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Distributed event-triggered control for multi-agent formation stabilization and tracking

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Abstract

This paper addresses the problem of formation control and tracking a of desired trajectory by an Euler-Lagrange multi-agent systems. It is inspired by recent results by Qingkai *et al.* and adopts an event-triggered control strategy to reduce the number of communications between agents. For that purpose, to evaluate its control input, each agent maintains estimators of the states of the other agents. Communication is triggered when the discrepancy between the actual state of an agent and the corresponding estimate reaches some threshold. The impact of additive state perturbations on the formation control is studied. A condition for the convergence of the multi-agent system to a stable formation is studied. Simulations show the effectiveness of the proposed approach.

Key words: Communication constraints, event-triggered control, formation stabilization, multi-agent system (MAS).

1 Introduction

Distributed cooperative control of a multi-agent system (MAS) usually requires significant exchange of information between agents. In early contributions, see, *e.g.*, [26, 42], communication is considered permanent. Recently, more practical approaches have been proposed. For example, in [43, 44, 45], communication is intermittent, alternating phases of permanent communication and of absence of communication. Alternatively, communication may only occur at discrete time instants, either periodically as in [13], or triggered by some event, as in [9, 11, 40, 48].

This paper proposes a strategy to reduce the number of communications for displacement-based formation control while following a desired reference trajectory. Agent dynamics are described by Euler-Lagrange models and include perturbations. This work extends results presented in [27] by introducing an event-triggered strategy, and results of [20, 36, 37] by addressing systems with more complex dynamics than a simple integrator. To obtain efficient distributed control laws, each agent uses an estimator of the state of the other agents. The proposed distributed communication triggering condition (CTC) involves the inter-agent displacements and the relative discrepancy between actual and estimated agent states. A single *a priori* trajectory has to be evaluated to follow the desired path. Effects of state perturbations on the formation and on the communications are analyzed. Conditions for the Lyapunov stability of the MAS have been introduced. The absence of Zeno behavior is proved.

This paper is organized as follows. Related work is detailed in Section 2. Some assumptions are introduced in Section 3 and the formation parametrization is described in Section 4. As the problem considered here is to drive a formation of agents along a desired reference trajectory, the designed distributed control law consists of two parts. The first part (see Section 4) drives the agents to some target formation and maintains the formation, despite the presence of perturbations. It is based on estimates of the states of the agents described in Section 4.3. The second part (see Section 5) is dedicated to the tracking of the desired trajectory. Communication instants are chosen locally by each agent using an event-triggered approach introduced in Section 6. A simulation example is considered in Section 7 to illustrate the reduction of the communications obtained by the proposed approach. Finally, conclusions are drawn in Section 8.

2 Related work

Event-triggered communication is a promising approach to save energy. It is well-suited to applications where communications should be minimized, *e.g.*, to improve furtivity, reduce energy consumption, or limit collisions between transmitted data packets. Application examples with such constraints are exposed, *e.g.*, in [18, 19] for the case of a fleet of vehicles, or in [4] where agents aim at merging local feature-based maps. The main difficulty consists in determining the CTC that will ensure the completion of the task assigned to the MAS, *e.g.*, reaching some consensus, maintaining a formation, *etc.* In a distributed strategy, the states of the other agents are not permanently available, thus each agent usually maintains estimators of the state of its neighbours to evaluate their control laws. Nevertheless, without permanent communication, the quality of the state estimates is difficult to evaluate. To address this issue, each agent maintains an estimate of its *own* state using only the information it has shared with its neighbours. When the discrepancy between this own state estimate and its actual state reaches some threshold, the agent triggers a communication. This is the approach considered, *e.g.*, in [9, 12, 14, 34, 39, 40, 49]. These works differ by the complexity of the agents' dynamics [12, 34, 49], the structure of the state estimator [9, 14, 39, 40], and the determination of the threshold for the CTC [34, 39].

Most of the event-triggered approaches have been applied in the context of consensus in MAS [9, 14, 34]. This paper focuses on distributed formation control, which has been considered in [20, 36, 37]. Formation control consists in driving and maintaining all agents of a MAS to some reference, possibly time-varying configuration, defining, *e.g.*, their relative positions, orientations, and speeds. Various approaches have been considered, such as behavior-based flocking [6, 25, 31, 33, 38], or formation tracking [5, 8, 10, 23, 30].

Behavior-based flocking [6, 25, 31, 33, 38] imposes several behavior rules (attraction, repulsion, imitation) to each agent. Their combination leads the MAS to follow some desired behavior. Such approach requires the availability to each agent of observations of the state of its neighbours. These observations may be deduced from measurements provided by sensors embedded in each agent or from information communicated by its neighbours. In all cases, these observations are assumed permanently available. In addition, if a satisfying global behavior may be obtained by the MAS, behavior-based flocking cannot impose a precise configuration between agents.

Different formation-tracking methods have been considered. In leader-follower techniques [5, 8, 10, 23], based on mission goals, a trajectory is designed only for some leader agent. The other follower agents, aim at tracking the leader as well as maintaining some target formation defined with respect to the leader. A virtual leader has been considered in [7, 8, 32] to gain robustness to leader failure. This requires a good synchronization among agents of the state of the virtual leader. Virtual structures have been introduced in [30, 41], where the agent control is designed to satisfy constraints between neighbours. Such approaches also address the problem of leader failure. In distance-based control, the constraints are distances between agents. In displacement-based control, relative coordinate or speed vectors between agents are imposed. In tensegrity structures [24, 27] additional flexibility in the structure is considered by considering attraction and repulsion terms between agents, as formalized by [3]. In addition to constraints on the structure of the MAS, [35] imposes some reference trajectory to each agent. In most of these works, permanent communication between agents is assumed.

Some recent works combine event-triggered approaches with distance-based or displacement-based formation control [20, 36, 37]. In these works, the dynamics of the agents are described by a simple integrator, with control input considered constant between two communications. The proposed CTCs consider different threshold formulations and require each agent to have access to the state of all other agents. A constant threshold is considered in [36]. A time-varying threshold is introduced in [20, 37]. The CTC depends then on the relative positions between agents and the relative discrepancy between actual and estimated agent states. These CTCs reduce the number of triggered communications when the system converges to the desired formation. A minimal time between two communications, named inter-event time, is also defined. Finally, in all these works, no perturbations are considered.

Logic-based control (LBC) techniques have been introduced in [2, 29, 46, 47] to reduce the number of communications in trajectory tracking problems. MAS with decoupled nonlinear agent dynamics are considered in [2, 29]. Agents have to follow parametrized paths, designed in a centralized way. CTCs introduced by LBC lead all agents to follow the paths in a synchronized way to set up a desired formation. Communication delays, as well as packet losses are considered. Nevertheless, if input-to-state stability conditions are established, absence of Zeno behavior is not analyzed.

q_i	vector of <i>coordinates</i> of Agent i in some global fixed reference frame \mathcal{R}
q	vector $\left[q_1^T \ q_2^T \ \dots \ q_N^T \right]^T \in \mathbb{R}^{N \cdot n}$, <i>configuration</i> of the MAS
x_i	state vector $\left[q_i^T, \dot{q}_i^T \right]^T$ of Agent i
\hat{q}_i^j	estimate of q_i performed by Agent j .
\hat{q}^j	estimate of q performed by Agent j .
\hat{x}_i^j	estimate of x_i performed by Agent j .
e_i^j	estimation error between q_i and \hat{q}_i^j .
r_{ij}	relative coordinate vector $r_{ij} = q_i - q_j$ between agents i and j .
r_{ij}^*	desired value for r_{ij} .
q_0	reference trajectory
q_i^*	reference trajectory for Agent i
ε_i	trajectory error for Agent i , $\varepsilon_i = q_i - q_i^*$
$t_{j,k}$	time at which the k -th message is sent by Agent j .
$t_{j,k}^i$	time at which the k -th message sent by Agent j is received by Agent i .

