmics and programming: notions of algorithm and program, variable, instructions, parallel computing or event-driven programming (that evokes but never quotes the

software Scratch¹³). It does not demand that specific algorithms or programs are taught, but gives examples of situations and activities that can be developed with the students to contribute to learning this general content. Like every discipline of the *collège*, computer science has to contribute to the interdisciplinary teaching and examples are proposed in this direction.

This new “Algorithmics and Programming” part of the curriculum raises many questions. It requires specific training for teachers, a reflection on the interactions between mathematics and computer science, and appropriate teaching resources that do not exist at this moment. It also questions the curriculum of algorithmics in the *Lycée*, as the approach of algorithmics and programming is very different in the two institutions and French students will soon be required to enter grade 10 with competencies in computer science.

The teaching of algorithmics in secondary school in France is not stabilized yet. Through the successive changes of the curricula, it is searching for its place, between mathematics and computer science, and we anticipate that many further evolutions will happen.

IMPLEMENTATION OF THE CURRICULUM: TEXTBOOKS AND TEACHING RESOURCES

In this section we focus on textbooks, including e-textbooks in France. We first present the general situation concerning textbooks, their design and use; then we focus on a specific e-textbook designed by a teacher association.

Textbooks and Open Educational Resources in France

In France, the publication and the use of textbooks is free. There is no authority controlling the textbooks published; teachers are free to choose which textbook will be used by their students (we call it “the class textbook” in what follows) and free in their use of textbooks. Most of the

¹³ Scratch (<https://scratch.mit.edu>) is an educational software for learning programming developed by the Massachusetts Institute of Technology and used in many countries for introducing children to programming.

time, the choice of the class textbook is made by the team of teachers for the grade concerned, for example, the team of mathematics teachers of grade 10 in a given upper secondary school. At primary school and lower secondary, the class textbook is bought by the school, but at upper secondary school it is bought by the students. At secondary school, this class textbook plays a central role in the teaching-learning activity; it is used in particular to work on exercises in class and to assign homework. In previous research (e.g., Gueudet, Pepin, & Trouche 2012; Pepin, Gueudet, & Trouche, 2013), we investigated the implementation of curriculum by teachers: selecting resources, transforming them, producing resources for students, and so on. We have documented that, in France, the textbook remains a central resource for mathematics teachers. On top of the class textbook used for the exercises, teachers use four or five other textbooks to prepare their course, to choose introductory activities, or to build assessments.

Naturally, in France like in other countries, mathematics teachers also have access to and use an abundance of teaching resources available on the Internet: lesson plans, various kinds of software, introductory activities with ready-made students' sheets, etc. The number of resources is constantly increasing; an important change occurred in 2016, linked once again with curriculum reform. The Ministry of Education selected a list of publishers to provide digital resources corresponding to the new curriculum. Teachers and students will use a range of online platforms offering these resources.

These evolutions linked with digital resources naturally also concern the textbook. All the textbooks on paper recently published in France are now associated with digital resources: a pdf version on a USB Key is offered to the teacher when his/her students buy the textbook; a website associated with the textbook offers free resources, like slides, various software files, a teacher guide, etc.; the students and the teacher can buy a "premium" digital version of the textbook,

which can be annotated, complemented with external resources, etc. Because the premium digital version is very expensive, it is not much used yet. The sales figure of “premium digital textbooks” was only 1% of the textbook sales figures in 2013 (Barbat-Layani, 2013). Nevertheless, we consider that the paper textbook is now linked with an e-textbook, which is a structured system of resources (Pepin et al., 2015).

These evolutions also yield changes in the design mode of textbooks (paper textbook and e-textbook). Usually in France, textbooks are written by specialists: inspectors, teacher educators, and so on. Since 2006, textbooks have also been published by an association of practicing teachers, Sésamath¹⁴ (Gueudet et al., 2016). The Sésamath paper textbooks are associated with a free e-textbook, and a free complete virtual environment, LaboMEP. In the following section, we focus on the theme of functions to analyze more precisely the Sésamath grade 10 e-textbook content.

