



## Portfolio management of mixed-species forests

Marielle Brunette, Arnaud Dragicevic, Jonathan Lenglet, Alexandra Niedzwiedz, Vincent Badeau, Jean-Luc Dupouey

### ► To cite this version:

Marielle Brunette, Arnaud Dragicevic, Jonathan Lenglet, Alexandra Niedzwiedz, Vincent Badeau, et al.. Portfolio management of mixed-species forests. [University works] auto-saisine. 2014, 20 p. hal-01589694

**HAL Id: hal-01589694**

**<https://hal.science/hal-01589694>**

Submitted on 18 Sep 2017

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution - ShareAlike 4.0 International License



Laboratoire d'**E**conomie **F**orestière



AgroParisTech  
ENGREF  
INSTITUT DES SCIENCES ET INDUSTRIES DU VIVANT ET DE L'ENVIRONNEMENT  
PARIS INSTITUTE OF TECHNOLOGY FOR LIFE, FOOD AND ENVIRONMENTAL SCIENCES

# Portfolio Management of Mixed-Species Forests

Marielle BRUNETTE  
Arnaud DRAGICEVIC  
Jonathan LENGLET  
Alexandra NIEDZWIEDZ  
Vincent BADEAU  
Jean-Luc DUPOUEY

*Août 2014*



**Document de travail**  
**n° 2014-09**

# Portfolio Management of Mixed-Species Forests

Marielle BRUNETTE<sup>1</sup>, Arnaud DRAGICEVIC<sup>2</sup>, Jonathan LENGLET<sup>3</sup>,  
Alexandra NIEDZWIEDZ<sup>4</sup>, Vincent BADEAU<sup>5</sup>, Jean-Luc DUPOUEY<sup>6</sup>

*August 2014*

Document de travail du LEF n°2014-09

## Abstract

We propose to test the portfolio selection theory on the historical data of tree species' productivities obtained from the French National Forest Inventory (IFN). We determine the optimal timber productivity-vulnerability arrangements out of the combinations of tree species and map the optimal compositions per administrative department in France. We also estimate the survivals of optimal portfolios using the species' probabilities of presence. Our results show that greater weights in the optimal portfolios correspond to higher probabilities of presence.

**Key words:** Forest Management, Portfolio Management, Mixed-Species Forests, Climate Change.

---

<sup>1</sup> UMR INRA – Agro ParisTech, Laboratoire d'Économie Forestière, 54042 Nancy Cedex, France.

Corresponding author : marielle.brunette@nancy.inra.fr

<sup>2</sup> Chaire Forêts pour Demain [Agro ParisTech–Office National des Forêts], 54042 Nancy Cedex, France

<sup>3</sup> UMR INRA – Agro ParisTech, Laboratoire d'Économie Forestière, 54042 Nancy Cedex, France.

<sup>4</sup> UMR INRA – Agro ParisTech, Laboratoire d'Économie Forestière, 54042 Nancy Cedex, France.

<sup>5</sup> UMR INRA – Université de Lorraine Écologie et Écophysiologie Forestières, 54280 Champenoux, France

<sup>6</sup> UMR INRA – Université de Lorraine Écologie et Écophysiologie Forestières, 54280 Champenoux, France

# 1 Introduction

In France, half of the forest areas are diversified in tree species, a proportion which remains valid in volume (Morneau et al., 2008). And yet, on a global scale, the industrial mixed-wood species plantations represent less than 0.1% of the forest cover. Industry and investors do not see great advantages in the mixture of species (Nichols et al., 2006; Knoke et al., 2008). Likewise, private forest owners and some forest economists consider that mixed forests are less profitable (Knoke et al., 2008). The additional costs incurring during the mixed-wood forest management (Nichols et al., 2006), the high resilience of monoculture (Powers, 1999) or the fact that monoculture does not show evidence of productivity decline (Powers, 1999 ; Evans, 2005), could explain such statement of facts.

Nevertheless, the literature also shows that benefits from mixed forests compared to monocultures are multiple: mixed forests enhance tree productivity (Morin et al., 2011), forest stability against natural hazards (Mayer et al., 2005; Schütz et al., 2006; Griess and Knoke, 2011), diseases and insect attacks (Bosu et al., 2006; Nichols et al., 2006); their biodiversity is richer (Vettenranta, 1996; Franklin et al., 1999; Halpern et al., 1999); they are also more aesthetically pleasing (Spiecker, 2000); they provide diversity in social demand (Clasen et al., 2011); and, under the risk of climate change, mixed-species stand structures enhance flexibility and adaptation (Jacobsen and Thorsen, 2003; Roman-Amat, 2007; Schou et al., 2012). Moreover, it has been shown that mixed forests can lead to higher financial returns (Kelty, 1992; Knoke et al., 2005).

The economic literature discussing the mixed-species forests has mainly focused on the impacts of management actions on the tree diversity (Buongiorno et al., 1995; Lu and Buongiorno, 1993) or the optimal composition of a forest in terms of species (Knoke et al., 2008; Roessiger et al., 2011; Neuner et al., 2013). This last issue was addressed through the theory of portfolio selection, where the tree species corresponded to financial assets. For instance, Thomson (1991) obtained the financial optimum from a mixture of tree species and Roessiger et al. (2011) highlighted the economic merits of mixed stand forests. They found that what they call the near-natural forestry is the best option for a forest owner of a small or medium property.

Knoke et al. (2008) conclude that mixed forests are more valuable to forest owners than monocultures. They prove it from the combinations of Norway spruce (*Picea abies*) and Common beech (*Fagus sylvatica*). Although Norway spruce shows dramatic fluctuations in returns, implying that the species is particularly exposed to risk, the authors observe that it yields the best financial return. They find that the blend which maximizes their equation is about 50% Norway spruce, 50% Common beech. Mixing a high-risk species with the low-risk common beech could therefore balance both risk and financial returns. The first study on mixing a large variety of tree species was led by Neuner et al. (2013). The latter focused on financial aspects by looking at the expected Net Present Value (NPV) of a forest owned by a private company in Bavaria, Germany. Using the portfolio selection, they determined the optimal percentage of each tree species in the forest stand. Species such as Norway spruce or Douglas-fir (*Pseudotsuga menziesii*) turned out to be sources of income, while others – including broad-leaved trees – mainly served to mitigate risks.

In order to predict yields and revenues, most of the studies aforementioned are based on Monte Carlo simulations. We propose a different approach, in that we simulate the portfolio selection from the French National Forest Inventory (IFN) historical data on the tree species' productivities. Thereby, instead of using the portfolio theory to calculate the income-risk optima, we determine the combinations of tree species that allow for an effective timber productivity-risk tradeoff. We define risk to be the tree species vulnerability, that is, the amplitude of variation around the productivity (Tilman et al., 1997). Not only the idea is to move away from the original application, but also to apply the methodology in a different context. Besides, the application is not disconnected from the forest management. As a matter of fact, what we study is the tolerance toward uncertainty, which can cover a broad spectrum of indicators, including those in

silviculture. In addition, we value the climate change impacts on the optimal compositions with a view to observe the survivability of the optima we obtain.

On the one hand, we recall the theoretical portfolio selection model that minimizes the portfolio risk. On the other hand, we develop a simulator computing these portfolios for each administrative department in France. We thus build a simulator that allows to determine and map the optimal forest compositions per department. Despite the obvious focus on the optimal compositions, we also compute the survival probabilities of the simulated portfolios using the species' probabilities of presence by years 2050 and 2100. Our study revolves around seven tree species: Pedunculate oak (*Quercus robur*), Downy oak (*Quercus pubescens*), Sessile oak (*Quercus petraea*), Holm oak (*Quercus ilex*), Common beech (*Fagus sylvatica*), Scots pine (*Pinus sylvestris*) and European larch (*Larix decidua*).

