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PAnTHErS: a Prototyping and Analysis Tool for Homomorphic Encryption Schemes.

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Abstract: Homomorphic Encryption (HE) enables third parties to process data without requiring a plaintext access to it. Its future is promising to solve Cloud Computing security issues. Still, HE is not yet usable in real cases due to complexity issues. For every new HE scheme, evaluation is of primary importance, but performances (execution time and memory cost) for various sets of parameters are currently difficult to estimate ahead of practical implementations. This paper introduces PAnTHErS, a Prototyping and Analysis Tool for Homomorphic Encryption Schemes that alleviates the need for implementation to estimate the performances of any new HE scheme. PAnTHErS supports parametric modeling of HE schemes and provides analysis features. In this paper, PAnTHErS is illustrated over some HE schemes and shows promising results.

1 Introduction

Homomorphic Encryption (HE) aims at answering security issues of Cloud Computing by allowing a user to delegate computations on confidential encrypted data to a third party. In 2009, Gentry (Gentry, 2009) constructed the first Fully Homomorphic Encryption (FHE), which is based on ideal lattices. He created a Somewhat Homomorphic Encryption (SHE) scheme and introduced a bootstrapping phase that permits to refresh the noise in ciphertexts. As bootstrapping is a costly operation, decryption circuit is simplified (squashing step) to have a lower multiplicative depth. Then, it is possible to evaluate the decryption circuit. A FHE scheme needs to have circular security. This means that it is safe to encrypt the private key under its own public key (Brakerski et al., 2012). Since then, a lot of HE schemes have been created. They are based on different hardness assumptions as *approximate-GCD* (Dijk et al., 2010), *Learning With Error* (LWE) (Lindner and Peikert, 2011), *Ring-LWE* (R-LWE) (Brakerski and Vaikuntanathan, 2011b) or *approximate-eigenvector* (Gentry et al., 2013). Several open-source implementations of HE are available. HElib (Halevi and Shoup, 2014) is the most known.

Despite all existing schemes and implementations, HE is still not usable in real world applications. One of the big challenges is that HE consumes a lot of memory resources. It implies large data transfers from the user to the server, due to the fact that the encrypted data is much larger than the plaintext. Moreover, computations on ciphertext exhibit an important complexity.

HE could solve security concerns in Cloud Computing. Nevertheless, no HE scheme fits every application efficiently. One possible alternative is to determine the best HE scheme given an application. Criterion are bounded by limitations of the server and application constraints like, among others, execution time (complexity), number of homomorphic operations that are processed, memory usage and security strength. As HE can be very memory and time consuming, analyzing every existing HE scheme by varying their input parameters would involve intensive software simulations.

In this work, we present a tool named PAnTHErS that aims to help analyzing and prototyping HE schemes. PAnTHErS workflow is illustrated in Figure 1: it proposes to build functional models of HE schemes (step (1)) which can be analyzed and configured regarding the application requirements (step (2)). Then, a set of schemes can be selected to be partially implemented on FPGA to provide hardware acceleration for HE computing (step (3), (4) and (5)). This paper focuses on steps (1) and (2) of the proposed flow.

The remainder of this paper is organized as follows. Section 2 introduces the modeling approach and Section 3 details analysis methods. Then, Section 4 describes the insight of PAnTHErS implementation

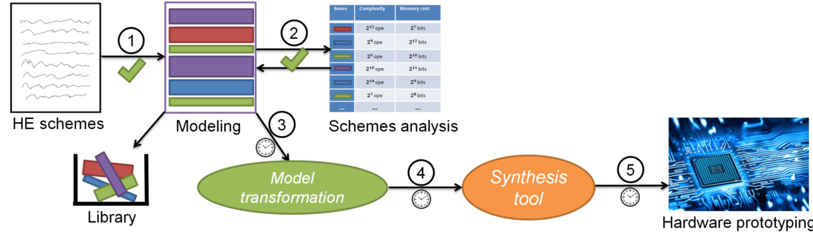


Figure 1: High-level illustration of PAnTHErS workflow.

and utilization. Section 5 presents PAnTHErS results on four HE schemes. Finally, Section 6 concludes and presents future work.

