

A Joint Segmentation and Reconstruction Algorithm For 3D Bayesian Computed Tomography Using Gauss-Markov-Potts Prior Model

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Summary: A good quality of volume reconstruction in 3D Computed Tomography (3D) has a key importance for non-destructive testing (NDT) in industry. Indeed, an artifact-free reconstruction with compact and well-distinguishable regions is desirable for controlling the volume. In order to obtain such a reconstruction, we propose an algorithm that enforces a Gauss-Markov-Potts prior model on the volume. Based on a Bayesian framework, the algorithm performs a joint maximization of the posterior distribution of all the unknowns. The algorithm iteratively updates estimation of the volume, its segmentation and any hyperparameters.

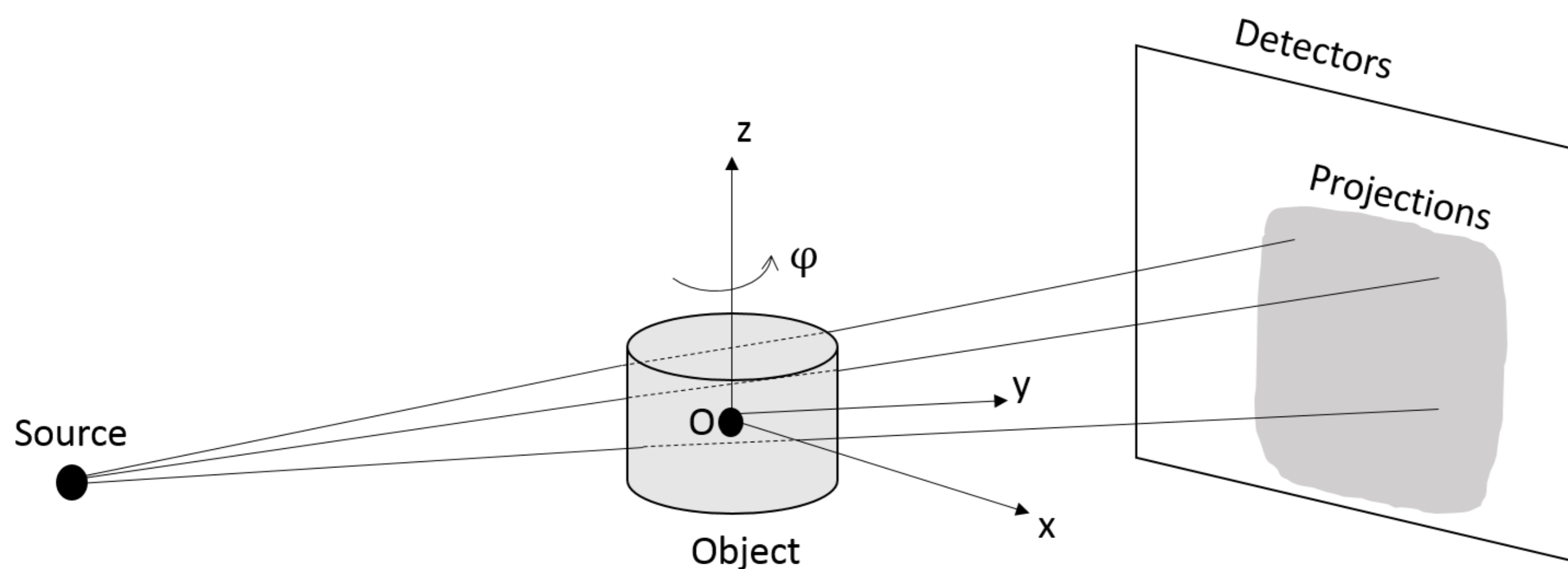
Forward model for 3D X-ray Computed Tomography

- **Objectives:** estimate the interior of an object using X-ray tomography to detect defaults. The object is rotated to have several projections.
- **Difficulties:** ill-posed problem, huge dimension
- **Forward model:** linear model based on Radon transform discretization and accounting for errors

$$\mathbf{g} = \mathbf{H}\mathbf{f} + \boldsymbol{\epsilon}$$

with

$$p(\epsilon_j | \mathbf{v}_{\epsilon_j}) = \mathcal{N}(\epsilon_j | 0, \mathbf{v}_{\epsilon_j}), \forall i$$