The paper is structured as follows. In Section 2, we develop the theoretical model of portfolio selection and efficient frontier. In Section 3, we describe the materials and the methods used to test the model. Section 4 presents the results while Section 5 discusses them and concludes.

## 2 Theoretical model

### 2.1 Model of portfolio selection (Markowitz, 1952, 1959)

The forest owner has to invest in a set of tree species for its future harvest of standing timber. She features a set of tree species  $a = 1, \dots, N$ . Holding the  $a$  tree asset yields the return of  $r_a$ . The return is subject to stochastic chocs due, for example, to climate change, making it random. The forest owner thus estimates the expected return and the covariance matrix  $\sigma$ , where  $\sigma_{ab}$  is the covariance of returns of tree species  $a$  and  $b$ . Indeed, if she considers the set of risky assets, she computes the covariance matrix of their returns.

The matrix is defined as of full rank and hence invertible.

$$\forall a, b \in \{1, \dots, N\} : \sigma_{ab} = \sigma(r_a, r_b) \quad (1)$$

By diversifying the tree species in her plot, the forest owner varies the portfolio return. She determines the efficient portfolios defined by their expected return and covariance matrix.

The variance of the diversified portfolio is

$$\sigma_p^2 = \sum_{a=1}^N \sum_{b=1}^N x_a x_b \sigma_{ab} \quad (2)$$

where  $x$  represents the weight of the species in the portfolio.

The forest owner seeks to minimize the variance of the portfolio, that is

$$\min_{x_a} \quad \frac{1}{2} \sum_{a=1}^N \sum_{b=1}^N x_a x_b \sigma_{ab} \quad (3)$$

subject to the objective return constraint

$$\sum_{a=1}^N x_a r_a \geq \bar{r} \quad (4)$$

and the budget constraint

$$\sum_{a=1}^N x_a = 1 \quad (5)$$

To solve the forest owner's minimization program, we form the Lagrangian

$$L = \frac{1}{2} \sum_{a=1}^N \sum_{b=1}^N x_a x_b \sigma_{ab} + \lambda \left( \bar{r} - \sum_{a=1}^N x_a r_a \right) + \mu \left( 1 - \sum_{a=1}^N x_a \right) \quad (6)$$

The first-order condition is

$$\lambda r_a - \sum_{b=1}^N x_b \sigma_{ab} = -\mu \quad (7)$$

Given that

$$\sum_{b=1}^N x_b \sigma_{ab} = \text{cov}(r_a, r_p) \quad (8)$$

We fall on

$$\lambda r_a - \text{cov}(r_a, r_p) = -\mu \quad (9)$$

$\lambda$  is the forest owner's return value relatively to the risk minimization;  $\mu$  is the marginal cost of investing or the opportunity cost of investment;  $\text{cov}(r_a, r_p)$  is the marginal risk, generated by the investment in a unit of species  $a$ , on the portfolio risk. We have

$$\frac{\partial \sigma_p^2}{\partial x_a} = 2 \sum_{b=1}^N x_b \sigma_{ab} = 2 \text{cov}(r_a, r_p) \quad (10)$$

The marginal investment in species  $a$  will increase the portfolio risk only if its return is positively correlated to that of the portfolio.

## 2.2 Efficient frontier

From first-order condition (??), we know that

$$\sum_{b=1}^N x_b \sigma_{ab} = \lambda r_a + \mu \quad (11)$$

We can write the vector product of the covariance matrix row  $a$  and of vector  $\mathbf{x}$ .

$$\sum_{b=1}^N x_b \sigma_{ab} = (\boldsymbol{\sigma}_a \cdot \mathbf{x}) \quad (12)$$

And thus

$$(\boldsymbol{\sigma} \cdot \mathbf{x}) = (\lambda \cdot \mathbf{r}) + (\mu \cdot \mathbf{1}) \quad (13)$$

where  $\mathbf{1}$  is the column vector with all components equal to 1. Since  $\boldsymbol{\sigma}$  is of full rank, it is invertible and thus

$$\mathbf{x} = \boldsymbol{\sigma}^{-1} (\lambda \cdot \mathbf{r} + \mu \cdot \mathbf{1}) = (\lambda \cdot \boldsymbol{\sigma}^{-1} \cdot \mathbf{r}) + (\mu \cdot \boldsymbol{\sigma}^{-1} \cdot \mathbf{1}) \quad (14)$$

where  $\boldsymbol{\sigma}^{-1} \cdot \mathbf{r}$  and  $\boldsymbol{\sigma}^{-1} \cdot \mathbf{1}$  depend on the covariance and the return of species. The optimal portfolio of mixed-species is thus a linear combination of those two vectors. The parameters

of this linear combination,  $\lambda$  and  $\mu$ , are two endogenous variables of the variance minimization program.

The budget constraint becomes

$$(\mathbf{1}^T \cdot \mathbf{x}) = 1 \quad (15)$$

which yields

$$(\lambda \cdot \mathbf{1}^T \cdot \boldsymbol{\sigma}^{-1} \cdot \mathbf{r}) + (\mu \cdot \mathbf{1}^T \cdot \boldsymbol{\sigma}^{-1} \cdot \mathbf{1}) = 1 \quad (16)$$

In parallel, the return constraint becomes

$$(\mathbf{r}^T \cdot \mathbf{x}) = r^* \quad (17)$$

And thus

$$(\lambda \cdot \mathbf{r}^T \cdot \boldsymbol{\sigma}^{-1} \cdot \mathbf{r}) + (\mu \cdot \mathbf{r}^T \cdot \boldsymbol{\sigma}^{-1} \cdot \mathbf{1}) = r^* \quad (18)$$

We can now compute the values of the parameters of the linear combination previously defined. The constraints yield

$$\begin{cases} \lambda = [(\mathbf{r}^T \cdot \boldsymbol{\sigma}^{-1} \cdot \mathbf{1}) - (\mathbf{1}^T \cdot \boldsymbol{\sigma}^{-1} \cdot \mathbf{1}) r^*] \left[ (\mathbf{r}^T \cdot \boldsymbol{\sigma}^{-1} \cdot \mathbf{1})^2 - (\mathbf{r}^T \cdot \boldsymbol{\sigma}^{-1} \cdot \mathbf{r}) (\mathbf{1}^T \cdot \boldsymbol{\sigma}^{-1} \cdot \mathbf{1}) \right]^{-1} \\ \mu = [(\mathbf{r}^T \cdot \boldsymbol{\sigma}^{-1} \cdot \mathbf{1}) r^* - (\mathbf{r}^T \cdot \boldsymbol{\sigma}^{-1} \cdot \mathbf{r})] \left[ (\mathbf{r}^T \cdot \boldsymbol{\sigma}^{-1} \cdot \mathbf{1})^2 - (\mathbf{r}^T \cdot \boldsymbol{\sigma}^{-1} \cdot \mathbf{r}) (\mathbf{1}^T \cdot \boldsymbol{\sigma}^{-1} \cdot \mathbf{1}) \right]^{-1} \end{cases} \quad (19)$$

By means of the first-order condition, we conclude that

$$\begin{aligned} \sigma_{opt}^2 &= \left[ (\mathbf{1}^T \cdot \boldsymbol{\sigma}^{-1} \cdot \mathbf{1}) r^{*2} - 2 (\mathbf{r}^T \cdot \boldsymbol{\sigma}^{-1} \cdot \mathbf{1}) r^* + (\mathbf{r}^T \cdot \boldsymbol{\sigma}^{-1} \cdot \mathbf{r}) \right] \\ &\times \left[ (\mathbf{r}^T \cdot \boldsymbol{\sigma}^{-1} \cdot \mathbf{r}) (\mathbf{1}^T \cdot \boldsymbol{\sigma}^{-1} \cdot \mathbf{1}) - (\mathbf{r}^T \cdot \boldsymbol{\sigma}^{-1} \cdot \mathbf{1})^2 \right]^{-1} \end{aligned} \quad (20)$$

**Proposition 1** *When all species are exposed to some level of risk, the efficient frontier of the mixed-species portfolio is the superior segment of a parabola originating from the linear combination.*

It represents, for each level of risk, the maximum return that the forest owner can expect from the portfolio. It also represents, for a level of return, the minimal risk to which the forest owner is exposed.

