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A novel differentiator: A compromise between super twisting and linear algorithms

M. Ghanes¹, J.P. Barbot², L. Fridman³ and A. Levant⁴

Abstract—Based on the frequency argument, a novel second order sliding mode differentiator with a variable exponent α is proposed in this article. The super twisting differentiator ($\alpha = 0,5$) is not sensible to perturbation but its accuracy is degraded when the signal is affected by the noise. The linear observer ($\alpha = 1$) has better property in the presence of noise but is less robust to perturbations. The goal of this paper is to propose a trade-off between the exact differentiator and linear observer. To reach this objective, the parameter α is made variable. In the absence of noise α goes to 0,5 and tends to 1 when the noise increases. In free-noise case and with or without perturbation, the novel differentiator behaves as a super twisting differentiator (exact differentiation). When the signal is affected by noise, only a practical stability of the differentiator is ensured. Finally simulation results are given to show that the novel differentiator has better performances compared to differentiators having α fixed.

I. INTRODUCTION

The problem of real-time signal differentiation in finite time has been investigated since at least last two decades [2], [9], [10], [11], [12], [13], [14], [17], [19], [22]. To the best of our knowledge, one of the first work in the framework of control theory, was proposed in [7] where the signal trajectory is approximated by a polynomial during a frame time and after that this polynomial is differentiated. Nowadays two main approaches are used, the algebraic one [15] and the second one based on sliding mode or homogeneous techniques. With respect to the differentiator design proposed by Levant [14], there exist several types of sliding mode or homogeneous differentiators (see for example [9] and [22]). The main advantage of such differentiators according to the algebraic [15] or polynomial [7] algorithms is to give an estimation without delay. Nevertheless, even if differentiators are not sensible to perturbations, their accuracy is degraded when the signal is affected by a noise. On another hand, it is well known that the linear differentiators have good properties with respect to the measurement noise but they are sensible to perturbations and does not ensure a finite-time convergence. To make a trade off between accuracy and noise sensibility, a first work dealing with a variable exponent law α was designed in [10] according to the output error, which means that any qualitative noise property is considered. Nevertheless, in [10] and the present work, it is important to highlight that the variable exponent gain α is between 0.5 (which corresponds to the exact differentiator)

and 1 (that corresponds to the linear differentiator). The main difference between our previous work [10] and the present contribution is that the exponent gain α is made variable with respect to the magnitude of the signal high frequency. Note that variable gain (not exponent gain) schemes of sliding mode controllers exist in the literature (see for example [1], [3], [4], [8], [16], [20], [21], [23], [25]). These schemes are introduced in order to minimize the amplitude variation of the control law able to reject the perturbation with reducing the energy consumption. It is important to note that in differentiators design, energy consumption is not relevant contrarily to the reduction of the noise measurement effect.

The remainder of the paper is organized as follows. Section II introduces a formal state space representation generating the signal to be differentiated. In section III, the dedicated differentiator is proposed. A convergence proof ensuring a practical stability of the differentiator is given when the signal to be differentiated is affected by a noise and an exact differentiation is obtained in a free-noise case. Simulation results are given in section IV illustrating the performances and the effectiveness of the proposed differentiator compared to cases when α is fixed.

II. SIGNAL GENERATION MODELING

Considering any ideal signal x_1 with at least a second bounded derivative. This signal can be represented by a double integrator with bounded unknown input u . In practice there exist always some noise measurement, then this ideal signal $y = x_1$ becomes a real one y_m , as it is given hereafter

$$\Sigma : \begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= u \\ y_m &= x_1 + w \end{cases} \quad (1)$$

where $x(t) \in R^2$ is the state of the system, $u(t) \in U \subset R$ is the unknown input, $y_m(t) \in R$ represents the output of the system and w refers to a white measurement noise.

Assumptions 1:

1. The unknown input $u \in U$ is supposed to be bounded, i.e., $|u(t)| \leq u_{max} \forall t \geq 0$, where u_{max} is the constant bound value of $u(t)$.
2. The measurement noise w is supposed to be bounded, i.e., $|w(t)| \leq w_{max} \forall t \geq 0$, where w_{max} is the constant bound value of $w(t)$.

