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Distribution of current density, temperature and mechanical deformation in YBCO bulks under Field-Cooling magnetization

Frederic Trillaud, Kévin Berger, Bruno Douine and Jean Lévêque

Abstract—Most of the latest models of HTS bulk magnetization take into account the thermal and electromagnetic coupling but still lack the implementation of the mechanical deformation under strong magnetic fields. The following work tries to fill up partly the gap by proposing a model of the dependence of the electrical conductivity upon mechanical deformation during Field-Cooling magnetization. This dependence is carried out by the critical current density as a function of induced thermal and mechanical strain in the bulk. As a first approximation and bearing in mind the obvious discrepancies, it is proposed to rely on a simple model inferred from HTS thin films and generic knowledge on ceramics. This first approach allows drawing some generalities on the expected thermal, magnetic and mechanical response of disk-shaped YBCO pellets during magnetization. The utter goal is to determine design parameters to insert those pellets in electrical rotatory machines.

Index Terms—magnetization, mechanical analysis, YBCO bulk modelling.

I. INTRODUCTION

THE magnetization of HTS pellets is a delicate process that involves cooling a single grown bulky ceramic compound from room temperature to cryogenic temperature and sweeping it with an external magnetic field to generate large induced currents. Ceramics being brittle materials, the first procedure is often the most critical one. Indeed, thermal induced mechanical deformations lead to the formation and/or the propagation of new and/or pre-existing cracks that ultimately limit the capability of the bulk to generate the required level of induced current to trap most of the applied external field [1], [2]. Then, during the magnetization, the combination of a large critical current density of the order of the critical current density and a large resulting magnetic flux density in the bulk induces local mechanical stresses which may propagate farther the cracks to degrade further the magnetic trapping [3]. The phenomena involved are fairly non-linear leading to a large increase in complexity of the multiphysics problem. The present work proposes a model simple enough to capture some first insights on the impact of the mechanical deformation on the distribution of current density and subsequently on the trapped magnetic field without having

to detail the generation and the propagation of cracks.

The proposed model describes the coupled electromagnetic, thermal and mechanical behaviour of a homogeneous YBCO pellet. The electromagnetic model which relies on the A-formulation of the Maxwell equations was presented in [4]. This model was modified to include both the dependence of the critical current density upon mechanical strain and temperature, in addition to the original dependence upon magnetic flux density, and the dependence of the n -value upon temperature and magnetic flux density as well. The latter improvement allowed exploring the behavior of the pellet under large magnetic flux densities where the n -value decreases to the point that the assumptions of the critical state model do not hold anymore (typically below $n < 15$). An ideal mechanical model of the bulk was inferred from generic knowledge on the stress-strain relation of ceramics [5], [6] and the strain dependence of superconducting wires and tapes [7], [8]. In this model, the bulk shows a linear elastic behaviour of the stress-strain curve until failure occurs at a characteristic fracture strength of the material. Then, the mechanical strain and the resulting stress are obtained from the Von Mises' theory [9].

II. MODELING OF THE PROBLEM

A. Geometry and assumptions

Fig. 1 shows the schematic dimensional drawing of the YBCO pellet and its surrounding. To simplify the problem, several basic assumptions were considered. The bulk is assumed to be homogeneous having orthotropic material properties. The mechanical behavior of the bulk can be entirely described by a functional dependence of J_c upon the equivalent strain ϵ_{eq} , the local temperature T and the resulting magnetic flux density B_r . Therefore, the model, which includes the bulk, the surrounding vacuum and the background electromagnet, was reduced to only 1/2nd of the axisymmetric initial one. The geometry and its mesh were carried out by the 3D modeller Gmsh [10]. The numerical coupled axisymmetric problem was solved by the finite element solver GetDP [11].

B. Coupled thermal, mechanical and electromagnetic model

Fig. 2 provides the coupling between the different equations involved in the multiphysics problem. A brief summary of the electromagnetic and thermal models and a more detailed description of the mechanical one are provided in the next paragraphs.

