A numerical method for the scattering by defects in axisymmetrical open elastic waveguides

Matthieu Gallezot, Fabien Treyssede, Laurent Laguerre

Available online at www.sciencedirect.com

ScienceDirect

Procedia Engineering 199 (2017) 1527–1532

Procedia Engineering

www.elsevier.com/locate/procedia

X International Conference on Structural Dynamics, EURODYN 2017

A numerical method for the scattering by defects in axisymmetrical open elastic waveguides

Matthieu Gallezot[a,*], Fabien Treyssède[a], Laurent Laguerre[a]

[a]*IFSTTAR, GERS, GeoEND, Route de Bouaye, Bouguenais F-44344, France*

Abstract

Guided wave modes can provide precise physical analyses of scattering phenomena. When the structure is surrounded by an infinite medium (open waveguide), the unboundedness of the transverse direction greatly complicates the numerical resolution of the problem and its analysis. Theoretically, the wave field can be obtained through modal expansion on trapped modes and on two continua of radiation modes. The latter are sometimes approximated by a discrete sum over leaky modes. However, leaky modes grow at infinity in the transverse direction and hence can approximate the wave fields only in a spatially limited region. A perfectly matched layer (PML) enables to reveal leaky modes in the modal basis and defines two continua of PML modes, discretized by the truncation of the PML. In this paper, the scattering properties of the modes of an elastic axisymmetric open waveguide are investigated with a hybrid method. The semi-analytical finite element (SAFE) method is combined with a PML and then coupled to a FE-PML computation around the inhomogeneity. The hybrid method is validated against literature results for the scattering of a guided wave at the junction between a closed and an open waveguide. Then, the diffraction of a high-frequency leaky mode by a notch is presented.

Peer-review under responsibility of the organizing committee of EURODYN 2017.

Keywords: open waveguide, perfectly matched layer, SAFE, diffraction, notch

1. Introduction

Guided wave modes can propagate over a wide distance. They allow non-destructive evaluation (NDE) of elongated structures. Moreover, these modes enable to precisely analyse scattering phenomena. But, when the structure is embedded in an infinite medium (open waveguide), the waves may leak into the surrounding medium and suffer from attenuation. The unboundedness of the transverse direction significantly complicates the numerical resolution of the problem and its analysis with a modal approach. Theoretically, the wave field can be obtained through modes superposition of trapped modes and of two continua of radiation modes [1]. The continua are yet difficult to handle mathematically and physically. They are sometimes approximated by a discrete sum over the so-called leaky modes [2, 3]. Leaky modes are able to provide a useful physical information: the attenuation of the waves as they propagate [4]. However, leaky modes grow at infinity in the transverse direction, and thereby, can approximate the wave fields only

* Corresponding author. Tel.: +033-2-40-84-56-20
E-mail address: matthieu.gallezot@ifsttar.fr

1877-7058
Peer-review under responsibility of the organizing committee of EURODYN 2017.
10.1016/j.proeng.2017.09.495

in a spatially limited region not far from the core of the waveguide [3]. In a recent work [5], the authors have shown that an infinite PML enables to naturally reveal the leaky modes such that the modal basis contains trapped modes (if any), revealed leaky modes and two rotated continua of radiation modes. The latter are discretized by the PML truncation [6,7], required for numerical purpose, and are called PML modes.

In this paper, the numerical computation of modal scattering properties in elastic open waveguides is investigated. Hybrid numerical methods have been proposed in the literature for the scattering analysis of closed elastic waveguides [8–11]. These methods couple a full computation in the near-field of the inhomogeneity with a modal solution, which can be obtained on the section of the waveguide for instance with the so-called semi-analytical finite element (SAFE) method [12]. Hybrid methods can handle arbitrary geometries while remaining computationally fast, contrary to transient finite element models [13], and avoid modal post-processing steps.

Herein, a hybrid method is proposed for the scattering analysis of open waveguides. The SAFE-PML modal solution is coupled to a FE-PML computation around the inhomogeneity. For simplicity, axisymmetric waveguides are considered. First, the numerical method is validated against literature results for the scattering of a guided wave at the junction between a closed and an open waveguide. Then, the diffraction of a leaky mode by a notch at high frequencies is studied, which is of interest for the NDE of embedded tendons or rock bolts.

