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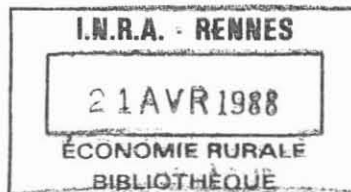
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DERIVATION OF LONG RUN FACTOR DEMANDS FROM SHORT RUN RESPONSES

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SUMMARY

The concept of the restricted cost function provides a dual approach to the analysis of short run technology. It allows also, under curvature restrictions, to infer the different possible equilibria, according to constraints on the firms. Moreover, in this paper, the properties of the restricted cost function are spelled out. Substitution possibilities related to the different regimes related to fixed factors or output are also derived from the restricted cost function.

This theoretical framework is applied to characterize the French cereal sector using a cross-section of farms.

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INTRODUCTION

The usefulness of empirical analysis of agricultural technology has been greatly enhanced by the use of flexible functional forms, based on cost or profit relationships rather than production functions. An important assumption underlying most cost function applications is that all inputs are in static equilibrium, according to marginal productivity pricing. This maintained hypothesis is very restrictive, especially for the agricultural sector, since certain inputs cannot be freely varied within the single period of observation. [Brown and Christensen, 1981 ; Mahé and Rainelli, 1987; Boutitie and al., 1987 ; Guyomard and Vermersch, 1987]. So, in this paper, we develop a short run Hicksian equilibrium model : only the variable inputs fully adjust to their cost minimizing levels, while the quasi-fixed inputs remain fixed.

We first provide a complete characterization of the theoretical short run Hicksian model, while paying close attention to the underlying assumptions (especially the assumptions of convexity of the restricted cost function in its arguments : variable input prices, quasi-fixed input levels, production level) and to the description of the different theoretical possible equilibria (short run and long run Hicksian equilibria, short run and long run Marshallian equilibria) given knowledge of short run environment.

Following Lau (1976) and reworking Sakai's decomposition (1974), we develop an analytical framework which allows to infer the long run compensated demand functions, the short run and long run uncompensated demand (and supply) functions, from the short run Hicksian equilibrium. In order to use this methodology,

sufficient curvature restrictions, which are not necessary verified at short run Hicksian equilibrium, are imposed : strict convexity of the restricted cost function with respect to, the quasi-fixed factors, the output, the quasi-fixed inputs and the output, respectively.

As a final objective, this conceptual framework is used in order to assess the technological characteristics of the French cereal sector. The study will use the restricted translog cost function, estimated by using a sample of farm accounts ($n = 208$) related to the year 1981, and will emphasize :

- substitution possibilities, own and cross-price elasticities, economies of scale in the short run and in the long run equilibria.
- shadow prices and optimal quantities of fixed inputs (family labor and land).

We examine especially the implications of taking into account the simultaneous quasi-fixity of two inputs : land and family labor. This application is attractive because the assumption of long run static equilibrium with respect to these factors has been at length questioned for French agriculture in the postwar period. We illustrate the theoretical relationships presented in section 2 for this particular case : two quasi-fixed factors and a restricted translog cost function.

1. A characterization of the restricted cost function

We consider a firm which uses $M + N$ inputs $(z_1, \dots, z_M, x_1, \dots, x_N) = (z, x)$, $x \geq 0$, $z \geq 0$ at prices (p_x, p_z) to produce one

output y , $y \geq 0$. The production possibilities set, Y , is supposed to have the following properties :

- (i) Y is closed and non-empty
- (ii) if $y \neq 0$ then $x \neq 0$
- (iii) for all $(x, y, z) \in Y$, if $x < \infty$ and $z < \infty$ then $y < \infty$
- (iv) There is free disposal of inputs and output ; i.e, for all $(x, y, z) \in Y$, the production plan (x', y', z') such that :
 $(x' \geq x ; z' \geq z ; y' \leq y)$ is possible, i.e
 $(x', y', z') \in Y$
- (v) $X(z, y) = \{x ; (x, y, z) \in Y\}$ is convex.

The restricted cost function is then defined by :

$$CR(p_x, y, z) = \min_x [p_x' \cdot x ; x \in X(y, z)] \quad (1)$$

With a strictly positive input prices vector p_x , hypothesis (i) ensures the existence of $CR(p_x, y, z)$: Furthermore, $CR(p_x, y, z)$ is non-negative, positive when y is non-zero, non-decreasing, positively linear homogeneous, concave and continuous in p_x [Mac Fadden, 1978, p.11].