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III. PROPOSED DIFFERENTIATOR

First of all, in our previous work [10] a differentiator is designed by making the exponent gain α variable according to the error output measurement. By doing this, it is not possible to distinguish clearly the effects of noise and perturbation respectively. Hereafter, a new type of signal differentiator is introduced by making the exponent gain α variable with respect to the magnitude of the high frequency signal. Roughly speaking, the high frequency signal, which is close¹ to the high frequency noise, is isolated and its magnitude is used to drive the exponent gain α . So if there is no noise, α is assigned to be equal to 0.5 (exact differentiator) and to tend to 1 (linear differentiator) when the noise increases.

The novel differentiator of y_m is proposed as follows:

$$D : \begin{cases} \dot{z}_1 &= z_2 + k_1 \mu |e_{1m}|^\alpha \text{sign}(e_{1m}) \\ \dot{z}_2 &= k_2 \alpha \mu^2 |e_{1m}|^{2\alpha-1} \text{sign}(e_{1m}) \\ \dot{z}_3 &= -\tau z_3 + \tau |y_{mhf}| \\ \alpha &= 0.5 \left(1 + \frac{z_3}{z_3 + \epsilon}\right) \\ y_{mhf} &= \frac{s'^4}{(s'^2 + 0.7654s' + 1)(s'^2 + 1.8478s' + 1)} y_m \\ s' &= \frac{s}{w_{cut}} \\ \hat{y} &= z_1 \\ e_{1m} &= y_m - z_1 \end{cases} \quad (2)$$

where z_1 and z_2 are respectively estimates of x_1 and x_2 . A mixed time-domain/Laplace-domain notation is adopted.

The differentiator (2) includes two filters.

The first filter (FF) is a high pass fourth order filter (Butterworth filter [6]) used to capture the magnitude of the high frequency signal (y_{mhf}). The second filter (SF) z_3 is a low pass first order filter of $|y_{mhf}|$.

The parameter α depends on the SF output in order to be variable with respect to the magnitude of the high frequency noise.

Remark 1: In (2), when α is fixed, according for example to [18], the degree of homogeneity $d = \alpha - 1$ and the weights r_1 and r_2 are equal to 1 and α respectively. In our case, the assumption of a fixed α is always closely satisfied because α is a slowly varying variable² and is satisfied in a steady-state case.

A. Gains and parameters tuning

- $k_i > 0$, $i = 1, 2$ are constants and refer to the gains of the linear part (see [17]). They are chosen such that the linear part of the derivative of the Lyapunov function candidate (see the proof of Theorem 1) has appropriate pole placement.

¹The decomposition of the unknown input u in Fourier series and its 2-times integration show that high frequency terms are divided by the square of the high frequency input. Consequently, as the input is bounded, the input high frequency effect on the signal is negligible.

²It is due to the fact that the SF is a slowly varying variable ($\dot{z}_3 = 0$) according to the tuning of τ .

- $\mu > 0$ is a constant parameter chosen large enough to cancel the perturbation (unknown input) effect (see the proof of theorem 1).

- ϵ is a constant parameter. It will be chosen greater than 1 to have $\alpha \in [0.5, 1]$. More precisely, when z_3 is close to zero (resp. close to ∞) α tends to 0, 5 (resp. tends to 1).

- $\tau > 0$ is a constant parameter. In order to fix the frequency range of the SF sufficiently low, this parameter will be chosen sufficiently small (singular perturbation argument).

- w_{cut} is the cutoff frequency of the FF. This frequency should be greater than the frequency of the ideal signal y (generated by system (1)).

- The damping ratio of the FF is given with 0.7654 and 1.8478. These values are obtained by developing the Butterworth polynomial (normalized denominator polynomial) $B_n(s') = \prod_k^n \frac{(s' - s'_k)}{w'_c}$ where $s'_k = w'_c \exp \frac{j(2k+n-1)\pi}{2n}$, with $k = 1, 2, 3, \dots, n$, $w'_c = 1$ and $n = 4$.