2. Numerical method

2.1. Elastodynamic variational formulation with a PML

Let us consider an open elastic waveguide in the cylindrical coordinate system $(\tilde{r}, \theta, z)$, where z is the axis of the waveguide, and $\tilde{r}$ is the radial direction (also called transverse direction in the following). The waveguide has a core made of one or several layers, embedded in an infinite surrounding medium. The waveguide may be locally inhomogeneous (defect), inside the core or at the interface with the surrounding medium. The waveguide and the defects are axisymmetric such that the displacement field is $\tilde{\boldsymbol{u}} = [\tilde{u}_r(\tilde{r}, z, \omega)\ \tilde{u}_z(\tilde{r}, z, \omega)]^T$ and $\partial(.)/\partial\theta = 0$. The superscript T denotes matrix transpose. A time-harmonic dependance $e^{-j\omega t}$ is assumed. The elastodynamic variational formulation is given by:

$$\int_{\tilde{\Omega}} \delta\tilde{\boldsymbol{\epsilon}}^T \tilde{\boldsymbol{\sigma}} \tilde{r} d\tilde{r} dz - \omega^2 \int_{\tilde{\Omega}} \tilde{\rho} \delta\tilde{\boldsymbol{u}}^T \tilde{\boldsymbol{u}} \tilde{r} d\tilde{r} dz = \int_{\tilde{\Omega}} \delta\tilde{\boldsymbol{u}}^T \tilde{\boldsymbol{f}} \tilde{r} d\tilde{r} dz, \tag{1}$$

where $\tilde{\boldsymbol{f}}$ is the vector of volumic acoustic forces. The strain and stress vectors are respectively $\tilde{\boldsymbol{\epsilon}} = [\tilde{\epsilon}_{rr}, \tilde{\epsilon}_{\theta\theta}, \tilde{\epsilon}_{zz}, 2\tilde{\epsilon}_{rz}]^T$ and $\tilde{\boldsymbol{\sigma}} = [\tilde{\sigma}_{rr}, \tilde{\sigma}_{\theta\theta}, \tilde{\sigma}_{zz}, \tilde{\sigma}_{rz}]^T$. The stress-strain relation is $\tilde{\boldsymbol{\sigma}} = \tilde{\boldsymbol{C}}\tilde{\boldsymbol{\epsilon}}$. $\tilde{\boldsymbol{C}}$ is the matrix of material properties, which can be elastic or viscoelastic. The surrounding medium is truncated by a PML of thickness h. The PML starts at a distance $r = d$ by an analytic continuation of Eq. (1) into the complex radial coordinate [14]:

$$\tilde{r}(r) = \int_0^r \gamma(\xi) d\xi, \tag{2}$$

where $\gamma(r)$ is a complex-valued function inside the PML such that outgoing waves in the surrounding medium are attenuated. Therefore, $\gamma(r) = 1$ if $r < d$, and $\mathrm{Im}[\gamma(r)] > 0$ if $d < r < d + h$. At the end of the PML, a Dirichlet condition is arbitrarily applied. For any function $\tilde{g}(\tilde{r})$ in Eq. (1), the change of variable $\tilde{r} \longrightarrow r$ is used to go back in the real transverse direction, with the following relations:

$$\tilde{g}(\tilde{r}) = g(r), \quad d\tilde{r} = \gamma(r) dr, \quad \frac{d\tilde{g}}{d\tilde{r}} = \frac{dg}{dr} \frac{1}{\gamma(r)}. \tag{3}$$