We can now compute the optimal portfolio that minimizes the risk, that is, the minimum variance portfolio

$$\begin{aligned} \frac{\partial \sigma_{opt}^2}{\partial r^*} &= \left[ 2 (\mathbf{1}^T \cdot \boldsymbol{\sigma}^{-1} \cdot \mathbf{1}) r^* - 2 (\mathbf{r}^T \cdot \boldsymbol{\sigma}^{-1} \cdot \mathbf{1}) \right] \\ &\times \left[ (\mathbf{1}^T \cdot \boldsymbol{\sigma}^{-1} \cdot \mathbf{1}) (\mathbf{r}^T \cdot \boldsymbol{\sigma}^{-1} \cdot \mathbf{r}) - (\mathbf{r}^T \cdot \boldsymbol{\sigma}^{-1} \cdot \mathbf{1})^2 \right]^{-1} \end{aligned} \quad (21)$$

By setting the derivative equal to zero, we find that the value of the minimum variance portfolio equals

$$\begin{cases} r^* = [(\mathbf{r}^T \cdot \boldsymbol{\sigma}^{-1} \cdot \mathbf{1})] [(\mathbf{1}^T \cdot \boldsymbol{\sigma}^{-1} \cdot \mathbf{1})]^{-1} \\ \sigma^* = [(\mathbf{1}^T \cdot \boldsymbol{\sigma}^{-1} \cdot \mathbf{1})]^{-\frac{1}{2}} \end{cases} \quad (22)$$

**Proposition 2** *There exists a mixed-species portfolio that minimizes the risk.*

From the results above, let us now simulate the optimal portfolios for each French administrative department.

## 3 Materials and methods

### 3.1 Data source

#### 3.1.1 Productivity and vulnerability

We study 7 tree species, which are representative of the French territory<sup>1</sup>. The study includes 4 species of oak: Pedunculate oak (*Quercus robur*), Downy oak (*Quercus pubescens*), Sessile oak (*Quercus petraea*) and Holm oak (*Quercus ilex*); it also includes Common beech (*Fagus sylvatica*), Scots pine (*Pinus sylvestris*) and European larch (*Larix decidua*). We assume the species to allow covering a large variety of tree behaviors and site conditions throughout France. All the data come from the French National Forest Inventory (IFN) historical database.

Since 2004, the IFN inventory is based on a systematic grid sampling method (Charru et al., 2010; Robert et al., 2010). The territorial networking consists of a 1 km x 1 km grid map. The measurements were made independently per administrative department at a nearly 8-year rate. In order to determine the main type of vegetation, the mapping was analyzed through aerial photography, followed by the field verification and measurement. The method began to be applied in 1971; the entire country was covered in 1980. By reason of the level of detail of the inventory, the time span between two measures in the same department was approximately 12 years. That point is clearly problematical to catch up with information such as the growth rate. In 2004, IFN decided to change its methodology for the sake of estimating the impact of events like droughts or storms. In consequence, the grid became larger (1 point for 10 km<sup>2</sup>); it moves by 2 km per year, providing a kilometer precision covered every 10 years. The new inventory method enables to cover annually the entire metropolitan territory. Furthermore, the territorial scale went from the administrative departments to the administrative regions composed of several departments.

The data collected by IFN are stand and tree characteristics as well as the environmental aspects. Given that we focus on productivity, we isolate the stand characteristics such as area, volume and increment. The area estimation is calculated using the field data and the photo-interpretation results. IFN estimates the sum of wood (more than 4 ha) and small wood (between 0.5 and 4 ha) while distinguishing the poplar plantations. Apropos of the volume calculation, the definition considers the volume over bark of the bole and a cutting top diameter of 7 cm. For a tree to be recorded, the minimum circumference has to be of 23.5 cm at 1.3 m height above the ground. Since the calculation is based on a 5-year rotation, the volume includes the volume of growing living trees as well as the volume of trees that died in the past 5 years. Finally, the increment is evaluated by means of two components: the volume of recruitment and the increment of previously counted trees. The recruitment corresponds to the number of trees that reach the minimum circumference threshold of 23.5 cm during the last 5 years. The increment or the tree growth is measured using an increment borer.

#### 3.1.2 Probabilities of presence

Probabilities of presence per tree species and per department (Badeau et al., 2010; Cheaib et al., 2012) are used to compute the probabilities of survival of the optimal portfolios. The distribution

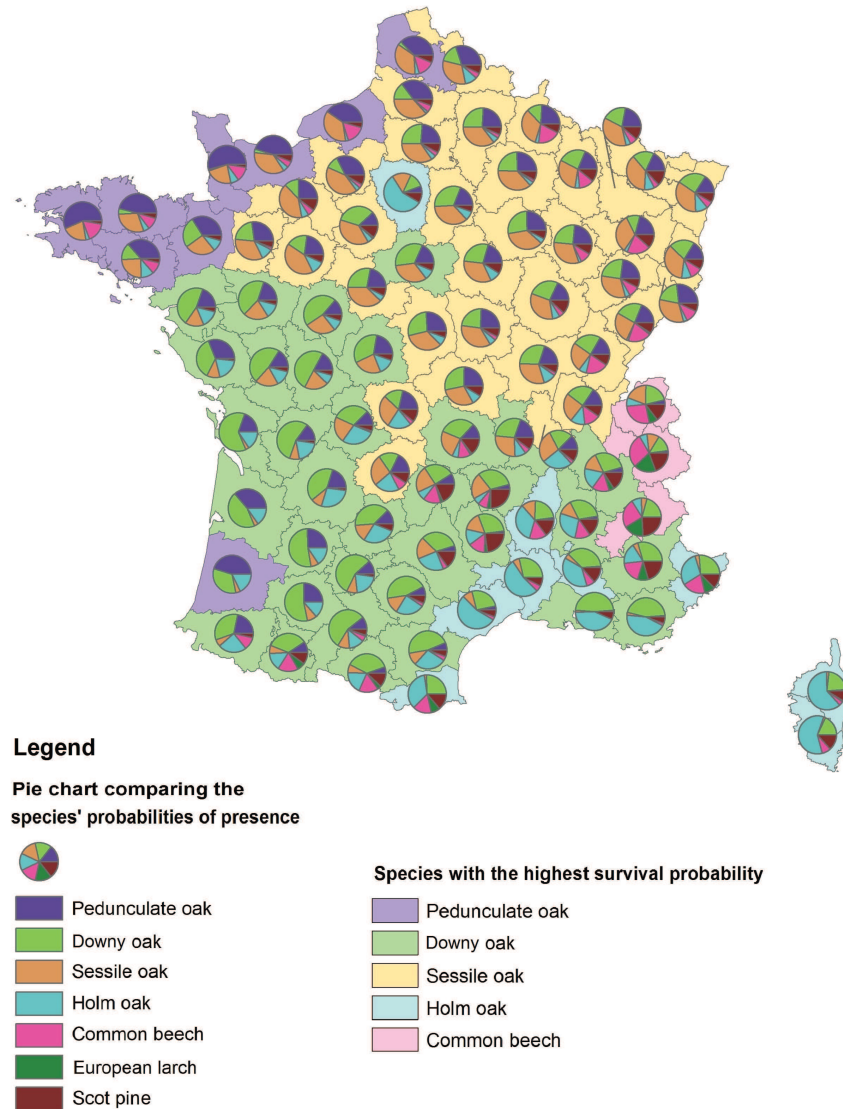
---

<sup>1</sup>Equally, the number of species has been compelled by the availability of probabilities of presence, as described hereafter.



of tree species and the probabilities of presence were extracted and aggregated from the IFN database. For each species, about 90 points were calculated in each department, ensuring the robustness of the results. These probabilities are given for two dates, respectively T2 and T3, which correspond to years 2050 and 2100. As presented in Fig. 1, we built a map representing the probabilities of presence<sup>2</sup> throughout the French metropolitan territory at the T2 horizon<sup>3</sup>.