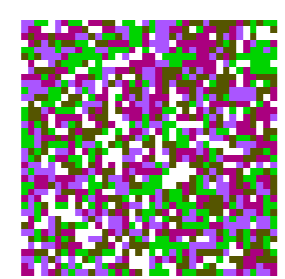


Gauss-Markov-Potts model

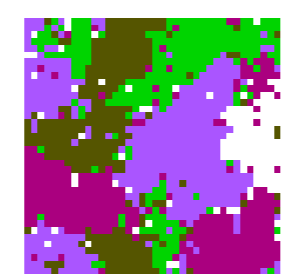
- The object is composed of K different materials which form compact regions \rightarrow hidden Gauss-Markov-Potts field \mathbf{z} .
 - **Homogeneity :** if $\mathbf{z}_j = k$, $p(\mathbf{f}_j | \mathbf{z}_j = k, \mathbf{m}_k, \mathbf{v}_k) = \mathcal{N}(\mathbf{f}_j | \mathbf{m}_k, \mathbf{v}_k)$
 - **Compacity :**
- $$p(\mathbf{z}_j | \mathbf{z}_i, i \in \mathcal{V}(j); \boldsymbol{\alpha}, \gamma_0) \propto \exp \left[\sum_{k=1}^K \alpha_k \delta(\mathbf{z}_j - k) + \gamma_0 \sum_{i \in \mathcal{V}(j)} \delta(\mathbf{z}_j - \mathbf{z}_i) \right]$$
- Potts coefficient γ_0 : huge influence on the granularity of the field.



