



HAL
open science

Private eradication of mobile public bads

Christopher Costello, Nicolas Querou, Agnès Tomini

► **To cite this version:**

Christopher Costello, Nicolas Querou, Agnès Tomini. Private eradication of mobile public bads. [University works] auto-saisine. 2016, 29 p. hal-01594067

HAL Id: hal-01594067

<https://hal.science/hal-01594067>

Submitted on 5 Jun 2020

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Private eradication of mobile public bads

Christopher Costello*, Nicolas Quérou†, Agnes Tomini ‡

January 25, 2016

Abstract

We consider analytically the non-cooperative behavior of many private property owners who each controls the stock of a public bad such as an invasive weed species, fire, or agricultural pest. The stock of the public bad can grow and disperse across a spatial domain of arbitrary size. In this setting, we characterize the conditions under which private property owners will control or eradicate, and determine how this decision depends on property-specific environmental features and on the behavior of other landowners. We show that high mobility or lower control by others result in lower private control. But when the marginal dynamic cost of the bad is sufficiently large, we find that complete eradication may be privately optimal (despite the lack of consideration of others' welfare) – in these cases, eradication arises in the non-cooperative game and is also socially optimal so there is, in effect, no externality. Finally, when property harboring the bad is not owned, or is owned in common, we derive the side payments required to efficiently control the mobile public bad.

JEL Classifications: H41, Q24, Q57, R12, R14

Key words: Public Bad, Spatial Externality, Invasive Species, Spread, Dispersal, Eradication

*Bren School, UC Santa Barbara, 4410 Bren Hall, Santa Barbara, CA 93117, USA, and NBER. E-mail: costello@bren.ucsb.edu

†CNRS, UMR 5474 LAMETA, F-34000 Montpellier, France. E-mail: querou@supagro.inra.fr

‡Aix-Marseille University (Aix-Marseille School of Economics): CNRS & EHESS, Centre de la Vieille Charité, 2 rue de la charité, F-13002, Marseille, France. E-mail: agnestomini@gmail.fr

1 Introduction

The management of public bad resources represents a ubiquitous challenge with real-world policy implications. Applications are numerous and include diverse resources such as fire, invasive species, antibacterial resistance, air pollution, noxious advertising, cyberspace viruses, aquaculture pathogens, infectious diseases, among other common nuisances. The problem is worsened by the mobility and renewability of these resources, since they grow and disperse to surrounding properties in subsequent periods, thus imposing future damages in other locations. While the literature often focuses on socially-optimal management,¹ issues that arise when individual property owners each make decentralized decisions has received little attention. Indeed, spatial connectivity induced by the mobility of the resource influences private decisions, which collectively can have important consequences for control or eradication across the spatial domain.²

This paper concerns the private management of spatially-distributed, mobile public bads. We focus on deriving the biological and economic conditions under which private property owners will find it privately optimal to control or eradicate the public bad; and how those decentralized decisions depend on property-specific features and on the control decisions of others. The entire analysis is analytical, so we seek general insights that can inform both positive and normative aspects of this empirically-extensive class of challenges.

A disparate literature contributes to the issue of public bad management from the spatial perspective. The literature on optimal control of infectious diseases commonly introduces a transmission parameter to capture the rate of spread, but typically does not model the spatial dimension of disease transmission (exceptions are noted in the review by Arino and den Driessche (2006)). The resource economics literature provides the closest related setting. First, it has become common recently to examine the optimal management of an exotic species that is spatially distributed across the landscape. A sole owner accounts for all spatial connections and optimizes her control efforts across space to characterize the optimal design of policies. In this setting some authors focus on the question of prevention vs. control: Leung et al. (2002) find, for zebra mussels, that ex-ante prevention is more efficient than ex-post control, while Burnett et al. (2008) use the real-world case of the Brown Tree snake in Hawaii, after having theoretically characterized the paths

¹Among others, Lichtenberg and Zilberman (1986) and Archer and Shogren (1996) seek to optimally control a pest population, other biological invasions (Shogren 2000; Olson and Roy 2002) or infectious diseases (Wiemer 1987; Gersovitz and Hammer 2004). Adda (2015) provides an evaluation of health-related policies relying on cost-benefit analysis.

²**For instance, Brito et al. (1991) or Geoffard and Philipson (1997) focus on the economics of vaccination (but not eradication) and abstract from issues raised by strategic interactions and heterogeneity.**

of expenditures and damages, to analyze the optimal integrated management of prevention and control. Others contrast long-run solutions from an optimal control system and solutions from a static optimization problem (Finnoff et al. 2010), or uniform vs spatially-optimized policy (Albers et al. 2010). Epanchin-Niell and Wilen (2012) numerically examine optimal policies over a range of spatial and ecological configurations, and emphasize the influence of these qualitative characteristics on policies. While some purely theoretical works exist (e.g. Blackwood et al. (2010)), most papers conduct numerical simulations either in stylized systems or in systems loosely parameterized by empirical observations because analysis tends to grow in complexity with the spatial domain. These focused numerical applications help establish insights in the settings they explore, but they also raise more general hypotheses that can be addressed by theory.

A second strand of literature explicitly introduces the non-cooperative nature of private property owners, and emphasizes mechanisms that can be used to induce cooperation. Grimsrud et al. (2008) show that coordination is more likely with low levels of invasion in a two-agent dynamic model. Epanchin-Neill and Wilen (2015) examine how different degrees of cooperation affect invasion and find that the degree of cooperation is related to control costs: less cooperation is required to achieve high control when costs are low relative to damages. Our analysis follows this line of research, but we rather provide a game-theoretical approach with many economic agents instead of conducting numerical analysis of particular system. Moreover, we focus on the role of heterogeneous landowners, for example with respect to costs, damages, and dispersal rates. We find that these sources of heterogeneity can significantly alter individual landowner incentives over control or eradication, suggesting that heterogeneity can play an important role in economic outcomes. Our theoretical approach allows us to home-in on the effects of different patterns of dispersal and infestation on non-cooperative outcomes.³ This helps to generalize previous numerical results.

Broadly speaking, the literature on network games provides interesting insights highlighting that players' behaviors are influenced by those around them (Jackson and Zenou 2014). Indeed, dispersal rates of a resource, disease transmission parameters, or a network structure represent an adjacency matrix linking agents and serve as a vector to impact their payoffs. This literature helps to characterize how individuals' decisions may depend on interactions across players. For example, biological invasions are often considered a "weakest-link" public good where the level of control is determined by the weakest contributor (Burnett 2006). Our theoretical framework also has ramifications for "reputation" spillovers, since reputations depend on networks of social relations between economic agents. Winfree and Mc-

³A decentralized model of global disease eradication is analyzed in Barrett (2003) within a static and homogeneous setting.

Cluskey (2005) analyze reputation as a dynamic common-property resource. They adopt the structure introduced by Shapiro (1982) to capture the spread of learning on firm quality. However, this literature largely uses experimental approaches (e.g. Palfrey and Prisbrey (1996)), and ignores the spatial dimension (an exception is Brandt et al. (2003)). Although informative, and suggestive of mechanisms that may be applied more broadly, these papers lack theoretical findings so general lessons, and the conditions under which they arise, are difficult to ascertain. The objective of this paper is therefore to provide a general analysis of decentralized public bad management in a spatial setting.

In particular, the aim of this paper is to analyze the private management of a spatially-distributed mobile public bad, and to examine the game theoretical interactions among non-cooperative property owners. We employ this apparatus to derive general results such as the conditions under which control and eradication will emerge, those under which non-cooperation results in a socially optimal pattern of control, and the effects of system characteristics (e.g. the rate of spread) on non-cooperative outcomes. Part of the literature stresses that there exists a spectrum of policy options to manage public bads: prevention and cure (Leung et al. 2002), detection and control (Kaiser and Burnett 2010), but very few focuses on eradication (Olson and Roy 2002; Burnett et al. 2006). This policy option is often considered to be too expensive or too complex (Gersovitz and Hammer 2005; Regan et al. 2006). We address this possibility by characterizing the conditions under which eradication is either socially optimal, or is an outcome of non-cooperative behavior across all spatially connected properties.

We develop an analytical model with an arbitrary number of spatially-distinct properties and discrete-time resource dynamics to analyze decentralized owners' incentives and the equilibrium behavior across those owners. In our theoretical model we also solve for the social planner's optimal control pattern across space and time. While we think this as a contribution in its own right, we regard it primarily as a benchmark case against which to compare decentralized equilibria across non-coordinating property owners. Aside from developing a tractable analytical framework for addressing mobile public bads, our results make three main analytical contributions. First, we show that the private trade-off between controlling the expansion of a public bad on one's own property and eradicating it depends on the magnitude of its spread. Furthermore, whether complete eradication over the entire domain emerges in the decentralized system depends on features such as the magnitude of patch connectivity. Second, in general we find the intuitive result that non-cooperative property owners will provide too little control of the public bad. This result accords with Fenichel et al. (2014) and is intuitive because private property owners will consider only their local costs and benefits of control, but will disregard the consequences of their actions on

adjacent owners. We also show analytically how the extent of this externality is driven by heterogeneity and other features of the problem. When the marginal dynamic cost inflicted by the stock is low, neither the social planner nor the non-cooperative private property owners will engage in much control, so little is to be gained from cooperation among private owners. In that case, private property delivers a near first-best outcome. But as the size of the dynamic cost increases, private property owners increase their control, but not as much as the social planner would have liked. Thus, as the marginal dynamic cost grows, so does society's benefit from cooperation among property owners. This intuitive finding suggests that as the size of the externality grows, so does the importance of government intervention (or private ordering) to internalize the externality. But we find that this result only holds for moderate levels of dynamic cost. If dynamic cost grows enough, then private property owners will eradicate on their own property; for example, you do not tend to observe poison ivy in urban backyards. We show that when eradication arises in the non-cooperative game, then it is also socially efficient. Thus, if marginal dynamic cost is sufficiently large, the cooperative and non-cooperative solutions converge, and there is no additional value from government intervention. This contribution suggests that government intervention may be justified (to coordinate the actions of private land owners), but only in cases of *intermediate* dynamic cost. Our third main contribution is to completely characterize the gains from inducing cooperative behavior among the non-cooperative property rights holders. Naturally, to the extent that properties are heterogeneous, the side payments required to achieve cooperation will differ across space. We derive the magnitude of these side payments as a function of damage, cost, spread, and growth.