Figure 1: Probabilities of presence per French administrative department



### 3.2 Raw data treatment

The IFN raw data contains detailed information such as areas, volumes, recruitment, increment, mortality and various ratios. The first step consisted in selecting the appropriate and relevant data. Inasmuch as we are interested in monitoring the production of wood in time, we separately

<sup>2</sup>Normalized to one.

<sup>3</sup>No important changes in tree species probabilities of presence occur between the two time-horizons.

extracted the yearly data on increment, recruitment and area per department and per region, for each tree species. The first difficulty dealt with the fact that the inventory data were discontinuous. We therefore made the assumption that no major event between two inventory dates occurred. Although this point is debatable, this option seemed acceptable regarding the data heterogeneity.

For each year, we computed the total relative productivity ( $P$ ). We define this productivity as the volume of wood produced in one year for a given surface. A forest stand is composed of one main species (with the largest total surface area) and potentially several secondary species. The total volume for a species was determined by adding the volume of newly countable trees (recruitment) and the increment in volume of the pre-existent trees (already counted in the last inventory) in areas where this species is considered as the main species. Forasmuch as most forests are not homogeneous, it is virtually impossible to obtain detailed surface data for one specific tree species. The considered surface thus corresponds to the area where the targeted species is the dominant one. Subsequently, the productivity can be seen as relative. In other words, the relative productivity for a given species refers to the total productivity of the area where this species is predominant. Algebraically, we have

$$P = \frac{\text{Recruitment} + \text{Increment}}{\text{Surface}} \text{ m}^3.\text{ha}^{-1}.\text{year}^{-1}$$

The average productivity and the variance per species and per department were computed over the whole period of study. In order to homogenize the commencement of the inventory, an inverse extrapolation was used to simulate past productivities back to 1978, year of the very first inventory (in Ariège, Midi-Pyrénées). We first tested the arithmetic progression and the polynomial regression, but finally opted for the geometric recurrence relation, which yielded lower jumps of variance. The formula is as follows

$$u_{n-1} = u_n \times q^{-1} \text{ for } n \in [1978, 2009]$$

where  $q = \bar{P} = \frac{1}{2009-k} \sum_{n=k}^{2009} \frac{P_n}{P_{n-1}}$  and  $k$  the year of the first inventory.

We then estimated the covariance matrix per department and per region. At last, using the surface data, we determined the species' proportions and thereby estimated the tree species' composition in each department.

### 3.3 Portfolio selection

The results obtained from the previous section were used as inputs in the portfolio selection model.

The average productivity was interpreted as the annual productivity and the variance as the species-specific vulnerability or risk. The covariance matrix was applied to compute the portfolio vulnerability. Three major statistic outputs were obtained: the portfolio average productivity, the vulnerability, and the slope signifying the portfolio  $\frac{\text{Productivity}}{\text{Vulnerability}}$  ratio.

The portfolio productivity corresponds to the sum of the productivities of all species weighted by their proportions in the forest stand and subtracted from the risk-free productivity. We defined the risk-free productivity as equal to the lowest productivity among the tree species. The underlying assumption is that the risk-free productivity, equivalent to the financial risk-free asset, is given by the lowest achievable productivity.

The first step consisted in calculating the productivity and the vulnerability of the current tree species' composition portfolio. This current portfolio is very likely not to be an optimal portfolio. We then minimized the portfolio vulnerability for a fixed portfolio target productivity; given the risk of climate change and the risk of recruitment failure, we chose the current portfolio productivity or status quo to be the anchor target. The outputs the model generates are the proportions of tree species. We ran the model for a range of portfolio target productivities. As

the productivity is expected to dwindle in the forthcoming years, 11 simulations were made with targets of 7 lower portfolio productivities and 3 higher portfolio productivities. The results were then used to build the efficiency frontier curve by plotting the expected productivities against the corresponding vulnerabilities. This curve represents the possibility for the forest owner to cope with the minimum risk for a given productivity.

Obviously, the tree species' composition varies in accordance with the simulated optimal portfolio. This variation ( $\Delta$ ) has also been estimated. The gap between the current portfolio and the optimal portfolio for the same productivities of tree species has been examined. In this manner, we could pay attention to the distance between the current portfolio and the optimal portfolio.

Once we obtained the range of optimal portfolios, we infused the species' probabilities of presence by years 2050 and 2100 and computed the survival probabilities of the portfolios. We assumed that the future portfolios' states only depended on the current state, and thus applied the probabilities through a Markov process. The latter corresponds to the product of a row vector  $(\mathbf{1}, \mathbf{n})$ , which gives the probability of presence for the  $\mathbf{n}$  tree species, and a column vector  $(\mathbf{n}, \mathbf{1})$ , reflecting the proportion of each species in the stand. To enable the comparison between the medium and the long terms, the survival probabilities have been computed at T2 and T3.

## 4 Results

Given the large numbers of territorial units (90 departments and 22 regions) and optimal portfolios' simulations, we decided to show the detailed results for the department of Meurthe-et-Moselle, and the global results for the French territory.

Meurthe-et-Moselle is one of the fourth departments of the Lorraine region. In the latter, the forested area covers 880 000 ha and represents 37% of the land surface (the national average is 29%). The forest sector in Lorraine is mainly subjected to the public management of the French National Forestry Office (ONF) and is one of the most important in terms of harvested volumes in France, that is, around 2.8 Mm<sup>3</sup> in 2011 (ONF, 2013). The region shows an important variety of climatic areas and tree species. This underlines its economic and ecological importance. In the Meurthe-et-Moselle department, the forested area covers 167 000 ha and represents 32% of the land surface.

Table 1: Average productivity (in m<sup>3</sup>.ha<sup>-1</sup>.year<sup>-1</sup>) and vulnerability (in %) in the Meurthe-et-Moselle department

Species	Mean productivity	Vulnerability
Pedunculate oak	1.80	5.5%
Sessile oak	2.18	1.9%
Common beech	3.26	4.3%
Scots pine	3.44	204.7%

Table 1 displays the average productivity and vulnerability of tree species present in the department and computed from the historical data. Despite the easy comparison between the productivities, a careful attention must be paid to the respective vulnerabilities. We can notice that productivity and vulnerability are not always proportional. Indeed, the Pearson's  $r$  correlation coefficient of 0.296 ( $p < 0.005$ ) implies an absence of linear correlation between the two factors. Hence, highly productive species can show low vulnerability and vice versa. In detail, Common beech is the most frequently encountered species, generally followed by Pedunculate oak. Scots pine, which is the most productive species, is also the riskiest.