2 Modeling

In PAnTHErS, a HE scheme is modeled into a set of functions which are stored in a library and shared among HE models.

Actually, HE schemes modeling (step (1)) produces both an executable model and an analysis model of a HE scheme. The first one is forwarded to model transformation (step (3)), while the second is used for HE schemes analysis (step (2)). While, the details regarding analysis functions is given in Section 4, this section details HE scheme modeling through *atomic*, *specific* and *HE basic* functions.

2.1 *Atomic* functions

In this paper, an *atomic* function represents a basic operation that can be operated in a HE scheme. Functions such as addition, multiplication, division, modulo, round and their variants for polynomials, matrix of integers and matrix of polynomials are some examples of *atomic* functions. These are basic blocks used to build more complex functions, *i.e.* *specific* functions. As an example, matrices addition and scalar matrix multiplication are stored as below in the library:

```
addMat(int[][] A, int[][] B, int[][] C) :
//adds two matrices.
    C = A + B

multScalMat(int a, int[][] B, int[][] C) :
//Multiplies a scalar with a matrix.
    C = a*B
```

2.2 *Specific* functions

A *specific* function is a set of *atomic* functions. Some schemes based on the same problem (*e.g.* R-LWE) use identical instructions. These instructions form

Algorithm 1 *distriLWE* (mathematical algorithm)

Require: q,n,m,k integers, χ a Gaussian distribution, R a ring, $\boldsymbol{s}$ a vector of size n.
Ensure: $distriLWE(q,n,m,k,\chi,R,\boldsymbol{s})$
  $\boldsymbol{A} \leftarrow R_q^{m\times n}$
  $\boldsymbol{b} \leftarrow \chi^m$
  $\boldsymbol{b} \leftarrow (\boldsymbol{A}.\boldsymbol{s} + k.\boldsymbol{b}) \mod q$
  return $\boldsymbol{A} \in R_q^{m\times n}, \boldsymbol{b} \in R_q^m$

then a *specific* function that is flushed to the library. For example, the following equation:

$$addTimes(\boldsymbol{A}, b, \boldsymbol{C}) = \boldsymbol{A} + b \times \boldsymbol{C} \quad (1)$$

with $\boldsymbol{A}, \boldsymbol{C}$ matrices and b an integer, can be modeled as a *specific* function using a set of *atomic* functions. Thus, it is stored in the library as:

```
addTimes(int[][] A, int b, int[][] C,
    int[][] D) :
    multScalMat(b,C,D) // D ← b × C
    addMat(A,D,D) // D ← A + D
```

Obviously, *specific* functions can be defined with both *atomic* and *specific* functions. Moreover, to be used in several HE schemes, they may be generalized. As an example, a function named *distriLWE*, presented in Algorithm 1, appears in (Fan and Vercauteren, 2012) using $k = 1$ and in (Brakerski and Vaikuntanathan, 2011a) using $k \neq 1$. Using functions available in the library, Algorithm 1 is rewritten as a set of *atomic* and *specific* functions resulting in function *distriLWE* written below. This new produced *specific* function is then integrated in the library for reusing purposes.

```
distriLWE(int q, int n, int m, int k, Set χ,
   Set R, poly[] s, poly[][] A, poly[] b) :
    rndMat(R, m, n, A) // A ← R^{m*n}
    rndMat(χ, m, 1, b) //b ← χ^m
    multMat(A, s, c) //c ← A.s
    addTimes(c, k, b, b) //b ← (c + k.b)
    modMat(b, q, b) //b ← b mod q
```

Table 1: Representation of memory and complexity after executing analysis functions of *distriLWE*. Parameter d is the maximal degree of a polynomial.