It is also possible to show that property (iv) implies that $CR(p_x, y, z)$ is non-decreasing in y and non-increasing in z (Guyomard and Vermersch, 1988). In addition, under the convexity hypothesis (v) of the section $X(y, z)$ of Y , the knowledge of the restricted cost function $CR(p_x, y, z)$ is sufficient to describe, in an exhaustive manner, the short run Hicksian technology which is employed (1). Finally, duality results state that :

$$X^*(y, z) = \{x \geq 0 ; p_x' \cdot x \geq CR(p_x, y, z), \text{ for all strictly positive } p_x\} = X(y, z) \quad (2)$$

Finally, assuming the restricted cost function $CR(p_x, y, z)$ is twice differentiable with respect to input prices, the derivative properties are:

- Shephard's lemma relates $CR(p_x, y, z)$ and the cost minimizing input demand functions through its partial derivatives :

$$\partial CR / \partial p_{x_n} = \bar{x}_n(p_x, y, z) \quad n = 1 \dots N \quad (3)$$

- The following Hessian matrix is symmetric, negative semi-definite, and of rank $N-1$:

$$\sum_{p_x p_x} = [\partial^2 CR / \partial p_{x_n} \partial p_{x_{n'}}] \quad \begin{matrix} n = 1 \dots N \\ n' = 1 \dots N \end{matrix}$$

Finally, under the assumption of differentiability of $CR(p_x, y, z)$ with respect to the quasi-fixed inputs and to the output, it is convenient to define the total Hessian matrix :

$$\Sigma = \begin{bmatrix} \sum_{p_x p_x} & \sum_{p_x z} & \sum_{p_x y} \\ \sum_{z p_x} & \sum_{zz} & \sum_{zy} \\ \sum_{y p_x} & \sum_{yz} & \sum_{yy} \end{bmatrix}$$

The properties of the production possibilities set and the twice differentiability of $CR(.)$ imply that the matrix $\sum_{p_x p_x}$ is negative semi-definite. They do not imply restrictions on the submatrices \sum_{zz} and \sum_{yy} .

2. *Inferring different theoretical equilibriums from the restricted cost function.*

The restricted (or variable) cost function, corresponding to the program of minimizing the cost of some subset of the inputs subject to the choice of the remaining inputs, provides a functional characterization of the technology at short run Hicksian equilibrium. Particularly, it is possible to calculate the short run Hicksian price elasticities of demand, for a variable factor :

$$\bar{\varepsilon}_{n,n} = \partial \text{Log } \bar{x}_n / \partial \text{Log } p_{x_n} \Big|_{p_{x_i}, z, y} \quad i \neq n$$

In long run Hicksian equilibrium, the total cost function may be written as :

$$\begin{aligned} \text{CT}(p_x, p_z, y) &= \min_{x, z} \left[p_x \cdot x + p_z \cdot z ; y = f(x, z) \right] \\ &= \min_z \left[\text{CR}(p_x, z, y) + p_z \cdot z ; y = f(x, z) \right] \\ &= \text{CR}(p_x, \bar{z}^h(p_x, p_z, y), y) + p_z \cdot \bar{z}^h(\cdot) \end{aligned}$$

The optimal long run Hicksian level of the input z_m $m = 1 \dots M$, is defined as $\bar{z}_m^h(p_x, p_z, y)$. The total cost function is obtained by first minimizing the restricted cost function conditional upon the level of z : and then minimizing total costs with respect to z , holding the variable inputs at their short run Hicksian levels $\bar{x}_n(p_x, z, y)$. A sufficient condition to use

this two-step decision rule is that the restricted cost function CR is strictly convex in z (in a domain which includes the observed and optimal long run Hicksian levels of the inputs z_m) ; that is the matrix \sum_{zz} is positive definite in this domain. So, it is sufficient to add the property of strictly convexity of the restricted cost function in z_m , $m = 1, \dots, M$, if we want to infer the long run Hicksian characteristics of the technology from the knowledge of the only restricted cost function CR (2). For example, the output-constant price elasticities of demand, or Hicksian price elasticities, can be derived from the restricted cost function and are conditional upon the optimal quantity $\bar{z}^h(p_x, p_z, y)$. These elasticities include both pure substitution and expansion effects, involved by quasi-fixed factor variations.

$$\begin{aligned}
 \epsilon_{nn}^h &= \partial \text{Log } \bar{x}_n^h / \partial \text{Log } p_x \quad \left| \begin{array}{c} p_{x_i}, p_z, y \\ i \neq n \end{array} \right. \\
 &= \partial \text{Log } \bar{x}_n^h(\bar{z}^h, y) / \partial \text{Log } p_{x_n} \quad \left| \begin{array}{c} p_{x_i}, z, y \\ i \neq n \end{array} \right. \quad A \\
 &+ \sum_{m=1}^M \partial \text{Log } \bar{x}_n^h(\bar{z}^h, y) / \partial \text{Log } z_m \cdot \partial \text{Log } \bar{z}_m^h / \partial \text{Log } p_{x_n} \quad \left| \begin{array}{c} p_{x_i}, p_z, y \\ i \neq n \end{array} \right. \quad B
 \end{aligned}$$