We organize the paper as follows: The analytical model is introduced in Section 2 and we derive the equilibrium strategies of non-cooperative property owners in Section 3. The social planner's problem is introduced and solved in Section 4, which puts us in a position to compare the decentralized solution with the social planner's in Section 5. We then calculate the gains from cooperation and discuss the cooperation-inducing side payments in Section 6. We conclude in Section 7. All proofs are found in the Appendix.

2 A spatially-connected model of a renewable public bad

The stock of a renewable public bad is spatially distributed. Space is divided into a set of I mutually exclusive and exhaustive "properties," each of which is assumed to be owned by a single profit maximizing owner. Properties may be heterogenous in

biology, and economics, but intra-property characteristics are homogenous. Using a discrete-time model, the stock residing on property i at the beginning of time period t is given by x_{it} and control efforts undertaken on property i will reduce the stock over the course of that time period. We denote the amount of stock removed on property i by h_{it} , which leaves a “residual stock” at the end of the period of $e_{it} \equiv x_{it} - h_{it}$. The residual stock grows according to a growth function $g(e_{it})$,⁴ and the resource stock is distributed across the landscape. The fraction of the resource stock that moves from property j to property i is given by D_{ji} , so $\sum_i D_{ji} \leq 1$.⁵ The equation of motion of the resource stock is:

$$x_{it+1} = \sum_{j=1}^I D_{ji} g(e_{jt}); \quad (1)$$

The resource stock on property i imposes damage on owner i , and the damage function may be property-specific (for example, a weed may cause more damage in an agricultural area than in an industrial area). We assume that damage is a function of post-harvest residual stock. If the residual stock in i is e_i , the marginal damage in i is $k_i(e_i)$, where $k_i'(e_i) > 0$.

The cost of control may also be property-specific (for example removing invasive mussels may be simpler in shallower water). The marginal cost of control in a property will also depend on the stock size in that property. This captures the so-called *stock effect* for which the marginal abatement cost is a decreasing function of the stock. We model the marginal control cost as $c_i(x_i)$, where $c_i'(x_i) < 0$. Taking all relevant economic variables into account, the period- t cost to owner i of stock, x_{it} , and control, h_{it} is:

$$\Phi_i(x_{it}, h_{it}) = \int_0^{x_{it}-h_{it}} k_i(s) ds + \int_{x_{it}-h_{it}}^{x_{it}} c_i(s) ds. \quad (2)$$

Following the identity $e_{it} \equiv x_{it} - h_{it}$, we can re-write Equation 3 as:

$$\Phi_i(x_{it}, e_{it}) = \int_0^{e_{it}} k_i(s) ds + \int_{e_{it}}^{x_{it}} c_i(s) ds \quad (3)$$

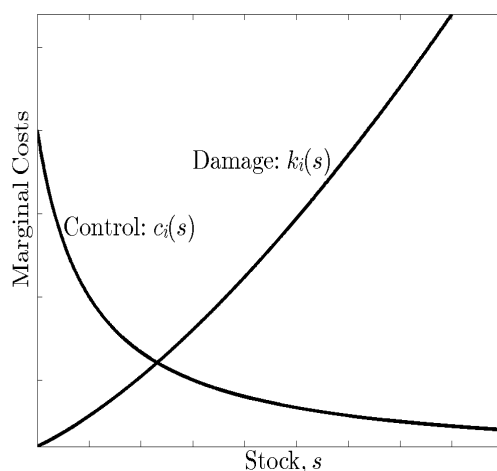
The first term on the right hand side of Equation 3 is the total damage cost on property i during period t and the second term is the total cost of control.

⁴We assume the usual conditions: $g(0) = 0$, $g'(e) > 0$, $g''(e) < 0$.

⁵This follows the recent literature from the natural sciences (see, e.g., Nathan et al. (2002), or Siegel et al. (2003)) who model dispersal of passive “Lagrangian particles.” An endogenous dispersion parameter may be a relevant alternative to account for density-dependent process, or situations where agents can affect that process. The latter case introduces an additional control variable (as in Rowthorn et al. (2009)), which has the benefit of introducing the containment policy to keep the bad within some specified area. While this has appeal in some settings, we focus instead on the problem of controlling (or eradicating) the stock of the bad on one’s property.

Management of the public bad will involve choosing the level of control to minimize the sum of control costs and damages. While the payoff to owner i depends only on the stock and control on property i , the stock itself will depend on past decisions in all other properties because the resource can move across space (see Equation 1). Thus, all properties are linked together and this is the sense in which we call this resource a *public bad*.

Figure 1: Illustration of marginal control cost ($c_i(s)$) and marginal damage cost ($k_i(s)$), for property i , as a function of the current stock of the public bad, s .



3 Property owners' strategies

We assume that each of the I property owners makes her own privately optimal decision about how much control to engage in each period. This is a complicated decision for owner i for two reasons. First, owner i 's strategy about how much to control may depend on all current stocks of the public bad and on the strategies applied by all other owners. Second, because the resource grows and moves, owner i 's strategy must account for the fact that less control today implies higher growth and thus higher damage (even on her own property) in future periods. This latter observation allows for a spectrum of management options for owner i , from doing nothing to full eradication. In this setting all owners simultaneously choose their level of control, $h_{it} \geq 0$. Equivalently, owner i can choose $e_{it} \geq 0$ (since $e_{it} \equiv x_{it} - h_{it}$ and x_{it} is known at the time of the decision) in order to minimize her

present value cost \hat{V}_{it} (taking all other variables as given) as follows:⁶

$$\hat{V}_{it}(x_{it}) = \min_{e_{it} \geq 0} \left(\Phi_i(x_{it}, e_{it}) + \delta \hat{V}_{it+1}(x_{it+1}) \right) \quad (4)$$

subject to Equation 1 which defines the state transitions as a function of all owners' controls.

In the following sections, we characterize the owners' optimal management strategies in order to derive the system-wide management outcomes, stock levels, and equilibrium payoffs in the decentralized system. We especially focus on the emergence of control and (partial or complete) eradication as equilibrium outcomes.

3.1 The strategy of partial control

First, we derive the conditions under which property owners optimally choose to control, but may not eradicate, the public bad. We thus focus on an interior equilibrium ($\hat{e}_{it} > 0 \forall i, t$), which is characterized as follows:

Proposition 1. *The interior equilibrium of the I-property public bad dynamic game is characterized by residual stock on property i (\hat{e}_{it}) given as follows:*

$$k_i(\hat{e}_{it}) = c_i(\hat{e}_{it}) - \delta c_i(\hat{x}_{it+1}) D_{ii} g'(\hat{e}_{it}) \quad (5)$$

Moreover, this interior equilibrium is state independent: \hat{e}_{it} is independent of \hat{x}_{it} for all i .

Here, \hat{e}_{it} and \hat{x}_{it} denote the residual stock and resource stock on property i , respectively.⁷ The level of control is simply $\hat{h}_{it} = \hat{x}_{it} - \hat{e}_{it}$. Proposition 1 shows that the equilibrium residual stock arises from a trade-off between the current marginal damage (on the LHS) and the long-run marginal control cost (on the RHS). Owner i will control the bad until the current marginal damage is equal to the current marginal cost of removing one additional unit of the stock, mitigated by the discounted future cost implied by an increased stock.

We note also that the strategy of owner i depends on \hat{x}_{it+1} (via its effect on future control costs), which suggests that owner i 's decision will depend on the past decisions of other owners j for whom $D_{ji} \neq 0$ (see Equation 1). If an adjacent owner engages in less control (and so leaves a larger \hat{e}_{jt-1}), how will owner i respond? We find a kind of "race to the bottom" emerges, in which less control by owner j implies less control by all connected owners. This consequence of strategic dynamic interactions among property owners is formalized as follows:

⁶Indeed, it turns out to be more mathematically convenient to keep track of the residual stock, e_{it} , rather than the explicit control, h_{it} , so we use e_{it} as the control variable for property owner i . The explicit amount of control, h_{it} , can then simply be backed out.

⁷The hat indicates the non-cooperative game equilibrium.