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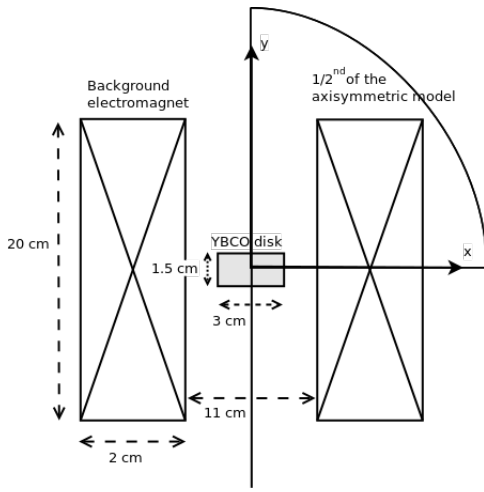


Fig. 1: Illustration of the experimental setup. In the actual experiment, the sample is surrounded by a Cu casing connected to a single-stage cryocooler to provide temperature homogenization and mechanical support.

a) *Electromagnetic model:* A detailed description of the electromagnetic model is given in [4]. This model expresses the Maxwell equations using the magnetic vector potential \mathbf{A} . The initial model was then modified to include the dependence of the electrical conductivity σ of the superconductor on temperature, magnetic flux density and equivalent strain. The general expression of the electrical conductivity is given by

$$\sigma = J_c E_c^{-n} \left(\left\| \frac{\partial \mathbf{A}}{\partial t} \right\| + 10^{-6} E_c \right)^{\frac{1-n}{n}} \quad (1)$$

where the critical electric field, E_c , is chosen equal to $1 \mu\text{V}/\text{cm}$. The critical current density J_c is divided in three terms as follows

$$J_c(T, B, \epsilon_{eq}) = J_{c,0} \times J_{c,TB} \times J_{c,m} \quad (2)$$

where $J_{c,0}(T_{ref}, SF, \epsilon_{eq} = 0)$ (equal to $11 \text{ kA}/\text{cm}^2$) is the self-field, strain-free critical current density at the reference temperature $T_{ref} = 77 \text{ K}$. $J_{c,TB}(T, B)$ is the temperature and magnetic flux density dependent strain-free critical current density given in the shape of a modified Kim-Anderson relation [4], [12], and $J_{c,m}(\epsilon_{eq})$ is the strain-dependent critical current density. Their respective expressions as well as the definition of the n -value are given below,

$$J_{c,TB} = \left(\frac{T - T_c}{T_{ref} - T_c} \right)^{0.96} \times \left(1 + \frac{B}{B_0(T)} \right)^{-0.7} \quad (3)$$

with,

$$B_0 = 1.73 \times \left[1 - \left(\frac{T}{T_{c0}} \right)^{2.17} \right] \quad (4)$$

and

$$J_{c,m} = \left[1 - \chi \left(\frac{\epsilon_{eq}}{\epsilon_c} \right)^2 \right] \times \left[\alpha + \left(\frac{1 - \alpha}{1 + \exp\left(\frac{\epsilon_{eq}}{\beta} - 1\right)} \right) \right] \quad (5)$$

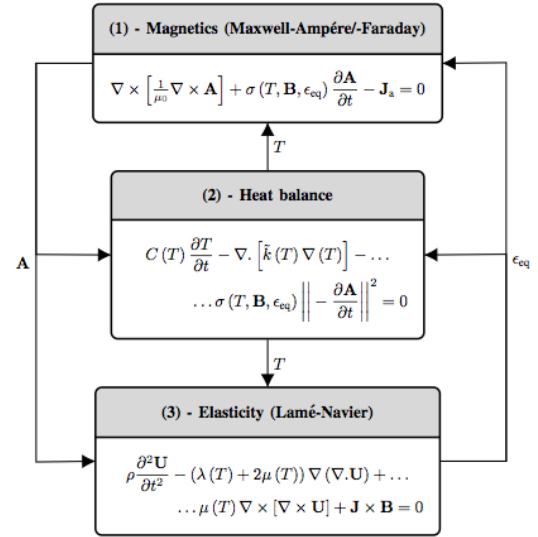


Fig. 2: Layout of the coupling between the electromagnetic (1), thermal (2) and mechanical (3) equations. The coupling is carried by the magnetic vector potential \mathbf{A} , the temperature T and the equivalent strain ϵ_{eq} . The magnetic flux density and the induced current are given by $\mathbf{B} = \nabla \times \mathbf{A}$ and $\mathbf{J} = -\sigma \partial \mathbf{A} / \partial t$, respectively. ρ and J_a are the mass density and the electromagnet current density, respectively.