[7a]

The generalization of this decomposition work is possible for all theoretical possible equilibria : short run and long run Hicksian equilibria, short run and long run Marshallian equilibria, and is summarized in table 1. The long run Marshallian situation, not necessarily achieved by the observed technology, can be approached by three equivalent ways : either by

cost minimization with respect to z given y and then by profit maximization with respect to y ; or by profit maximization with respect to y given z and then by cost minimization with respect to z , or by profit maximization with respect to z and to y . In order to infer the different possible equilibriums from the short run Hicksian equilibriums, the restricted cost function must verify the following sufficient curvature restrictions (3) :

- $\sum_{zz}(z^m)$ must be positive definite to infer the long run Hicksian demand equations from the short run Hicksian demand equations.

- $\sum_{yy}(y)$ must be positive definite to infer the Marshallian short run demand equations from the Hicksian short run demand equations.

$$- \begin{bmatrix} \sum_{zz}(z^m, y^m) & \sum_{zy}(z^m, y^m) \\ \sum_{yz}(z^m, y^m) & \sum_{yy}(y^m, y^m) \end{bmatrix} \text{ must be positive definite to}$$

infer the Marshallian demand equations from the Hicksian short run demand equations.

For example, under the assumption of strict convexity of the restricted cost function $CR(p_x, y, z)$ with respect to z and y , the long run Marshallian demand function of factor x_n may be written as :

$$x_n^m(p_x, p_z, p_y) = \bar{x}_n(p_x, \bar{z}^h(p_x, p_z, y^m(p_x, p_z, p_y)), y^m(p_x, p_z, p_y)) \quad [4]$$

$$= \bar{x}_n(p_z, z^m(p_x, p_z, p_y), y(p_x, z^m(p_x, p_z, p_y), p_y)) \quad [5]$$

$$= \bar{x}_n(p_x, z^m(p_x, p_z, p_y), y^m(p_x, p_z, p_y)) \quad [6]$$

Before proceeding to the empirical section, it is useful to express [4], [5] and [6] in terms of price elasticities of demand. For example, using the decomposition [4], the long run Marshallian elasticities matrix may be written as :

$$\begin{aligned} [\varepsilon_{nn}^m] &= [\bar{\varepsilon}_{nn}^m] && \left. \begin{aligned} &+ [x_{nm}] [z_{mn}] \\ &+ [x_{nm}] [z_{m1}] [y_{1m}] \\ &+ [x_{n1}] [y_{1n}] \end{aligned} \right\} \begin{array}{l} A \\ B \\ C \end{array} \\ & && [7b] \end{aligned}$$

where :

$$[x_{nn}] = [\partial \text{Log } \bar{x}_n(\bar{z}^h(y^m), y^m) / \partial \text{Log } z_m] \quad (N \times M)$$

$$[z_{mn}] = [\partial \text{Log } \bar{z}_m^h(y^m) / \partial \text{Log } p_{x_n}] \quad (M \times N)$$

$$[z_{m1}] = [\partial \text{Log } \bar{z}_m^h(y^m) / \partial \text{Log } y] \quad (M \times 1)$$

$$[y_{1m}] = [\partial \text{Log } y^m / \partial \text{Log } p_{x_n},] \quad (1 \times N)$$

$$[x_{n1}] = [\partial \text{Log } \bar{x}_n(z^n(y^m), y^m) / \partial \text{Log } y] \quad (N \times 1)$$

Therefore, the total change in x_n^m , in response to a change in p_{x_n} , may be decomposed into three effects :

- a pure substitution effect along the old isoquant : A
- an expansion effect due to the quasi-fixed factors variation until the Hicksian optimal level : B
- an expansion effect due to the change in the output level along the new expansion path associated with inputs prices : C.

3. The econometric model

In this section, we describe the econometric model used to characterize the structure of French cereal sector : the data concern a cross-section of farms. We specify a translog functional form for the restricted cost function with an additive error term ε_{CT} (4) :

$$\ln CR(y, p_i, z_j) = a_0 + a_1 (\ln y) + \frac{1}{2} a_2 (\ln y)^2$$

$$+ \sum_{i=1}^4 c_i (\ln p_i) + \frac{1}{2} \sum_{i=1}^4 \sum_{j=1}^4 d_{ij} (\ln p_i) (\ln p_j)$$

$$+ \sum_{h=1}^2 f_h (\ln z_h) + \frac{1}{2} \sum_{h=1}^2 \sum_{k=1}^2 g_{hk} (\ln z_h) (\ln z_k) +$$

$$\sum_{i=1}^4 \sum_{k=1}^2 k_{ih} (\ln p_i) (\ln z_h) + \varepsilon_{CT} \quad (4) \quad [8]$$

where CR is restricted cost ; y is output ; p_i are variable input prices, as for fuel and oil, fertilizer, capital, hired labor ; z_j are the quasi-fixed inputs : family labor and land.