((a)) Memory table.

Name	$\boldsymbol{A}$	$\boldsymbol{c}$	$\boldsymbol{b}$
Object	POLY	POLY	POLY
Dimensions	(n,m)	$(m,1)$	$(m,1)$

((b)) Complexity table: *operations*.

	Mult	Add	Div	Mod	Rnd	Round
INT	$m \times d$	0	0	0	0	0
POLY	$n \times m$	$n \times m$	0	m	$(n+1) \times m$	0

2.3 *HE basic* functions

A HE scheme is composed of five functions: key generation (KeyGen), encryption (Enc), decryption (Dec), addition (Add) and multiplication (Mult). In this paper, these are referred as *HE basic* functions which are built using *atomic* and *specific* functions from the library. In option, a function which "refreshes" a HE scheme can be added to the *HE basic* functions and so, can be modeled too. Contrary to *atomic* and *specific* functions, *HE basic* functions are not stored in the library: they are only created for one particular HE scheme.

To summarize: *Atomic*, *specific* and *HE basic* functions enable modeling any kind of HE schemes. More generally, the modeling process can be used in another cryptography context or even in a mathematical context. In our work, 25 *atomic* and 31 *specific* functions were produced and included in the library. These functions made possible the modeling of 14 HE schemes of the literature. To model future schemes, other *atomic* or *specific* functions can be created and added to the library if necessary. In this section, HE scheme executable modeling has been explained. This modeling enables analysis modeling which is described in the next section.

3 Library functions analysis

Previous section shows how HE schemes can be modeled into sets of *atomic*, *specific* and *HE basic* functions. To analyze a modeled HE scheme, each *atomic* and *specific* function of the library is linked to analysis functions: one for memory and one for complexity. This section explains how these two functions are created. *HE basic* functions possess also their proper memory and complexity analysis functions which are created using the same construction model as *specific* functions.

3.1 Memory analysis function

Memory cost analysis function evaluates the maximal amount of integers and polynomials that need to be stored at the same time during the execution of *atomic* and *specific* functions. The memory is represented by a table that keeps parameter names, dimensions and objects they contain (integers or polynomials). For instance, Table 1(a) shows how temporary variables and outputs of *distriLWE* are saved.

All variables of HE schemes are stored in the memory table. A variable can be either temporary or an output. At the end of each *atomic*, *specific* or *HE basic* function, variables created during the function are sorted. That way, it is possible to see memory evolution through the execution. This memory evolution permits to return the maximal memory needed for a HE scheme.

Below, the function Memory.*multScalMat* is the memory analysis function of *multScalMat* which is an *atomic* function. Memory table is filled thanks to Memory.*new* function call.

```
Memory.multScalMat(int a, int[][] B,
   int[][] C) : // adds outputs of multScalMat
   // in Memory table.
    n = Memory.rows(B) //# of rows of B
    m = Memory.cols(B) //# of columns of B
    Memory.new(C,INT,n,m) //adds C to memory
      //table or changes its dimensions
```

As a *specific* function is a set of *atomic* functions, its associated memory analysis functions constitute then a set of related memory analysis functions. Memory.*addTimes* shows an example of memory evaluation for the *specific* function *addTimes* presented in Section 2.2.

```
Memory.addTimes(int[][] A, int b, int[][] C,
     int[][] D) :
     Memory.multScalMat(b,C,D)
     Memory.addMat(A,D,D)
```

3.2 Complexity analysis function

In this paper, complexity represents the number of operations executed. It is determined on the basis of six operations: multiplication, addition, division, modulo, random and round. Those operations exhibit different complexities if they are used with integers or polynomials only. Complexity is calculated for integers on one hand and for polynomials on the other hand.

A table *operations* is conceived to store complexity in those terms. For an evaluated function, the table is updated with the total of each type of operations performed. The *operations* table is represented in Table 1(b) after calling complexity function of *distriLWE*.