2.2. Hybrid FE-modal formulation

The hybrid formulation presented here follows the same steps than in closed waveguides [10,11]. Firstly, the general domain $\tilde{\Omega}$ is restricted to an arbitrary small area Ω (see Fig. 1) around the inhomogeneity (junction, crack...). The waveguide is assumed homogeneous and infinitely long outside Ω. The finite element discretization of Eq. (1) on Ω yields the classical system:

$$\delta\boldsymbol{U}^T(\boldsymbol{K} - \omega^2\boldsymbol{M})\boldsymbol{U} = \delta\boldsymbol{U}^T\boldsymbol{F}, \tag{4}$$

where $\boldsymbol{K}$ and $\boldsymbol{M}$ are the stiffness and mass matrices, $\boldsymbol{F}$ and $\boldsymbol{U}$ are the vectors of nodal forces and displacements. Note that both $\boldsymbol{K}$ and $\boldsymbol{M}$ are complex because of the introduction of the PML. The system is partionned into degrees of freedom belonging to the cross-sections $\Sigma = \Sigma_1 \cup \Sigma_2$ (see Fig. 1) and those that remain as follows:

$$\boldsymbol{U} = \begin{pmatrix} \boldsymbol{U}_\Sigma \\ \boldsymbol{U}_{\overline{\Sigma}} \end{pmatrix}, \quad \boldsymbol{F} = \begin{pmatrix} \boldsymbol{F}_\Sigma \\ \boldsymbol{F}_{\overline{\Sigma}} \end{pmatrix}, \tag{5}$$

where $\overline{\Sigma} = \Omega \setminus \Sigma$. The fields $\boldsymbol{U}_\Sigma$ and $\boldsymbol{F}_\Sigma$ are expanded on the modal basis of each cross-section:

$$\boldsymbol{U}_\Sigma = \sum \alpha_m^- \boldsymbol{U}_m^- + \sum \alpha_m^+ \boldsymbol{U}_m^+, \quad \boldsymbol{F}_\Sigma = \sum \alpha_m^- \boldsymbol{F}_m^- + \sum \alpha_m^+ \boldsymbol{F}_m^+, \quad (m = 1 \dots M). \tag{6}$$

where $\boldsymbol{U}_m$ is the vector of modal displacement for a given mode m and $\boldsymbol{F}_m$ is the vector of modal force computed with SAFE-PML (see section 2.3). α_m are the modal coefficients. The superscript + stands for outgoing reflected modes (unknown), and the superscript − denotes ingoing modes (known). The main fundamental difference between hybrid methods in closed and in open waveguides lies in the modes involved in Eq. (6). Indeed, as far as open waveguides with PML are concerned, the modal basis contains trapped modes, leaky modes and PML modes [5] instead of guided modes, inhomogeneous modes and evanescent modes. The known and unknown variables are denoted as follows:

$$\boldsymbol{U}^- = \begin{pmatrix} \boldsymbol{\alpha}^- \\ \boldsymbol{F}_{\overline{\Sigma}} \end{pmatrix}, \quad \boldsymbol{U}^+ = \begin{pmatrix} \boldsymbol{\alpha}^+ \\ \boldsymbol{U}_{\overline{\Sigma}} \end{pmatrix}. \tag{7}$$

From Eqs. (7) and (6), the fields $\boldsymbol{U}$ and $\boldsymbol{F}$ are rewritten as:

$$\boldsymbol{U} = \boldsymbol{G}_u^- \boldsymbol{U}^- + \boldsymbol{G}_u^+ \boldsymbol{U}^+, \quad \boldsymbol{F} = \boldsymbol{G}_f^- \boldsymbol{U}^- + \boldsymbol{G}_f^+ \boldsymbol{U}^+, \tag{8}$$

with the following matrices:

$$\boldsymbol{G}_u^- = \begin{pmatrix} \boldsymbol{B}_u^- & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{pmatrix}, \quad \boldsymbol{G}_u^+ = \begin{pmatrix} \boldsymbol{B}_u^+ & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} \end{pmatrix}, \quad \boldsymbol{G}_f^- = \begin{pmatrix} \boldsymbol{B}_f^- & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} \end{pmatrix}, \quad \boldsymbol{G}_f^+ = \begin{pmatrix} \boldsymbol{B}_f^+ & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{pmatrix}. \tag{9}$$