Without loss of generality, symmetry is imposed on the parameters d_{ij} and g_{hk} . Shephard's lemma gives the cost share equations, on which we add a disturbance term, ε_i , to reflect errors in optimization :

$$M_i = p_i x_i / CR = c_i + \sum_{j=1}^4 d_{ij} (\ln p_j) + \sum_{h=1}^2 k_{ih} (\ln z_h) + \varepsilon_i \quad [9]$$

$$i = 1, 2, 3, 4$$

By construction, $\sum_{i=1}^4 M_i = 1$. This additivity constraint implies :

$$\left[\begin{array}{l} \sum_{i=1}^4 M_i = 1 \\ \sum_{i=1}^4 k_{ih} = 0 \end{array} \right. , \quad \left[\begin{array}{l} \sum_{i=1}^4 d_{ij} = 0 \\ \forall h \end{array} \right. \quad \forall j$$

Symmetry and additivity constraints ensure the theoretical restriction of homogeneity of degree one in inputs prices.

The set of equations [8] and [9] will be used to estimate the parameters of CR, from which the short run Hicksian price elasticities of demand will be derived :

$$E_{ij}^{CT} = (d_{ij} + M_i M_j) / M_i \quad \forall i, \forall j, j \neq i$$

$$E_{ii}^{CT} = (d_{ii} + M_i^2 - M_i) / M_i$$

By solving equations [10] with respect to the quasi-fixed factors and using the procedure developed by Brown and Christensen [1981, p. 212-214]; we can compute the long run Hicksian price elasticities of demand, not only for a variable factor (x_n , $n = 1 \dots N$), but also for a quasi-fixed factor (z_m , $m = 1 \dots M$).

$$\partial CR [p_x, z, y] / \partial z_m + p_{z_m} = 0 \quad m = 1, \dots, M \quad [10]$$

In the most recent studies, only one quasi-fixed factor is considered [Brown and Christensen, 1981 ; Halvorsen and Smith, 1986; Squires, 1987]. For the specific case with two

quasi-fixed inputs, and assuming that the matrix $\sum_{zz} (\bar{z}^h)$ is positive definite, we obtain the following expressions for the long run Hicksian price elasticities (5).

$$E_{ij}^{LT(h)} = E_{ij}^{CT} - (1/M_i)(A(i,j)/B(h,k))$$

$$E_{ih}^{LT(h)} = (M_k/M_i) \cdot (C(i, h, k)/B(h,k))$$

$$E_{hi}^{LT(h)} = -C(i, h, k)/B(h,k)$$

$$E_{hk}^{LT(h)} = -M_k(g_{kh} + M_h \cdot M_k) / B(h,k)$$

$$E_{hh}^{LT(h)} = M_h(g_{kk} + M_k^2 - M_k) / B(h,k)$$

$$M_h = -p_h \cdot \bar{z}_h^h / CR(y, p_x, \bar{z}^h)$$

$$M_k = -p_k \cdot \bar{z}_k^h / CR(y, p_x, \bar{z}^h)$$

$$\begin{aligned} A(i,j) = & (M_i M_k + k_{ik}) [(g_{hh} + M_h^2 - M_h)(M_j \cdot M_k + k_{jk}) - \\ & (g_{hk} + M_k \cdot M_h)(M_j M_h + k_{jh})] + \\ & (M_i M_h + k_{ih}) [(g_{kk} + M_k^2 - M_k)(M_j \cdot M_h + k_{jh}) - \\ & (g_{hk} + M_k M_h)(M_j M_k + k_{jk})] \end{aligned}$$

$$B(h,k) = (g_{kk} + M_k^2 - M_k)(g_{hh} + M_h^2 - M_h) - (g_{hk} + M_h \cdot M_k)^2$$

$$C(i, h, k) = (g_{kk} + M_k^2 - M_k)(M_i M_h + k_{ih}) - (g_{hk} + M_h M_k)(M_i M_k + k_{ik})$$

i and j refer to a variable factor

h and k refer to a quasi-fixed factor.