with $\chi = 0.1$, $\beta = 0.025$ and $\alpha = 10\%$. Fig. 3 shows the resulting dependence of $J_{c,m}$ as a function of the normalized equivalent strain. The critical strain ϵ_c is defined by the fracture strength ϑ_F so that $\epsilon_c = \vartheta_F / E$, where E is the Young's modulus of the bulk. For YBCO bulks at 77 K , the fracture strength is of the order of 90 MPa [13] and the Young's modulus is about 150 GPa [14]. The model tries to catch up the actual behaviour of the bulk by showing a degradation before reaching rupture. To that end, the $J_{c,m}$ starts degrading at about 90% of the fracture strength which can be considered as an optimistic value compared to the reported ultimate tensile strength of the order of 10 to 20 MPa [6]. The n -value does not depend upon the strain. However, a temperature and magnetic flux density dependence is taken into account as proposed in [15],

$$n = \left(n_1 + \frac{n_1 - n_0}{1 + \frac{B}{B_{00}}} \right) \times \left(\frac{2}{1 + \frac{T}{T_{ref}}} \right) \quad (6)$$

with $n_0 = 30$, $n_1 = 5$, $B_{00} = 0.5 \text{ T}$.

b) *Thermal model:* The thermal model which is coupled to the electromagnetic model through the dissipating term $Q = \sigma \left\| \partial \mathbf{A} / \partial t \right\|^2$ includes temperature-dependent material properties. The volumetric heat capacity C and the diagonal tensor of thermal conductivity

$$\tilde{k} = \begin{pmatrix} k_{ab} & 0 \\ 0 & k_c \end{pmatrix} \quad (7)$$

are fitted from averaged results found in [16]–[19]. Two thermal boundaries are considered, thermal conduction through the Cu casing connected to a single-stage cryocooler and adiabatic conditions. The latter was used to reproduce the

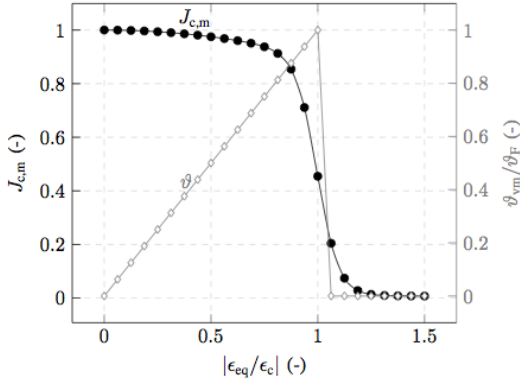


Fig. 3: $J_{c,m}$ as a function of the absolute value of ϵ_{eq} normalized to ϵ_c . The normalized stress-strain corresponding curve is also provided for an ideal case.

fig:data

experimental data whereas the former allowed extrapolating the behaviour of the bulk to greater applied magnetic fields.

c) *Thermoelastic model*: The mechanical model is coupled to the electromagnetic model through the Lorentz force $\mathbf{J} \times \mathbf{B}$ whereas the thermal induced stress is derived from the hooke's law. The space distribution and time evolution of the deformation vector $\mathbf{U}(x, y, t) = \{u_x, u_y, u_{xy}\}$ is related to the stress vector $\boldsymbol{\vartheta} = \{\vartheta_x, \vartheta_y, \vartheta_{xy}\}$ by the following linear equation

$$\boldsymbol{\vartheta} = \tilde{E}(\boldsymbol{\epsilon}_m - \boldsymbol{\epsilon}_{th}) \quad (8)$$

where $\boldsymbol{\epsilon}_m = \mathcal{D}(\mathbf{U})$ is the mechanical strain and $\boldsymbol{\epsilon}_{th} = \{\alpha_{ab}\Delta T, \alpha_c\Delta T, 0\}$ is the thermal induced strain for a local increase in temperature of $\Delta T = T - T_{ref}$ [20]. α_{ab} and α_c are the directional linear thermal expansion coefficients of the bulk in K^{-1} [21]. \mathcal{D} is the symmetric gradient which is expressed as

$$\mathcal{D} = \begin{pmatrix} \frac{\partial}{\partial x} & 0 & \frac{\partial}{\partial x} \\ 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial y} \\ 0 & 0 & 0 \end{pmatrix}^T \quad (9)$$