$\boldsymbol{B}_u^\pm = [\boldsymbol{U}_1^\pm \ \boldsymbol{U}_2^\pm \ \dots \ \boldsymbol{U}_M^\pm]$ and $\boldsymbol{B}_f^\pm = [\boldsymbol{F}_1^\pm \ \boldsymbol{F}_2^\pm \ \dots \ \boldsymbol{F}_M^\pm]$ are respectively the matrices of modal displacements and modal forces. Finally, Eq. (4) and Eq. (8) yield for any arbitrary field $\delta \boldsymbol{U}^T = [\delta \boldsymbol{\alpha}^+ \delta \boldsymbol{U}_{\overline{\Sigma}}]^T$:

$$\boldsymbol{G}_u^{+T} (\boldsymbol{D}\boldsymbol{G}_u^+ - \boldsymbol{G}_f^+) \boldsymbol{U}^+ = \boldsymbol{G}_u^{+T} (\boldsymbol{G}_f^- - \boldsymbol{D}\boldsymbol{G}_u^-) \boldsymbol{U}^-, \tag{10}$$

where $\boldsymbol{D} = (\boldsymbol{K} - \omega^2 \boldsymbol{M})$. The system (10) is solved for each angular frequency ω.

2.3. Modes of an open waveguide with the SAFE-PML method

The SAFE-PML method is used to obtain the modes involved in the modal expansion (6). It starts from the source-free variational formulation written on Σ, assuming an e^{jkz} dependence for $\boldsymbol{u}$ (e^{-jkz} for $\delta \boldsymbol{u}$), where k is the axial wavenumber. The transverse direction is discretized with one-dimensional finite elements such that $\boldsymbol{u}^e(r, \omega) = \boldsymbol{N}^e(r) \boldsymbol{U}^e(\omega)$. $\boldsymbol{N}^e$ is the matrix of interpolating functions and $\boldsymbol{U}^e$ is the vector of nodal displacements. The strain-displacement relation is written $\boldsymbol{\epsilon} = \boldsymbol{B}_r \boldsymbol{U}^e + jk \boldsymbol{B}_z \boldsymbol{U}^e$(resp. $\delta \boldsymbol{\epsilon} = \boldsymbol{B}_r \boldsymbol{U}^e - jk \boldsymbol{B}_z \boldsymbol{U}^e$), where:

$$\boldsymbol{B}_r = \left[\frac{1}{\gamma(r)} \boldsymbol{L}_r \frac{\partial}{\partial r} + \boldsymbol{L}_0 \frac{1}{\tilde{r}} \right] \boldsymbol{N}^e, \quad \boldsymbol{B}_z = \boldsymbol{L}_z \boldsymbol{N}^e \quad \text{with} \quad \boldsymbol{L}_r = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad \boldsymbol{L}_0 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \boldsymbol{L}_z = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}. \tag{11}$$

This yields the following quadratic eigenvalue problem [7,12]:

$$\left(\boldsymbol{K}_1 - \omega^2 \boldsymbol{M}_S + jk(\boldsymbol{K}_2 - \boldsymbol{K}_2^T) + k^2 \boldsymbol{K}_3 \right) \boldsymbol{U} = \boldsymbol{0}, \tag{12}$$

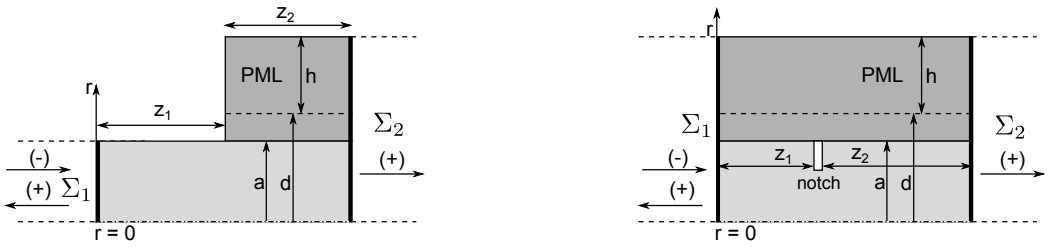

Fig. 1: Finite element domain Ω for (a) validation case (junction of a closed and an open waveguide) and (b) high-frequency application (diffraction by a notch in an open waveguide). Parameters and materials are given in the text.