Characteristics of parameters created and/or modified are stored and updated if needed through complexity analysis. Indeed, to calculate complexity of each function, dimensions of objects used in that function are needed. For that, dimensions required for the complexity evaluation are extracted from parameter characteristics. Then, cells of the table *operations*, containing global complexity, are incremented by the number of operations executed in the evaluated algorithm. After that, characteristics of output parameters affected by current operation are updated. As an example, the function Complexity.*multScalMat*, written below, evaluates computational complexity of *multScalMat*.

```
Complexity.multScalMat(int a, int[][] B,
   int[][] C) :
     n = B.rows()
     m = B.cols()
     t = B.type() //returns type of B
     operations[INT][MULT] += n * m
     C.update(t, n, m) //updates info about C
```

To evaluate complexity of *specific* functions, an associated complexity analysis function is created by identifying *atomic* functions used and calling their related complexity analysis function. Most of the functions are as simple as *distriLWE* which is a set of *atomic* functions. However, complexity evaluation remains difficult for few *specific* functions. The main issue comes with *while* loops with a non-deterministic condition or with conditions based on another function like finding a prime number. In this work, the worst case is considered.

To summarize: in the end of the modeling phase, the library contains *atomic* and *specific* functions and their associated memory and complexity analysis functions. As *specific* and *HE basic* functions templates are similar to their associated analysis functions, these last ones can be automatically generated at the creation of a *specific* or *HE basic* function. It enables fast analysis of modeled HE schemes which is one of the main goals of PAnTHErS.

4 PAnTHErS implementation and application

At this stage, the library can be filled with *atomic*, *specific* and their corresponding analysis functions. PAnTHErS can be used to evaluate HE schemes. This section gives information about PAnTHErS implementation and describes its easy utilization from a user and a HE expert point of view.

4.1 Implementation

PAnTHErS is implemented in Python using Sage. Each type of functions (*e.g.* *atomic*) is defined by a class (*e.g.* AtomicFunction class) allowing creating HE schemes executable. Moreover, each type of functions has also two associated classes corresponding to complexity and memory cost analysis (*e.g.* AtomicFunctionComplexity and AtomicFunctionMemory classes). A design pattern Visitor can be used to generate analysis functions automatically (*e.g.* *Memory.addTimes* and *Complexity.multScalMat* functions presented in Section 3). A Visitor is an operation performed on elements of an object structure of a class without changing the class itself (Lasater, 2007).

In addition, for each HE scheme that has to be model a distinct class is created. *HE basic* functions are implemented in each HE scheme class. Other functions can be added in those classes for optional calculations. For instance, a function called *Depth* was added in HE scheme classes for our case studies in Section 5. This function calculates the multiplicative depth: the number of operations which can be done homomorphically. A mathematical equation is needed to compute the depth of a HE scheme. Finally, a Main class is created to represent the application. This class models the application *i.e.* the succession of *HE basic* functions. As an example, the application begins by KeyGen and made the following operations: three Enc, one Add and two Mult. And, the application finishes with one Dec.

It is important to point out that complexity/memory cost are first expressed in number of operations/polynomials in some ring R_q. However, functions implemented in PAnTHErS permits to convert complexity as number of multiplications and memory cost as number of 32-bit integers stored.

Converting all operation complexities as number of multiplications allows having one global complexity. Operations on polynomials in some ring R_q are converted first in operations on integers in R_q. Then, by comparing the execution time of the six operations, ratios are found. After calculating several execution times for each operation, the mean (m) and the median (M) of those execution times are computed. Then, the *mean* between m and M is calculated ($mean = \frac{m+M}{2}$). A normalization process is performed on means to take multiplication operation execution time as refer-

Table 2: Ratios calculated between different operations.