Proposition 2. *A larger residual stock in one property causes an increase in the optimal residual stock in all connected properties: $\frac{\partial \hat{e}_{it}}{\partial e_{jt}} > 0$, where $D_{ji} \neq 0$.*

Proposition 2 is consistent with results in Fenichel et al. (2014): The incentive to control the public bad increases with the control effort of other owners. As one property owner reduces the stock of public bad on her property, adjacent (or otherwise spatially connected) owners will follow suit. Public bad control is thus a strategic complement, and the strategic reaction to each others' decisions may induce a kind of domino effect. Consequently, this particular game of strategic complements is a spatial analog of the “weaker-link” problem (Cornes 1993): The level of control in the entire spatial domain is not determined by the lowest individual level effort, but lower control by a single owner will trigger the spread of a spatially mobile public bad. This eventually leads to a loss in welfare across the entire spatial domain. These strategic interactions may suggest that eradication is unlikely to emerge as an equilibrium outcomes. Yet, as highlighted in the next section, there are cases where full or partial eradication will emerge from private management.

3.2 The emergence of eradication

Will private owners ever eradicate the public bad? The majority of the literature focuses on partial control, and thus neglects whether it is optimal for private agents to eradicate. Indeed, eradication is often considered unrealistic (Simberloff (2009)). For instance, vaccination efforts by some agents may reduce incentives of others to immunize (Anderson and May 1991), even though it may be socially desirable. Naturally, though, the decision of whether to completely eradicate a public bad will depend on adjacent owners' actions because if they lack control on their property, the likelihood of future infestation may be very high. We thus expect strategic interactions to play an important role in individual decisions.

In order to proceed, we will dissect the optimality condition in Equation 5. That Euler equation defines the first order condition for an interior dynamic optimum. The term $k_i(\hat{e}_{it}) - c_i(\hat{e}_{it}) + \delta c_i(\hat{x}_{it+1})D_{ii}g'(\hat{e}_{it})$ can be thought of as the *marginal dynamic cost* of the public bad to property owner i . Naturally, the property owner would like to set the marginal dynamic cost equal to zero. But nontrivial cases exist when it is not possible for that expression to equal zero. We use this fact to analyze the circumstances under which eradication is an outcome of decentralized decisions by property owners.⁸ For example, if even very small stocks impose large damage costs, it turns out that the optimal decision for owner

⁸In the remainder of the analysis, when we mention eradication outcomes, we refer to situations where the public bad is eradicated starting at time $t = 0$. We will briefly discuss the implicit effect of initial conditions at the end of the analysis.

i can be to completely eradicate the stock on her property. If this is the case for all property owners, then complete eradication across the entire spatial domain will arise from non-cooperative behavior. These results are summarized as follows:

Proposition 3. *Complete eradication across the entire spatial domain arises from non-cooperative behavior of property owners if and only if:*

$$\min_{i \in I} [k_i(0) - (1 - \delta D_{ii} g'(0)) c_i(0)] > 0. \quad (6)$$

Realistic cases exist in which full eradication in fact does arise, as is suggested by Proposition 3. Full eradication requires that $[k_i(0) - (1 - \delta D_{ii} g'(0)) c_i(0)] > 0$. To gain some intuition, first consider the case in which marginal damage from a small stock ($k_i(0)$) is large. Provided it is not too costly to eradicate the last units of the stock (so $c_i(0)$ is not too large), eradication will be optimal. But even if marginal damage from a small stock is small (so the stock must build up before it inflicts any significant damage), it may still be optimal to eradicate. To see that, suppose $k_i(0) = 0$. Then eradication emerges provided that $1 < \delta D_{ii} g'(0)$, so decentralized eradication is more likely when self-retention (D_{ii}) is large, intrinsic growth is large ($g'(0)$), or the discount factor is large (δ). This interesting result is a consequence of the foresight by the property owner. If she fails to eradicate now, the stock will grow and cause much more damage in subsequent periods.

Cases also exist in which eradication arises on some properties, but not on others; we refer to this as “partial eradication.” For example, if property i has high marginal damage from a weed infestation (perhaps it is a native plant nursery) and property j has low marginal damage (perhaps it is rangeland), then this analysis suggests that owner i may find it privately optimal to eradicate the weed on her property while owner j does not. To characterize the conditions under which partial eradication emerges as an equilibrium outcome, we separate the owners into two distinct groups: A group E of $n_e > 0$ owners who fully eradicate on their own properties, and a group of remaining owners who optimally choose to only partially control on their properties. As long as some owners fail to completely eradicate, the public bad will still reside in some of the areas because of growth and spread originating from the partially-controlled properties. The emergence of partial eradication as an equilibrium of non-cooperative behavior is characterized as follows:

Proposition 4. *Partial eradication arises from non-cooperative behavior if and only if there exists a group of owners $E \subset I$ such that, in any period t :*

$$\min_{i \in E} [k_i(0) - c_i(0) + \delta D_{ii} g'(0) c_i(\hat{x}_{it+1})] > 0 \quad (7)$$

and the remaining property owners control the public bad such that:

$$k_j(\hat{e}_{jt}) = c_j(\hat{e}_{jt}) - \delta c_j(\hat{x}_{jt+1}) D_{jj} g'(\hat{e}_{jt}) \quad \forall j \notin E. \quad (8)$$

Note that for any owner $i \in E$ the corresponding stock level $\hat{x}_{it+1} = \sum_{j \notin E} D_{ji} g(e_{jt}^e)$ is positive as long as it is connected to another property whose owner who only partially controls the public bad. Intuitively, it is possible that the partial eradication situation described in Proposition 4 leaves room for self-consistent transfer payments, where (some) owners from group E are willing to compensate (some) owners from the other group to increase their levels of control. However, this intuition relies on the assumption that complete eradication would be socially optimal, which has yet to be analyzed.

4 Socially optimal management of a mobile public bad

In the spatial dynamic game, property owners consider only the payoffs on their own properties when making optimal decisions. By contrast, a sole owner must account for the entire spatial domain when managing the public bad. The sole owner must optimize the spatial and temporal control to minimize the present value of the sum of costs to all properties, subject to the resource dynamics. Written as a dynamic programming equation, the sole owner's problem is to minimize the present value cost V_t , and the problem is defined as follows:

$$V_t(\mathbf{x}_t) = \min_{\{e_{1t}, e_{2t}, \dots, e_{Nt}\} \geq 0} \sum_i \Phi(x_{it}, e_{it}) + \delta V_{t+1}(\mathbf{x}_{t+1}) \quad (9)$$

subject to the Equation 1, and where the bold notation \mathbf{x}_t indicates the vector $\mathbf{x}_t \equiv [x_{1t}, x_{2t}, \dots, x_{Nt}]$. This appears to be an incredibly complicated problem to solve, particularly as I gets large, because it involves an I dimensional decision where each decision is connected over time via the spread and growth dynamics. But it turns out that this problem can be solved analytically, and that the optimal spatial-temporal control policy can be completely characterized.

4.1 Control as a socially optimal management strategy

We first consider the case in which partial control is optimal for the social planner. The socially optimal partial control policy is given as follows:

Proposition 5. *The sole owner's optimal partial control strategy has residual stocks, $\bar{e}_t > 0$, characterized as follows*

$$k_i(\bar{e}_{it}) = c_i(\bar{e}_{it}) - \delta \sum_j c_j(\bar{x}_{jt+1}) D_{ij} g'(\bar{e}_{it}). \quad (10)$$

In a manner similar to the decentralized result (Proposition 1), the sole owner's optimal residual stock results from a trade-off between marginal damage (on the LHS) and the marginal control cost (on the RHS). Again, the marginal cost of control is composed of the current marginal control cost and the sum of the discounted marginal control cost in the future.

4.2 Is eradication socially optimal?

In a manner similar to the decentralized property owners, we can determine the conditions under which complete eradication is socially optimal. Intuitively, if damage is very high, or if eradication costs are very low, then it may pay to bear the one-time costs of eradication rather than bear an infinite stream of damages (and costs) in perpetuity. If Condition 10 cannot be met for any $\bar{e}_{it} > 0$, then it is optimal to eradicate the entire resource stock on all properties. There are also conditions under which it is socially optimal to eradicate on some properties but not all. These results are formalized below:

Proposition 6. (a) *Complete eradication across the entire spatial domain is socially optimal if and only if:*

$$\min_{i \in I} \left[k_i(0) - (1 - \delta D_{ii} g'(0)) c_i(0) + \delta g'(0) \sum_{j \neq i} D_{ij} c_j(0) \right] > 0. \quad (11)$$

(b) *Eradication across part of the spatial domain is socially optimal if and only if there exists a set of properties $\bar{E} \subset I$ such that, in any period*

$$\min_{i \in \bar{E}} \left[k_i(0) - c_i(0) + \delta g'(0) \sum_j D_{ij} c_j(\bar{x}_{it+1}) \right] > 0 \quad (12)$$

and the remaining property owners control the public bad such that:

$$k_j(\bar{e}_{jt}) = c_j(\bar{e}_{jt}) - \delta \sum_k c_k(\bar{x}_{kt+1}) D_{jk} g'(\bar{e}_{jt}) \quad \forall j \notin \bar{E}. \quad (13)$$

The result on full eradication (Proposition 6a) is similar to the case of decentralized management except that here, spatial externalities between all properties are accounted for by the social planner. Proposition 6b characterizes conditions under which it is socially optimal to only partially eradicate. Again, notice that, in any property $i \in \bar{E}$, the stock level $\bar{x}_{it+1} = \sum_{j \notin \bar{E}} D_{ji} g(\bar{e}_{jt})$ is positive at the beginning of period $t + 1$ provided that it is connected to a property in which it was socially optimal to only partially control the public bad. While the residual

stock level is the same under both management regimes for properties where the public bad is eradicated, it may differ in others.