where the elementary matrices are:

$$\boldsymbol{K}_1^e = \int_{\Sigma^e} \boldsymbol{B}_r^T \boldsymbol{C} \boldsymbol{B}_r \tilde{r}\gamma dr, \quad \boldsymbol{K}_2^e = \int_{\Sigma^e} \boldsymbol{B}_r^T \boldsymbol{C} \boldsymbol{B}_z \tilde{r}\gamma dr, \quad \boldsymbol{K}_3^e = \int_{\Sigma^e} \boldsymbol{B}_z^T \boldsymbol{C} \boldsymbol{B}_z \tilde{r}\gamma dr, \quad \boldsymbol{M}_S^e = \int_{\Sigma^e} \rho \boldsymbol{N}^{eT} \boldsymbol{N}^e \tilde{r}\gamma dr. \tag{13}$$

Eq. (12) can be linearized into a generalized eigenvalue problem in order to be solved with standard eigensolvers [15]. $\boldsymbol{K}_1$, $\boldsymbol{M}_S$ and $\boldsymbol{K}_3$ are complex-valued symmetric matrices due to the PML. Then for a given angular frequency ω, there are two sets of eigensolutions [7], i.e. M symmetric pairs denoted (k_m^+, U_m^+) and (k_m^-, U_m^-), $(m = 1 \dots M)$. From Eq. (12) and eigenvectors, the modal forces are defined [11]:

$$\boldsymbol{F}_m^{\pm} = (\boldsymbol{K}_2^T + jk_m^{\pm}\boldsymbol{K}_3)\boldsymbol{U}_m^{\pm}. \tag{14}$$

3. Numerical results

3.1. Validation test case

The scattering of a guided wave at the junction between a closed and an open waveguide [16] is considered as a validation test case – see Fig. 1a. The core of the waveguide is a steel cylinder of radius $a = 1$ mm ($\rho = 7932$ kg/m^3, $c_l = 5960$ m/s, c_s=3260 m/s), partly embedded in epoxy ($\rho = 1170$ kg/m^3, $c_l = 2610$ m/s, c_s= 1000 m/s). The L(0,1) guided mode is incident from the closed waveguide, with a unit modal coefficient. Its reflection at the junction is studied.

Figure 2 shows a good agreement between the hybrid FE-SAFE-PML method and the reference result. Accurate comparisons have been made with other results of Ref. [16] as well, not shown here for conciseness.

In the closed part of the waveguide, the modal expansions are performed only on the L(0,1) guided mode. As stated in Ref. [16], it has been checked that evanescent and inhomogeneous modes modify the results only by a negligible amount, even close to the junction. In the open part of the waveguide, the inclusion of PML modes slightly improves the accuracy of the results. Hence, 10 modes (1 leaky mode and 9 PML modes) have been included in the modal expansion in the open waveguide. Note that at low frequencies, the large wavelength implies that the parameter $\hat{\gamma}h = \int_d^{d+h} \gamma(\xi)d\xi$ has to be high enough for the PML to be accurate. A parabolic profile $\gamma(r) = 1 + 3(\hat{\gamma} - 1)((r-d)/h)^2$ has been used for the attenuation function in order to improve the accuracy of the computation [7].

3.2. Diffraction by a notch at high frequencies

Pavlakovic et al. [4] have studied open waveguides where all the modes propagate while leaking energy into the surrounding medium. Therefore, the attenuation rates can be very high and then reduce the distance of propagation. However, dispersion curves show that there is a set of longitudinal modes that present a series of local attenuation minima in a limited range of high frequencies. These modes are quite interesting for NDE applications. For a viscoelastic steel waveguide ($\eta_l = 0.003$ Np/wavelength, $\eta_s = 0.008$ Np/wavelength), of radius $a = 10$ mm and embedded in grout ($\rho = 1600$ kg/m^3, $c_l = 2810$ m/s, c_s=1700 m/s, $\eta_l = 0.043$ Np/wavelength, $\eta_s = 0.1$ Np/wavelength), Pavlakovic et