Ope.	×	+	/	%	Rand	≈
Ratios	1	2.32	0.18	1.56	0.18	0.38

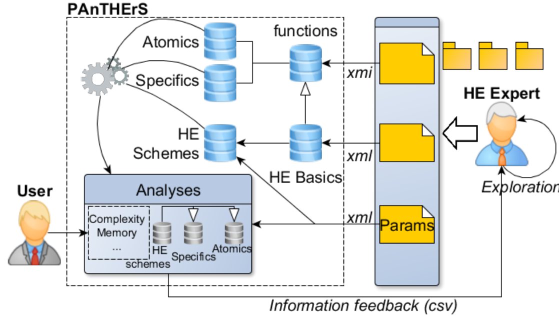


Figure 2: PAnTHErS utilization workflow.

ence. So, each *mean* is divided by the *mean* of multiplication to get the ratio. For each operation, we produce a list of execution times on an Intel Core-i5 machine using Sage. From those lists, ratios, presented in Table 2, were found taking multiplication as reference. For instance, time execution of 1 multiplication equals to time execution of 2.32 additions. In PAnTHErS, a user can change these ratios with custom values adapted to his own architecture.

4.2 PAnTHErS usage

Figure 2 shows PAnTHErS utilization workflow by a HE expert and a user. A HE expert, who can be also a user, can interact with PAnTHErS in order to make an exploration of his HE schemes. PAnTHErS usage depends on the number of *specific* functions available in the library. PAnTHErS library contains all *atomic* functions but the library does not necessarily have any *specific* functions. Here, the expert starts with a library empty of *specific* functions to illustrate how PAnTHErS is populated.

First of all, the expert creates *specific* functions to fill the library (*e.g. addTimes* function in Section 2). He starts creating a *specific* function by giving its name, input and output parameters. The function is composed of *atomic* functions where the expert has specified their inputs and outputs. The description of the *specific* function is given in a XMI file. Validating a function can permits the automated creation of its analysis functions. This generation, possible with a Visitor, is represented by gearwheels on Figure 2. He repeats this operation for every *specific* function he needs for his HE scheme.

Now, the HE expert has to model the *HE basic* functions of his HE scheme whose skeletons are written down in XML file. As for creating a *specific* function, the expert uses functions in the library to write *HE basic* functions and specifies their inputs and outputs.

Once *HE basic* analysis functions are created, the expert enters sets of input parameters (a range and a step for each one) to analyze HE schemes; Moreover, he fixes the number of *HE basic* functions executed and their execution order *i.e.* his application modeled. This sequence is analyzed regarding each set of parameter and final results are returned in CSV format to the expert. Final results correspond to memory cost, computational complexity and depth. Finally, he can do an exploration of all of his HE schemes and their possible input parameters.

PAnTHErS feedback contains useful information for the HE expert as maximal computational complexity and sum of all memory cost of *HE basic* functions. The study of these analyses enables the expert choosing the best parameters to fit its application.

Equally important, a user can interact on analysis part by adding other analysis calculations for HE schemes. Indeed, he has access to *atomic* and *specific* functions which are visible to anyone. This way, PAnTHErS is extended by various analyses that are interesting for future HE experts' exploration. Moreover, he takes part in making implementation choices to improve HE schemes execution. Having analysis results on a particular HE scheme, he knows, for instance, where it is interesting to do parallelization or hardware acceleration.

To summarize: knowing PAnTHErS utilization, a HE expert can easily model and analyze any HE scheme. By varying their input parameters, several analyses are produced for each HE scheme. Thanks to these analyses, the expert is able to select the most interesting one and a set of parameter guaranteeing a computational complexity, a memory cost and a depth adapted to his application. Also, PAnTHErS can be extended by other analyses implemented by a designer.