An e-Textbook in France: Sésamath Grade 10, the Case of Functions

Sésamath is an association of mathematics teachers in France, most of whom teach at secondary school. The association was created in 2001, with the project of designing and publishing free resources for teaching and learning mathematics. The Sésamath websites receive more than 15 million hits each year. Around 20,000 teachers subscribe to the teachers’ website, Sesaprof, and more than 1 million students are subscribed to LaboMEP. The use of Sésamath’s online resources has been a very large scale phenomenon in France for several years.

We focus here on the Sésamath e-textbook for grade 10. It is freely accessible online, and associated with a paper textbook that is not free, but whose price is half of the other textbooks’ prices because the Sésamath association does not pay royalties.

¹⁴ <http://www.sesamath.net>

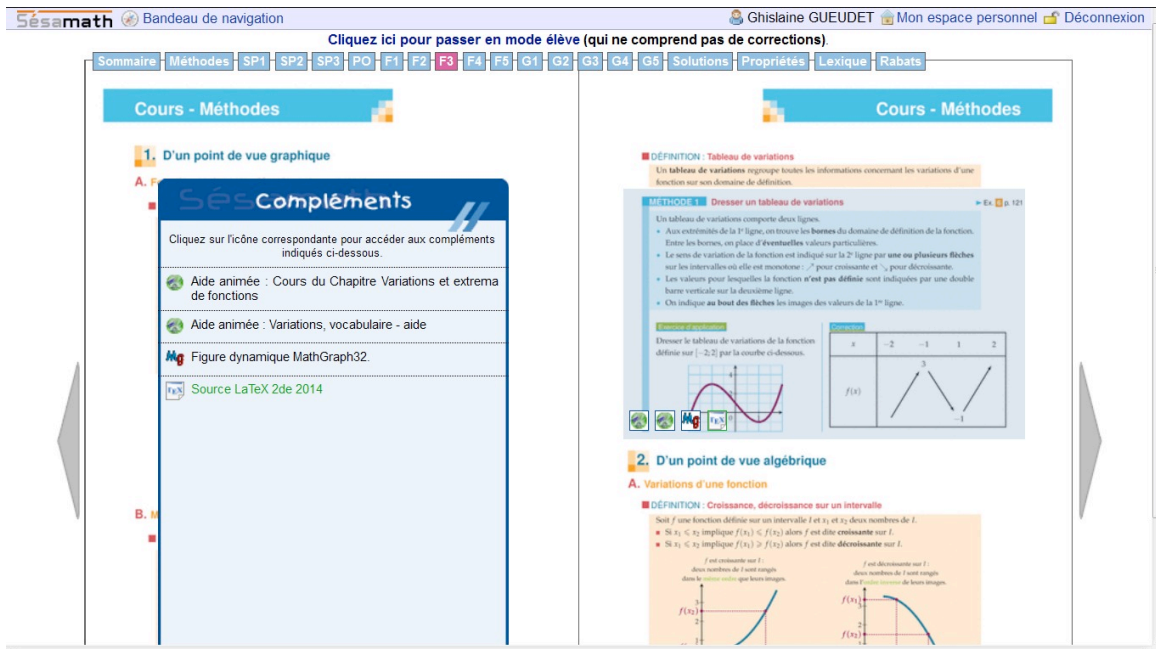


Figure 3.4. The Sésamath grade 10 e-textbook

Figure 3.4 displays a double page of the e-textbook about the variations of functions. The e-textbook contains all the content of the paper textbook, and possibilities of navigation in this content; on the top of Figure 3.4, we can observe an “F3” in purple color (top border strip), for chapter 3 on functions, where we are. We can change for other chapters on geometry or statistics, and for some tools like user guides for various software. Moreover, when browsing the page with the mouse, some “complements” windows appear, offering different tools: animated helps, dynamic figures, etc. This window also offers the source file of the page: the teachers can download it to make all the modifications they wish to introduce. Concerning algorithmics, the Sésamath textbook offers exercises involving algorithms in all the chapters, including the chapters dedicated to functions. The teachers can associate these exercises with files, for example AlgoBox files, using LaboMEP.