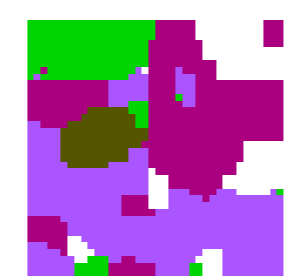
$\gamma_0 = 0.5$



$\gamma_0 = 0.7$

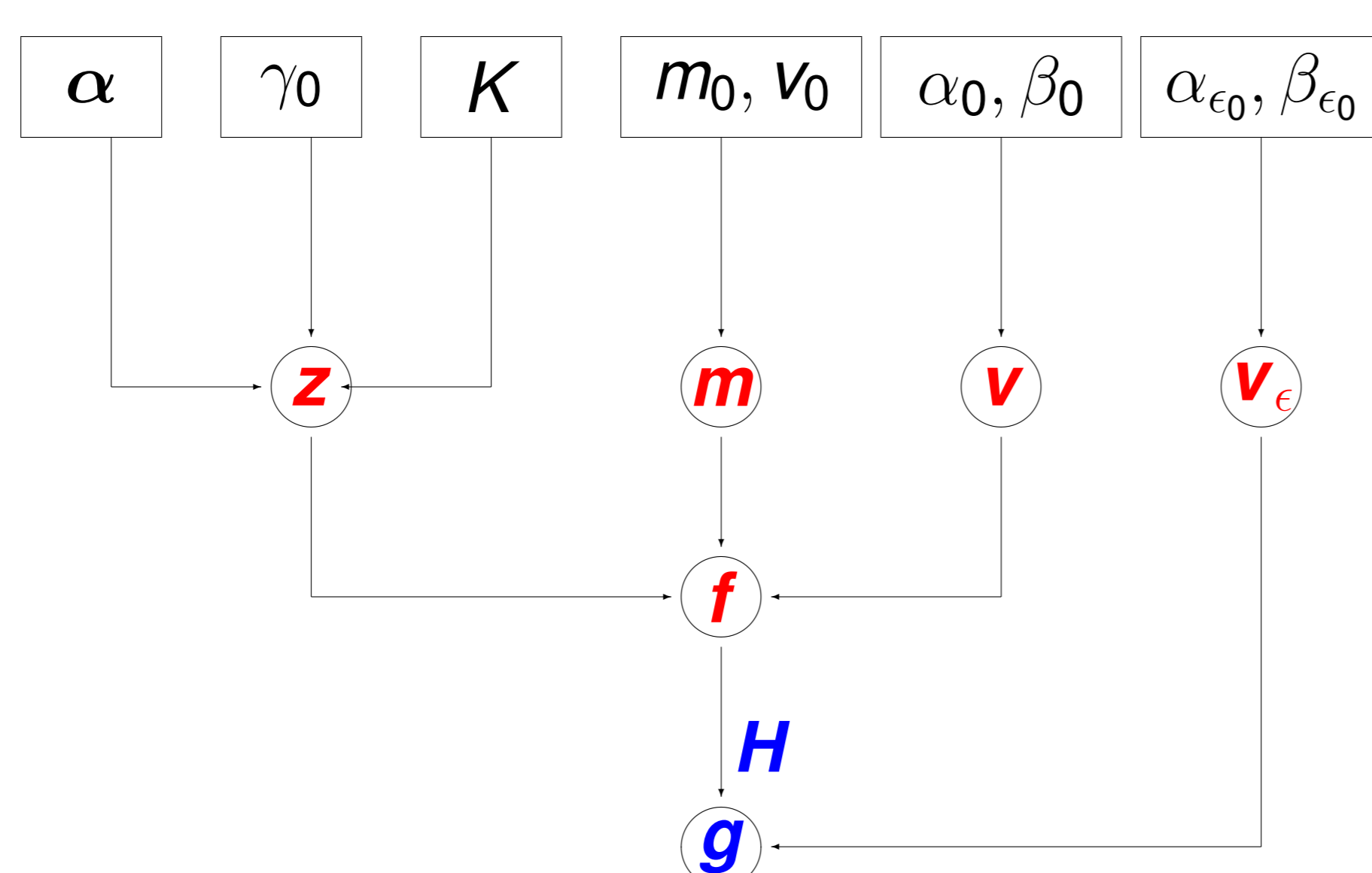


$\gamma_0 = 0.8$



$\gamma_0 = 1.6$

Hierarchical model



- Fix the hyperparameters $\boldsymbol{\alpha}, \gamma_0, \mathbf{m}_0, \mathbf{v}_0, \alpha_0, \beta_0, \alpha_{\epsilon_0}$ and β_{ϵ_0} .
- Estimate \mathbf{f}, \mathbf{z} and the hyperparameters $\mathbf{v}_{\epsilon}, \mathbf{m}$ and \mathbf{v} .

JMAP algorithm

- Maximize joint posterior distribution

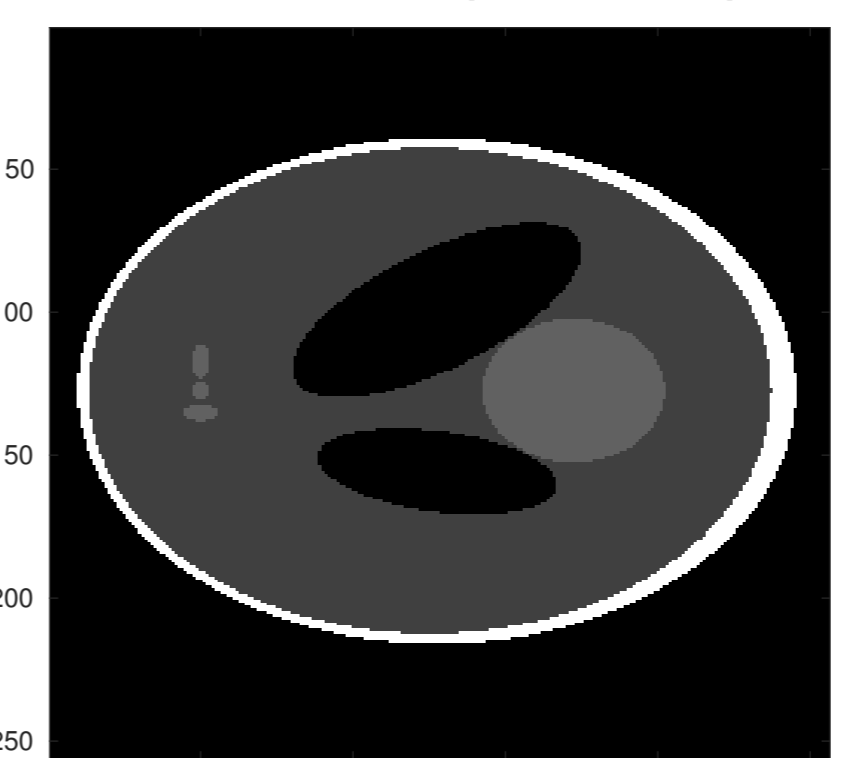
$$(\hat{\mathbf{f}}, \hat{\mathbf{z}}, \hat{\mathbf{v}}_{\epsilon}, \hat{\mathbf{m}}, \hat{\mathbf{v}}) = \arg \max_{\{\mathbf{f}, \mathbf{z}, \mathbf{v}_{\epsilon}, \mathbf{m}, \mathbf{v} | \mathcal{G}; \mathcal{M}\}} \{p(\mathbf{f}, \mathbf{z}, \mathbf{v}_{\epsilon}, \mathbf{m}, \mathbf{v} | \mathcal{G}; \mathcal{M})\}$$