B. Stability analysis

Let us define the state estimation error as

$$\begin{cases} e_1 = x_1 - z_1 \\ e_2 = x_2 - z_2. \end{cases}$$

The following change of coordinates is being useful to prove the finite time convergence as it was introduced in [17]

$$\begin{cases} \Lambda_1 = \mu |e_1|^\alpha \text{sign}(e_1) \\ \Lambda_2 = e_2. \end{cases} \quad (3)$$

In the sequel, the derivative of α will be needed

$$\begin{aligned} \dot{\alpha} &= 0.5 \frac{\dot{z}_3(z_3 + \epsilon) - \dot{z}_3 z_3}{(z_3 + \epsilon)^2} \\ &= 0.5 \frac{\epsilon}{(z_3 + \epsilon)^2} \dot{z}_3. \end{aligned} \quad (4)$$

By exploiting expression (4) the derivative of (3) is

$$\begin{aligned} \dot{\Lambda}_1 &= \mu \alpha |e_1|^{\alpha-1} (\Lambda_2 - k_1 \mu |e_{1m}|^\alpha \text{sign}(e_{1m})) \\ &\quad + \mu \ln(|e_1|) |e_1|^\alpha \text{sign}(e_1) 0.5 \frac{\epsilon}{(z_3 + \epsilon)^2} \dot{z}_3 \\ \dot{\Lambda}_2 &= u - k_2 \alpha \mu^2 |e_{1m}|^{2\alpha-1} \text{sign}(e_{1m}). \end{aligned} \quad (5)$$

In the next theorem, conditions to have the practical stability of the state estimation error (3) are given in the presence of noise. In a free-noise case, an exact differentiation is obtained (Levant's like differentiator).

Theorem 1: If system (1) satisfies assumptions 1-1 and 1-2, then (2) is a second order sliding mode differentiator of (1) with a variable gain exponent α . Moreover, the state estimation error (3) converges to a ball whose radius is function of $|w|$ and $|u|$.

Proof:

Consider the following Lyapunov function candidate

$$V = \Lambda^T P \Lambda$$

whose time derivative along the trajectories of (5) is

$$\dot{V} = \dot{\Lambda}^T P \Lambda + \Lambda^T P \dot{\Lambda} \quad (6)$$

where $\Lambda = (\Lambda_1; \Lambda_2)$ and $P = P^T$ is a positive definite constant matrix.

Dynamics error (5) can be simplified as follows

$$\begin{aligned} \dot{\Lambda}_1 &= \mu \alpha |e_1|^{\alpha-1} (\Lambda_2 - k_1 \mu |e_{1m}|^\alpha \text{sign}(e_{1m})) \quad (7) \\ \dot{\Lambda}_2 &= u - k_2 \alpha \mu^2 |e_{1m}|^{2\alpha-1} \text{sign}(e_{1m}) \end{aligned}$$

according to the singular perturbation argument (see [24]). In fact \dot{z}_3 in (5) can be considered equal to zero due to the requirement on the low-pass filter (SF) given in subsection III-A. Moreover, it can be seen that in $\dot{\Lambda}_1$ of (5) when e_1 tends to zero, the second term is multiplied by 0 ($\lim_{|e_1| \rightarrow 0} \ln(|e_1|) |e_1|^\alpha = 0$) while the first term is multiplied by ∞ ($\lim_{|e_1| \rightarrow 0} |e_1|^{\alpha-1} = \infty$).

Now, in order to rewrite (7) in terms of e_1 instead of e_{1m} , the following inequalities are used

- When $e_1 + w \geq 0$ with $[e_{1m}]^\beta = |e_1 + w|^\beta \text{sign}(e_1 + w)$, then

$$|e_1|^\beta \text{sign}(e_1) - 2|w|^\beta \leq [e_{1m}]^\beta \leq |e_1|^\beta \text{sign}(e_1) + 2|w|^\beta$$

- When $e_1 + w \leq 0$, then

$$|e_1|^\beta \text{sign}(e_1) + 2|w|^\beta \geq [e_{1m}]^\beta \geq |e_1|^\beta \text{sign}(e_1) - 2|w|^\beta.$$

Thus a function $f(e_1, w) \in [-2, 2]$ can be deduced

$$[e_{1m}]^\beta = |e_1|^\beta \text{sign}(e_1) + f(e_1, w) |w|^\beta \quad (8)$$

which allows to rewrite (7) in function of e_1 and w separately

$$\begin{aligned} \dot{\Lambda}_1 &= \alpha \mu |e_1|^{\alpha-1} (\Lambda_2 - k_1 \Lambda_1) + k_1 \mu f_1(e_1, w) |w|^\alpha \\ \dot{\Lambda}_2 &= u \\ &\quad - \alpha \mu |e_1|^{\alpha-1} (k_2 \Lambda_1 - k_2 \mu |e_1|^{1-\alpha} f_2(e_1, w) |w|^{2\alpha-1}) \end{aligned} \quad (9)$$

where in (8), $f = f_1$ for $\beta = \alpha$ and $f = f_2$ for $\beta = 2\alpha - 1$.