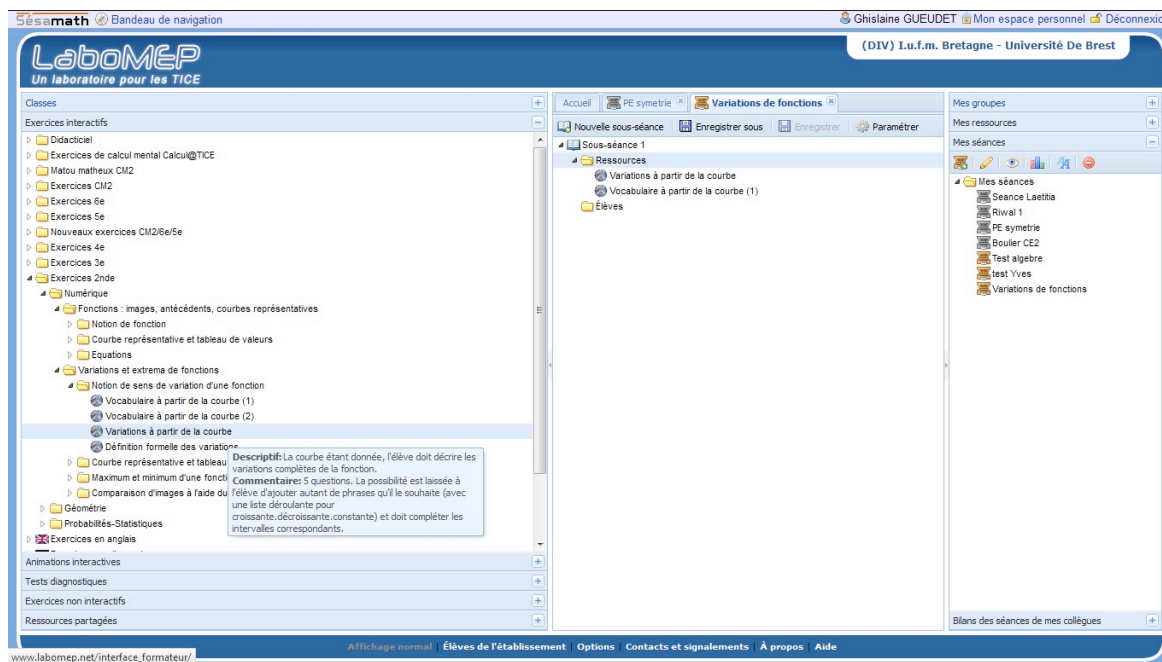
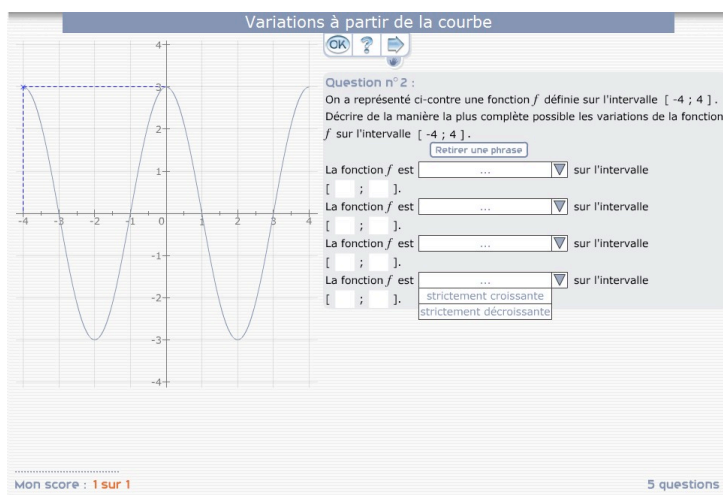


Figure 3.5. LaboMEP, the virtual learning environment of the Sésamath association

The Sésamath e-textbook is directly linked with LaboMEP, which is a full virtual environment developed by the Sésamath association. The LaboMEP window (Figure 3.5) is split in three columns. In the left column, we find the resources proposed by Sésamath, in particular interactive exercises, called “Mathenpoche” (for “Math in the Pocket”). When the mouse is on a particular exercise, a small window (in blue) appears with the description of the Mathenpoche exercise. (The translation of the text in the blue window is as follows: “*The graph of a function is given, students must describe the function variations. 5 questions. The student can choose the number of sentences he/she needs. Then he/she must choose in a pulldown menu – increasing, decreasing, constant – and complement for the corresponding intervals.*”) In this left column, the teacher can also have access to resources shared with his/her colleagues. This possibility of collective work is central for the members of the Sésamath association, and is not offered by other publishers yet. The Sésaprof website also offers various forums where teachers can

collaborate with their colleagues and with the textbook authors. In LaboMEP, the central column is the “working column” for the teacher, where he/she can build his/her lessons, choosing groups of students and designing for them a lesson with exercises, extracts of the e-textbook but also external (not designed by Sésamath) resources or weblinks. The right column displays all the lessons prepared by the teacher, and also gives access to the students’ work on the exercises.