5 Case studies

This section shows PAnTHErS results considering four HE schemes. PAnTHErS is applied on FV (Fan and Vercauteren, 2012), YASHE (Bos et al., 2013), F-NTRU (Doröz and Sunar, 2016) and SHIELD (Khedr et al., 2016) which are all based on R-LWE. To model the first two schemes, we consider using PAnTHErS with a library filled of *atomic* functions only. Then, each modeled scheme takes benefit of the previous models leading to rapid modeling. Schemes are then

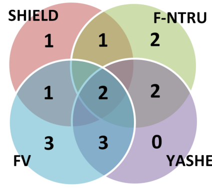


Figure 3: *Specific* functions distribution between FV, YASHE, F-NTRU and SHIELD schemes.

analyzed regarding several sets of input parameters. Finally, PAnTHErS draws curves which show evolution of computational complexity, memory cost and multiplicative depth.

5.1 Modeling

Figure 3 gives the distribution of *specific* functions between FV, YASHE, F-NTRU and SHIELD schemes. In this figure, each circle pictures a HE scheme. When a number is in an intersection of circles, it represents the number of shared functions between the HE schemes. Figure 3 shows that, from four modeled schemes, 60 % of their *specific* functions are used in at least two schemes. Reusing *specific* functions from the library makes modeling easier. Starting from scratch to model FV and YASHE, 11 *specific* functions are created but already five are shared between the two schemes. Then, three new *specific* functions are needed to model F-NTRU and finally, only one new is required to model SHIELD.

5.2 Experimental setup

Each considered HE scheme has been modeled as described in Section 2. In addition, a function to calculate multiplicative depth was added to each class except for SHIELD. Indeed, depth calculation is not fully detailed in (Khedr et al., 2016). To compute the depth of FV and YASHE, the bound of noise is given in (Lepoint and Naehrig, 2014). Two depth formulas are explicitly written for F-NTRU in (Doröz and Sunar, 2016): the first one for the theoretical depth and the second for depth in average. Both were implemented in F-NTRU class. Both were implemented in F-NTRU class.

For the proposed experimentations, the analysis step has been configured to cover one execution of KeyGen, Enc, Dec, Add, Mult and Depth. In this case, each ciphertext is considered "refreshed" in Mult function after the multiplication. In the end, PAnTHErS returns computational complexity, memory cost of each *HE basic* function and depth depending of input parameters, by summing up partial contributions, besides, with no need of time consuming evaluation.

Table 3: Time execution of all PAnTHErS analysis expressed in minutes.

Schemes	FV	YASHE	F-NTRU	SHIELD
Time	6.279	9.864	3.731	0.598

Table 4: Time execution of one PAnTHErS analysis versus time execution of real HE scheme execution expressed in seconds.

Schemes	FV	YASHE	F-NTRU	SHIELD
Analysis	0.058	0.088	0.079	0.069
Execution	6.44	35.13	53.64	48.80

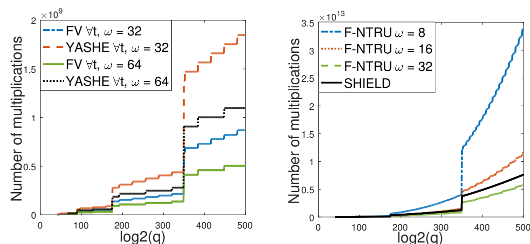
Before performing any analysis, input parameters must be configured. For each set of parameters, each scheme provides 80-bit of security considering input parameters given by (Migliore et al., 2017). In all HE schemes, computations are made in $R = \mathbb{Z}[X]/(\Phi_d(X))$ where $\Phi_d(X)$ is the irreducible dth cyclotomic polynomial. In F-NTRU and SHIELD, d is a power of 2. Polynomials of R have a maximal degree of $n = \varphi(d)$. All polynomial operations are located in $R_q = R/qR$ with q the modulus. In FV and YASHE, the plaintext to cipher is in $R_t = R/tR$. An integer base w is provided in FV, YASHE and F-NTRU; it is used in some functions to decompose words in base w. All schemes need two Gaussian distributions χ_{key} and χ_{err} bounded by respectively B_{key} and B_{err}.