Finally a compact form of (9) is obtained

$$\dot{\Lambda} = \phi(A\Lambda + N) + \mathcal{U} \quad (10)$$

where $\mathcal{U} = (0; u)$, $\phi = \alpha \mu |e_1|^{\alpha-1}$,

$$N = \begin{bmatrix} k_1 \mu f_1(e_1, w) |w|^\alpha \\ k_2 \mu |e_1|^{1-\alpha} f_2(e_1, w) |w|^{2\alpha-1} \end{bmatrix}, \text{ and } A = \begin{pmatrix} -k_1 & 1 \\ -k_2 & 0 \end{pmatrix}.$$

Note that the matrix P is chosen such that

$$A^T P + P A = -Q \quad (11)$$

where Q is a constant definite positive matrix and the eigenvalues of A are equal to $\frac{-k_1 \pm \sqrt{k_1^2 - 4k_2}}{2}$. For an appropriate choice of k_1 and k_2 , A is a Hurwitz matrix.

Replacing now (10) in (6) and using (11), the derivative of V becomes

$$\begin{aligned} \dot{V} &\leq -\phi \Lambda^T Q \Lambda + 2\phi N^T P \Lambda + 2\mathcal{U}^T P \Lambda \\ &\leq -\lambda_{\min}(Q) \phi \|\Lambda\|^2 + 2\phi \lambda_{\max}(P) \|\Lambda\| \|N\|_{\max} \\ &\quad + 2u_{\max} \lambda_{\max}(P) \|\Lambda\|. \end{aligned} \quad (12)$$

If $\|\Lambda\|$ verifies the following inequality

$$\|\Lambda\| \geq \frac{2\lambda_{\max}(P)}{\lambda_{\min}(Q)} (\|N\|_{\max} + \frac{u_{\max}}{\phi}) := \rho,$$

\dot{V} (12) is negative, then this ensures that the error with respect to $\|\Lambda\|$ converges to a ball whose radius equal or smaller to ρ . Consequently $|e_2|$ (equal to $|\Lambda_2|$) is smaller or equal than this radius when t tends to ∞ . ■

Corollary 1: In the absence of noise, i.e. $w = 0$ the radius of the ball tends to zero.

Proof: When $w = 0$, z_3 in (2) tends to zero asymptotically, then α goes to 0, 5 and the differentiator (2) becomes Levant's like differentiator (super twisting differentiator) (13). In this case the perturbation due to u is directly canceled by the sign function for sufficiently large μ .

$$D_{\text{exact}} : \begin{cases} \dot{z}_1 &= z_2 + k_1 \mu |e_{1m}|^{0,5} \text{sign}(e_{1m}) \\ \dot{z}_2 &= k_2 0,5 \mu^2 \text{sign}(e_{1m}) \\ \hat{y} &= z_1 \\ e_{1m} &= y_m - z_1. \end{cases} \quad (13)$$

IV. SIMULATION RESULTS

In this section, simulations results of the proposed differentiator are presented. The designed differentiator was implemented in Matlab Simulink environment where an ode1 (Euler) solver is used with a fixed-step size equal to 0.0005.

The parameters of the signal generation modeling (1) are fixed as follows:

- Initial conditions: $x_1(0) = 0$, $x_2(0) = -10$.

- Unknown input:

$$u(t) = \begin{cases} 0 & \text{if } t \in [150s, 300s], [450s, 600s], [750s, 900s] \\ \sin(0.05t) & \text{else.} \end{cases}$$

- Noise: $w(t)$ is a white noise of power 0.5, sample time 0.0005 (fixed step-size), starting seed for random number generator 23541. This noise is physically supposed bounded (its magnitude is physically saturated) and it is multiplied

- by 0 for $t \in [300, 500s]$ and $[900, 1000s]$

- by $\frac{0,1}{35}$ between 500 and 600 s

- by $\frac{0,15}{35}$ for $t \in [0, 300s]$, $[800s, 900s]$

- and finally by $\frac{0,25}{35}$ for $t \in [600s, 800s]$.