The results obtained in Sections 3 and 4 imply an interesting dependence of spatial connectivity on the differences between decentralized and socially optimal management; this is further explored in what follows.

5 Tragedy of the commons, inefficient coordination and spatial connectivity

The literature on decentralized common pool resource management often emphasizes its shortcomings compared to socially optimal management, and these comparisons are often restricted to quantitative comparisons of outcomes. By contrast, we adopt two lines of comparison. First, we analyze the conditions under which the tragedy of the commons emerges (where each owner chooses positive, yet sub-optimally low control of the public bad). Second, we characterize cases in which property owners fail to coordinate on the socially-optimal strategies: i.e. they only partially control, while the social planner would fully eradicate the mobile public bad. Inspecting Conditions 5 and 10 or Conditions 6 and 7, it is intuitive that spatial connectivity will play an important role in these comparisons.

We first assess how “spread” may exacerbate the tragedy of the commons beyond the deleterious effects arising from strategic interactions, which we have already examined. When no spatial externality exists, $D_{ii} = 1$ (so $D_{ji} = 0$ for all $j \neq i$), and we would expect the decentralized solution, $\hat{e}_{it} > 0$, to equal the socially optimal solution, $\bar{e}_{it} > 0$. If an externality exists, $D_{ji} \neq 0$, and we find that control levels differ under a decentralized management, as is summarized below:

Proposition 7. *In any period t :*

- (a) *In the absence of spread (so $D_{ij} = 0 \forall i \neq j$), the decentralized equilibrium is equivalent to the socially optimal policy for all properties, $\hat{e}_{it} = \bar{e}_{it}$.*
- (b) *When property i is a pure source, i.e. $D_{ii} = 0$, the Nash equilibrium is strictly higher than the optimal policy for each property, $\hat{e}_{it} > \bar{e}_{it}$.*
- (c) *When $D_{ii} \in (0, 1)$, we have, for any property i , $\hat{e}_{it} \geq \bar{e}_{it}$.*

Proposition 7 confirms that the tragedy of commons emerges under private management, except (intuitively) in the limiting case when there is no spatial connectivity. When a property that harbors the bad is spatially connected to another property, *laissez-faire* tends to yield a suboptimal level of control; this echoes Fenichel et al. (2014). Moreover, the magnitude of spatial connectivity does affect owner i 's optimal choice. In the presence of spread, the degree to which owner i 's strategy depends on decisions by owner j will depend both on strategic

interactions and on spatial features such as the magnitude of connectivity. Here we examine the impact of the spread parameters in order to identify how changes in residual stock on property i are driven by changes in the self-retention rates and off-property spread rates. For instance, assume a two-patch sink-source system, where patch i is the source (so $D_{ii} = 0$ and $D_{ij} > 0$) and patch j a sink (so $1 \geq D_{jj} > 0$ and $D_{ji} = 0$). Owner i will thus optimize her control within her patch ignoring the mobility of the public bad towards the second patch, and thus under control the amount of public bad. On the contrary, owner j will behave like a sole-owner disconnected from her neighbor. By extrapolation to a system with multiple sources and multiple sinks, we may conjecture that the Nash equilibrium should be higher than the optimal policy for sources, but that outcomes should be similar in sinks. This unidirectional flow system has the advantage that higher control effort levels occur where the spread of contamination is high.

To sharpen intuition, we continue with the two-property case ($N = 2$) and focus on the case where control emerges as the decentralized equilibrium. In that case, there are two self-retention parameters: D_{ii} , and D_{jj} , and two dispersal parameters: D_{ij} , and D_{ji} . The impact of self-retention on property i , D_{ii} , and the spread from property j to i , D_{ji} can both be interpreted as a higher quantity of the public bad so intuition on their effects will be straightforward to garner. A more nuanced question is: How will owner i 's optimal strategy depend on spread to, and self-retention on, property j (that is, how does \hat{e}_i depend on D_{ij} and D_{jj})? These parameters affect stock on property j , and thus due to strategic interactions, will indirectly affect residual stock on property i . More specifically, if j responds by engaging in less control, then by Proposition 2, owner i may also respond by engaging in less control. All results on the dependence of owner i 's residual stock on the spread of the public bad are summarized as follows:

Proposition 8. *There exist $0 \leq \bar{D}_i < 1$ and $0 \leq \bar{D}_j < 1$ such that, if either $D_{ii} \geq \bar{D}_i$ or $D_{jj} \geq \bar{D}_j$, an increase in off-property spread (either D_{ji} or D_{ij}), results in a larger residual stock level on property i :*

$$\frac{\partial \hat{e}_{it}}{\partial D_{ji}} > 0; \quad \frac{\partial \hat{e}_{it}}{\partial D_{ij}} > 0.$$

A higher value of self-retention, respectively D_{ii} and D_{jj} , results in a lower residual stock level on property i if and only if the respective marginal cost is inelastic:

$$\begin{aligned} \frac{\partial \hat{e}_{it}}{\partial D_{ii}} < 0 &\Leftrightarrow 1 > \varepsilon_1 \quad \text{with } \varepsilon_1 = -D_{ii}g(\hat{e}_{it})\frac{c'_i(\hat{x}_{it+1})}{c_i(\hat{x}_{it+1})} > 0 \\ \frac{\partial \hat{e}_{it}}{\partial D_{jj}} < 0 &\Leftrightarrow 1 > \varepsilon_2 \quad \text{with } \varepsilon_2 = -D_{jj}g(\hat{e}_{jt})\frac{c'_j(\hat{x}_{jt+1})}{c_j(\hat{x}_{jt+1})} > 0 \end{aligned}$$

The first part of Proposition 8 shows that residual stock on property i is increasing in both off-property spread parameters (D_{ji} and D_{ij}). The intuition is that an increase in D_{ji} is *as if* owner j now engages in less control, since more resource moves toward property i . This consequently entices owner i to raise the stock on her own property. The effect of higher D_{ij} on optimal stock in i is more surprising since it describes a higher movement from i to j which may suggest a decrease in the residual stock of property i . But the strategic interactions between owners outweighs this effect, following Proposition 2, so an increase in D_{ij} causes owner i to raise the stock on her own property.

Analyzing the effects of self-retention (D_{ii} and D_{jj}) also yield insights, though this becomes more complicated. Here, whether \hat{e}_{it} will increase or decrease in response to a rise in D_{ii} (or D_{jj}) will depend on the nature of control costs. If the marginal cost of control is relatively flat (so $c'(\cdot) \approx 0$), then owner i will engage in more control if D_{ii} is larger. This makes intuitive sense: If a pest population is more likely to persist on one's property, then it seems intuitive that the owner would engage in more control compared to a case in which it is likely to quickly move off of one's property. As a consequence of strategic dynamic interactions, a similar result emerges regarding owner i 's response to an increase in D_{jj} . But these results can be flipped if marginal cost is sufficiently steep. The proof of Proposition 8 (in the Appendix) spells this out in detail and also suggests that they are likely to hold under conditions even more general than are considered here.

If it is socially optimal to eradicate and eradication emerges under *laissez-faire*, then there is no tragedy of the commons. However, whether another form of the tragedy of the commons may exist, where private owners under *laissez-faire* coordinate on the “wrong” strategy (the suboptimal one), is an open question. We now turn to the issue of comparing the type of optimal strategies induced by decentralized and socially optimal management. We investigate the conditions under which eradication can emerge in both the decentralized and socially optimal settings, and how these conditions depend on spatial connectivity. A useful first result is summarized below:

Lemma 1. *If complete eradication emerges as a decentralized solution, then complete eradication is socially optimal.*

Lemma 1 shows that it is possible that there is consistency between the control by decentralized private owners and the optimal control by a social planner. If decentralized property owners all find it privately optimal to eradicate the public bad (e.g. because the damages they faced were sufficiently large to justify the cost of eradication), then complete spatial eradication is also socially optimal. It turns out that this will always be the case for sufficiently large values of $D_{ii}g'(0)$ (when $1 < \delta D_{ii}g'(0)$), summarized by the following corollary:

Corollary 1. *If self retention is sufficiently high on all properties, that is:*

$$\min_{i \in I} D_{ii} \geq \frac{1}{\delta g'(0)}, \quad (14)$$

then complete eradication is socially optimal and will emerge from non-cooperative behavior.