All the computed covariances were assembled to form the covariance matrix (Table 2). The higher the covariance, the riskier the investment. Species with covariances close to zero are the most effective in reducing the overall portfolio vulnerability. The weight in the portfolio corresponds to the proportion of the species in the department, that is, the part of the forested area occupied by this species. In finance, this would correspond to the weight of an asset in the portfolio. The weighted sum of covariances is calculated for each species. The sum of sub-totals indicates the whole portfolio vulnerability.

Table 2: Meurthe-et-Moselle covariance matrix

Species	Weights	30.92%	15.96%	0%	3.12%
Pedunculate oak	30.92%	0.0531357	0.0233746	0.0407517	−0.061749
Sessile oak	15.96%	0.0233746	0.0180579	0.0243981	−0.113869
Common beech	0%	0.0407517	0.0243981	0.0417561	−0.186863
Scots pine	3.12%	−0.061749	−0.113869	−0.186863	1.982599
Total	100%	0.0092	0.0103	0.0000	−0.0010

Fixing a range of portfolio productivity targets yields a range of optimal portfolios. The results are gathered in Table 3. The latter includes the average portfolio productivities and vulnerabilities as well as the slope of the efficient frontier. For each optimal portfolio, the optimizing standing timber composition is given in percentage. Because a decrease in productivity is expected in the forthcoming years, most of the simulations correspond to scenarios of decreases in productivity.

Two columns in Table 4 are of key importance. The Current column does not show an optimal portfolio: it shows the portfolio profile with the current composition. The Optimum column reconfigures the weights of tree species in order to minimize the portfolio vulnerability while maintaining the current productivity. The comparison between the two is given by  $\Delta$ . For instance, the portfolio vulnerability in the Meurthe-et-Moselle department could be reduced by increasing the weights of Sessile oak and Scots pine. In contrast, Pedunculate oak and Common beech should be less favored.

Table 3: Current and optimal portfolios in the Meurthe-et-Moselle department

	Current	Optimum	$\Delta$
Mean productivity	0.6945	0.6945	0%
Vulnerability	2.82%	1.13%	−1.69%
Pedunculate oak	32.29%	0%	−32.29%
Sessile oak	27.24%	71.68%	44.44%
Common beech	38.45%	21.46%	−16.99%
Scots pine	2.02%	6.86%	4.84%

The deviation of the tree species' composition from the current state indicates the distance to the optimum. The greater the gap, the less efficient the on-going tree species' distribution.

According to Proposition 1, the range of optimal portfolios enables to build the efficient

frontier curve (Fig. 2)<sup>4</sup>. It illustrates the variations of the productivity-vulnerability ratios and, by extension, the opportunity set that offers the highest portfolio productivity for a given level of risk. The inflection point on the left point locates the minimum variance portfolio; it is not necessarily the portfolio with the minimum productivity. The choice of a particular optimal portfolio depends on the degree of risk aversion that is specific to the decision-maker. A risk-averse individual will tend to favor a portfolio in the vicinity of the inflection point.

The current portfolio is not represented for the reason of its excessive vulnerability. This implies that significant changes need to be undertaken so as to obtain efficient combinations.

Figure 2: Meurthe-et-Moselle efficient frontier with average productivities (ordinates) at different levels of vulnerabilities (abscissa)

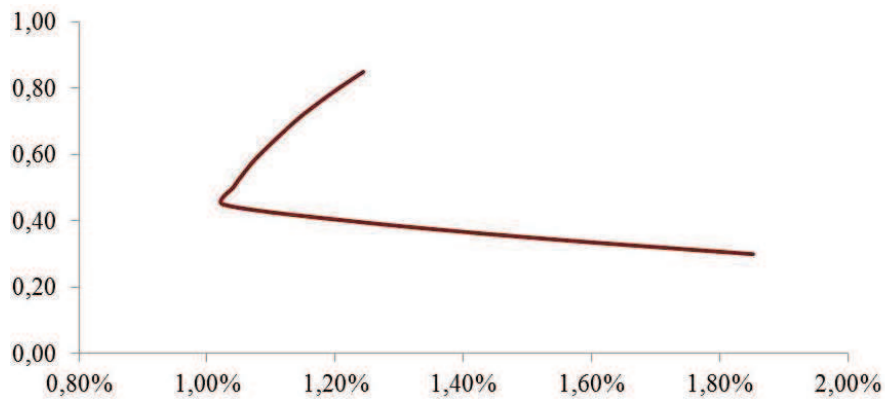
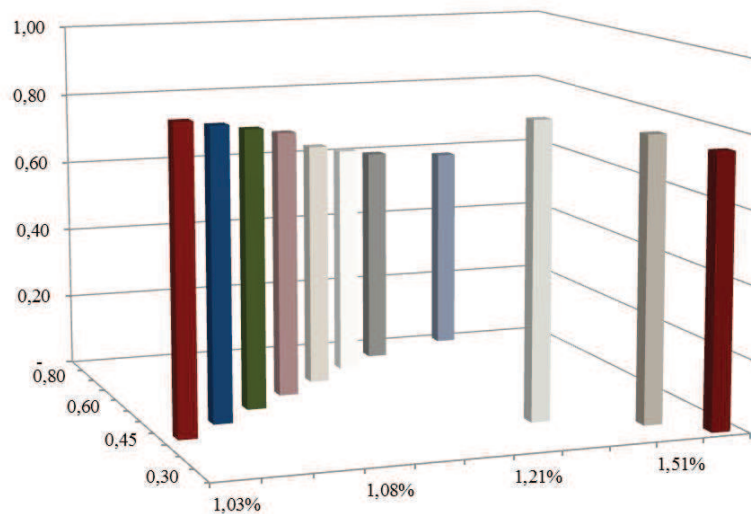


Table 3 also gives the levels of portfolios' survival at T2 and T3 as a complementary decision-aid tool. The probabilities are closely linked to the species' proportions and vary within regions. In some cases, the maximum probability of survival corresponds to the minimum vulnerability. Nevertheless, in most of the cases, it does not. Fig. 3 provides an illustration of T2.

Figure 3: Portfolios survivals at T2 in the Meurthe-et-Moselle department with survival probabilities (applicates) of average productivities (ordinates) at different levels of vulnerabilities (abscissa)