According to section (III-A), differentiator (2) parameters are selected as follows:

- Initial conditions: $z_1(0) = z_2(0) = z_3(0) = 0$.

- $k_1 = 10$, $k_2 = 40$, $\mu = 1$, $\tau = \frac{1}{3}$, $w_{\text{cut}} = 10$.

As it can be noticed, the proposed differentiator scheme with a variable exponent α gives good performances (see figures 1, 2 and 3. In these figures (1, 2 and 3), the estimation error e_2 between x_2 and z_2 is better reduced when α is variable compared to the cases when it is all time fixed. Results comparison are made with Levant's differentiator case ($\alpha = 0,5$), linear differentiator case ($\alpha = 1$) and the average between both cases, i.e. $\alpha = 0,75$. With or without perturbation (unknown input) and in the presence of noise, the estimation error goes to a ball whose radius is better reduced than the noted cases (i.e. α is equal to 0.5 or 0.75 or 1) and an exact differentiation is obtained ($\alpha = 0,5$) in a free-noise situation under perturbation (unknown input). This exact differentiation is also obtained at initial conditions (considered also by the differentiator as perturbations). This better result is possible thanks to the proposed variable law α that is displayed in figure 4. The latter takes the better choice. It tends to 1 when the noise (figure 6) increases (Linear differentiator case) and takes 0,5 value (Levant's differentiator case) under unknown input (figure 5) or different initial conditions with a free-noise case. As it was expected, the response of the low pass filter (SF) z_3 displayed in figure 4 increases when the noise increases (α increases) and goes to zero in the free-noise case (α decreases in order to have 0,5 value). Moreover, the results of the second state x_2 and its observation z_2 when α is variable and when it is fixed to $\alpha = 0,5$ are shown in figures 7 and 8, respectively. These results confirm the well founded of the proposed differentiator.