Corollary 1 implies that if all self-retention parameters are sufficiently large, then there will be no tension between socially optimal and private incentives. Conversely, it may often be the case that complete eradication is socially optimal, but does not arise from decentralized owners' decisions. This could be the case either if all owners privately choose partial control strategies, or if eradication arises on some properties, but not on others. For example, if property i has high marginal damage from an invasive weed and property j has low marginal damage, then this analysis suggests that owner i may find it privately optimal to exterminate the weed on her property while owner j does not. But even in that case (when one decentralized owner eradicates and another does not), it may be *socially optimal* to fully eradicate on all properties (for example if i is downwind of j). This result could arise because failing to eradicate on property j eventually causes damage on property i , which diminishes social welfare. These results are summarized in the following proposition:

Proposition 9. *Assume that complete eradication is socially optimal (Condition 11 is satisfied), then it will never arise from decentralized management if:*

$$\max_{i \in I} [k_i(0) - c_i(0) (1 - \delta D_{ii} g'(0))] \leq 0. \quad (15)$$

Proposition 9 characterizes situations under which a tension arises between socially optimal management and private incentives: while it is socially optimal to eradicate in all regions, private owners may choose a different policy. This result will enable us to delve into the cases for which eradication is biologically and/or economically feasible; we focus on the role of spatial parameters, summarized as follows:

Proposition 10. *Suppose complete eradication is socially optimal, and denote by i a property with sufficiently low self-retention ($D_{ii} < \frac{1}{\delta g'(0)}$). Then the effect of spatial parameters on the emergence of eradication is summarized as follows:*

- (a) *An increase in dispersal D_{ij} (where $i \neq j$) makes the emergence of complete eradication less likely under decentralized management.*
- (b) *Provided that self retention remains lower than $\frac{1}{\delta g'(0)}$, an increase in self retention makes the emergence of complete eradication less likely under decentralized management.*

The effect of off-property spread is straightforward. An increase in out-dispersal increases the incentive for the social planner to eradicate, but does not alter the incentives of decentralized property owners. Thus, this enlarges the set of cases where a tension arises between the two types of management. But the effect of self retention is harder to intuit. Corollary 1 and Proposition 9 imply that when all properties except one (say i) are characterized by sufficiently high values of self retention, then an increase in the value of self retention on property i may have two opposing effects. First, if the increase is such that self retention is now higher than the threshold value, then Corollary 1 implies that it has a positive effect as it removes the potential tension between socially optimal and private incentives. However, if the initial value is so low that the increase is not sufficient to move it over the threshold, then the effect is negative as it enlarges the set of cases where complete eradication fails to emerge under decentralized management. These results can be used to assess whether we might expect strong consistency between socially and decentralized management or tensions arising due to strategic behavior among property owners.

6 Cooperation with side-payments

So far, we have analyzed the decentralized decisions of property owners who are harmed by a public bad that moves across space, and we have contrasted that case with the optimal solution of a social planner who can perfectly anticipate the spatial migration of the public bad, and who can perfectly target control efforts across space. An obvious result is that there are cases in which a tension arises between these two types of management. This naturally begs the question of what institutions can help transition from the decentralized solution to the socially optimal solution.⁹ The case where partial eradication arises from decentralized behavior, but where complete eradication is socially optimal, provides a convenient case to assess the potential of monetary transfers to achieve the socially-optimal outcome. Indeed, in that case, we need only provide incentives to a restricted set of owners. The focus of this section is to assess the impact of spatial characteristics on the size of the benefit from cooperation.

To sharpen the findings, we focus on the case of two properties (i and j). The main point is to assess when a monetary transfer from owner i to owner j is Pareto-Improving, that is, it makes both owners better off compared to the (no-transfer) decentralized management outcome. While we have not explicitly modeled transaction costs, it seems reasonable to assume that the larger is the potential benefit from cooperation, the more likely it is that transaction costs can

⁹One mechanism that holds promise for public *goods* (but to our knowledge has never been analyzed for public *bads*) is “unitization” (Kaffine and Costello 2011).

be overcome (Demsetz 1967). Thus, we would like to explore the conditions under which we might expect a large, or small, benefit from complete cooperation over the control of this spatially-mobile public bad.

We first re-write Conditions 7 and 8 for partial eradication to occur in the case of two regions. The stock will be partially eradicated on property i if and only if:

$$k_i(0) - c_i(0) + \delta g'(0) D_{ii} c_i(D_{ji} g(\hat{e}_{jt})) > 0 \quad (16)$$

where $\hat{e}_{jt} > 0$ is characterized implicitly by the following equation:

$$k_j(\hat{e}_{jt}) - c_j(\hat{e}_{jt}) + \delta D_{jj} g'(\hat{e}_{jt}) c_j(D_{jj} g(\hat{e}_{jt})) = 0. \quad (17)$$

Condition 17 means that owner j chooses to control, leaving a residual stock $\hat{e}_{jt} > 0$. Condition 16 means that owner i has incentives to eradicate the public bad each time period.¹⁰ Since $c'_i(\cdot) < 0$ and $D_{ji} > 0$, the incentives of owner i are weaker than under complete eradication defined by Condition 6.

Since owner i eradicates every time period, while owner j only partially controls the public bad on her property, the corresponding payoffs $\hat{\Pi}_i$ and $\hat{\Pi}_j$ for owners i and j are defined as follows:

$$\begin{aligned} \hat{\Pi}_i &= - \left[\int_0^0 k_i(s) ds + \int_0^{x_{i0}} c_i(s) ds + \frac{\delta}{1-\delta} \left(\int_0^0 k_i(s) ds + \int_0^{D_{ji} g(\hat{e}_j)} c_i(s) ds \right) \right] \quad (18) \\ \hat{\Pi}_j &= - \left[\int_0^{\hat{e}_j} k_j(s) ds + \int_{\hat{e}_j}^{x_{j0}} c_j(s) ds + \frac{\delta}{1-\delta} \left(\int_0^{\hat{e}_j} k_j(s) ds + \int_{\hat{e}_j}^{D_{jj} g(\hat{e}_j)} c_j(s) ds \right) \right] \quad (19) \end{aligned}$$

Conversely, as the socially optimal outcome is characterized by complete eradication, the corresponding payoffs of each owner are defined as follows:

$$\bar{\Pi}_i = - \int_0^{x_{i0}} c_i(s) ds \quad (20)$$

$$\bar{\Pi}_j = - \int_0^{x_{j0}} c_j(s) ds \quad (21)$$

Now, consider the possibility of owner i making a payment to owner j to reduce the residual stock in her property. The transfer will be feasible if there exist positive gains from cooperation, that is, if the sum of payoffs (20) and (21) resulting from cooperation exceeds the sum of payoffs (18) and (19) resulting from decentralized management. This is indeed the case, as highlighted by the following proposition:

¹⁰The re-writing follows from Proposition 4. Due to the expressions 16 and 17 and the fact that economic costs, growth and dispersal are time independent, the optimal choice of owner j is time and state independent (see Proposition 1). In other words \hat{e}_{jt} can simply be written \hat{e}_j .

Proposition 11. *Assume that there are two properties (i and j), that complete eradication is socially optimal (that is, the cooperative outcome), and that Conditions 16 and 17 are satisfied. Then, there are positive gains from cooperation.*

Proposition 11 shows that, under certain conditions with decentralized management, the owner of property i has incentives to compensate the owner of property j to reduce the residual stock on her property. The actual amount to be transferred depends on the spatial characteristics. The next result examines how the surplus from cooperation depends on these features.

Proposition 12. *Assume that there are two properties (i and j), that complete eradication is socially optimal (that is, the cooperative outcome), and that Conditions 16 and 17 are satisfied. Then, the following conclusions hold:*

(a) *The gains from cooperation are increasing in dispersal from property j to property i (D_{ji}).*

(b) *The gains from cooperation are increasing in self-retention (D_{jj}) provided marginal cost is elastic.*

Proposition 12 seems to suggest that compensation should be oriented toward the weakest-link, that is, the property j for which:

$$k_j(0) - (1 - \delta D_{jj} g'(0)) c_j(0) = \min_{l \in I} k_l(0) - (1 - \delta D_{ll} g'(0)) c_l(0).$$

Indeed, using Equations 16 and 17, it follows that:

$$k_i(0) - c_i(0) + \delta g'(0) D_{ii} c_i(0) \geq k_i(0) - c_i(0) + \delta g'(0) D_{ji} c_i(D_{ji} g(\hat{e}_j)) > 0$$

and

$$k_j(0) - [1 - \delta D_{jj} g'(0)] c_j(0) < k_j(\hat{e}_j) - c_j(\hat{e}_j) + \delta D_{jj} g'(\hat{e}_j) c_j(D_{jj} g(\hat{e}_j)) = 0.$$

An appropriate transfer payment might be used to induce owner j to engage in additional control, thus lowering her residual stock level, because it would benefit the adjacent owners. Moreover, the larger is dispersal from property j to others, the larger is the surplus resulting from potential cooperation, thus potentially increasing the willingness of others to compensate the weakest-link.

7 Conclusion

We have developed and analyzed a model of a renewable public bad resource, such as an invasive species, antibacterial resistance, or aquaculture disease, that can move across space. Decentralized property owners undertake costly control to

reduce damage on their own properties, and because the resource is mobile, this control has consequences for all other property owners. The resulting externality induces a spatial-temporal game between the property owners who will each act strategically given the behavior of other owners. Our first contribution is to completely characterize the equilibrium strategy of each owner and the resulting effects on stock and control of the public bad across space. We also solve for the socially optimal level of control across space and show that it always (weakly) exceeds the level of control undertaken by decentralized owners.

A key focus of our analysis is on the conditions under which eradication is undertaken by decentralized owners and/or is desired by the social planner. We find that there is often consistency between these - realistic cases exist in which all decentralized owners will eradicate the stock on their properties; in these cases the social planner would choose the same level of control, so no policy intervention is warranted. But cases also exist in which one or more decentralized owner fails to completely eradicate (even though it is socially desirable). In such cases, side-payments can induce appropriate control, and we have characterized the features of the problem that lead to large or small potential gains from this kind of side payment.

