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3D metric-based anisotropic mesh adaptation for the fast multipole accelerated boundary element method in acoustics

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Abstract

We introduce a metric-based anisotropic mesh adaptation strategy for the fast multipole accelerated boundary element method (FM-BEM) applied to exterior boundary value problems of the three-dimensional Helmholtz equation. The present methodology is independent of discretization technique and iteratively constructs meshes refined in size, shape and orientation according to an “optimal” metric reliant on a reconstructed Hessian of the boundary solution. The resulting adaptation is anisotropic in nature and numerical examples demonstrate optimal convergence rates for domains that include geometric singularities such as corners and ridges.

Keywords: acoustic scattering, boundary element methods, anisotropic mesh adaptation, fast multipole method

1 Introduction

We consider the scattering of an incident acoustic field $u^i(\mathbf{x})$ (characterized by the wavenumber k) by a bounded domain $\Omega \subset \mathbb{R}^3$ with boundary $\Gamma = \partial\Omega$ and unit normal \mathbf{n} outward to Ω . The corresponding scattered field $u^s(\mathbf{x})$ exterior to the obstacle is a solution to the time-harmonic scalar wave equation

$$\nabla^2 u^s + k^2 u^s = 0 \quad \mathbf{x} \in \mathbb{R}^3 \setminus \Omega \quad (1)$$

satisfying either Dirichlet (i.e. $u^s = -u^i$) or Neumann (i.e. $\partial u^s / \partial \mathbf{n} = -\partial u^i / \partial \mathbf{n}$) boundary conditions on Γ , as well as the Sommerfeld radiation condition at infinity. Problems can be formulated as boundary integral equations (BIEs) whose corresponding numerical solutions are constructed by boundary element methods (BEMs). The main advantages of BEMs is that their formulations exactly account for the radiation conditions and restrict the discretization of the domain to that of the boundary alone. Standard BEMs, however, lead to dense and (possibly) nonsymmetric linear systems whose solutions become prohibitively expensive for large-scale

problems. Fast multipole methods (FMM) overcome this drawback by enabling drastic reduction in solution time and memory requirements.

Further improvements of accuracy and computation time can be made by employing adapted meshes. Such strategies optimize the placement of the degrees of freedom to better capture solutions with anisotropic features as well as discontinuities in the acoustic field near geometric singularities such as corners or ridges. Fewer studies on these strategies have been made for BEMs, and most current BEM adaptation strategies, like those relying on Dörfler marking, have been confined to isotropic techniques. These methods are unable to recover optimal orders of accuracy and have been restrictive to Galerkin discretization techniques as well as the particular underlying equations. The focus of this work is to introduce and extend an anisotropic mesh adaptivity strategy [3] in the context of FM-BEM that addresses these issues by using a metric-based error estimator whose effectiveness has been demonstrated for volumetric (finite element) methods but not for BEMs.

2 Metric-based mesh adaptation

To find an optimal mesh that achieves a desired level of accuracy and convergence we use the following iterative procedure:

Coarse initialization step. Generate an initial uniform mesh $\mathcal{T}_i = \mathcal{T}_0$ with $N_i = N_0$ vertices for the surface Γ . The parameter N_0 can be chosen, for example, by requiring elements to have widths of approximately $\lambda/2$ (where $\lambda = 2\pi/k$).

Step 1. Compute a BEM approximation $u_{N_i}^s$ on the mesh with boundary element basis functions $\{\psi_j\}_{j=1}^{N_i}$.

Step 2. Associate with \mathcal{T}_i a Riemannian metric space $\mathbf{M} = (\mathcal{M}(\mathbf{x}))_{\mathbf{x} \in \Gamma}$, where \mathcal{M} is the metric tensor whose value at each vertex dictates the size and orientation of adjacent elements upon adaptation. Defining \hat{u}^s to be the second-

order Taylor expansion of the exact solution u^s around a mesh vertex, $\Pi_N u^s$ the linear interpolant of u^s on the mesh \mathcal{T}_i with elements K , we extend to the use of boundary solutions the results of [3] so that the total interpolation error

$$\sum_{K \in \mathcal{T}_i} \|\hat{u}^s - \Pi_N u^s\|_{L^2(K)} \leq 2 \int_{\Gamma} \text{trace} \left(\mathcal{M}^{-\frac{1}{2}} H \mathcal{M}^{-\frac{1}{2}} \right) d\Gamma \quad (2)$$

is minimized by a mesh generated by the metric

$$\mathcal{M}_{L^2} = N \left(\int_{\Gamma} \det(|H|)^{\frac{1}{3}} \right)^{-1} \det(|H|)^{-\frac{1}{6}} |H|. \quad (3)$$

Here, H is a symmetric matrix representing the Hessian of u^s and is computed at a vertex from the approximate solution u_N^s by the expression

$$(H)_{ij} = -\frac{3}{|K|} \sum_{K \in \mathcal{T}_i} \left(\frac{\partial u_N^s}{\partial x_i} \right)_K \int_K \frac{\partial \psi_{k \in K}}{\partial x_j} dx. \quad (4)$$

Step 3. Construct a new mesh \mathcal{T}_{i+1} with vertices $N_{i+1} = 2N_i$ that is *quasi-unit* with respect to the optimal metric computed from (3), i.e. seek triangles K with edges $\{\mathbf{e}_i\}_{i=1}^3$ such that $\frac{1}{\sqrt{2}} \leq \|\mathbf{e}_i\|_{\mathcal{M}} \leq \sqrt{2}$, $i = 1, 2, 3$ and $|K|_{\mathcal{M}} \simeq \frac{\sqrt{3}}{4}$.

Step 4. Iterate over Steps 1-4 until a specified maximum number of vertices N is surpassed.

3 Validation of the adaptive mesh strategy with numerical examples

The proposed strategy constructs adapted meshes that can recover optimal convergence rates for domains with corners and ridges. Figure 1 shows a mesh after four adapting iterations—with clear refinement at the edges—for the exterior Dirichlet problem of the scattering of an incident plane-wave ($k = 5$) by a cube with a cavity. The Hessian and metric tensor of Step 2 are computed by METRIX [3] and the mesh construction of Step 3 by the AMG library [4]. The approximate solution in Step 1 employs a \mathbb{P}_1 -element discretization for a BEM whose efficient solution is facilitated by the fast multipole method [5]. The relative L^2 -errors for the scattered field with varying degrees of freedom, depicted in Figure 2, indicate a reduced convergence order for a uniform refinement due to edge singularities of the obstacle. On the other hand, the anisotropic refinement is shown to recover the optimal convergence rate of $\mathcal{O}(n^{-1})$.

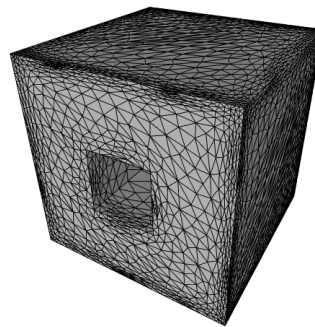


Figure 1: Adapted mesh at the fourth step of adaptation for the cube with cavity.

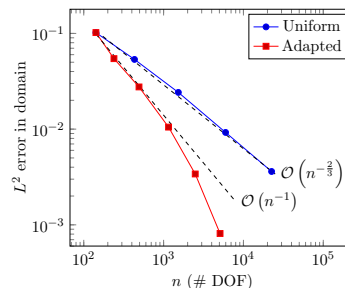


Figure 2: Relative L^2 -errors for uniform and anisotropic refinement of the example.

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