Table 1

Main notations

3 Notations and hypotheses

Table 1 summarizes the main notations used in this paper.

Consider a MAS consisting of a network of N agents which topology is described by an undirected graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$. $\mathcal{N} = \{1, 2, \dots, N\}$ is the set of nodes and $\mathcal{E} \subset \mathcal{N} \times \mathcal{N}$ the set of edges of the network. The set of neighbours of Agent i is $\mathcal{N}_i = \{j \in \mathcal{N} \mid (i, j) \in \mathcal{E}, i \neq j\}$. N_i is the cardinal number of \mathcal{N}_i . For some vector $x = \left[x_1 \ x_2 \ \dots \ x_n \right]^T \in \mathbb{R}^n$, we define $|x| = \left[|x_1| \ |x_2| \ \dots \ |x_n| \right]^T$ where $|x_i|$ is the absolute value of the i -th component of x . Similarly, the notation $x \geq 0$ will be used to indicate that each component x_i of x is non negative, *i.e.*, $x_i \geq 0 \ \forall i \in \{1 \dots n\}$. A continuous function $\beta(r, s) : [0, a] \times [0, \infty) \rightarrow [0, \infty)$ is said to belong to class \mathcal{KL} if for each fixed s , the function $\beta(\cdot, s)$ is strictly increasing and $\beta(0, s) = 0$, and for each fixed r , the function $\beta(r, \cdot)$ is decreasing and $\lim_{s \rightarrow \infty} \beta(r, s) = 0$.

Let $q_i \in \mathbb{R}^n$ be the vector of *coordinates* of Agent i in some global fixed reference frame \mathcal{R} and let $q = \left[q_1^T \ q_2^T \ \dots \ q_N^T \right]^T \in \mathbb{R}^{N \cdot n}$ be the *configuration* of the MAS. The dynamics of each agent is described by the Euler-Lagrange model

$$M_i(q_i) \ddot{q}_i + C_i(q_i, \dot{q}_i) \dot{q}_i + G = \tau_i + d_i, \quad (1)$$

where $\tau_i \in \mathbb{R}^n$ is some control input described in Section 4.2, $M_i(q_i) \in \mathbb{R}^{n \times n}$ is the inertia matrix of Agent i , $C_i(q_i, \dot{q}_i) \in \mathbb{R}^{n \times n}$ is the matrix of the Coriolis and centripetal term on Agent i , G accounts for gravitational acceleration supposed to be known and constant, and d_i is a time-varying state perturbation satisfying $\|d_i(t)\| < D_{\max}$. The state vector of Agent i is $x_i^T = [q_i^T, \dot{q}_i^T]$. Assume that the dynamics satisfy the following assumptions:

- A1)** $M_i(q_i)$ is symmetric positive and there exists $k_M > 0$ satisfying $\forall x, x^T M_i(q_i) x \leq k_M x^T x$.
- A2)** $M_i(q_i) - 2C_i(q_i, \dot{q}_i)$ is skew symmetric or negative definite and there exists $k_C > 0$ satisfying $\forall x, x^T C_i(q_i, \dot{q}_i) x \leq k_C \|\dot{q}_i\| x^T x$.
- A3)** There exists $\dot{q}_{\max} \in \mathbb{R}_+^n$ and $\ddot{q}_{\max} \in \mathbb{R}_+^n$ such that $|\ddot{q}_i| \leq \ddot{q}_{\max}$ and $|\dot{q}_i| \leq \dot{q}_{\max}$.
- A4)** The left-hand side of (1) can be linearly parametrized as

$$M_i(q_i) x_1 + C_i(q_i, \dot{q}_i) x_2 = Y_i(q_i, \dot{q}_i, x_1, x_2) \theta_i \quad (2)$$

for all vectors $x_1, x_2 \in \mathbb{R}^n$, where $Y_i(q_i, \dot{q}_i, x_1, x_2)$ is a regressor matrix with known structure and θ_i is a vector of unknown but constant parameters associated with the i -th agent.

- A5)** For each $i = 1, \dots, N$, θ_i is such that $\theta_{\min, i} < \theta_i < \theta_{\max, i}$, with known $\theta_{\min, i}$ and $\theta_{\max, i}$.

Assumptions A1, A2, A3 and A4 have been previously considered, *e.g.*, in [21, 22, 23].

Moreover, one assumes that

- A6)** each Agent i is able to measure without error its own state x_i ,
- A7)** there is no packet losses or communication delay between agents.

In what follows, the notations M_i and C_i are used to replace $M_i(q_i)$ and $C_i(q_i, \dot{q}_i)$.

4 Formation control problem

This section aims at designing a decentralized control strategy to drive a MAS to a desired target formation in some global reference frame \mathcal{R} , while reducing as much as possible the communications between agents. The target formation is first described in Section 4.1. The potential energy of a MAS with respect to the target formation is introduced to quantify the discrepancy between the target and current formations. The proposed distributed control, introduced in Section 4.2, tries to minimize the potential energy. To evaluate the control input of each agent despite the communications at discrete time instants only, estimators of the coordinate vectors of all agents are managed by each agent, as presented in Section 4.3. The presence of perturbations increases the discrepancy between the state vector and their estimates. A CTC is designed to limit this discrepancy by updating the estimators as described in Section 6.

4.1 Formation parametrization

Consider the relative coordinate vector $r_{ij} = q_i - q_j$ between two agents i and j and the target relative coordinate vector r_{ij}^* for all $(i, j) \in \mathcal{N}$. A target formation is defined by the set $\{r_{ij}^*, (i, j) \in \mathcal{N}\}$.

The *potential energy* $P(q, t)$ of the formation represents the disagreement between r_{ij} and r_{ij}^*

$$P(q, t) = \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N k_{ij} \|r_{ij} - r_{ij}^*\|^2 \quad (3)$$

where the $k_{ij} = k_{ji}$ are some spring coefficients, which can be positive or null, and where $k_{ii} = 0$. $P(q, t)$ has been introduced for tensegrity formations in [24, 27]. The minimum number of non-zero coefficients k_{ij} , $i, j \in \mathcal{N}$ to properly define a target formation is $N - 1$. Indeed, for a given r^* , all target relative coordinate vectors r_{ij}^* between any pair of agents i and j can be expressed from components of r^* . Nevertheless, a number of non-zero k_{ij} larger than $N - 1$ introduces robustness in the formation, in particular with respect to the loss of an agent. The values of the k_{ij} s that make a given r^* an equilibrium formation may be chosen using the method developed in [27].

Definition 1 *The MAS asymptotically converges to the target formation with a bounded error iff there exists some $\epsilon_1 > 0$ such as*

$$\lim_{t \rightarrow \infty} P(q, t) \leq \epsilon_1. \quad (4)$$

A control law designed to reduce the potential energy $P(q, t)$ allows a bounded convergence of the MAS. To describe the evolution of $P(q, t)$, one introduces as in [27]

$$g_i = \frac{\partial P(q, t)}{\partial q_i} = \sum_{j=1}^N k_{ij} (r_{ij} - r_{ij}^*) \quad (5)$$

$$\dot{g}_i = \sum_{j=1}^N k_{ij} (\dot{r}_{ij} - \dot{r}_{ij}^*) \quad (6)$$

$$s_i = \dot{q}_i + k_p g_i \quad (7)$$

where g_i and \dot{g}_i characterize the evolution of the discrepancy between the current and target formations and k_p is a positive scalar design parameter.