Our results also imply an interesting result about the role of initial conditions on decentralized and optimal control of a spatially-connected public bad. If the initial invasion is sufficiently large on all properties, then all property owners will control to their optimal levels immediately and *the resulting level of residual stock will be independent of the initial invasion size*. But if the initial invasion is large on some properties and small on others, we can obtain a striking result. Consider the two-property case and assume the initial invasion occurs only on property A (not on property B). In that case, owner A will control more than she would have had the invasion also extended to property B . Thus, reasonable conditions exist under which *it is optimal to aggressively control (or eradicate) a “small” initial invasion, even though it would be optimal to only weakly control a “large” invasion*. The same result can be obtained in under *laissez-faire*.

To obtain sharp analytical results of this spatial-temporal game has required making some simplifying assumptions. We modeled marginal damage on property i as a function of resource stock on property i , which depends on the previous period’s stock in all properties and the spread from those properties to property i . A more complicated version of damage would allow for damage in period t to depend on how much damage had been caused in previous periods; though we do not analyze that case here. We modeled the marginal control cost as a decreasing function of the stock of the bad; the higher is the local stock of the bad, the smaller is the marginal cost of abatement. While this follows the literature and seems to fit most applications, an extension could allow for marginal cost to

also depend explicitly (not just implicitly) on the quantity removed. Regarding the spread of the stock, we have assumed that the fraction of the stock that spreads from property i to property j is constant. An extension could allow for the spread to depend on the density of the stock in both areas. While these changes would complicate the solution to our model, we think they are unlikely to overturn the main findings of this paper. But these are fertile opportunities for empirical applications of this work.

Our approach fundamentally assumes that the resource is a public *bad* for all property owners. An interesting extension would allow the resource to be a public *good* for a subset of owners. For example, wolves may be a “bad” for ranchers and a “good” for conservationists. This type of public good has been analyzed by Weitzman (2015). This will enable us to consider conflicts of interest between those who want to conserve the resource, and those who will impede the provision of the public good. Moreover, at a first glance, we expect that the structure of strategic interaction of the group benefiting from the public good might be different, such that the control strategy (Proposition 2) might become a strategic substitute, thus inducing interesting, and yet unexplored, dynamics.

Overall, our results suggest an interesting general result about the gain from coordination among decentralized property owners. If the marginal dynamic cost of the public bad is small, then decentralized owners choose a level of control that is lower than, but approximately equal to, the control that would be chosen by a social planner. In those cases, the gain from coordination of decentralized owners is likely to be small. If the marginal dynamic cost is moderate, an interior solution is likely to obtain under which some control will be undertaken by the decentralized property owners, but that this control will fall well short of what would be chosen by the social planner. In these cases, the gains from coordination are large. But when the marginal dynamic cost is sufficiently large, decentralized owners will choose to eradicate on their own property. We proved that in those cases, complete eradication is also socially optimal. In such cases, there is no gain from coordination. Taken together, these results suggest that the gain from coordination among decentralized owners is largest for an intermediate public bads, which may run counter to intuition and may be suggestive of cases when government intervention or coordination schemes are most economically relevant.

References

- Adda, J. (2015). Economic activity and the spread of viral diseases: Evidence from high frequency data. *Quarterly Journal of Economics* forthcoming.
- Albers, H. J., C. Fischer, and J. N. Sanchirico (2010). Invasive species management in a spatially heterogeneous world: Effects of uniform policies. *Resource*

- and *Energy Economics* 32, 483–499.
- Anderson, R. M. and R. M. May (1991). *Infectious diseases of humans: dynamics and control*. Oxford University Press.
- Archer, D. and J. Shogren (1996). Endogenous risk in weed control management. *Agricultural Economics* 14, 103–122.
- Arino, J. and P. V. den Driessche (2006). Disease spread in metapopulations. *Nonlinear dynamics and evolution equations* 48, 1–13.
- Barrett, S. (2003). Global disease eradication. *Journal of the European Economic Association* 1(2-3), 591–600.
- Blackwood, J., A. Hastings, and C. Costello (2010). Cost-effective management of invasive species using linear-quadratic control. *Ecological Economics* 69(3), 519–527.
- Brandt, H., C. Hauert, and K. Sigmund (2003). Punishment and reputation in spatial public goods games. *Proceedings of the Royal Society of London B: Biological Sciences* 270(1519), 1099–1104.
- Brito, D. L., E. Sheshinski, and M. Intriligator (1991). Externalities and compulsory vaccinations. *Journal of Public Economics* 45(1), 69–90.
- Burnett, K. (2006). Introductions of invasive species: Failure of the weaker link. *Agricultural and Resource Economics Review* 35, 21–28.
- Burnett, K., S. D’Evelly, B. Kaiser, P. Nantamanasikarn, and J. Roumasset (2008). Beyond the lamppost: optimal prevention and control of the brown tree snake in hawaii. *Ecological Economics* 67(1), 66–74.
- Burnett, K., B. Kaiser, B. Pitafi, and J. Roumasset (2006). Prevention, eradication, and containment of invasive species: Illustration from hawaii. *Agricultural and Resource Economics Review* 35(1), 63–77.
- Cornes, R. (1993). Dyke maintenance and other stories: Some neglected types of public goods. *Quarterly Journal of Economics* 108(1), 259–271.
- Demsetz, H. (1967). Toward a theory of property rights. *The American economic review*, 347–359.
- Epanchin-Neill, B. and J. Wilen (2015). Individual and cooperative management of invasive species in human-mediated landscapes. *American Journal of Agricultural Economics* 97(1), 180–198.
- Epanchin-Niell, R. S. and J. E. Wilen (2012). Optimal spatial control of biological invasions. *Journal of Environmental Economics and Management* 63(2), 260–270.

- Fenichel, E. P., T. J. Richards, , and D. W. Shanafelt (2014). The control of invasive species on private property with neighbor-to-neighbor spillovers. *Environmental and Resource Economics*, 1–25.
- Finnoff, D., A. Potapov, and M. Lewis (2010). Control and the management of a spreading invader. *Resource and Energy Economics* 32, 534–550.
- Geoffard, P.-Y. and T. Philipson (1997). Disease eradication: Private versus public vaccination. *The American Economic Review* 87(1), 222–230.
- Gersovitz, M. and J. Hammer (2004). The economical control of infectious diseases. *The Economic Journal* 114, 1–27.
- Gersovitz, M. and J. Hammer (2005). Tax/subsidy policies toward vector-borne infectious diseases. *Journal of Public Economics* 89(4), 647–674.
- Grimsrud, K., J. Chermak, J. Hansen, J. Thacher, and K. Krause (2008). A two-agent dynamic model with an invasive weed diffusion externality: an application to yellow starthistle (*Centaurea solstitialis* L.) in new mexico. *Journal of Environmental Management* 89, 322–335.
- Jackson, M. O. and Y. Zenou (2014). Games on networks. *Handbook of game theory* 4.
- Kaffine, D. T. and C. Costello (2011). Unitization of spatially connected renewable resources. *The BE Journal of Economic Analysis & Policy* 11(1).
- Kaiser, B. and K. Burnett (2010). Spatial economic analysis of early detection and rapid response strategies for an invasive species. *Resource and Energy Economics* 32, 566–585.
- Leung, B., D. M. Lodge, D. Finnoff, J. F. Shogren, M. A. Lewis, and G. Lamberti (2002). An ounce of prevention or a pound of cure: bioeconomic risk analysis of invasive species. *Proceedings of the Royal Society of London. Series B: Biological Sciences* 269(1508), 2407–2413.
- Lichtenberg, E. and D. Zilberman (1986). The econometrics of damage control: why specification matters. *American Journal of Agricultural Economics* 68, 261–273.
- Nathan, R., G. G. Katul, H. S. Horn, S. M. Thomas, R. Oren, R. Avissar, S. W. Pacala, and S. A. Levin (2002). Mechanisms of long-distance dispersal of seeds by wind. *Nature* 418(6896), 409–413.
- Olson, L. and D. Roy (2002). The economics of controlling a stochastic biological invasion. *American Journal of Agricultural Economics* 84, 1311–1316.
- Palfrey, R. and J. Prisbrey (1996). Altruism, reputation and noise in linear public goods experiments. *Journal of Public Economics* 61, 409–427.

- Regan, T., M. McCarthy, P. Baxter, F. Panetta, and H. Possingham (2006). Optimal eradication: when to stop looking for an invasive plant. *Ecological Letters* 9, 759–766.
- Rowthorn, R. E., R. Laxminarayan, and C. Gilligan (2009). Optimal control of epidemics in metapopulations. *Journal of the Royal Society Interface* 6(41), 1135–1144.
- Shapiro, C. (1982). Consumer information, product quality, and seller reputation. *The Bell Journal of Economics* 13, 20–35.
- Shogren, J. (2000). Risk reductions strategies against the "explosive invader". In W. M. D. S. Perrings, C. (Ed.), *The Economics of Biological Invasions*, pp. 56–69. Edward Elgar, Northampton, MA.
- Siegel, D., B. Kinlan, B. Gaylord, and S. Gaines (2003). Lagrangian descriptions of marine larval dispersion. *Marine Ecology Progress Series* 260, 83–96.
- Simberloff, D. (2009). We can eliminate invasions or live with them. successful management projects. *Biological Invasions* 11, 149–157.
- Weitzman, M. (2015). A voting architecture for the governance of free-driver externalities, with application to geoengineering. *Scandinavian Journal of Economics* (Forthcoming).
- Wiemer, C. (1987). Optimal disease control through the combined use of preventive and curative measures. *Journal of Development Economics* 25(2), 301–319.
- Winfree, J. and J. McCluskey (2005). Collective reputation and quality. *American Journal of Agricultural Economics* 87, 206–213.