Outside the long run Hicksian optimum, the shadow price (or dual price) for each quasi-fixed factor can be evaluated as :

$$p_{z_m}^* = - [\partial CR(.) / \partial z_m] \quad [11]$$

If the ex-post shadow price and the ex-ante observed price of a quasi-fixed factor are equal, then the actual level z_m corresponds to the long run Hicksian level z_m^h . Moreover, the concept of shadow price allows us to characterize, in a dual and simple way, the disequilibrium induced by fixity. [Kulatilaka, 1985]. Indeed, assuming that the restricted cost function is strictly convex in z_m , $m = 1 \dots M$; it is easy to show [Guyomard, Vermersch, 1987].

$$\left[\begin{array}{l} p_{z_m}^* \leq p_{z_m} \iff z_m^h \leq z_m \\ p_{z_m}^* \geq p_{z_m} \iff z_m^h \geq z_m \end{array} \right.$$

Finally, a measure of short run returns to scale can be defined as :

$$ECH^{CT} = [\partial \ln CR / \partial \ln y]^{-1} \Big|_{p_x, z} \quad [12]$$

Caves, Christensen and Swanson [1981], followed by Halvorsen and Smith [1986], suggest a different measure which takes into account not only changes of the variable inputs but also changes of the quasi-fixed inputs

$$ECH^O = [1 - \sum_{m=1}^M (\partial \ln CR / \partial \ln z_m)] / [\partial \ln CR / \partial \ln y] \Big|_{p_x} \quad [13]$$

This measure doesn't represent returns to scale along the expansion path where total cost is minimized at every level of output and generally differs from a long run measure as :

$$ECH^{LT} = [\partial \ln CT / \partial \ln y]^{-1} \Big|_{p_x, p_z} \quad [14]$$

However, ECH^0 et ECH^{LT} give equal measures of returns to scale at the long run Hicksian equilibrium [Guyomard, Vermersch, 1987]. Consequently, considering the restricted cost function CR and the optimal long-run Hicksian levels \bar{z}^h , it is possible to infer :

$$ECH^{LT} = [1 - \sum_{m=1}^M (\partial \ln CR(\bar{z}^h) / \partial \ln z_m)] / [\partial \ln CR(\bar{z}^h) / \partial \ln y] \Big|_{p_x} \quad [15]$$

4. Data and empirical results

The model is estimated using data from a sample of farm accounts ($n = 208$) related to the year 1981 ; for each farm the cereal production represents at least eighty percent of the total production. Four variable inputs and two quasi-fixed factors are taken into account : the model is specified with prices for fuel and oil, fertilizers, capital and hired labor ; acreage under cultivation and family labor are included as fixed inputs. For the capital input, it is assumed that the service flow from the stock of capital is proportional to this stock and just like in Dormont and Sevestre (1986) the user cost of capital is only the apparent interest rate. The level of output is measured by cash sales and variable costs are defined as the value of variable inputs : the

factor shares are only the values of this inputs divided by the restricted cost.

The restricted cost function is jointly estimated with the cost share equations which sum to unity : so, one of them is dropped. The system of equations is estimated by the maximum likelihood method whose corresponding estimates are invariant to choice of deleted equation [Barten, 1969].

The parameter estimates for the final form of the model are shown in table 2 together with their estimated standard errors. For each point, the estimated cost shares are positive and the concavity of the restricted cost function in inputs prices is verified at the sample average. If z_h and z_k represent respectively the levels of family labor and land, at the sample average, the fitted cost function is convex in z_h and concave in z_k : the wrong sign of parameter f_k is probably derived from multicollinearity between z_k and y , the level of output. However, f_k is reestimated by a production function model relating y (the level of output) to z_k (the level of the quasi-fixed factor), in order to solve the optimal level \bar{z}_k^h : in this case, the multicollinearity problems are replaced by simultaneity problems.

Estimates of short run Hicksian price elasticities of demand, evaluated at sample average, are shown in table 3 with their estimated t ratios. Except for hired labor, all the estimated own-price elasticities are significant at the 5 % level. Demands for energy and capital are the most price-responsive ; significant short run net substitution possibilities between capital and hired labor, capital and fertilizers are found. At the sample average, $ECH^{CT} = 2,63$: this figure may be viewed as too high but multicollinearity problems or omitted factors can distort this result. Nevertheless, the result is consistent with other measures of returns to scale based on a total cost function where all

inputs are variable [Soutitie and al., 1987] : both studies show that there are scale economies.

Assuming that observed levels of quasi-fixed factors, are long run Hicksian levels, estimates of long run price elasticities of demand are shown in table 4 : these are consistent with Le Chatelier principle. Long run net substitution possibilities appear between family labor and hired labor, family labor and capital, land and energy, land and capital, land and fertilizers.

Table 5 shows the long run Hicksian price elasticities in the case where only fixity of land is relaxed : the previous relations of substitutability with land also appear. The long run Hicksian price elasticities of demand in the case where the two constraints upon family labor and land can be relaxed are shown in table 6. These two last long run Hicksian price elasticities (tables 5 and 6) can't be compared, in the view of the Le Chatelier principle, with the previous short run Hicksian price elasticities, because, in each case, the point of approximation is different:

$(z_h, \bar{z}_k^h(z_h)) ; (\bar{z}_h^h, \bar{z}_k^h) ; (z_h, z_k) ;$ respectively.