In each scheme, parameters n and q are interdependents on each other. To choose n with regards to q, there is a maximum $\log_2(q)$. We took n and $\log_2(q)$ presented in (Migliore et al., 2017). Our tests cover all $\log_2(q) \in \{40, 48, ..., 500\}$. Making sure that $w < q$, we took $\log_2(w) \in \{2, 32, 64, 128\}$ for FV and YASHE analysis and $\log_2(w) \in \{1, 8, 16, 32\}$ for F-NTRU analysis. Finally, for FV and YASHE, we vary t by taking $t \in \{2, 8, 32, 64\}$. And, we set $B_{key} = 1$ and $B_{err} = 9.2 \times 2\sqrt{n}$ to calculate depth.

To evaluate PAnTHErS efficiency, a benchmark of 100 executions has been performed. Table 3 recaps time execution of PAnTHErS for each scheme depending on the number of evaluated sets of parameters. Varying parameters as explained before imply 6904 analyses for FV and YASHE, 1840 for F-NTRU and 460 for SHIELD. Table 4 compares one analysis execution time versus one real execution time. All these executions were made using Sage, version 7.6.

5.3 Results

This section presents and analyzes the results obtained for the considered HE schemes. One of the main objectives of the proposed approach is to determine a set of adequate HE schemes and their associ-



((a)) FV and YASHE complexity. ((b)) SHIELD and F-NTRU complexity.

Figure 4: Evolution of computational complexity in function of $\log_2(q)$ expressed in number of multiplications. We fix $\omega = \log_2(w)$.

ated input parameters which fit for requirements of an application. When taking each scheme individually, there is no way to decide which one best fits to an application since this choice is driven by the application requirements. Analysis must target these features to select an interesting candidate. If several schemes match the application, thanks to tests and results, they can be compared to detect the most interesting one.

Figures 4, 5 and 6 show analysis results i.e. evolution of complexity, memory cost and multiplicative depth of the four HE schemes in function of $\log_2(q)$. Breaks, visible in each figure, correspond to the change of n.

When complexities and memory costs of the four schemes are drawn together on the same graph, we notice that the scale difference is too important to be well displayed. To ensure good graph readability, we choose to focus on two algorithms comparisons only at a time, resulting on two sets graphs, comparing respectively FV with YASHE and F-NTRU with SHIELD.

From Figure 4(a), it is clear that w impacts on FV and YASHE complexity. For an application with computational complexity constraints, a user will prefer use a bigger w which implies a lower complexity. FV is the most interesting because it is the less complex. Additional analyses show that the impact of t on computational complexity is non-existent. From Figure 4(b), SHIELD seems a better candidate than F-NTRU (with a small w) for an application with complexity constraints. Nonetheless, F-NTRU tends to have a lower complexity while w increases.

Figure 5(a) shows that, for FV and YASHE, Mult memory cost falls as w grows up. Moreover, this Figure illustrates that for $\log_2(w) = 64$, YASHE is less memory consuming than FV. If $\log_2(w) = 32$, YASHE is more interesting until around $\log_2(q) = 400$. However, Figure 5(b) shows that Add function of FV consumes more memory than Add function of YASHE for all q. Additional analyses illustrate that the same variations than Mult function exist for KeyGen function but that w and t have no influence on Enc and Dec functions for both schemes. Among F-NTRU *HE basic* functions, the Dec function consumes the less of memory. Analyses show that Mult and Add are identical and that they are the most memory consuming. Figure 5(c) illustrates that, despite a high w, Enc function of SHIELD remains the less consuming in term of memory than Enc function of F-NTRU. Figure 5(d) shows it is the contrary for KeyGen function.