\tilde{E} is a symmetric tensor lumping the mechanical properties of the sample whose expression is then given by

$$\tilde{E} = \begin{pmatrix} \lambda + 2\mu & \lambda & 0 \\ \lambda & \lambda + 2\mu & 0 \\ 0 & 0 & \mu \end{pmatrix} \quad (10)$$

The Lamé's coefficients λ and μ are given by $\frac{\nu E}{(1-\nu)(1+2\nu)}$ and $\frac{\nu E}{(1+2\nu)}$, respectively [22]. In this representation, μ is equal to the shear modulus G , E is the Young's modulus and ν is the Poisson's ratio equal to 0.255 [23]. The azimuthal component of the strain, u_x/x , is computed at the post-processing stage.

III. RESULTS AND DISCUSSION

The proposed model was successfully cross-checked with available experimental data obtained during Field-Cooling

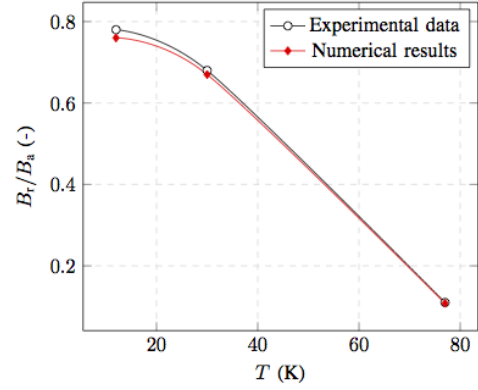


Fig. 4: Comparison of experimental data and results of the numerical model (see Table I). B_r and B_a are the resulting and applied magnetic flux densities, respectively.

TABLE I: Comparison between experimental data (*exp*) and numerical results (*num*) at a ramp rate of 0.003 T/s in conduction cooling.

T_{op} (K)	B_a (T)	B_r^{exp}/B_a (%)	B_r^{num}/B_a (%)	J_{max}^{num}/J_c (%)	ΔT_{max}^{num} (K)	ϑ_{vm}^{av} (MPa)
12	3.4	78	76	96.2	9.3	10
30	3.08	68	67	95.8	2.5	7
77	3.4	10.8	11	94	0	0.2

TABLE II: Numerical results obtained for $T_{op} = 30$ K at a ramp of 0.003 T/s for $\vartheta_F = 90$ MPa in adiabatic conditions.

B_a (T)	B_r/B_a (%)	J_{max}/J_c (%)	ΔT_{max} (K)	$\vartheta_{vm}^{av}/\vartheta_F$ (%)
3.08	67	95.8	2.5	11
10	20	95.6	5.4	18
15	13	114	6.4	22

magnetization at a ramp rate of 0.003 T/s. For this verification, the Cu casing and its connection to the cold head was modelled. Fig. 4 in conjunction with Table I provides a comparison between the experimental data and the numerical results. A good matching was achieved. Due to the magnitude of B_a , the incidence of the mechanical deformation on the results was found negligible. Starting from the experimental result obtained at 30 K and considering adiabatic conditions, greater B_a were explored. Table II summarizes some of the results for different B_a and a fracture strength ϑ_F equal to 90 MPa. In that particular case, the average stress ϑ_{vm}^{av} remains well below the fracture strength. Therefore, the magnitude of the trapped magnetic flux density is determined by the heat dissipation generated by the induced current in the bulk. Consequently, as the background field is increased, the electrical conductivity and the fraction of total magnetic flux density over background magnetic flux density B_r/B_a lower. Fig. 5 shows the expected distributions of the normalized critical current density J/J_c , the temperature T and the deformation vector \mathbf{U} . The results were extracted at time t_i corresponding to the initial magnetization as shown in Fig. 6d and at the final time t_f when steady-state regime was achieved. During the initialization of the magnetization, the current density penetrates from the external corners of the bulk to fill up gradually the sample. In steady state regime, the current density diffuses throughout