4.2 Distributed control

The control law proposed in [27] is defined as $\tau_i = \tau_i(q_i, \dot{q}_i, q)$ and aims at reducing $P(q, t)$, thus making the MAS converge to the target formation in case of permanent communication. In this approach, each agent evaluates its control input using the state vectors of its neighbours obtained via permanent communication.

Here, in a distributed context with limited communications between agents, agents cannot have permanent access to q . Thus, one introduces the estimate \hat{q}_j^i of q_j performed by Agent i to replace the missing information in the control law. The MAS configuration estimated by Agent i is denoted as $\hat{q}^i = [\hat{q}_1^{iT} \dots \hat{q}_N^{iT}]^T \in \mathbb{R}^{N.n}$. The way \hat{q}_j^i is evaluated is described in Section 4.3.

In a distributed context with limited communications, with the help of \hat{q}^i , Agent i is able to evaluate

$$\bar{g}_i = \sum_{j=1}^N k_{ij} (\bar{r}_{ij} - r_{ij}^*) \quad (8)$$

$$\bar{s}_i = \dot{q}_i + k_p \bar{g}_i \quad (9)$$

with $\bar{r}_{ij} = q_i - \hat{q}_j^i$ and $\dot{\bar{r}}_{ij} = \dot{q}_i - \dot{\hat{q}}_j^i$. Using \bar{g}_i and \bar{s}_i , Agent i is able to evaluate the following adaptive distributed control input to be used in (1)

$$\tau_i(q_i, \dot{q}_i, \hat{q}^i, \dot{\hat{q}}^i) = -k_s \bar{s}_i - k_g \bar{g}_i + G - Y_i(q_i, \dot{q}_i, k_p \dot{\hat{q}}_i, k_p \bar{g}_i) \bar{\theta}_i, \quad (10)$$

$$\dot{\bar{\theta}}_i = \Gamma_i Y_i(q_i, \dot{q}_i, k_p \dot{\hat{q}}_i, k_p \bar{g}_i)^T \bar{s}_i \quad (11)$$

with $k_g > 0$, $k_s \geq 1 + k_p(k_M + 1)$ a design parameter and Γ_i an arbitrary symmetric positive definite matrix.

Section 4.3 introduces the estimator \hat{q}_j^i of q_j needed in (10).

4.3 Communication protocol and estimator dynamics

In what follows, the time instant at which the k -th message is sent by Agent j is denoted $t_{j,k}$. Let $t_{j,k}^i$ be the time at which the k -th message sent by Agent j is received by Agent i . In this paper, we assume that there is no communication delay between agents. Therefore, $t_{j,k}^i = t_{j,k}$ for all $i \in \mathcal{N}_j$. When a communication is triggered at $t_{i,k}$ for Agent i , it broadcasts a message containing $t_{i,k}$, $q_i(t_{i,k})$, $\dot{q}_i(t_{i,k})$ and its estimated matrix $\bar{\theta}_i(t_{i,k})$. Once a message is received by neighbours of Agent i , its content is used to update their estimate of the state of Agent i as presented in the next section.

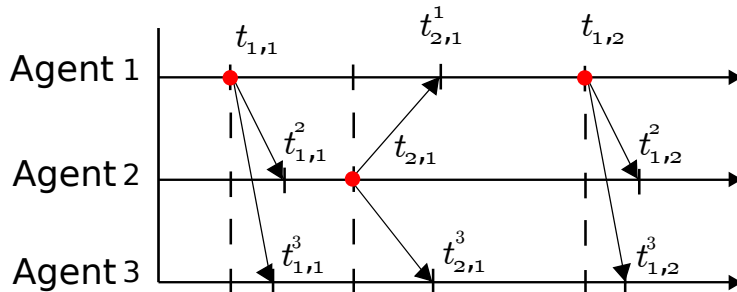


Fig. 1. Example of transmission times $t_{i,k}$ by Agent i of k -th message and reception times $t_{i,k}^j$ of k -th message by Agent j .

4.3.1 Estimator dynamics

Following the idea of [39, 40], the estimate \hat{q}_j^i of q_j made by Agent i is evaluated considering

$$\hat{M}_j^i(\hat{q}_j^i) \ddot{\hat{q}}_j^i + \hat{C}_j^i(\hat{q}_j^i, \dot{\hat{q}}_j^i) \dot{\hat{q}}_j^i + G = \hat{\tau}_j^i, \forall t \in [t_{j,k}^i, t_{j,k+1}^i[\quad (12)$$

$$\hat{q}_j^i(t_{j,k}^i) = q_j(t_{j,k}^i) \quad (13)$$

$$\dot{\hat{q}}_j^i(t_{j,k}^i) = \dot{q}_j(t_{j,k}^i), \quad (14)$$

where $\hat{M}_j^i(\hat{q}_j^i)$ and $\hat{C}_j^i(\hat{q}_j^i, \dot{\hat{q}}_j^i)$ are estimates of M_j and C_j computed from $Y_j(\hat{q}_j^i, \dot{\hat{q}}_j^i, x, y)$ and $\bar{\theta}_j(t_{j,k}^i)$ using

$$\hat{M}_j^i(\hat{q}_j^i) x + \hat{C}_j^i(\hat{q}_j^i, \dot{\hat{q}}_j^i) y = Y_j(\hat{q}_j^i, \dot{\hat{q}}_j^i, x, y) \bar{\theta}_j(t_{j,k}^i). \quad (15)$$

The estimator (12) managed by Agent i requires an estimate $\hat{\tau}_j^i$ of τ_j evaluated by Agent j . This estimate, used by Agent i , is evaluated as

$$\hat{\tau}_j^i = -k_s \hat{s}_j^i - k_g \hat{g}_j^i + G - Y_j(\hat{q}_j^i, \dot{\hat{q}}_j^i, k_p \hat{g}_j^i, k_p \hat{g}_j^i) \hat{\theta}_j^i \quad (16)$$

$$\dot{\hat{\theta}}_j^i = \Gamma_j Y_j(\hat{q}_j^i, \dot{\hat{q}}_j^i, k_p \hat{g}_j^i, k_p \hat{g}_j^i)^T \hat{s}_j^i \quad (17)$$

$$\hat{\theta}_j^i(t_{j,k}^i) = \bar{\theta}_j(t_{j,k}^i) \quad (18)$$

where $\hat{s}_j^i = \dot{\hat{q}}_j^i + k_p \hat{g}_j^i$, $\hat{g}_j^i = \sum_{k=1}^N k_{jk} (\hat{r}_{jk}^i - r_{jk}^*)$, $\dot{\hat{g}}_j^i = \sum_{k=1}^N k_{jk} (\dot{\hat{r}}_{jk}^i - \dot{r}_{jk}^*)$, $\hat{r}_{jk}^i = \hat{q}_j^i - \hat{q}_k^i$, and $\hat{\theta}_j^i$ is the estimate of $\bar{\theta}_j$.

Errors appear between q_i and its estimate \hat{q}_i^j obtained by an other Agent j due to the presence of state perturbations, the non-permanent communication, and the mismatch between θ_i , $\bar{\theta}_i$, and $\hat{\theta}_i$. The errors for the estimates performed by Agent j are expressed as

$$e_i^j = \hat{q}_i^j - q_i, \quad j \in \mathcal{N} \quad (19)$$

$$e^j = \hat{q}^j - q. \quad (20)$$

These errors are used in Section 6 to trigger communications when e_i^i and \dot{e}_i^i become too large. Figure 2 summarizes the overall structure of the estimator and controller.