In order to evaluate the importance of taking into account the quasi-fixity of some inputs, we also estimated a translog total cost function with only four inputs : fuel and oil, capital, hired labor and fertilizers. It is interesting to note the relationships between the elasticities (short run and long run Hicksian elasticities) derived from the restricted cost function and those would be provided by a cost function including only the variable inputs. If the $[(x_i), (z_j)]$ production function weak separability is valid (6), the elasticities estimated with the last model would reflect net substitution elasticities, i.e. along a four inputs

isoquant. Therefore, they would be upward biased estimates of net long run elasticities calculated along a six inputs isoquant. Following Berndt and Wood (1979) it is possible to evaluate this upward bias using the following equality.

$$\begin{aligned}\varepsilon_{nn'}^{h(CV)} &= \partial \text{Log } x_n^h / \partial \text{Log } p_{x_n} \Big|_y \\ &= \partial \text{Log } x_n^h / \partial \text{Log } p_{x_n} \Big|_v \\ &\quad + [\partial \text{Log } x_n^h / \partial \text{Log } v \cdot \partial \text{Log } v / \partial \text{Log } p_v \cdot \partial \text{Log } p_v / \partial \text{Log } p_{x_n}] \Big|_v\end{aligned}$$

where v (respectively p_v) is the output (respectively the price) of the weakly separable subfunction $(x_i); i = 1, \dots, 4$.

Since the terms in brackets is negative, $\varepsilon_{nn'}^{h(CV)}$ is always inferior to $\varepsilon_{nn'}^{h(CT4)} = \partial \text{Log } x_n^h / \partial \text{Log } p_{x_n} \Big|_v$. Empirically,

this inequality is rejected by the data for only two out of the sixteen cases (tables 6 and 7).

It is not possible to compare, analytically and theoretically, the short run Hicksian elasticity of demand derived from the six inputs restricted cost function and the long run Hicksian elasticity of demand calculated with the four inputs total cost function. This mainly comes from the fact that, in both cases, the point of approximation is different and the second term B in equation [7a] may be either positive or negative. Note that some variable inputs (for example seeds) are not included in the model because no measure of unit price could be computed from the available data. As a result, even if these inputs are weakly

separable from the others, all the elasticity estimates would be upward biased for the same reason as before.

Finally, the long run measures of returns to scale give :

$$ECH^{LT}(z_h, \bar{z}_k^h(z_h)) = 3.69$$

$$ECH^{LT}(\bar{z}_h^h, \bar{z}_k^k) = 4.05$$

where h corresponds to family labor and k to land.

So, at the sample average, there exists economies of scale, whatever measure you utilize.

Concluding remarks

The use of a restricted cost function permits the estimation of the characteristics of the French cereal technology in a framework of short run static Hicksian equilibrium. Significant short run net substitution possibilities between capital and hired labor, capital and fertilizers are found. Moreover, at the sample average, technology exhibits positive economies of scale.

The use of a restricted cost function allows also the characterization of the other theoretical possible equilibria (long run Hicksian equilibrium, short run and long run Marshallian equilibria), under the sufficient assumptions of strict convexity of the function CR in $\bar{z}^h(y)$ (long run Hicksian demand of the

quasi-fixed inputs), in $\bar{y}^m(z)$ (short run Marshallian supply of the output), and in z^m et y^m (long run Marshallian demand of the quasi-fixed inputs and long run Marshallian supply of the output), respectively.

Practically, the data used in this study make possible to infer the characteristics concerning only the long run Hicksian equilibrium. The fixity hypothesis of family labor and land can be relaxed and some insights on long run Hicksian price elasticities of demand are derived : input substitutability is still valid in the long run.

The restricted cost function can also be used to evaluate others characteristics of the short run Hicksian technology. It is especially efficient to measure the capacity utilization that has an explicit economic theoretical foundation, and to calculate the capacity output elasticities with respect to input prices. The methodology used in this paper and the theoretical works of Morrison [1985, 1986] offer a fruitful approach for further research in this area.

Table 1. Characterization of the different theoretical equilibriums from the knowledge of the only restricted cost function $CR(p_x, z, y)$.