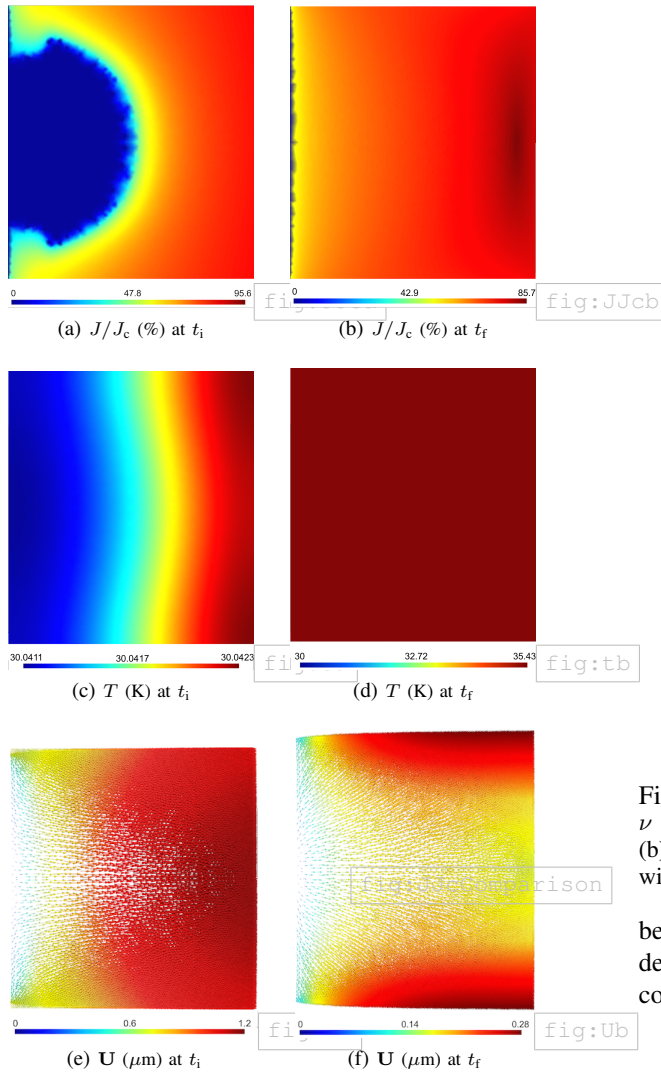


Fig. 5: Distribution of J/J_c , temperature T and displacement vector U , whose magnitude is in μm , for $T_{\text{op}} = 30\text{ K}$, $B_a = 10\text{ T}$ and $\nu = 0.003\text{ T/s}$ in adiabatic conditions.

the bulk with a peak value at its outer edge. The temperature is nearly homogeneous during the process due to the slow magnetization rate. The peak of deformation occurs during the magnetization to relax when the background magnetic field vanishes. Fig. 6a and 6b compare the patterns drawn by the current density at time t_i for both a strain-free J_c and a strain-dependent J_c , respectively. Both figures show a sharp diffusion front of the current density although the pattern is distinct. Due to the axisymmetry of the problem, the stress which is obtained from the gradient of mechanical deformation according to (8) tends to increase moving away from the outer stress-free surface. At the axis of symmetry, it reaches its maximum value thereby explaining the loss of the elliptical front of the current density as it diffuses to the center of the bulk. This difference can be clearly appreciated in Fig 6c which shows the current density distribution in the bulk at t_i for a lesser fracture strength equal to 20 MPa. For this case, the solution appears noisy, presenting convergence issues. However, it is apparent that the front of current diffusion at nearly J_c has