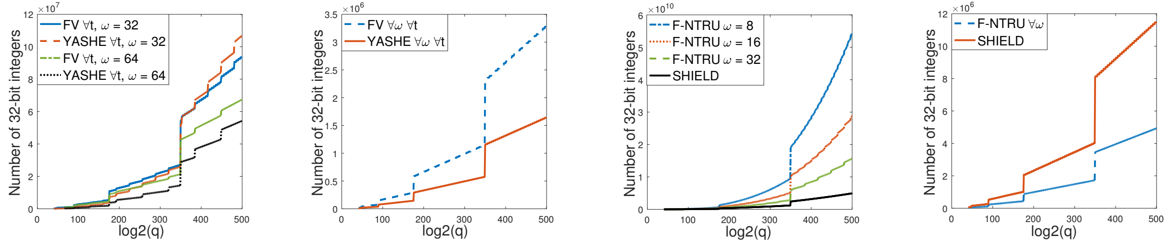
Theoretical multiplicative depth is represented by an integer. For FV, YASHE and F-NTRU, growing w implies a lower complexity and a lower memory cost, however, it implies also a lower depth. Figure 6(a) illustrates that FV tends to have a greater depth comparing to YASHE by taking the same input parameters. Theoretically, depth curves of FV, YASHE and F-NTRU are closed: F-NTRU depth is lower. Nonetheless, depth of FV and YASHE is slightly smaller if t increases. Analyses show that the difference between depths in function of t seems to become more important as q raises up. In practice, it is possible to have a greater depth. For instance, Figure 6(b) shows that F-NTRU depth is usually 1.5 times greater in average than theoretically. For a fixed depth, a user will choose a HE scheme less complex and less consuming in memory: FV and YASHE seems more interesting.

To summarize: these case studies show PAnTHErS utilization on four HE schemes of the literature. Moreover, this section demonstrates that as functions are shared between HE schemes, the modeling is faster. Thanks to several analyses, we were able to detect two kinds of schemes. Among results showed in this section, SHIELD seems to have a lower memory cost than F-NTRU and FV is clearly less complex than YASHE.

The proposed approach realizes a fast analysis of various HE schemes and display comparative results, enabling HE experts to select viable candidates for their application. PAnTHErS helps them to focus on analysis and development of the best HE schemes matching their needs and their application constraints.

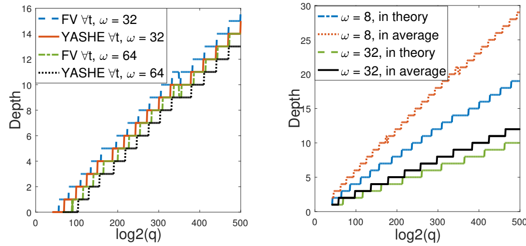
6 Conclusion and future works

This paper presents PAnTHErS, a tool that provides a way of evaluating HE schemes. Besides, this approach offers scalability and incremental design. It dispenses with the need for software implementation and simulations of HE schemes. The schemes are modeled as sets of reusable functions that are stored



((a)) FV and YASHE Mult memory cost.
((b)) FV and YASHE Add memory cost.
((c)) SHIELD and F-NTRU Enc memory cost.
((d)) F-NTRU and SHIELD KeyGen memory cost.

Figure 5: Evolution of memory cost in function of $\log_2(q)$ expressed in number of 32-bit integers stored. We fix $\omega = \log_2(w)$.



((a)) FV and YASHE depth. ((b)) F-NTRU depth in theory and in average.

Figure 6: Evolution of multiplicative depth in function of $\log_2(q)$. We fix $\omega = \log_2(w)$.

in the library. After the modeling phase, PAnTHErS returns valuable information about HE schemes in terms of computational complexity, memory cost and multiplicative depth. This analysis is a lightweight operation as the functions of the library have already been analyzed. Evaluating PAnTHErS results enables to determine if the scheme is an interesting candidate for a particular application using HE.

Future works will focus on optimizing PAnTHErS. The analysis step will be extended with new metrics. Then, an extra feature of PAnTHErS will address automated generation of hardware accelerators targeting a FPGA implementation for HE schemes. This will rely on an open-source high-level synthesis environment.

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