- At iteration t :

1. $\mathbf{f}^{(t)} \leftarrow \arg \max_{\mathbf{f}} \{p(\mathbf{f}, \mathbf{z}^{(t-1)}, \mathbf{v}_{\epsilon}^{(t-1)}, \mathbf{m}^{(t-1)}, \mathbf{v}^{(t-1)} | \mathcal{G}; \mathcal{M})\}$: estimation
 2. $\mathbf{z}^{(t)} \leftarrow \arg \max_{\mathbf{z}} \{p(\mathbf{f}^{(t)}, \mathbf{z}, \mathbf{v}_{\epsilon}^{(t-1)}, \mathbf{m}^{(t-1)}, \mathbf{v}^{(t-1)} | \mathcal{G}; \mathcal{M})\}$: segmentation
 3. $\mathbf{v}_{\epsilon}^{(t)} \leftarrow \arg \max_{\mathbf{v}_{\epsilon}} \{p(\mathbf{f}^{(t)}, \mathbf{z}^{(t)}, \mathbf{v}_{\epsilon}, \mathbf{m}^{(t-1)}, \mathbf{v}^{(t-1)} | \mathcal{G}; \mathcal{M})\}$
 4. $\mathbf{m}^{(t)} \leftarrow \arg \max_{\mathbf{m}} \{p(\mathbf{f}^{(t)}, \mathbf{z}^{(t)}, \mathbf{v}_{\epsilon}^{(t)}, \mathbf{m}, \mathbf{v}^{(t-1)} | \mathcal{G}; \mathcal{M})\}$
 5. $\mathbf{v}^{(t)} \leftarrow \arg \max_{\mathbf{v}} \{p(\mathbf{f}^{(t)}, \mathbf{z}^{(t)}, \mathbf{v}_{\epsilon}^{(t)}, \mathbf{m}^{(t)}, \mathbf{v} | \mathcal{G}; \mathcal{M})\}$
- } hyperparameters optimization

First results in simulation

- 3D Shepp-Logan phantom (256³ voxels) and its original segmentation ($K = 5$)