Remark 2 *The structure of the estimator for $\hat{\tau}_j^i$ is chosen so as to get an accurate estimate for q in order to keep the e_i^j s and \dot{e}_i^j s small. In absence of perturbations, i.e., when $D_{\max} = 0$ and if θ_i is perfectly known, i.e., $\bar{\theta}_i = \hat{\theta}_i = \theta_i$, the estimation error e_i^i introduced in (19) vanishes. The price to be paid for the use of this estimator structure for $\hat{\tau}_j^i$ is that every agent needs to maintain an estimator of the state of all other agents.*

4.3.2 Communication protocol

When a communication is triggered at $t_{i,k}$ for Agent i , it broadcasts a message containing $t_{i,k}$, $q_i(t_{i,k})$, $\dot{q}_i(t_{i,k})$ and its estimated $\bar{\theta}_i(t_{i,k})$. We assume that this message is received by all other agents, either directly when the network is fully connected, or after several hops when the network is connected. The latter case requires the use of a flooding protocol [15, 28]. Since communications have been assumed without delay, one has $\hat{q}_i^i(t) = \hat{q}_i^j(t)$ for all $(i, j) \in \mathcal{N}^2$. This simplifies the stability study in Appendix 9.1.

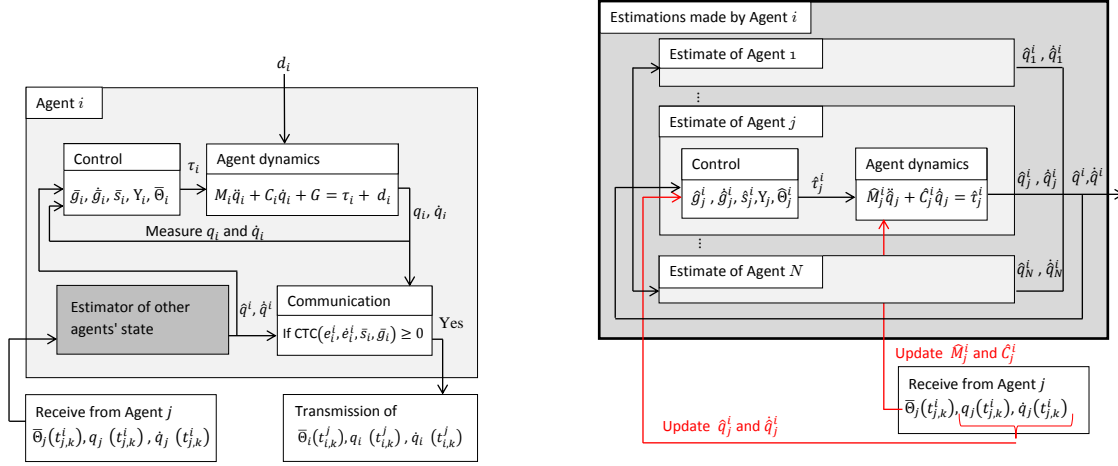


Fig. 2. Formation control system architecture

5 Time-varying formation and tracking

Consider, without loss of generality, the first agent as a reference agent¹ and introduce the target relative configuration vector $r^* = [r_{11}^{*T} \dots r_{1N}^{*T}]^T$ which may be time-varying. In this section, the MAS has to follow some reference trajectory $q_1^*(t)$, while remaining in a desired formation. Agent 1, taken as the reference agent, aims at following $q_1^*(t)$. It is assumed that all agents have access to $q_1^*(t)$. Moreover, assume that the target formation can be time-varying and is represented by the relative configuration vector $r^*(t)$. Therefore the reference trajectory of each agent can be expressed as $q_i^*(t) = q_1^*(t) + r_{i1}^*(t)$.

To guarantee that individual reference trajectories can be tracked by each agent, it is assumed that for $i = 1, \dots, N$,

$$|\dot{q}_i^*| < \dot{q}_{\max} \quad (21)$$

$$|\ddot{q}_i^*| < \ddot{q}_{\max}. \quad (22)$$

Definition 3 The MAS reaches its tracking objective iff there exists $\epsilon_1 > 0$ and $\epsilon_2 > 0$ such that (4) is satisfied and

$$\lim_{t \rightarrow \infty} \|q_1(t) - q_1^*(t)\| \leq \epsilon_2, \quad (23)$$

i.e., iff the reference agent asymptotically converges to the reference trajectory, and the MAS asymptotically converges to the target formation with bounded errors.

A distributed control law is designed to satisfy this target. Introduce the trajectory error terms

$$\begin{aligned} \varepsilon_i &= q_i - q_i^* \\ \hat{\varepsilon}_i^j &= \hat{q}_i^j - q_i^*. \end{aligned}$$

The terms $g_i, \bar{g}_i, \hat{g}_i^j, \bar{s}_i$ and \hat{s}_i^j introduced in Sections 4 are now redefined as follows to address the trajectory tracking

¹ a virtual agent may also be considered.

problem

$$g_i = \sum_{j=1}^N k_{ij} (r_{ij} - r_{ij}^*) + k_0 \varepsilon_i \quad (24)$$

$$\bar{g}_i = \sum_{j=1}^N k_{ij} (\bar{r}_{ij} - r_{ij}^*) + k_0 \varepsilon_i \quad (25)$$

$$\hat{g}_i^j = \sum_{j=1}^N k_{ij} (\hat{r}_{ij}^j - r_{ij}^*) + k_0 \hat{\varepsilon}_i^j \quad (26)$$

$$s_i = \dot{q}_i - \dot{q}_i^* + k_p g_i \quad (27)$$

$$\bar{s}_i = \dot{q}_i - \dot{q}_i^* + k_p \bar{g}_i \quad (28)$$

$$\hat{s}_i^j = \dot{\hat{q}}_i^j - \dot{q}_i^* + k_p \hat{g}_i^j \quad (29)$$

where $k_0 \geq 0$ is a positive design parameter which may be used to control the tracking error with respect to the reference trajectory. When no reference trajectory is considered, $k_0 = 0$.

From these terms, a new distributed control input to be used in (1) is defined for Agent i as

$$\tau_i = -k_s \bar{s}_i - k_g \bar{g}_i + G - Y_i(q_i, \dot{q}_i, \dot{\bar{p}}_i, \bar{p}_i) \bar{\theta}_i \quad (30)$$

$$\dot{\bar{\theta}}_i = \Gamma_i Y_i(q_i, \dot{q}_i, \dot{\bar{p}}_i, \bar{p}_i)^T \bar{s}_i \quad (31)$$

where $\bar{p}_i = k_p \bar{g}_i - \dot{q}_i^*$ and $\dot{\bar{p}}_i = k_p \dot{\bar{g}}_i - \ddot{q}_i^*$.