The “Mathenpoche” interactive exercises are the most popular resources produced by Sésamath (see Figure 3.6). Each “exercise” comprises 5 to 10 questions. The student must answer, and is allowed two attempts before losing one point, if he/she fails at the question. An animated help is associated with each exercise.



Question 2

A function f is represented on the interval $[-4, 4]$. Describe as completely as possible the variations of f on $[-4, 4]$. Possibility to add or withdraw a sentence. Choose in the pulldown menu between “strictly increasing” and strictly decreasing”; complement the interval.

The score is 1 over 1: the student succeeded at question 1.

Figure 3.6. Mathenpoche interactive exercise, variations from the graph

The involvement of teachers in the design of resources coordinated by Sésamath, the rich network of resources offered by this association, and the success of these resources are major phenomena that influence the implementation of the curriculum in France. It invites teachers to be involved – through platforms and other digital means – in the collective design of their teaching resources.

CONCLUSION

In this conclusion, we synthesize the central facts presented concerning the mathematics curriculum and its recent evolutions in France. This includes discussions of curriculum content and structure; curriculum design; and curriculum implementation, resources, and teachers' work.

Curriculum Content and Structure

We have described the evolution of the French curriculum over a long period as well as more recent changes. At all these different stages, rigorous proof remains an important aim of the teaching, which can be interpreted as a remnant of the “Modern Maths” reform, where the “Modern Maths” now represents the traditional mathematics. Although the “rigor” philosophy is still present, the mathematics taught has also evolved and encompasses more statistics, more probability and analyses of variability, and some computer science, reflecting the evolution of contemporary mathematicians' actual work. The links between mathematics and other disciplines are emphasized; the experimental aspects and the inquiry approach are promoted. These last evolutions correspond to international trends, which also have an important influence on the French situation. In particular, the recommendations for educational policy at the European level (European Parliament, 2006) strongly influenced the choices in France, in general and also for mathematics.

Curriculum Design

Concerning the design of the official curriculum, we observed a progressive evolution towards the involvement of more actors in it. Not only the inspectors representing the institution, but also researchers in mathematics education intervene; large consultations are organized about the curriculum project, and the creation of the Superior Council for Programs constitutes a major step in the French history of curriculum design.

Curriculum Implementation, Resources, and Teachers' Work

Teachers in France do not align with a single textbook; they do significant work to design their teaching. For several years, they have had access to an abundance of resources on the web, designed not only by the institution but also by individuals or associations. The use of online resources, available on different platforms, will certainly increase in the next years, in particular because the Ministry of Education has launched a call for production of online resources associated with the new 2016 curriculum. Teachers are more expected to act as designers of their mathematics courses, including now computer science and interdisciplinary aspects. At the same time, little inservice teacher education is offered. Hence, we can expect the curriculum implementation to be misaligned with the official curriculum.

Designing the curriculum and the teaching of mathematics to support learning for all students (at least in the compulsory education) remains a major challenge in France, considering poor results on national and international assessments (Keskpaik & Salles, 2013). Although France has a brilliant, internationally recognized community of mathematicians, many students encounter difficulties in mathematics, starting at primary school. This paradoxical situation led to the publication by the Ministry of Education in December 2014 of a “mathematical strategy” (Ministère de l'Éducation Nationale, 2014b), aiming to promote mathematics in and out of schools, and organized around three axes: new curriculum (2016); better trained teachers (this has not yet been followed by concrete decisions); changing the image of mathematics, in particular thanks to playful approaches, and out-of-school activities. Thus, further analyses of the French curriculum, of its design and implementation, will certainly be needed.

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