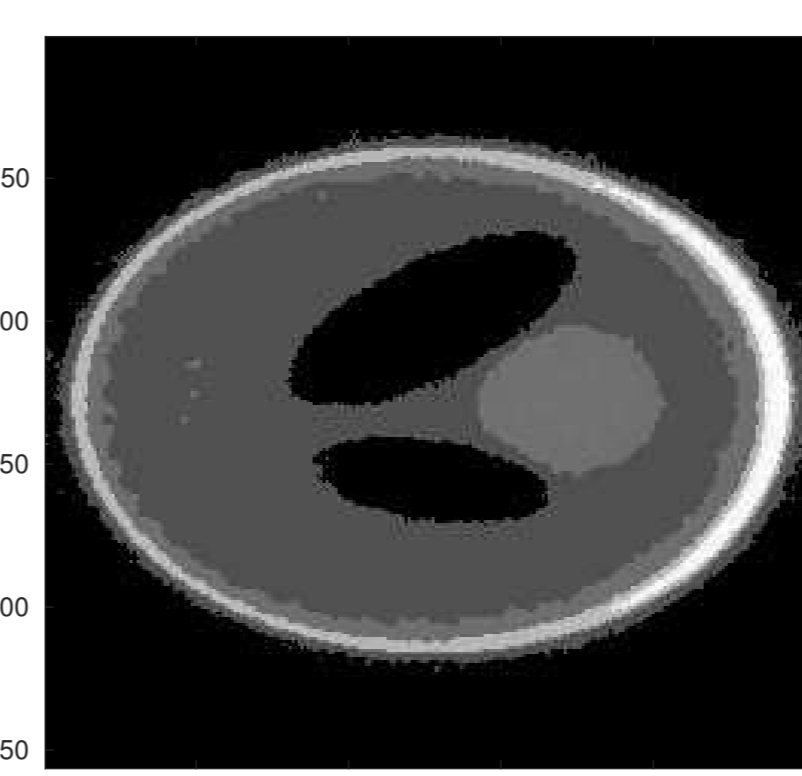


Shepp-Logan phantom (middle slice)



Original segmentation (middle slice)

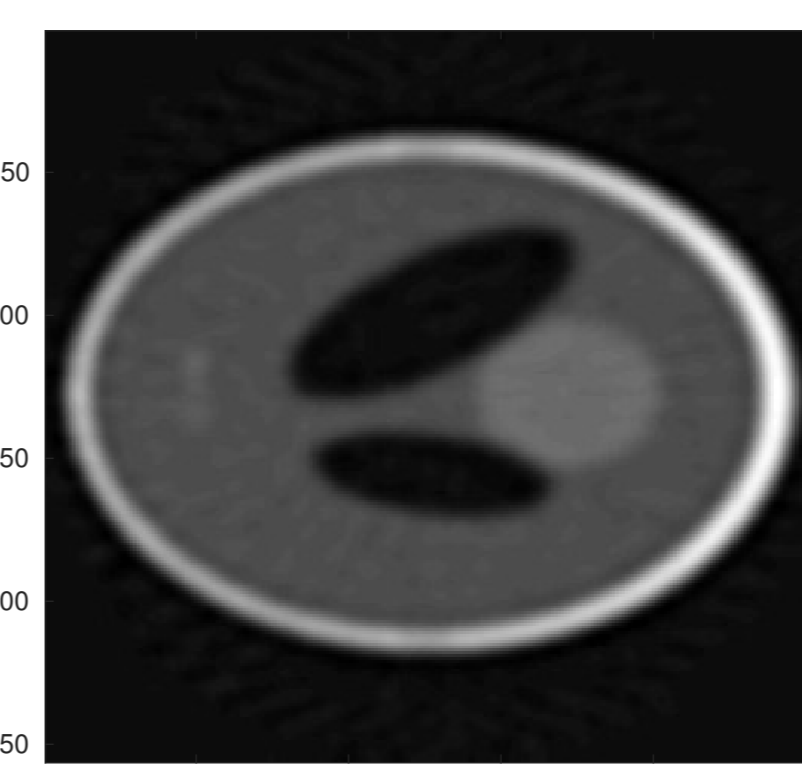
- Comparison of our JMAP algorithm ($\gamma_0 = 3$, SNR=20 db) with Total Variation Minimization (TV) with 64 projections.



JMAP reconstruction



Joint segmentation by JMAP



TV reconstruction



Posterior segmentation of TV reconstruction

Conclusion and prospects

- Satisfying results for our method, which outperforms TV minimization
- Thanks to the prior model, we know what means each parameter. That makes the parameters easy to fix for applying the algorithm.
- Prospects : tests on real data, GPU implementation.