The estimators maintained by Agent i are defined with the same dynamics as 12 but the evaluation of the estimate $\hat{\tau}_j^i$ of τ_j is now evaluated as

$$\hat{\tau}_j^i = -k_s \hat{s}_j^i - k_g \hat{g}_j^i + G - Y_j(\hat{q}_j^i, \dot{\hat{q}}_j^i, \dot{\hat{p}}_j^i, \hat{p}_j^i) \hat{\theta}_j^i \quad (32)$$

$$\dot{\hat{\theta}}_j^i = \Gamma_j Y_j(\hat{q}_j^i, \dot{\hat{q}}_j^i, \dot{\hat{p}}_j^i, \hat{p}_j^i)^T \hat{s}_j^i \quad (33)$$

where $\hat{s}_j^i = \dot{\hat{q}}_j^i - \dot{q}_j^* + k_p \hat{g}_j^i$, $\hat{p}_j^i = k_p \hat{g}_j^i - \dot{q}_j^*$ and $\dot{\hat{p}}_j^i = k_p \dot{\hat{g}}_j^i - \ddot{q}_j^*$.

The communication protocol introduced in Section 4.3.2 remains the same. The way the estimator (12-14) for the state of all agents is defined with the control input (32,33) and the absence of communication delays ensure that $\hat{x}_i^i = \hat{x}_i^j$ for all pair of agents i and j in the network.

6 Event-triggered communications

Theorem 4 introduces a CTC used to trigger communications to ensure a bounded asymptotic convergence of the MAS to the target formation. The initial value of the state vectors are considered to be known by all agents. In practice, this condition can be satisfied by triggering a communication from all agents at time $t = 0$ to initialize the estimates of the state of the neighbours of all agents.

Let $k_{\max} = \max_{\ell=1 \dots N} (k_{\ell j})$ and $k_{\min} = \min_{\ell=1 \dots N} (k_{\ell j} \neq 0)$, $\alpha_i = \sum_{j=1}^N k_{ij}$, $\alpha_{\min} = \min_{i=1, \dots, N} \alpha_i$ and

$\alpha_{\max} = \max_{i=1, \dots, N} \alpha_i$. Define also for $\bar{\theta}_i \in \mathbb{R}^p$ and $\bar{\theta}_i = [\bar{\theta}_{i,1}, \dots, \bar{\theta}_{i,p}]^T$

$$\Delta \theta_{i,\max} = \begin{bmatrix} \max \{ |\bar{\theta}_{i,1} - \theta_{\min,i,1}|, |\bar{\theta}_{i,1} - \theta_{\max,i,1}| \} \\ \vdots \\ \max \{ |\bar{\theta}_{i,p} - \theta_{\min,i,p}|, |\bar{\theta}_{i,p} - \theta_{\max,i,p}| \} \end{bmatrix} \quad (34)$$

and $\Delta\theta_i = \bar{\theta}_i - \theta_i$.

Theorem 4 Consider a MAS with agent dynamics given by (1) and the control law (30). Consider some design parameters $\eta \geq 0$, $\eta_2 > 0$, $0 < b_i < \frac{k_s}{k_s k_p + k_g}$,

$$c_3 = \frac{\min \left\{ 1, k_1, k_p, k_0, 2k_0 \left(2k_0 + \frac{\alpha_{\min} k_{\min}}{k_{\max}} \right) \right\}}{\max \{1, k_M\}}$$

and $k_1 = k_s - 1 + k_p(k_M + 1)$. In absence of communication delays, the system (1) is input-to-state practically stable (ISpS) and the agents can be driven to some target formation such that

$$\lim_{t \rightarrow \infty} \sum_{i=1}^N k_0 \|\varepsilon_i\|^2 + \frac{1}{2} P(q, t) \leq \xi \quad (35)$$

with

$$\xi = \frac{N}{k_g c_3} [D_{\max}^2 + \eta + c_3 \Delta_{\max}] \quad (36)$$

where $\Delta_{\max} = \max_{i=1:N} (\sup_{t>0} (\Delta\theta_i^T \Gamma_i^{-1} \Delta\theta_i))$, if the communications are triggered when one of the following conditions is satisfied

$$\begin{aligned} k_s \bar{s}_i^T \bar{s}_i + k_p k_g \bar{g}_i^T \bar{g}_i + \eta &\leq \alpha_M^2 (k_e e_i^T e_i + k_p k_M \dot{e}_i^T \dot{e}_i) \\ &+ \alpha_M k_C^2 k_p \|e_i^i\|^2 \sum_{j=1}^N k_{ji} \left[\|\dot{q}_j^i\| + \eta_2 \right]^2 + k_g b_i \|\dot{q}_i - \dot{q}_i^*\|^2 \\ &+ k_p \|e_i^i\| \left[\alpha_M^2 \left(1 + \|Y_i\| \Delta\theta_{i,\max} \right)^2 + \frac{\|Y_i\| \Delta\theta_{i,\max} \|^2}{\left(1 + \|Y_i\| \Delta\theta_{i,\max} \right)^2} \right] \end{aligned} \quad (37)$$

$$\|\dot{q}_i\| \geq \|\dot{q}_i^i\| + \eta_2 \quad (38)$$

with $k_e = k_s k_p^2 + k_g k_p + \frac{k_g}{b_i}$, and $Y_i = Y_i(q_i, \dot{q}_i, \hat{p}_i, \bar{p}_i)$.

The proof of Theorem 4 is given in Appendix 9.1.

Corollary 5 Consider a MAS with agent dynamics given by (1) and the control law (30). For any Agent i , let $t_{i,k}$ and $t_{i,k+1}$ be two consecutive communication instants at which the CTC of Theorem 4 have been satisfied. Then $t_{i,k+1} - t_{i,k} > 0$.

The proof of Corollary 5 is provided in Appendix 9.2.

The CTCs proposed in Theorem 4 are analyzed assuming that the estimators of the state of the agents and the communication protocol is such that $\forall (i, j) \in \mathcal{N} \times \mathcal{N}$,

$$\hat{x}_i^i(t) = \hat{x}_i^j(t) \quad (39)$$

$$\hat{x}_i^i(t_{i,k}) = x_i^i(t_{i,k}), \quad (40)$$

where (39) is called the *estimate synchronization condition* and (40) the *estimator reset condition*. Theorem 4 is valid independently of the way the estimate \hat{x}_i^i of x_i is evaluated provided that (39) and (40) are satisfied.

From (35) and (37), one sees that η can be used to adjust the trade-off between the bound ξ on the formation and tracking errors and the amount of triggered communications. If $\eta = 0$, there is no perturbation and θ_i is perfectly known, the system converges asymptotically.

The CTC (38) is related to the discrepancy between \dot{q}_i and $\hat{\dot{q}}_i$. Choosing a small value of η_2 may lead to frequent communications. On the contrary, when η_2 is large, (37) is more likely to be satisfied. A value of η_2 that corresponds to a trade-off between the two CTCs (37) and (38) has thus to be found to minimize the amount of communications.

The CTCs (37) and (38) mainly depend on e_i^i and \dot{e}_i^i . A communication is triggered by Agent i when the state estimate \hat{x}_i^i of its own state vector x_i is not satisfying, *i.e.*, when e_i^i and \dot{e}_i^i becomes large. To reduce the number of triggered communications, one has to keep e_i^i and \dot{e}_i^i as small as possible. This may be achieved by increasing the accuracy of the estimator, as proposed in Section 4.3, but possibly at the price of a more complex structure for the estimator.

The perturbations have a direct impact on e_i^i and \dot{e}_i^i , and, as a consequence, on the frequency of communications. (36) shows the impact of D_{\max} and η on the formation and tracking errors: in presence of perturbations, the formation and tracking errors cannot reach a value below a minimum value due to the perturbations. At the cost of a larger formation and tracking errors, η can reduce the number of triggered communications and so can reduce the influence of perturbations on the CTC (37).