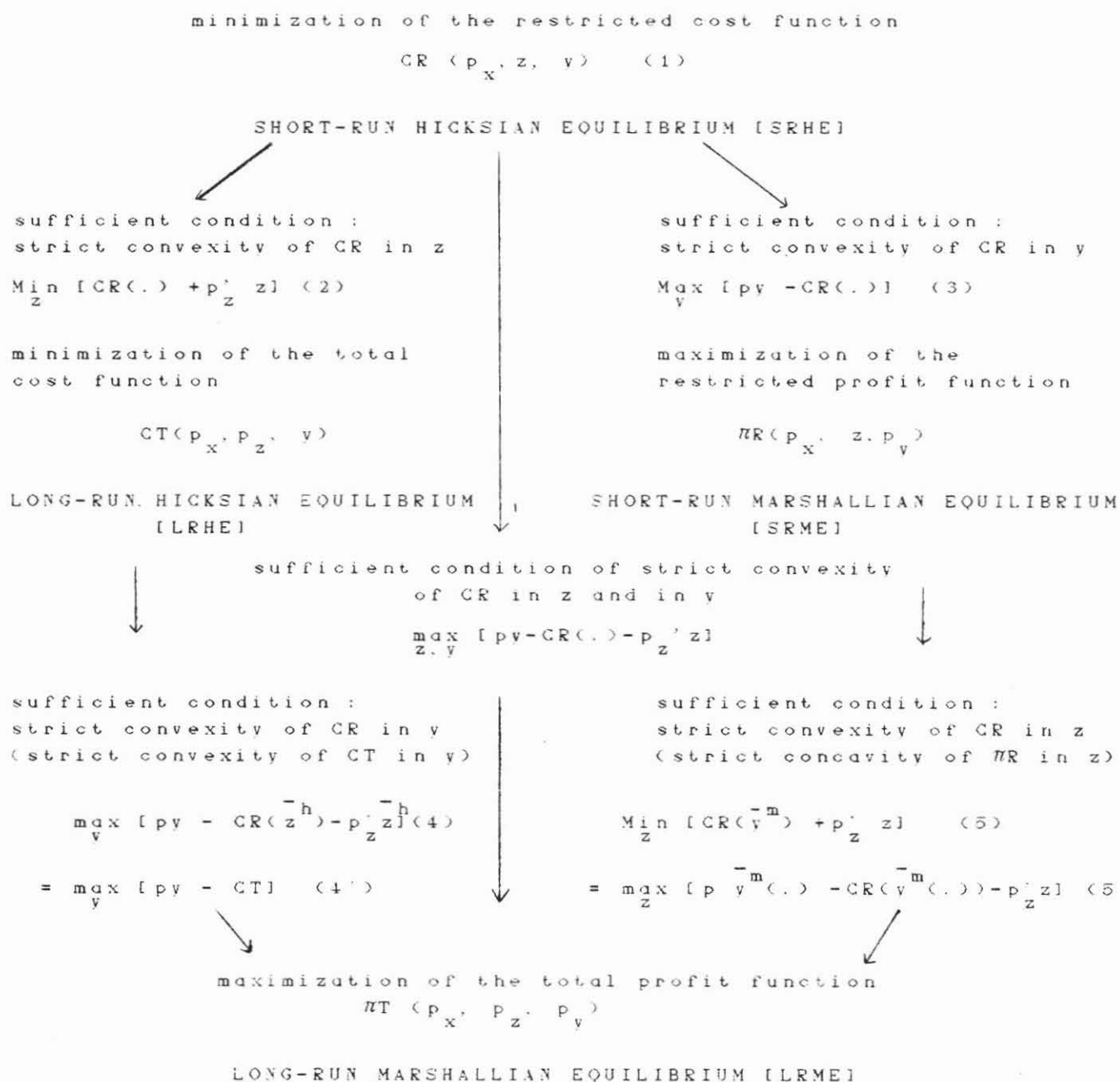


Table 2. Parameter estimates

Parameter	Estimate	Standard error
a_0	12.146	0.035
a_1	0.379	0.121
a_2	0.055	0.117
c_1	0.109	0.004
c_2	0.306	0.011
c_3	0.139	0.013
d_{11}	0.027	0.015
d_{12}	- 0.026	0.007
d_{13}	- 0.022	0.011
d_{22}	0.107	0.016
d_{23}	0.002	0.018
d_{24}	- 0.084	0.018
d_{34}	- 0.081	0.031
d_{33}	0.101	0.033
d_{44}	0.145	0.039
f_1	- 0.137	0.086
f_2	0.730	0.144
g_{11}	- 0.080	0.136
g_{12}	0.011	0.204
g_{22}	- 0.223	0.311
k_{11}	0.004	0.100
k_{12}	- 0.002	0.009
k_{21}	0.035	0.029
k_{22}	0.034	0.025
k_{31}	- 0.095	0.033
k_{32}	0.009	0.030