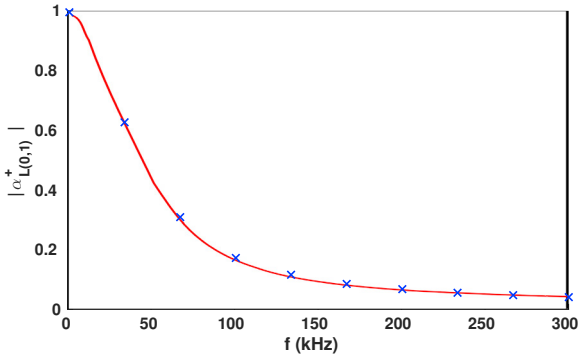

Fig. 2: Amplitude of the modal coefficient of the reflected L(0,1) mode at the junction. Line: Ref. [16] ; cross: hybrid FE-SAFE-PML method. Parameters: $z_1 = z_2 = 0.25a$; $d = 1.05a$; $\hat{\gamma} = 4 + 4i$, $h = 2a$.

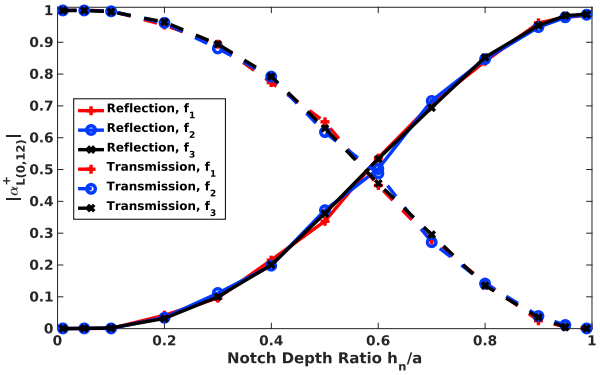

Fig. 3: Amplitude of the modal coefficients of the reflected (solid line) and the transmitted (dashed line) L(0,12) leaky mode by a notch of various depth h_n. Parameters: $z_1 = z_2 = 0.25a$; $d = 1.05a$; $\hat{\gamma} = 1 + 2i$, $h = a$. Frequencies: $f_1 = 2.093$ MHz, $f_2 = 2.288$ MHz and $f_3 = 2.484$ MHz.

al. [4] have shown that the L(0,12) leaky mode has the lowest attenuation at f = 2.3 MHz. This is the first mode that presents a series of attenuation minima.

The steel cylinder is damaged by a notch of varying depth h_n/a and of negligible width compared to the wavelength – see Fig. 1b. The L(0,12) leaky mode is launched with a unit amplitude coefficient on the notch for the first three minima, at frequencies $f_1 = 2.093$ MHz, $f_2 = 2.288$ MHz and $f_3 = 2.484$ MHz. One is interested in the amplitude of the modal coefficients of the reflected and of the transmitted L(0,12) leaky mode when the depth of the notch varies. Owing to viscoelasticity and leakage losses, the value of the modal coefficients depends on the location along the waveguide. The solution of Eq. (10) gives the modal coefficients at cross-sections Σ, and hence, they depend on the arbitrary distances z_1 and z_2. They can be retropropagated [17,18], in order to get reference modal coefficients at the notch, such that:

$$\alpha^+_m|_{notch} = \alpha^+_m|_\Sigma e^{-jk_m z_i}, \quad i = (1, 2) \tag{15}$$

The modal coefficients can then be obtained at any location in the waveguide.

For all frequencies the modal coefficient of the reflected mode increase with the notch depth – see Fig. 3. Conversely, the modal coefficient of the transmitted mode decreases with the notch depth. That is to say that the L(0,12) leaky mode is sensitive to deep notch rather than surface defects. Indeed, the mode shape shows that when attenuation minima occur, most of the energy is concentrated in the centre of the waveguide and not at the interface [4]. Despite a large number of modes occurring at these frequencies, results (not presented here for conciseness) show that mode conversion is low and limited to only a few modes such as the L(0,13) and the L(0,14). These modes suffer from a greater attenuation that the L(0,12). Hence, the L(012) prevails for long range inspection.

In order to reach convergence, 75 modes have been included in the modal expansion of each cross-section. Since the frequencies are high, the wavelength is quite small. Therefore, the PML parameter $\hat{\gamma}h$ remains small and so does the size of the matrices in practice.