The discrepancy between the actual values of M_i and C_i and of their estimates \hat{M}_i^i and \hat{C}_i^i determines the accuracy of $\bar{\theta}_i$, so $\Delta\theta_{i,\max}$, and the estimation errors. Even in absence of state perturbations, due to the linear parametrization, it is likely that $\hat{M}_i^i \neq M_i$, $\hat{C}_i^i \neq C_i$ and $\Delta\theta_{i,\max} > 0$, which leads to the satisfaction of the CTCs at some time instants. Thus, the CTC (37) leads to more communications when the model of the agent dynamics is not accurate, requiring thus more frequent updates of the estimate of the states of agents.

The choice of the parameters α_M , k_g , k_p and b_i also determines the number of broadcast messages. Choosing the spring coefficients k_{ij} such that $\alpha_i = \sum_{j=1}^N k_{ij}$ is small leads to a reduction in the number of communication triggered due to the satisfaction of (37).

7 Simulation results

The performance of the proposed algorithm is evaluated considering a set of $N = 6$ agents. Two models will be considered to describe the dynamics of the agents.

7.1 Models of the agent dynamics and estimator

7.1.1 Double integrator with Coriolis term (DI)

The first model consists in the dynamical system

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i = \tau_i + d_i$$

with $q_i \in \mathbb{R}^2$ and where

$$M_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad C_i(\dot{q}_i) = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} \|\dot{q}_i\|. \quad (41)$$

Then the vectors $\bar{\theta}_i(0) = \hat{\theta}_i^j(0)$, $i = 1, \dots, N$ are obtained using (2). In place of the estimator in Section 4.3 a first less accurate estimate of x_j made by Agent i , is evaluated as

$$\hat{q}_j^i(t) = q_j(t_{j,k}^i) \quad (42)$$

$$\hat{\dot{q}}_j^i(t) = \dot{q}_j(t_{j,k}^i). \quad (43)$$

This estimator allows one to better observe the tradeoff between the potential energy of the formation and the communication requirements.

For this dynamical model, the parameters of the control law (30) and the CTC (37) have been selected as: $k_M = \|M_i\| = 1$, $k_C = \|C_i\| = 0.1$, $k_p = 1$, $k_g = 15$, $k_s = 1 + k_p(k_M + 1)$, $b_i = \frac{1}{k_g}$, and $k_0 = 2$.

7.1.2 Surface ship (SS)

The second model considers surface ships with coordinate vectors $q_i = [x_i \ y_i \ \psi_i]^T \in \mathbb{R}^3$, $i = 1 \dots N$, in a local earth-fixed frame. For Agent i , (x_i, y_i) represents its position and ψ_i its heading angle. The dynamics of the agents is described by the surface ship dynamical model taken from [17], assumed identical for all agents, and expressed in the body frame as

$$M_{b,i} \dot{v}_i + C_{b,i}(v_i) v_i + D_{b,i} v_i = \tau_{b,i} + d_{b,i}, \quad (44)$$

where $v_i = [u_i \ v_i \ r_i]^T$ is the velocity vector in the body frame, $\tau_{b,i}$ is the control input, $d_{b,i}$ is the perturbation, and

$$M_{b,i} = \begin{bmatrix} 25.8 & 0 & 0 \\ 0 & 33.8 & 1.0115 \\ 0 & 1.0115 & 2.76 \end{bmatrix}$$

$$C_{b,i}(v_i) = \begin{bmatrix} 0 & 0 & -33.8v_i - 1.0115r_i \\ 0 & 0 & 25.8u_i \\ 33.8v_i + 1.0115r_i & -25.8u_i & 0 \end{bmatrix}$$

$$D_{b,i} = \begin{bmatrix} 0.72 & 0 & 0 \\ 0 & 0.86 & -0.11 \\ 0 & -0.11 & -0.5 \end{bmatrix}.$$

At $t = 0$, one assumes that Agent i has access to estimates $\hat{M}_{b,i}^i$ of $M_{b,i}$, $\hat{C}_{b,i}^i$ of $C_{b,i}$, and $\hat{D}_{b,i}^i$ of $D_{b,i}$ described as

$$\begin{aligned} \hat{M}_{b,i}^i &= (\mathbf{1}_{3 \times 3} + 0.1 \Xi_i^M) \odot M_{b,i} \\ \hat{C}_{b,i}^i &= (\mathbf{1}_{3 \times 3} + 0.1 \Xi_i^C) \odot C_{b,i} \\ \hat{D}_{b,i}^i &= (\mathbf{1}_{3 \times 3} + 0.1 \Xi_i^D) \odot D_{b,i}, \end{aligned}$$

where $\mathbf{1}_{3 \times 3}$ is the 3×3 matrix of ones, Ξ_i^M , Ξ_i^C , and Ξ_i^D are matrices which components are independent and identically Bernoulli random variables with values in $\{-1, 1\}$, and \odot is the Hadamard product. These estimates are transmitted at $t = 0$ to all other agents. As a consequence, the estimates of $M_{b,i}$ and $C_{b,i}$ made by all agents at $t = 0$ are all identical.

The model (44) is expressed with the coordinate vectors q_i in the local earth-fixed frame using the transform

$$\begin{aligned} \dot{q}_i &= J_i(\psi_i) v_i \\ J_i(\psi_i) &= \begin{bmatrix} \cos \psi_i & -\sin \psi_i & 0 \\ \sin \psi_i & \cos \psi_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

where $J_i(\psi_i)$ is a simple rotation around the z -axis in the earth-fixed coordinate. Define $J_i^{-T} = (J_i^{-1})^T$. Then, (44) can be rewritten as

$$J_i^{-T} M_{b,i} J_i^{-1} \ddot{q}_i + J_i^{-T} [C_{b,i}(v) - M_{b,i} J_i^{-1} \dot{J}_i + D_{b,i}] J_i^{-1} \dot{q}_i = J_i^{-T} \tau_b + J_i^{-T} d_{b,i}$$

and so

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i = \tau_i + d_i$$

where

$$M_i(q_i) = J^{-T} M_b J^{-1},$$

$$C_i(q_i, \dot{q}_i) = J_i^{-T} \left[C_{b,i}(J_i^{-1}\dot{q}_i) - M_{b,i} J_i^{-1} \dot{J}_i + D_{b,i} \right] J^{-1},$$

and τ_i is the control input in earth-fixed coordinates as defined in (30).

The vectors $\bar{\theta}_i(0) = \hat{\theta}_i^j(0)$, $i = 1, \dots, N$ are obtained using (2). The estimator described in Section 4.3 is employed.

For this dynamical model, the parameters of the control law (30) and the CTC (37) have been selected as: $k_M = \|M_i\| = 33.8$, $k_C = \|C_v(1_N)\| = 43.96$, $k_p = 6$, $k_g = 20$, $k_s = 1 + k_p(k_M + 1)$, $b_i = \frac{1}{k_g}$, and $k_0 = 1.5$.