Table 3. Short run Hicksian price elasticities of demand, evaluated at sample average, z_h and z_k (h: family labor, k : land)

price	fuel and oil	capital	hired labor	fertili- zers
quantity				
fuel and oil	- 0.638 (4.63)	0.075 (1.24)	- 0.065 (0.62)	0.628 (4.53)
capital	0.026 (1.24)	- 0.344 (6.68)	0.145 (3.91)	0.173 (2.98)
hired labor	- 0.051 (0.62)	0.321 (3.91)	- 0.136 (0.56)	- 0.135 (0.6)
fertilizers	0.153 (4.53)	0.118 (2.98)	- 0.042 (0.6)	-0.23 (2.65)

Table 4. Long run Hicksian price elasticities of demand, evaluated at sample average z_h and z_k (h : family labor, k : capital)

price quantity	fuel and oil	capital	hired labor	fertili- zers	family labor	land
fuel and oil	- 0.706	-0.047	-0.16	0.307	0.119	0.492
capital	- 0.0168	-0.425	0.065	-0.022	0.130	0.268
hired labor	- 0.129	0.144	-0.496	-0.337	0.961	-0.143
fertilizers	0.075	-0.015	-0.105	-0.634	-0.039	0.718
family labor	0.034	0.106	0.355	-0.047	-1.09	0.64
land	0.170	0.262	-0.063	1.025	0.763	-2.16

Table 5. Long run Hicksian price elasticities of demand, evaluated at sample average, z_h and \bar{z}_k^h (h : family labor, k : land)

price quantity	fuel and oil	capital	hired labor	fertili- zers	land
fuel and oil	- 0.734	- 0.192	- 0.205	0.228	0.89
capital	- 0.07	- 0.606	0.012	-0.22	0.37
hired labor	- 0.156	0.030	- 0.283	- 0.575	0.966
fertilizers	0.06	- 0.15	- 0.13	- 0.63	0.895
land	0.013	0.036	0.018	0.054	-0.119

Table 6. Long run Hicksian price elasticities of demand, evaluated at sample average, \bar{z}_h^h et \bar{z}_k^h (h : family labor, k : land)

price quantity	fuel and oil	capital	hired labor	fertili- zers	family labor	land
fuel and oil	- 0.734	-0.175	-0.247	0.256	0.013	0.856
capital	- 0.06	-0.545	-0.16	-0.119	0.04	0.859
hired labor	- 0.193	-0.35	0.845	-1.18	-0.244	1.10
fertilizers	0.06	-0.082	-0.368	-0,521	0.044	0.86
family labor	0.066	0.587	-1.61	0,94	0.36	-0.34
land	0.011	0.031	0.018	0,045	-8.410 ⁻⁴	-0.105

Table 7. Long run Hicksian price elasticities of demand, evaluated at sample average with a four inputs total cost.

price	fuel and oil	capital	hired labor	fertili- zers
quantity				
fuel and oil	-0.65	0.18	-0.12	0.60
capital	0.07	-0.61	0.34	0.20
hired labor	-0.10	0.71	0.0085	-0.63
fertilizers	0.14	0.13	-0.19	-0.08

NOTES

(1) The Hicksian short run technology, at level z_0 , can be defined by : $Y^0 = \{(x, y) ; (x, y, z_0) \in Y\}$.

(2) By direct application of the relationships between the Hessian matrices of CR and CT at the long run Hicksian equilibrium point, it follows [Guyomard, Vermersch, 1987] :

$$\partial^2 CR(.) / \partial \bar{z}^2 = - [\partial \bar{z}^h / \partial p_z^2]^{-1} = - [\partial^2 CT / \partial p_z^2]^{-1}$$

The concavity of the total cost function in all the inputs prices implies that the matrix $[\partial^2 CT(.) / \partial p_z^2]$ is negative semi-definite. So a sufficient condition to infer the long run Hicksian equilibrium from the restricted cost function is that the matrix $[\partial^2 CR(.) / \partial \bar{z}^2]$ is positive definite, and so that the restricted cost function is strictly convex in \bar{z}^h .

(3) These sufficient conditions can be clearly proved by using the relationships which exist between the Hessian matrices of CR, CT, π_R et π_T at the respective optimum levels and by using a analogous method as noted in footnote (2).

(4) A full quadratic expansion of the restricted cost function around the approximation point would include quadratic terms for y and z_j ; for y and p_i . However, the likelihood ratio test for the hypothesis saying that the corresponding coefficients equal zero is not rejected at conventional level (5 %).

(5) Note that the generalization of this derivation to the case of M ($M \geq 3$) quasi-fixed factors is not straightforward. It is

theoretically possible to compute the short run and long run Marshallian price elasticities of demand (and of supply) using the parameters of the only restricted cost function [Guyomard, 1988]. Unfortunately the function $CR(.)$ is not convex in y and so this calculation is not correct.

(6) The hypothesis that the restricted cost function is logarithmically separable into a function of variable inputs and quasi fixed inputs is not rejected at a 5 % level (test based on the likelihood ratio method).

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