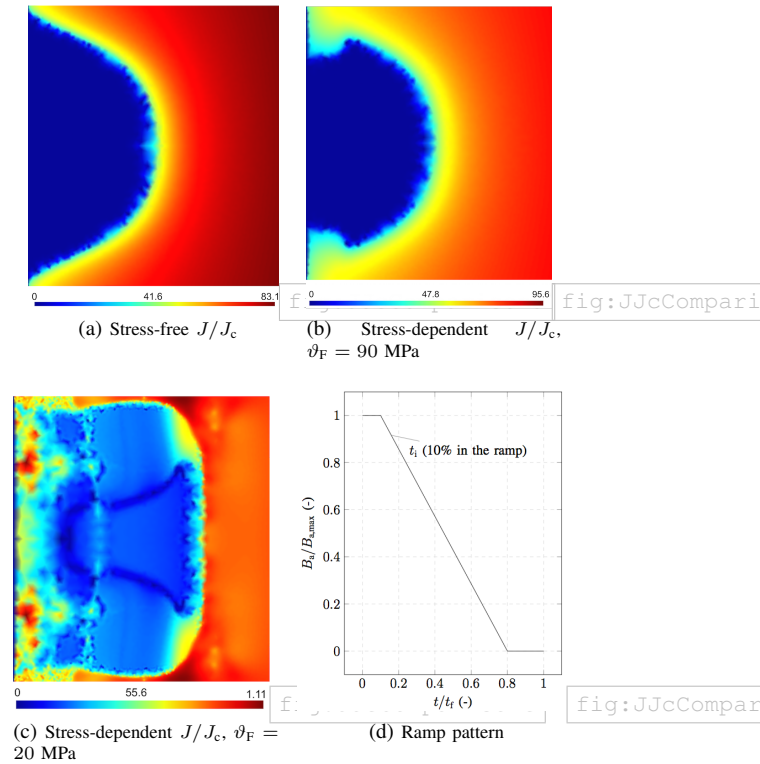


Fig. 6: Distribution of J/J_c (%) for $T_{\text{op}} = 30\text{ K}$, $B_a = 10\text{ T}$ and $\nu = 0.003\text{ T/s}$ in adiabatic conditions. (a) strain-free J_c ($J_{c,m} = 1$), (b) strain-dependent J_c with $\vartheta_F = 90\text{ MPa}$, (c) strain-dependent J_c with $\vartheta_F = 20\text{ MPa}$, and (d) ramp pattern for the magnetization.

been expelled towards the edges of the bulk and the current density in the inner portion has been drastically reduced in comparison of Fig. 6a and 6b.

IV. CONCLUSION

A coupled thermal, mechanical and electromagnetic axisymmetric model of YBCO pellets was implemented using finite element method and cross-checked with available experimental data obtained during FC magnetization. It led to first insights of the impact of the mechanical stress on the distribution of induced current density at different magnitudes of background magnetic fields, operating temperatures and ramp rates. It was shown that the magnitude of the strength fracture should be assessed when it is of the order of the stress generated in the bulk since it impacts strongly the distribution of current density. It is rather a complicated task since it should be performed at cryogenic temperatures with enough statistics to represent the variability from sample to sample. Indeed, it is related to the pre-existence of defect and cracks during the growth of the sample and generated during the initial cooling. When the induced stress is of the order of the fracture strength, the numerical model becomes unstable, an issue that should be dealt with to allow studying any limiting cases arising from the chosen ideal approach. Indeed, the actual stress-strain curve is expected to show a slight deviation from pure elastic behaviour and the critical current density is likely to degrade at a lower critical strain.