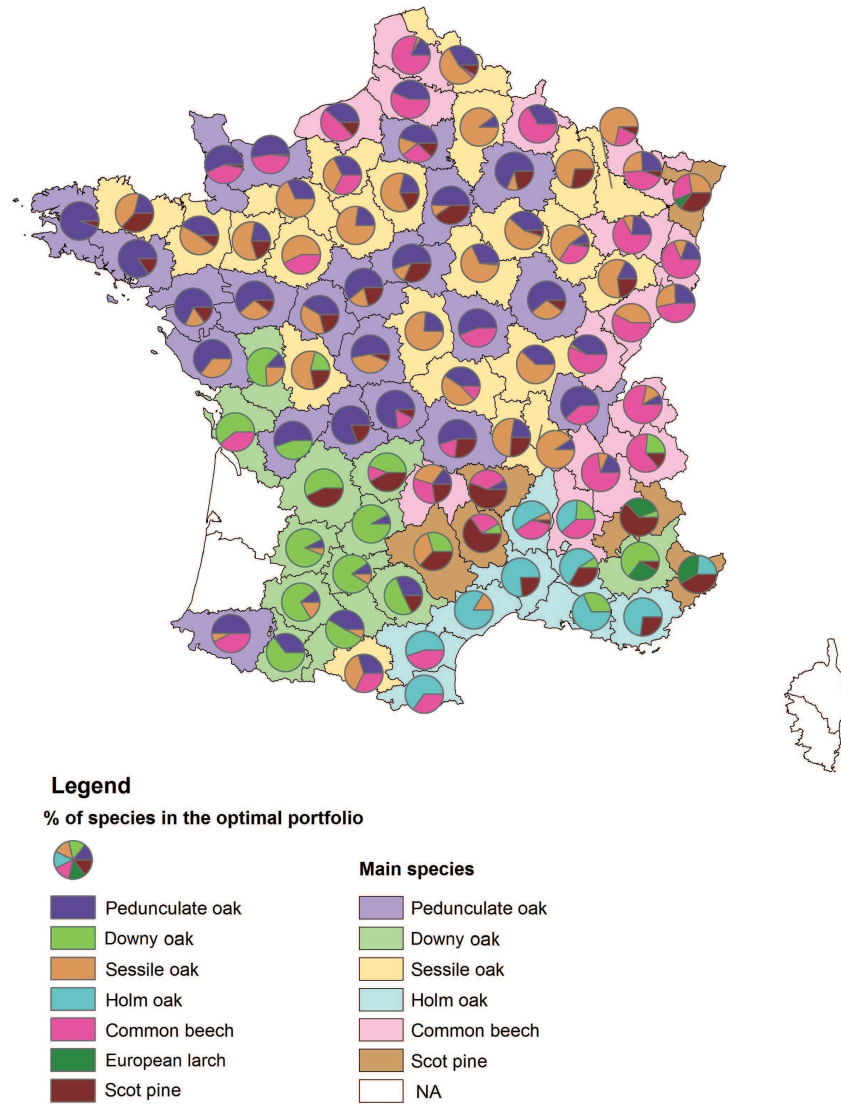
<sup>4</sup>The least vulnerable portfolio stands at the inflection point illustrating Proposition 2.





The same methodology is applied to each French department in order to obtain the optimal species' compositions. We then map our results in Fig. 4. Some French departments are characterized by monocultures, as the departments of Landes and Gironde in the South-West, where only Maritime Pine appears. In such a case, no optimal output can be yielded.

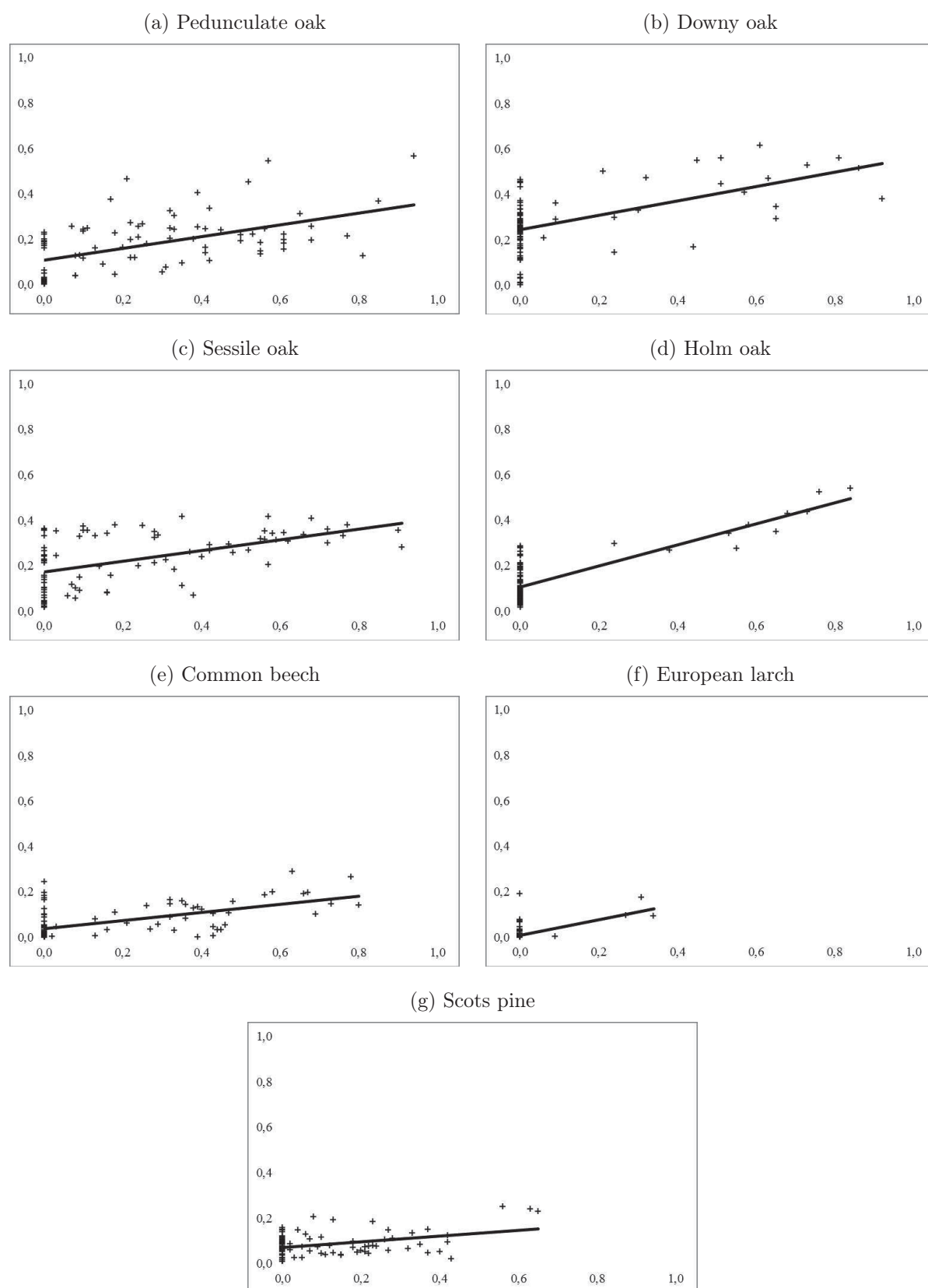
Figure 4: Optimal portfolio per French administrative department



The nationwide results presented in Fig. 4 corroborate the previous studies and the current recommendations. On the long term, the presence of the historically implanted Common beech is to be called into question (Hanewinkel et al., 2012). As indicated by Badeau et al. (2010), this species may observe a radical decrease in its natural range by 2100 and will only subsist in the north-eastern part of France or in mountain areas. Furthermore, the balance between Sessile oak and Pedunculate oak should be modified in favor of Sessile oak, which has proven to be more adaptive and drought-resistant (Bréda et al., 1993; Vivin et al., 1993).

Let us now see whether there exist linear relationships between the species' optimal portfolio weights and their probabilities of presence (Table 5). The clouds of dots are in Fig. 5.

Figure 5: Clouds of dots situating the weights of the optimal portfolios (abscissa) and the probabilities of presence (ordinates)





We observe significant positive Pearson’s correlations for an estimate of  $p < 0.01$  for all species, such that greater weights in the optimal portfolios correspond to higher probabilities of presence in 2050. In greater detail, the Holm oak’s portfolio weights are highly correlated to its probabilities of presence throughout the French territory. The weights of Pedunculate oak, Downy oak, Sessile oak and Common beech are moderately correlated to their respective probabilities of presence, whereas the Scots pine’s portfolio weights and probabilities are weakly correlated.

Table 5: Pearson  $r$ -correlation coefficient

Species	$r$ -coefficient
Pedunculate oak	0.532**
Downy oak	0.569**
Sessile oak	0.522**
Holm oak	0.804**
Common beech	0.584**
European larch	0.612**
Scots pine	0.392**

\*\* Correlation is significant at the 0.01 level.