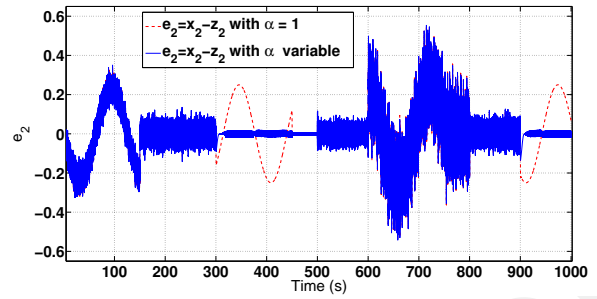


Fig. 3. Estimation error e_2 with α variable and fixed to 1

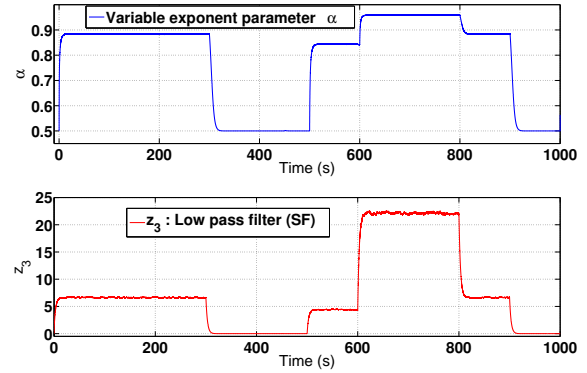


Fig. 4. Exponent variable α and low pass filter z_3

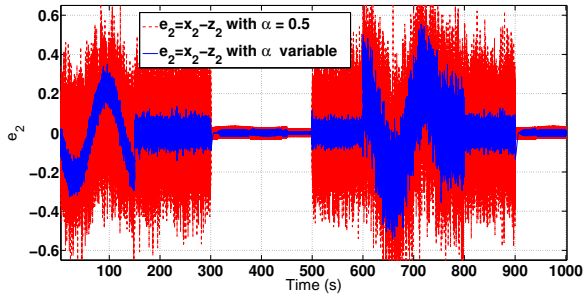


Fig. 1. Estimation error e_2 with α variable and fixed to 0,5

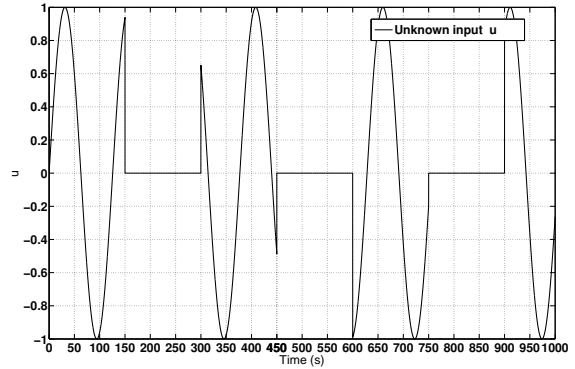


Fig. 5. Unknown u

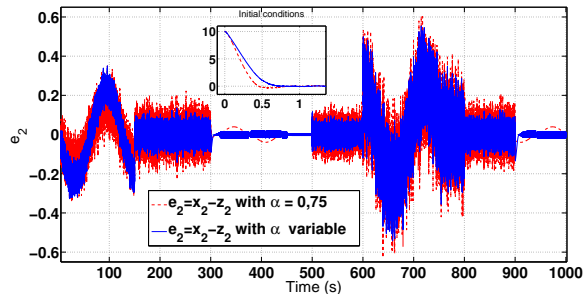


Fig. 2. Estimation error e_2 with α variable and fixed to 0,75

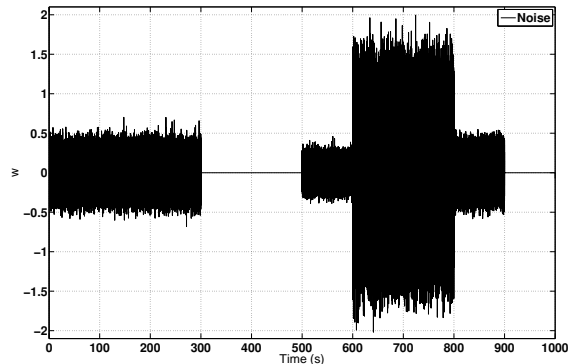


Fig. 6. Noise (added to the state x_1)

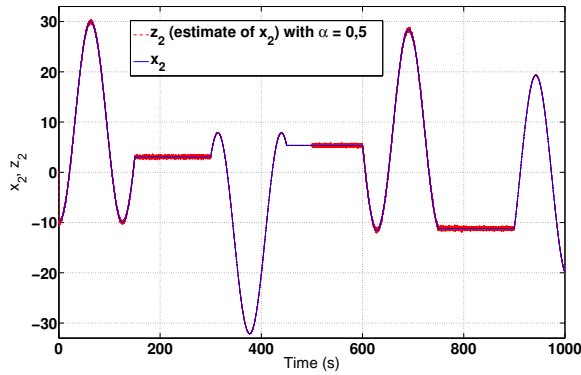


Fig. 7. State x_2 and its estimate z_2 with $\alpha = 0,5$

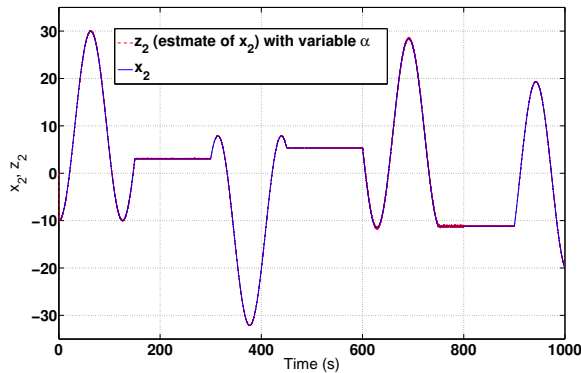


Fig. 8. State x_2 and its estimate z_2 with variable α

V. CONCLUSION

In this paper, a novel differentiator has been presented. Usually, in a free-noise situation an exact differentiator (super twisting differentiator) is used under perturbation while the linear differentiator is employed when the signal to be differentiated is affected by noise. Roughly speaking, both properties, mainly accuracy (exact differentiation) and less sensitivity to noise (linear differentiator) are combined in the proposed differentiator by making the exponent parameter α variable with respect to the high frequency magnitude of the signal. If the signal to be differentiated is affected by noise (resp. not affected by noise), α tends to 1 (resp. to 0,5) and the proposed differentiator behaves as a linear differentiator (resp. as a super twisting differentiator). The dedicated differentiator has been tested in simulation. The obtained results demonstrated that it performs well compared to others differentiators when α is considered fixed. Our on going work will focus on an experimental validation and a more general stability analysis of the designed differentiator.

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