7.1.3 Simulation parameters

One chooses the components of the initial value $x(0)$ of the state vector as

$$q(0) = \left[\begin{array}{c} \left[\begin{array}{c} -0.35 \\ -1.11 \\ 0 \end{array} \right]^T \\ \left[\begin{array}{c} 4.59 \\ -4.59 \\ 0 \end{array} \right]^T \\ \left[\begin{array}{c} 4.72 \\ 2.42 \\ 0 \end{array} \right]^T \\ \left[\begin{array}{c} 0.64 \\ 1.36 \\ 0 \end{array} \right]^T \\ \left[\begin{array}{c} 3.53 \\ 1.56 \\ 0 \end{array} \right]^T \\ \left[\begin{array}{c} -1.26 \\ 3.36 \\ 0 \end{array} \right]^T \end{array} \right]^T,$$

and $\dot{q}(0) = 0_{N \times 1}$. The vector of relative target configurations corresponds to a hexagonal formation

$$r^* = \left[\begin{array}{c} \left[\begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right]^T \\ \left[\begin{array}{c} 2 \\ 0 \\ 0 \end{array} \right]^T \\ \left[\begin{array}{c} 3 \\ \sqrt{3} \\ 0 \end{array} \right]^T \\ \left[\begin{array}{c} 2 \\ 2\sqrt{3} \\ 0 \end{array} \right]^T \\ \left[\begin{array}{c} 0 \\ 2\sqrt{3} \\ 0 \end{array} \right]^T \\ \left[\begin{array}{c} -1 \\ \sqrt{3} \\ 0 \end{array} \right]^T \end{array} \right]^T.$$

Using the approach developed in [27], the following matrix $K = [k_{ij}]_{\substack{i=1 \dots N \\ j=1 \dots N}}$ can be computed from r^*

$$K = 0.1 \begin{bmatrix} 0 & 1.85 & 0 & 0.926 & 0 & 1.85 \\ 1.85 & 0 & 1.85 & 0 & 0.926 & 0 \\ 0 & 1.85 & 0 & 1.85 & 0 & 0.926 \\ 0.926 & 0 & 1.85 & 0 & 1.85 & 0 \\ 0 & 0.926 & 0 & 1.85 & 0 & 1.85 \\ 1.85 & 0 & 0.926 & 0 & 1.85 & 0 \end{bmatrix}$$

and $\alpha_i = \sum_{j=1}^N k_{ij} = 0.463$, for all $i = 1, \dots, N$ and $\alpha_M = 0.463$.

A fully-connected communication graph is considered. The simulation duration is $T = 2s$. Matlab's ode45 integrator is used with a step size $\Delta t = 0.01$ s. Since time has been discretized, the minimum delay between the transmission of two messages by the same agent is set to Δt . The perturbation $d_i(t)$ is assumed of constant value over each interval of the form $[k\Delta t, (k+1)\Delta t]$. The components of $d_i(t)$ are independent realizations of zero-mean uniformly distributed

noise $U\left(-\frac{D_{\max}}{\sqrt{3}}, \frac{D_{\max}}{\sqrt{3}}\right)$ and are thus such that $\|d_i(t)\| \leq D_{\max}$. Let N_m be the total number of messages broadcast during a simulation. The performance of the proposed approach is evaluated comparing N_m to the maximum number of messages that can be broadcast $\bar{N}_m = NT/\Delta t \geq N_m$. The percentage of residual communications is defined as $R_{\text{com}} = 100 \frac{N_m}{\bar{N}_m}$. R_{com} indicates the percentage of time slots during which a communication has been triggered.

When a tracking has to be performed, one considers the target trajectory of the first agent

$$q_1^*(t) = \begin{bmatrix} 4 \sin(0.4t) \\ 4 \cos(0.4t) \\ 0.4t \end{bmatrix},$$

the other agents having to remain in formation. Define the tracking error $\varepsilon_0 = q_1 - q_1^*$.

7.2 Formation control with DI

Figure 3 shows the evolution of the communication ratio R_{com} and of the potential energy at $t = T$. For all simulations, one has $P(q, T) \leq \xi$ for the different values of D_{\max} and η .

In Figure 3 (a), the number of communications obtained once the system has converged increases as the level of perturbations becomes more important, as expected. Increasing η in the CTC 37 helps reducing R_{com} . Nevertheless, increasing η also increases the potential energy $P(q, T)$ of the formation, as can be seen in Figure 3 (b). In Figure 3 (b), when $\eta \geq 3$, one observes that the potential energy starts to decrease with the level of perturbation D_{\max} to increase again when D_{\max} gets large. To explain this surprising behavior, Figure 3 (c) shows that there exists a threshold $\bar{R}_{\text{com}} = 2.25$ below which the potential energy significantly increases to ensure proper convergence. Therefore η should be chosen such that R_{com} remains above this threshold. Even large values of D_{\max} can be tolerated provided that η is chosen large enough to provide a sufficient amount of communications.

7.3 Formation control with ship dynamical model

Figure 4 shows the trajectories of the agents when the control (30) is applied and the communications are triggered according to the CTC of Theorem 4. Figure 4 (a) illustrates the results obtained using the accurate estimator (12), Figure 4(b) illustrates results obtained using the simple estimator (42). The agents converge to the desired formation with a limited number of communications, even in presence of perturbations.

Figure 5 shows the evolution of R_{com} and of $P(q, T)$ parametrized by η for different values of D_{\max} . For all simulations, one has $P(q, T) \leq \xi$ for the different values of D_{\max} and η . As expected and shown in Section 7.2, the potential energy obtained once the system has converged increases with D_{\max} . It can also be observed that increasing η reduces the number of messages broadcast, without a significant impact on $P(q, T)$, contrary to what was observed with the DI with simple estimator.

7.4 Tracking control with DI

The simulation duration is $T = 3.5$ s.

Figures 6 and 7 show the evolution of the communication ratio R_{com} , the potential energy and the tracking error at $t = T$.

In Figure 6 (a), the number of communications obtained once the system has converged decreases as the level of perturbation becomes more important, especially when η is small, which was not expected. Such behavior is not observed with the accurate estimator (12), where R_{com} increases when the perturbations become more important, as illustrated in Figure 9 (a) with the ship model. This behavior can be explained by the fact a large D_{\max} makes $\|\bar{g}_i\|$ and $\|\bar{s}_i\|$ larger, which reduces the number of times the CTC (37) is satisfied, even if the error $\|e_i^z\|$ is also affected. Difference with accurate estimator is the error e_i^z is keeping small by the estimator, so the influence of perturbations is more significant on e_i^z than on $\|\bar{g}_i\|$ or $\|\bar{s}_i\|$, which leads to a larger number of communications triggered.