8 Appendix

Proof of Proposition 1

The first result follows from the first order conditions. To prove state independence, use Equation 4. Then the necessary conditions for an interior solution to agent i 's problem yields Expression 5. The necessary condition is also sufficient given the assumption of convexity of returns in the strategy (residual stock level). The term on the left hand side in Expression 5 does not depend on the vector of stock levels x_t by inspection. The right hand side in Expression 5 depends on the period $t + 1$ state, but is independent of the period t state. For an interior solution we have $e_{it} < x_{it}$ and, using the equation of motion 1, we can conclude that x_{it+1} depends on e_{it} but not on x_{it} . Thus, the right hand side in Expression 5 does not depend on the vector of stock level x_t . This implies that the period t game problem has state independent strategies. Moreover, since economic costs, growth and dispersal do not vary with time, e_{it} will not depend on time (but does depend on spatial fundamentals).

Proof of Proposition 2

Denote $\Psi_i \equiv k_i(e_{it}) - c_i(e_{it}) + \delta c_i(x_{it+1})D_{ii}g'(e_{it}) = 0$ the first order condition (FOC) which defines the best response of the owner of property i , i.e. the set of best responses for different residual stock levels of the other owners in the other properties. Let $e_{it}(e_{jt}, \dots, e_{lt})$ denote owner i 's reaction function. According to the FOC, we know that $\Psi_i[e_{it}(e_{jt}, \dots, e_{lt}), e_{jt}, \dots, e_{lt}] \equiv 0$.

Total differentiating this expression (omitting the time argument), we get:

$$\frac{\partial \Psi_i}{\partial e_i} \frac{\partial e_i}{\partial e_j} + \frac{\partial \Psi_i}{\partial e_j} = 0 \Leftrightarrow \frac{\partial e_i}{\partial e_j} = -\frac{\frac{\partial \Psi_i}{\partial e_j}}{\frac{\partial \Psi_i}{\partial e_i}}$$

with
$$\frac{\partial \Psi_i}{\partial e_i} = k'_i(\hat{e}_{it}) - c'_i(\hat{e}_{it}) + \delta D_{ii} \left[c_i(\hat{x}_{it+1})g''(\hat{e}_{it}) + c'_i(\hat{x}_{it+1})D_{ii}(g'(\hat{e}_{it}))^2 \right] = SOC_i > 0$$

$$\frac{\partial \Psi_i}{\partial e_j} = \delta D_{ii}D_{ji}c'(x_{it+1})g'(e_{it})g'(e_{jt}) < 0 \quad \text{since } c'(\cdot) < 0 \text{ and } g'(\cdot) > 0$$

So $\frac{\partial e_{it}}{\partial e_{jt}} > 0$.

Proof of Proposition 3

The result follows from the first order conditions ensuring a corner solution $\hat{e}_i = 0$ for any property i .

Proof of Proposition 4

Since Condition 6 is not satisfied, we can deduce from the convexity of the payoff functions that there exists at least one property for which the owner has incentives to increase the residual stock level compared to full harvest (provided that other owners harvest the entire stock in their properties). This rules out complete eradication as a non cooperative equilibrium outcome. Now, if the owner of property $j \notin E$ decides at period t to increase the residual stock level (again assuming that the residual stock level is zero in any property $i \in E$, and positive in any other property $l \notin E$) her optimal choice is given by \hat{e}_{jt} (as characterized in Expression 8). Then Condition 7 implies that the owner of property $i \in E$ will find it optimal to maintain the

residual stock level in her property at zero (assuming that the owner of property $j \notin E$ chooses \hat{e}_{jt}). Thus, at the Nash equilibrium, the residual stock level will be zero on property $i \in E$ and positive on property $j \notin E$.

Proof of Proposition 5

The characterization follows from the first order conditions ensuring an interior policy in any property i .

Proof of Proposition 6

The characterization of full eradication (Condition 11) follows from the first order conditions ensuring a corner policy $\bar{e}_i = 0$ in any property i .

The proof of the characterization of partial eradication as a socially optimal outcome is similar than that of Proposition 4.

Proof of Proposition 7

If $D_{ii} = 1$, then Equations 5 and 10 are identical. If $D_{ii} = 0$, then Equation 5 becomes $c_i(\hat{e}_{it}) - k_i(\hat{e}_{it}) = 0$, while Equation 10 becomes $c_i(\bar{e}_{it}) - k_i(\bar{e}_{it}) = \delta \sum_{j \neq i} D_{ij} c_j(\bar{x}_{jt+1}) D_{ij} g'(\bar{e}_{it}) > 0$. We observe that the LHS of these two equalities are similar. Since $c'_i(e_{it}) - k'_i(e_{it}) < 0$, for these two equalities to hold, we must have $\bar{e}_{it} < \hat{e}_{it}$.

We now examine the case where $D_{ii} \in (0, 1)$ by comparing Equations 5 and 10. Rewriting gives:

$$k_i(\hat{e}_{it}) = c_i(\hat{e}_{it}) - \delta c_i(\hat{x}_{it+1}) D_{ii} g'(\hat{e}_{it}) \quad (22)$$

$$k_i(\bar{e}_{it}) = c_i(\bar{e}_{it}) - \delta c_i(\bar{x}_{it+1}) D_{ii} g'(\bar{e}_{it}) - \underbrace{\delta \sum_{j \neq i} c_j(\bar{x}_{jt+1}) D_{ij} g'(\bar{e}_{it})}_{\mathcal{L}}. \quad (23)$$

The first two terms on the right hand side of Equations 22 and 23 are identical. Since $k_i(\cdot)$ is increasing in e , and because the underbraced term $\mathcal{L} > 0$, it is clear that $\hat{e}_{it} \geq \bar{e}_{it}$.

Proof of Proposition 8

In a case with two properties i and j , assuming interior equilibria, we have:

$$k_i(e_i) = c_i(e_i) - \delta c_i(x_i) D_{ii} g'(e_i) \quad (24)$$

$$k_j(e_j) = c_j(e_j) - \delta c_j(x_j) D_{jj} g'(e_j) \quad (25)$$

We omit subscript t in Expressions 24 and 25. They imply that e_i and e_j are the solution to the above system (since $x_k = \sum_l D_{lk} g(e_l)$ for $k = i, j$), which in turn implies that e_i and e_j are both functions of $\theta = \{D_{ii}, D_{jj}, D_{ij}, D_{ji}\}$. These two first order conditions can thus be written as a function of the parameter, θ , as $\Psi_i(e_i(\theta), e_j(\theta), \theta) \equiv 0$ and $\Psi_j(e_i(\theta), e_j(\theta), \theta) \equiv 0$. We can thus totally differentiate both conditions:

$$\begin{cases} \frac{\partial \Psi_i}{\partial e_i} \frac{\partial e_i}{\partial \theta} + \frac{\partial \Psi_i}{\partial e_j} \frac{\partial e_j}{\partial \theta} + \frac{\partial \Psi_i}{\partial \theta} = 0 \\ \frac{\partial \Psi_j}{\partial e_i} \frac{\partial e_i}{\partial \theta} + \frac{\partial \Psi_j}{\partial e_j} \frac{\partial e_j}{\partial \theta} + \frac{\partial \Psi_j}{\partial \theta} = 0 \end{cases} \quad (26)$$

Solving this system, we get that $\frac{\partial e_i}{\partial \theta}$ (and symmetrically $\frac{\partial e_j}{\partial \theta}$):

$$\frac{\partial e_i}{\partial \theta} = \frac{-\frac{\partial \Psi_i}{\partial \theta} \frac{\partial \Psi_j}{\partial e_j} + \frac{\partial \Psi_j}{\partial \theta} \frac{\partial \Psi_i}{\partial e_i}}{\frac{\partial \Psi_i}{\partial e_i} \frac{\partial \Psi_j}{\partial e_j} - \frac{\partial \Psi_i}{\partial e_j} \frac{\partial \Psi_j}{\partial e_i}} \quad (27)$$

$$\text{with } \begin{aligned} \frac{\partial \Psi_i}{\partial e_i} &= SOC_i > 0, & \frac{\partial \Psi_i}{\partial e_j} &= \delta D_{ii} D_{ji} g'(e_i) g'(e_j) c'_i(x_i) < 0 \\ \frac{\partial \Psi_j}{\partial e_j} &= SOC_j > 0, & \frac{\partial \Psi_j}{\partial e_i} &= \delta D_{jj} D_{ij} g'(e_i) g'(e_j) c'_j(x_j) < 0 \end{aligned}$$

And the following derivatives:

Table 1: Partial derivatives

θ	$\frac{\partial \Psi_i}{\partial \theta}$	$\frac{\partial \Psi_j}{\partial \theta}$
D_{ii}	$\delta g'(e_i) [c_i(x_i) + D_{ii} c'_i(x_i) g(e_i)]$	0
D_{ji}	$\delta D_{ii} g'(e_i) c'_i(x_i) g(e_j)$	0
D_{jj}	0	$\delta g'(e_j) [c_j(x_j) + D_{jj} c'_j(x_j) g(e_j)]$
D_{ij}	0	$\delta D_{jj} g'(e_j) c'_j(x_j) g(e_i)$

Observe that for any θ the denominator of Equation 27 is

$$SOC_i SOC_j - \delta^2 D_{ii} D_{jj} D_{ij} D_{ji} (g'(e_i))^2 (g'(e_j))^2 c'_i(x_i) c'_j(x_j).$$

If either D_{ii} or D_{jj} is sufficiently large, the denominator is positive. Notice that this condition is sufficient, but not necessary. For instance, if either D_{ij} or D_{ji} is sufficiently large while optimal residual stock levels remain positive, then the denominator is positive too. The same conclusion follows if the discount factor is small enough or, obviously, if either cost function is linear (that is, if either $c'_i(\cdot) = 0$ or $c'_j(\cdot) = 0$).