4. Conclusion

A hybrid method has been presented for the scattering analysis in axisymmetric open waveguides. It combines a modal decomposition based on a SAFE-PML method with a FE-PML computation around the inhomogeneity. The method has been validated against literature results, considering the scattering of a guided mode at the junction between a closed and an open waveguide. Then, the scattering of a high-frequency leaky mode on a notch in a fully embedded waveguide has been considered. Though limited to axisymmetric open waveguides, the hybrid method presented here could be generalised to arbitrary geometries and defects. Contrary to full transient finite element model, it provides a computationally fast method without requiring modal post-processing.

Acknowledgements

The authors wish to thank Région Pays de la Loire for its financial support.

References

[1] V. Maupin, The radiation modes of a vertically varying half-space: a new representation of the complete green's function in terms of modes, Geophysical Journal International 126 (1996) 762–780.
[2] R. Collin, Field theory of guided waves, IEEE Press, Wiley, 1991.
[3] J. Hu, C. R. Menyuk, Understanding leaky modes: slab waveguide revisited, Adv. Opt. Photon. 1 (2009) 58–106.
[4] B. Pavlakovic, M. Lowe, P. Cawley, High-frequency low-loss ultrasonic modes in imbedded bars, Journal of Applied Mechanics - Transactions of the ASME 68 (2001) 67–75.
[5] M. Gallezot, F. Treyssède, L. Laguerre, Contribution of leaky modes in the modal analysis of unbounded problems with perfectly matched layers, J. Acoust. Soc. Am. 141 (2017) EL16–EL21.
[6] F. Olyslager, Discretization of continuous spectra based on perfectly matched layers, SIAM Journal on Applied Mathematics 64 (2004) 1408–1433.
[7] K. L. Nguyen, F. Treyssède, C. Hazard, Numerical modeling of three-dimensional open elastic waveguides combining semi-analytical finite element and perfectly matched layer methods, Journal of Sound and Vibration 344 (2015) 158–178.
[8] N. Rattanawangcharoen, W. Zhuang, A. Shah, S. Datta, Axisymmetric guided waves in jointed laminated cylinders, Journal of Engineering Mechanics-ASCE 123 (1997) 1020–1026.
[9] J. Galan, R. Abascal, Numerical simulation of lamb wave scattering in semi-infinite plates, International Journal for Numerical Methods in Engineering 53 (2002) 1145–1173.
[10] F. Benmeddour, F. Treyssède, L. Laguerre, Numerical modeling of guided wave interaction with non-axisymmetric cracks in elastic cylinders, International Journal of Solid and Structures 48 (2011) 764–774.
[11] F. Treyssède, Mode propagation in curved waveguides and scattering by inhomogeneities: Application to the elastodynamics of helical structures, J. Acoust. Soc. Am. 129 (2011) 1857–1868.
[12] A. Marzani, Time-transient analysis response for ultrasonic guided waves propagating in damped cylinders, International Journal of Solid and Structures (2008).
[13] D. Alleyne, M. Lowe, P. Cawley, The reflection of guided waves from circumferential notches in pipes, Journal of Applied Mechanics - Transactions of the ASME 65 (1998) 635–641.
[14] W. C. Chew, W. H. Weedon, A 3d perfectly matched medium from modified maxwells equations with stretched coordinates, Microwave and Optical Technology Letters 7 (1994) 599–604.
[15] F. Tisseur, K. Meerbergen, The quadratic eigenvalue problem, SIAM Rev. 43 (2001) 235–286.
[16] T. Vogt, M. Lowe, P. Cawley, The scattering of guided waves in partly embedded cylindrical structures, J. Acoust. Soc. Am. 113 (2003) 1258–1272.
[17] M. V. Predoi, M. Castaings, L. Moreau, Influence of material viscoelasticity on the scattering of guided waves by defects, J. Acoust. Soc. Am. 124 (2008) 2883–2894.
[18] B. Hosten, L. Moreau, M. Castaings, Reflection and transmission coefficients for guided waves reflected by defects in viscoelastic material plates, J. Acoust. Soc. Am. 121 (2007) 3409–3417.