We thus obtain an intuitive result, in the sense that the weights of the most resilient species are the ones that should be privileged in the optimal portfolios. Thereby, forest ecology and forest economics lead to convergent conclusions, which confirms the soundness of our pluridisciplinary approach.

## 5 Discussion and conclusions

The objective of this paper was to see whether it was possible to determine the optimal composition of standing timber using the Markowitz portfolio theory. In this sense, we answer the call by Legay et al. (2008) who question the benefits obtained from the mixture of forest species in terms of forest management. The use of a well-tested financial method in the interest of answering the technical forestry issues is a way to strengthen the bond between the two disciplines. Indeed, the use of non-financial values dependent on biotic and abiotic factors made possible to establish a parallel between finance and forest science. Having said that, we must acknowledge that our approach is not supposed to be an absolute decision-aid tool; forest management is a science in which many different aspects are taken into account. Since historical data could not be used alone to predict the future states of the forests, the ecological probabilities of presence have been included. The latter support the soundness of the simulations.

We used the historical data from IFN, which has been collected over a long period and originally not designed for these kinds of simulations. For example, the wood production data only concern the main species. However, the forest stand is not monospecific and several sub-species are frequently found. It is thus impossible to measure or calculate the area covered by one single species. Since the volume of wood produced by one species is divided between the areas occupied by several species, a larger or smaller underestimation of productivity may have occurred. Notwithstanding, this issue has been bypassed by considering large surfaces such as administrative departments and regions.

Another issue dealt with the time span. Data from the early 70’s are available but cannot be used due to the changes in the IFN inventory methodology. These old data are not computable

via the current extraction tool used by IFN. We have decided to solve the problem by simulating past productivities using a geometric series formula.

Still, the great advantage of these data is their availability. The inventory method varied in time, but not from the geographical point of view. It means that the results are fully comparable between the regions.

## 5.1 The portfolio selection theory in a forest context

If the portfolio selection has proven to be efficient in a forest context (Neuner et al., 2013; Knoke et al., 2005, 2008; Knoke, 2008), this is the first time that the tree productivity has been completely assimilated to a financial return. Despite being innovative, this approach also yields new questions and limitations. The first concerns the number of species used in the simulations. Since the successive simulations make the proportion of tree species variable, it is irrelevant to conduct simulations in departments with less than two species. For a two-species portfolio, only one combination to obtain the targeted productivity is suitable; the minimization of the portfolio vulnerability proves to be impossible. A solution could be found by considering more species on the national scale, or by regionalizing the choice of species. For example, in Bouches-du-Rhône, it would make sense to consider Aleppo pine (*Pinus halepensis*, Mill.) which covers 87.5% of the forested area.

Another important limitation concerns the planted species. In France, some species are artificially planted and presently cover large surfaces. This is particularly noteworthy in the South-West Landes forest, which represents the largest forest of planted Maritime pine (*Pinus pinaster*, Aiton) in Europe. Douglas fir (*Pseudotsuga menziesii* and, Mirb.) and Poplars (*Populus* genus), with numerous cultivars, are also concerned. Most of the time, these species are planted with the purpose to create an even-aged stand. After a clear cut, the productivity falls down to zero and this drop would be interpreted by the portfolio model as a critical risk. This phenomenon could be attenuated via uneven-aged forests. Likewise, the bias could be counteracted by considering larger surfaces provided that plantations are not minority.

In the same register, natural hazards have a real influence on the tree species' assessment. The first problem is related to geography, for the impact of natural disasters such as storms is not fairly distributed across the country. As an example, the 1999 Lothar storm hit the north of France; the southern part of the country was not affected. Therefore, the same tree species will be considered as riskier in the north. Moreover, if the wind sensibility could be regarded as a common factor of vulnerability, this is no longer true for the forest fires. Mortality due to forest fires does not depend on the tree characteristics, but is only related to location: fires are much more common in the south of France. These limitations cannot be avoided.

## 5.2 Adequacy with forest management

Although the present study is not intended to give definitive guidelines for forest management in the French departments, its results could be consulted on a larger scale such as the ecological regions. In all cases, the aim was to show the potentiality of the portfolio selection theory in a forest context. Although being promising, the methodology needs improvement.

Only seven species were used to run the simulations. Even if these species are among the most represented, they are not representative of the forests on a regional scale. Taking account of the regional species would allow obtaining more accurate results. Unfortunately, the probabilities of presence of these species lack, which prevents from assessing the portfolios' survivals.

Moreover, many studies have shown that changes in environmental conditions lead to different species' distributions (Walker et al., 2002; Iverson et al., 1999). Southern species may become adapted to northern conditions. For instance, the Holm oak tree range limit could be pushed northward within 50 years (Hanewinkel et al., 2012). Even if the probabilities of presence of some

tree species are known by 2050 and 2100, it would be impossible to compute an optimal portfolio. The calculation of both the productivity and the vulnerability is based on historical data, yet they do not exist if the species have not been previously settled. This portfolio calculation method suits the well-established species; it could become a predictive model of the forests' compositions only if the projections of productivities were available.

Finally, this opening model could be supplemented with additional inputs. The growth rate of a tree is determined by a myriad of factors such as the inter- and intra-specific competition, soil potential or water availability. The additional factors would lead to the calculation of composite indices of productivity and variability. The extension would be especially justified on the scale of a single forest. Conditions vary dramatically from one forest to another; to avoid biased results and irrelevant conclusions, detailed inputs are needed to analyze small forest areas. In sum, this work could be the foundation for more extensive investigations.

The results would have been different had we focused on financial returns. Thereby, a species may show a low variance in productivity while being subjected to highly fluctuating timber prices. The priority was given to the preservation of the means of production, that is, the expected amount of wood: a goal for which the model proves to be suitable.

## Acknowledgment

This work was financially supported by a grant overseen by the French National Research Agency through the Laboratory of Excellence ARBRE, a part of the Investments for the Future Program (ANR 11 – LABX-0002-01). It was also supported by the French National Forestry Office through the Forests for Tomorrow International Teaching and Research Chair. The authors are indebted to Edwin van der Werf (Wageningen University) for his comments and suggestions toward this work.