Using derivatives in Table 1, we deduce the following derivatives:

$$\frac{\partial e_i}{\partial D_{ii}} = \frac{\delta g'(e_i) [c_i(x_i) + D_{ii} c'_i(x_i) g(e_i)] SOC_j}{SOC_i SOC_j - \delta^2 D_{ii} D_{jj} D_{ij} D_{ji} (g'(\hat{e}_i))^2 (g'(\hat{e}_j))^2 c'_i(\hat{x}_i) c'_j(\hat{x}_j)} \quad (28)$$

$$\frac{\partial e_i}{\partial D_{ji}} = -\frac{\delta D_{ii} g'(e_i) g(e_j) c'_i(x_i) SOC_j}{SOC_i SOC_j - \delta^2 D_{ii} D_{jj} D_{ij} D_{ji} (g'(e_i))^2 (g'(e_j))^2 c'_i(x_i) c'_j(x_j)} \quad (29)$$

$$\frac{\partial e_i}{\partial D_{jj}} = \frac{\delta^2 D_{ii} D_{ji} g'(e_i) [g'(e_j)]^2 c'_i(x_i) [c_j(x_j) + D_{jj} c'_j(x_j) g(e_j)]}{SOC_i SOC_j - \delta^2 D_{ii} D_{jj} D_{ij} D_{ji} (g'(\hat{e}_i))^2 (g'(\hat{e}_j))^2 c'_i(\hat{x}_i) c'_j(\hat{x}_j)} \quad (30)$$

$$\frac{\partial e_i}{\partial D_{ij}} = \frac{\delta^2 D_{ii} D_{jj} D_{ji} g'(e_i) [g'(e_j)]^2 c'_i(x_i) c'_j(x_j) g(e_i)}{SOC_i SOC_j - \delta^2 D_{ii} D_{jj} D_{ij} D_{ji} (g'(\hat{e}_i))^2 (g'(\hat{e}_j))^2 c'_i(\hat{x}_i) c'_j(\hat{x}_j)} \quad (31)$$

Now in cases where the denominator is positive, we conclude that $\frac{\partial e_i}{\partial D_{ji}} > 0$ and $\frac{\partial e_i}{\partial D_{ij}} > 0$. When the marginal cost is quite inelastic, i.e. $1 > -D_{ii}g'(e_i)\frac{c'(x_i)}{c(x_i)} > 0$, then $\frac{\partial e_i}{\partial D_{ii}} < 0$. The sign of $\frac{\partial e_i}{\partial D_{jj}}$ depends also on the marginal cost elasticity. If the marginal cost is quite inelastic, i.e. $1 > -D_{jj}g'(e_j)\frac{c'(x_j)}{c(x_j)} > 0$, then $\frac{\partial e_i}{\partial D_{jj}} < 0$.

Proof of Lemma 1

If complete eradication is a Nash equilibrium outcome, then for any property i , due to Proposition 3 and Condition 11 in Proposition 6 we have $k_i(0) - [1 - \delta D_{ii}g'(0)]c_i(0) > 0$. Since $g'(0)$ is positive and all dispersal parameters are non negative, this implies that the following condition holds:

$$k_i(0) - [1 - \delta D_{ii}g'(0)]c_i(0) + \delta g'(0) \sum_{j \neq i} D_{ij}c_j(0) \geq k_i(0) - (1 - \delta D_{ii}g'(0))c_i(0) > 0.$$

Using Propositions 3 and 6 implies that complete eradication is socially optimal.

Proof of Corollary 1

The result follows immediately from Condition 6 in Proposition 3 and Condition 11 in Proposition 6.

Proof of Proposition 9

The result follows immediately from Condition 6 in Proposition 3 and Condition 11 in Proposition 6.

Proof of Proposition 10

From proposition 9 we deduce that the length of the interval characterizing values of marginal abatement costs (on property i) over which tensions arise between socially optimal and decentralized management is given by

$$\Delta_i = \frac{k_i(0) + \delta g'(0) \sum_{j \neq i} D_{ij}c_j(0)}{1 - \delta D_{ii}g'(0)} - \frac{k_i(0)}{1 - \delta D_{ii}g'(0)} = \frac{\delta g'(0) \sum_{j \neq i} D_{ij}c_j(0)}{1 - \delta D_{ii}g'(0)}.$$

This implies the following conclusions:

1. We have $\frac{\partial \Delta_i}{\partial D_{ij}} = \frac{\delta g'(0)c_j(0)}{1 - \delta D_{ii}g'(0)} > 0$ since $1 - \delta D_{ii}g'(0) > 0$ and provided $c_j(0) > 0$, which implies that the length of the interval increases as dispersal increases. This concludes the proof of the first claim.
2. Again, differentiating with respect to the self retention rate, we obtain:

$$\frac{\partial \Delta_i}{\partial D_{ii}} = \frac{(\delta g'(0))^2 \sum_{j \neq i} D_{ij}c_j(0)}{1 - \delta D_{ii}g'(0)} > 0$$

since $1 - \delta D_{ii}g'(0) > 0$ and provided $\sum_{j \neq i} D_{ij}c_j(0) > 0$, which implies that the length of the interval increases as self retention increases (provided that self retention remains lower than the threshold value $\frac{1}{\delta g'(0)}$). This concludes the proof of the second claim.

Proof of Proposition 11

We compute the gains from cooperation by the following expression:

$$\begin{aligned} S &= \bar{\Pi}_i + \bar{\Pi}_j - \hat{\Pi}_i - \hat{\Pi}_j \\ &= \frac{\delta}{1-\delta} \left[\int_0^{D_{ji}g(\hat{e}_j)} c_i(s)ds + \int_0^{\hat{e}_j} k_j(s)ds + \int_{\hat{e}_j}^{D_{jj}g(\hat{e}_j)} c_j(s)ds \right] - \int_0^{\hat{e}_j} c_j(s)ds + \int_0^{\hat{e}_j} k_j(s)ds, \end{aligned} \quad (32)$$

which is positive by inspection. Moreover, it is easily checked that:

$$\bar{\Pi}_i - \hat{\Pi}_i = \frac{\delta}{1-\delta} \int_0^{D_{ji}g(\hat{e}_j)} c_i(s)ds > 0 \quad (33)$$

$$\bar{\Pi}_j - \hat{\Pi}_j = - \int_0^{\hat{e}_j} c_j(s)ds + \int_0^{\hat{e}_j} k_j(s)ds + \frac{\delta}{1-\delta} \left[\int_0^{\hat{e}_j} k_j(s)ds + \int_{\hat{e}_j}^{D_{jj}g(\hat{e}_j)} c_j(s)ds \right] \quad (34)$$

Using Expression 34 and the convexity of the payoff function, we conclude that

$$\bar{\Pi}_j - \hat{\Pi}_j < \frac{\hat{e}_j}{1-\delta} [k_j(\hat{e}_j) - c_j(\hat{e}_j) + \delta D_{jj}g'(\hat{e}_j)c_j(D_{jj}g(\hat{e}_j))]. \quad (35)$$

The right hand side of Condition 35 is equal to zero by Condition 17, which enables us to conclude that the difference on the left hand side is negative. Thus, owner i gains from cooperation, while owner j would lose from it. Since gains from cooperation are positive overall, this implies that owner i would be willing to compensate owner j , so that he would lower her residual stock level. This concludes the proof of the first statement of the proposition.

Proof of Proposition 12

We first differentiate the Expression 32 with respect to parameter D_{ji} , and we obtain (keeping in mind that $\frac{\partial \hat{e}_j}{\partial D_{ji}} = 0$):

$$\frac{\partial S}{\partial D_{ji}} = \frac{\delta}{1-\delta} g(\hat{e}_j) c_i(D_{ji}g(\hat{e}_j)) > 0,$$

which concludes the proof of the first statement of the proposition.

Second, differentiating with respect to D_{jj} , we obtain:

$$\frac{\partial S}{\partial D_{jj}} = \frac{\delta}{1-\delta} \left[D_{ji}g'(\hat{e}_j)c_i(D_{ji}g(\hat{e}_j)) \frac{\partial \hat{e}_j}{\partial D_{jj}} + g(\hat{e}_j)c_j(D_{jj}g(\hat{e}_j)) \right], \quad (36)$$

with

$$\frac{\partial \hat{e}_j}{\partial D_{jj}} = \frac{-\delta g'(\hat{e}_j) [g'(\hat{e}_j)D_{jj}c_j'(D_{jj}g(\hat{e}_j)) + c_j(D_{jj}g(\hat{e}_j))]}{SOC_j}.$$

Now, as in Proposition 8, we deduce that $\frac{\partial \hat{e}_j}{\partial D_{jj}} > 0$ if and only if the marginal cost is quite elastic. This concludes the proof.