REFERENCES

- [1] S. Haindl, H.W. Weber, M. Sefciková, K. Zmorayová, P. Diko, N. Hari Babu, D. A. Cardwell, L. Shlyk, G. Krabbes, "Studies of cracking behavior in melt-processed YBCO bulk superconductors," *Journal of Physics: Conference Series*, vol. 43, pp. 429–433, 2006.
- [2] Y. Ren, R. Weinstein, J. Liu, R.P. Sawh, C. Foster, "Damage caused by magnetic pressure at high trapped field in quasi-permanent magnets composed of melt-textured Y-Ba-Cu-O superconductor," *Physica C*, vol. 251, pp. 15–26, 1995.
- [3] T.H. Johansen, "Flux-pinning-induced stress and magnetostriction in bulk superconductors," *Superconductor Science and Technology*, vol. 13, pp. R121–R137, 2000.
- [4] F. Trillaud, K. Berger, B. Douine, J. Lévêque, "Comparison between modeling and experimental results of magnetic flux trapped in YBCO bulks," *IEEE Transactions on Applied Superconductivity*, vol. 26, no. 3, p. 6800305 (5pp), 2016.
- [5] R. Davidge, *Mechanical behaviour of ceramics*. Cambridge Solid State Science Series, Cambridge University Press, 1979.
- [6] M. Tomita, M. Murakami and K. Yoneda, "Improvements in the mechanical properties of bulk YBCO superconductors with carbon fibre fabrics," *Superconductor Science and Technology*, vol. 15, pp. 803–807, 2002.
- [7] M. Sugano, T. Nakamura, T. Manabe, K. Shikimachi, N. Hirano and S. Nagaya, "The intrinsic strain effect on critical current under a magnetic field parallel to the c axis for a MOCVD-YBCO-coated conductor," *Superconductor Science and Technology*, vol. 21, p. 115019 (8pp), 2008.
- [8] D.C. van der Laan, T.J. Haugan, P.N. Barnes, D. Abrahimov, F. Kameani, D.C. Labalestier and M.W. Rupich, "The effect of strain on grains and grain boundaries in $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ coated conductors," *Superconductor Science and Technology*, vol. 23, p. 014004 (7pp), 2010.
- [9] T. Anderson, *Fracture mechanics - fundamentals and Applications*. CRC Press, 3 ed., 2006.
- [10] C. Geuzaine and J.-F. Remacle, "Gmsh: a three-dimensional finite element mesh generator with built-in pre- and post-processing facilities," *International Journal for Numerical Methods in Engineering*, vol. 79, no. 11, pp. 1309–1331, 2009.
- [11] P. Dular and C. Geuzaine and F. Henrotte and W. Legros, "A general environment for the treatment of discrete problems and its application to the finite element method," *IEEE Transactions on Magnetics*, vol. 34, no. 5, pp. 3395–3398, 1998.
- [12] F. Inanir, S. Yildiz, K. Oxturk, and S. Celebi, "Magnetization of Gd diffused $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ superconductor: Experiment and theory," *Chinese Physics B*, vol. 22, no. 7, p. 077402 (5pp), 2013.
- [13] A. Murakami, H. Teshima, M. Morita, T. Kudo and A. Iwamoto, "Effects of RE compositional boundaries on fracture strength at 77 K in large single-grained RE-Ba-Cu-O bulk 150 mm in diameter," *Journal of Physics: Conference series*, vol. 507, p. 012034 (4pp), 2014.
- [14] K. Konstantopoulou, Y.H. Shi, A.R. Dennis, J.H. Durrell, J.Y. Pastor, and D.A. Cardwell, "Mechanical characterization of GdBCO/Ag and YBCO single grains fabricated by top-seeded melt growth at 77 and 300 K," *Superconductor Science and Technology*, vol. 27, no. 11, p. 115011 (11pp), 2014.
- [15] K. Berger, J. Lévêque, D. Netter, B. Douine, and A. Rezzoug, "Influence of Temperature and/or Field Dependences of the $E - J$ Power Law on Trapped Magnetic Field in Bulk YBaCuO ," *IEEE Transactions on Applied Superconductivity*, vol. 17, no. 2, pp. 3028–3031, 2007.
- [16] T.R. Ravindran, V. Sankaranarayanan, and R. Srinivasan, "Thermal conductivity of $\text{YBa}_2\text{Cu}_3\text{O}_7$," *Pramana - Journal of Physics*, vol. 39, no. 2, pp. 109–116, 1992.
- [17] H. Fujishiro, M. Ikebe, T. Naito, K. Noto, S. Kohayashi and S. Yoshizawa, "Anisotropic thermal diffusivity and conductivity of YBCO(123) and YBCO(211) mixed crystals. I," *Japanese Journal of Applied Physics*, vol. 33, no. 9A, pp. 4965–4970, 1994.
- [18] R. Jha, P. Rani, V.P.S. Awana, "Revisiting heat capacity of bulk polycrystalline $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$," *Journal of Superconductivity and Novel Magnetism*, vol. 27, pp. 287–291, 2014.
- [19] M.D. Ainslie and H. Fujishiro, "Modelling of bulk superconductor magnetization," *Superconductor Science and Technology*, vol. 28, p. 053002 (20pp), 2015.
- [20] M. Niffenegger, K. Reichin, "The proper use of thermal expansion coefficients in finite element calculations," *Nuclear Engineering and Design*, vol. 243, pp. 356–359, 2012.
- [21] I. Parinov, *Microstructure and Properties of High-Temperature Superconductors*. Technology & Engineering, Springer Science & Business Media, 2013.
- [22] A. Adamowitz, "Axisymmetric FE model to analysis of thermal stresses in a brake disk," *Journal of theoretical and applied mechanics*, vol. 53, no. 2, pp. 357–370, 2015.
- [23] P. Diko, "Growth-related microstructure of melt-grown $\text{REBa}_2\text{Cu}_3\text{O}_y$ bulk superconductors," *Superconductor Science and Technology*, vol. 13, no. 3, pp. 1202–1213, 2000.