# References

- [1] Badeau, V., Dupouey, J.L., Cluzeau, C., Drapier, J., Lebas, C. (2010), "Climate change and the biogeography of French tree species: first result and perspectives ". In *Forests, carbon cycle and climate change*, Editions Quae, c/o INRA; Versailles; France, p.231-252.
- [2] Bosu, P., Cobbinah, J R., Nichols, J.D., Nkrumah, E.E., Wagner, M.R. (2006), "Survival and growth of mixed plantations of *Milicia excelsa* and *Terminalia superba* nine years after planting in Ghana", *Forest Ecology and Management* doi:10.1016/j.foreco.2006.05.032.
- [3] Bréda, N., Granier, A., Dreyer, E., Cochard, H. (1993) "Field comparison of transpiration, stomatal conductance and vulnerability to cavitation of *Quercus petraea* and *Quercus robur* under water stress", *Annales des Sciences Forestières* 50: 571-582.
- [4] Buongiorno, J., Peyron, J.L., Houllier, F., Bruciamacchie, M. (1995), "Growth and management of mixed-species, uneven-aged forests in the French Jura: implications for economic returns and tree diversity ", *Forest Science* 41(3): 397-429.
- [5] Charru, M., Seynave, I., Morneau, F., Bontemps, J.D. (2010), "Recent changes in forest productivity: An analysis of national forest inventory data from common beech (*Fagus sylvatica* L.) in north-eastern France ", *Forest Ecology and Management* 260: 864-874.
- [6] Cheaib, A., Badeau, V., Boe, J., Chuine, I., Delire, C., Dufrêne, E., François, C., Gritti, E.S., Legay, M., Pagé, C., Thuiller, W., Viovy, N., Leadley, P. (2012), "Climate change impacts on tree ranges : model intercomparison facilitates understanding and qualification of uncertainty ", *Ecology Letters* 15: 533-544.
- [7] Clasen, C., Griess, V.C., Knoke, T. (2011), "Financial consequences of losing admixed tree species: A new approach to value increased financial risks by ungulate browsing ", *Forest Policy and Economics* 13: 503-511.
- [8] Evans, J. (2005), "Growth rates over four rotations of pine in Swaziland ", *International Forestry Review* 7(4): 305-310.
- [9] Franklin, J.F., Norris, L.A., Berg, D.R., Smith, G.A. (1999), "The history of DEMO: an experiment in regeneration harvest of Northwestern forest ecosystems", *Northwest Science* 73: 3-11.
- [10] Griess, V.C., Knoke, T. (2011), "Growth performance, wind-throw, insects - meta-analyses of parameters influencing performance of mixed Species stands in boreal and northern temperate biomes", *Canadian Journal of Forest Research* 41: 1141-1159.
- [11] Halpern, C.B., Evans, S.A., Nelson, C.R., McKenzie, D., Liguori, D.A. (1999), "Response of forest vegetation to varying levels and patterns of green-tree retention: an overview of a long-term experiment", *Northwest Science* 73: 27-44.
- [12] Hanewinkel, M., Cullmann, D., Schelhaas, M.J., Nabuurs, G.J., Zimmermann, N. (2012), "Climate change may cause severe loss in the economic value of European forest land ", *Nature Climate Change, Letters*, 1687, 5 pp.
- [13] Iverson, L.R., Prasad, A.M., Schwartz, M.W. (1999), "Modeling potential future individual tree-species distribution in the eastern United States under a climate change scenario: a case study with *Pinus virginiana* ", *Ecological Modelling* 115: 77-93.
- [14] Jacobsen, J.B., Thorsen, B.J. (2003), "A Danish example of optimal thinning strategies in mixed-species forest under changing growth conditions caused by climate change", *Forest Ecology and Management* 180: 375-388.

- [15] Kelty, M.J. (1992), “Comparative productivity of monocultures and mixed-species stands”. In: *The Ecology and Silviculture of Mixed-Species Forests*, Kluwer Academic Publishers, 31 mars 1992, 308p.
- [16] Knoke, T., Stimm, B., Ammer, C., Moog, M. (2005), “Mixed forests reconsidered: A forest economics contribution on an ecological concept”, *Forest Ecology and Management* 213: 102-116.
- [17] Knoke T., Ammer C., Stimm B., Mosandl R. (2008), “Admixing broadleaved to coniferous tree species: A review on yield, ecological stability and economics”, *European Journal of Forest Research* 127: 89-101.
- [18] Knoke, T. (2008), “Mixed forests and finance – methodological approaches”, *Ecological Economics* 65: 590-601.
- [19] Legay, M., Cordonnier, T., Dhôte J.F. (2008), “Des forêts mélangées pour composer avec les changements climatiques”, *Revue Forestière Française* LX 2: 181-190.
- [20] Lu, H.C., Buongiorno, J. (1993), “Long and short term effects of alternative cutting regimes on economic returns and ecological diversity in mixed-species forests ”, *Forest Ecology and Management* 58(3-4): 173-192.
- [21] Markowitz, H. (1952), “Portfolio Selection”, *Journal of Finance* 7: 77-91.
- [22] Markowitz, H. (1959), “Portfolio Selection: Efficient diversification of investments”, Wiley, New York, NY.
- [23] Mayer, P., Brang, P., Dobbertin, M., Hallenbarter, D., Renaud, J.P., Walthert, L., Zimmermann, S. (2005), “Forest storm damage is more frequent on acidic soils ”, *Annals of Forest Science* 62: 303-311.
- [24] Morin X., Fahse L., Scherer-Lorenzen M., Bugmann H. (2011), “Tree species richness promotes productivity in European temperate forests through a strong complementarity effect ”, *Ecology Letters* 14: 1211-1219.
- [25] Morneau, F., Duprez, C., Hervé, J.C. (2008), “Les forêts mélangées en France métropolitaine, caractérisation à partir des résultats de l’Inventaire Forestier National”, *Revue Forestière Française* 2: 107-120.
- [26] Neuner, S., Beinhofer, B., Knoke, T. (2013), “The optimal tree species composition for a private forest enterprise - applying the theory of portfolio selection ”, *Scandinavian Journal of Forest Research* 28: 38-48.
- [27] Nichols, J.D., Bristow, M., Vanclay K.K. (2006), “Mixed-species plantations : prospects and challenges”, *Forest Ecology Management* 233: 383-390.
- [28] Office National des Forêts (2013), “Chiffres-clés en Lorraine”. ONF. [Online]. Available : <http://www.onf.fr/lorraine/sommaire/onf/chiffres-cles/@@index.html>. [2013, August 19].
- [29] Powers, R.F. (1999), “On the sustainable productivity of planted forests ”, *New Forests* 17: 263-306.
- [30] Robert, N., Vidal, C., Colin, A., Hervé, J.C., Hamza, N., Cluzeau, C. (2010), “France. In: Tomppo, E., Gschwantner, T., Lawrence, M., McRoberts, R.E. (Eds.), *National forest inventories pPathways for common reporting*. Springer, Heidelberg, Dordrecht, London, New York, pp. 207-221.

- [31] Roessiger, J., Griess, V. C., Knoke, T. (2011), “May risk aversion lead to near-natural forestry? A simulation study”, *Forestry* 84(5): 527-537.
- [32] Roman-Amat, B. (2007), “Préparer les forêts françaises au changement climatique”. Rapport à MM. Les Ministères de l’Agriculture et de la Pêche et de l’Ecologie, du Développement et de l’Aménagement Durables, December.
- [33] Schou, E., Bredhal Jacobsen, J., Lokke Kristensen K. (2012), “An economic evaluation of strategies for transforming even-aged into near-natural forestry in a conifer-dominated forest in Denmark”, *Forest Policy and Economics* 20: 89-98.
- [34] Schütz, J.P., Götz, M., Schmid, W., Mandallaz, D. (2006), “Vulnerability of spruce (*Picea abies*) and beech (*Fagus sylvatica*) forest stands to storms and consequences for silviculture ”, *European Journal of Forest Research* 125: 291-302.
- [35] Spiecker, H. (2000), “Growth of Norway spruce under changing environmental conditions in Europe”, Emil Klimo, Herbert Hager and Jiri Kylhavy (eds.), *Spruce Monocultures in Central Europe - Problems and Prospects*, EFI Proceedings 33.
- [36] Tilman, D., Lehman, C. and Thomson, K. (1997), “Plant diversity and ecosystem productivity: theoretical considerations ”, *Proceedings of the National Academy of Sciences* 94: 1857-1861.
- [37] Thomson, T.A. (1991), “Efficient combinations of timber and financial market investments in single-period and multiperiod portfolios ”, *Forest Science* 37: 461-480.
- [38] Vettenranta, J. (1996), “Effect of species composition on economic return in a mixed stand of Norway spruce and Scots pine”, *Silva Fennica* 30(1): 47-60.
- [39] Vivin, P., Aussenac, G., Levy, G. (1993), “Differences in drought resistance among 3 deciduous oak species grown in large boxes”, *Annals of Forest Science* 6: 571-582.
- [40] Walker, K.V., Davis, M.B., Sugita, S. (2002), “Climate change and shifts in potential tree species range limits in the Great Lakes Region”, *Journal of Great Lakes Research* 28: 555-567.