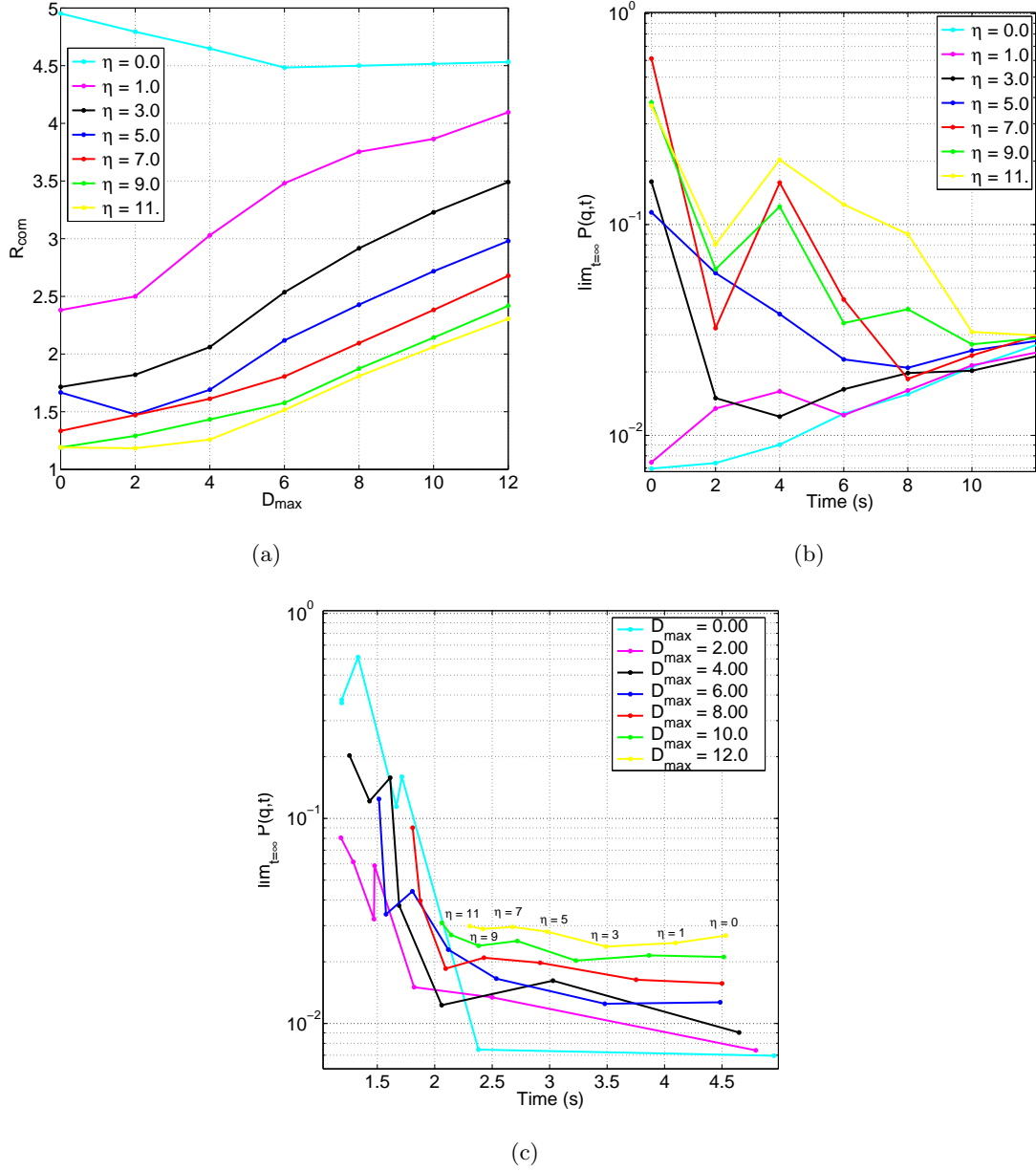


Fig. 3. Evolution of R_{com} and $P(q,t)$ for different values of $D_{\text{max}} \in \{0, 2, 4, 6, 8, 10, 12\}$, $\eta \in \{0, 1, 3, 5, 7, 9, 11\}$, and $\eta_2 = 7.5$. The DI model and the simple estimator (42)-(43) are considered.

Figure 6 (a) illustrates that the parameter η in the CTC (37) can help reducing R_{com} . It can be seen that there exists for R_{com} a threshold ($R_{\text{com}} = 7$) which R_{com} cannot reach : we can deduce a minimal number of communications is required for system converge with the constant estimator (42)-(43).

Figures 6 (b) and (c) show that the potential energy of the formation $P(q,t)$ and the tracking error ε_0 increase when the perturbation level increases. The influence of parameter η is also illustrated: Figure 7 shows that a larger value of η leads to an increase of $P(q,t)$, but reduces ε_0 . Indeed, the less communications, the more difficult it is for some Agent i to be synchronized with the others agents to reach the target formation. However, be less synchronized with the other agents allows Agent i to be more synchronized with its target trajectory q_i^* , inducing a small tracking error ε_0 . Thus, a trade off between the $P(q,t)$ and ε_0 has to be reached.

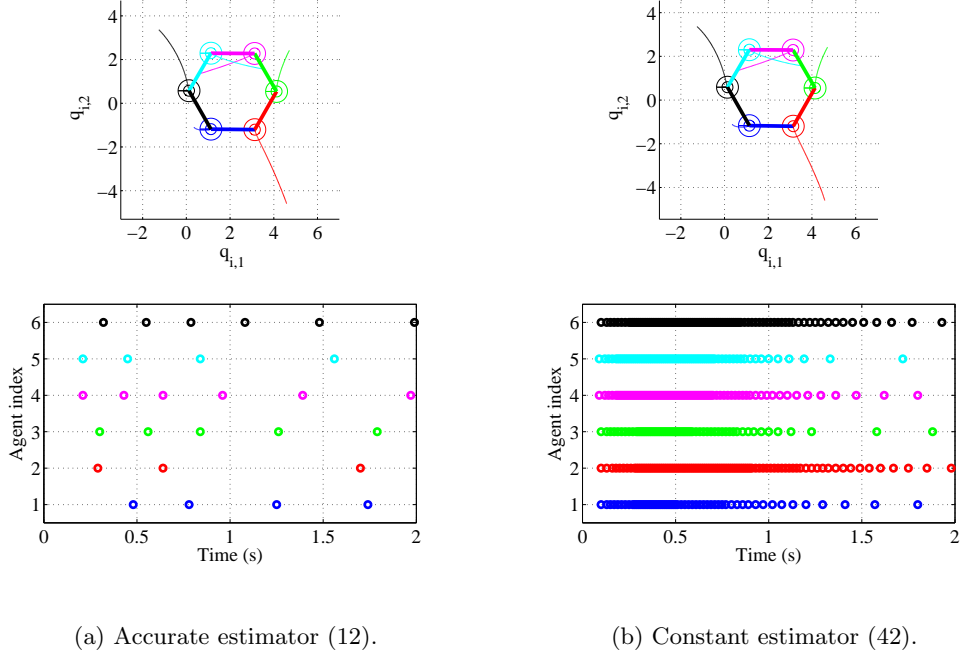


Fig. 4. Hexagonal formation with $D_{\max} = 20$, $\eta = 20$ and $\eta_2 = 7.5$. Agents are represented by circles. In (a), $R_{\text{com}} = 2.61\%$ and $P(q, T) = 0.001$. In (b) $R_{\text{com}} = 18.25\%$ and $P(q, T) = 0.001$.

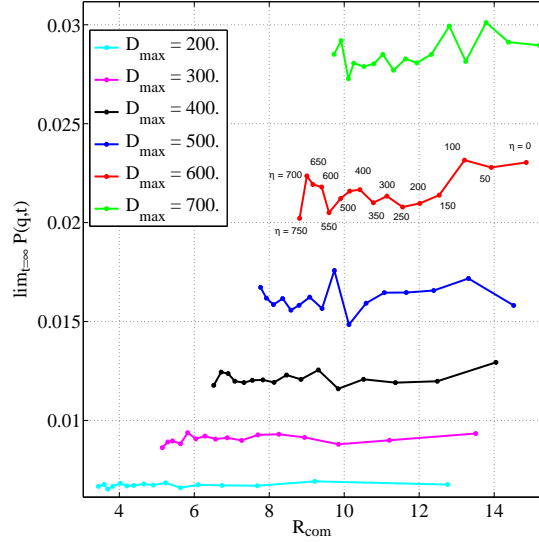


Fig. 5. Evolution of R_{com} and $P(q, t)$ for different values of $D_{\max} \in \{200, 300, \dots, 700\}$, $\eta \in \{0, 50, 100, \dots, 750\}$ and $\eta_2 = 7.5$. Model (44) and accurate estimator (12) are considerate.

7.5 Tracking with surface ship model

The simulation duration is $T = 2.5$ s.

Figures 9 and 10 show the evolution of the communication ratio R_{com} , the potential energy and the tracking error at $t = T$.

In Figure 9 (a), the number of communications obtained once the system has converged increases as the level of

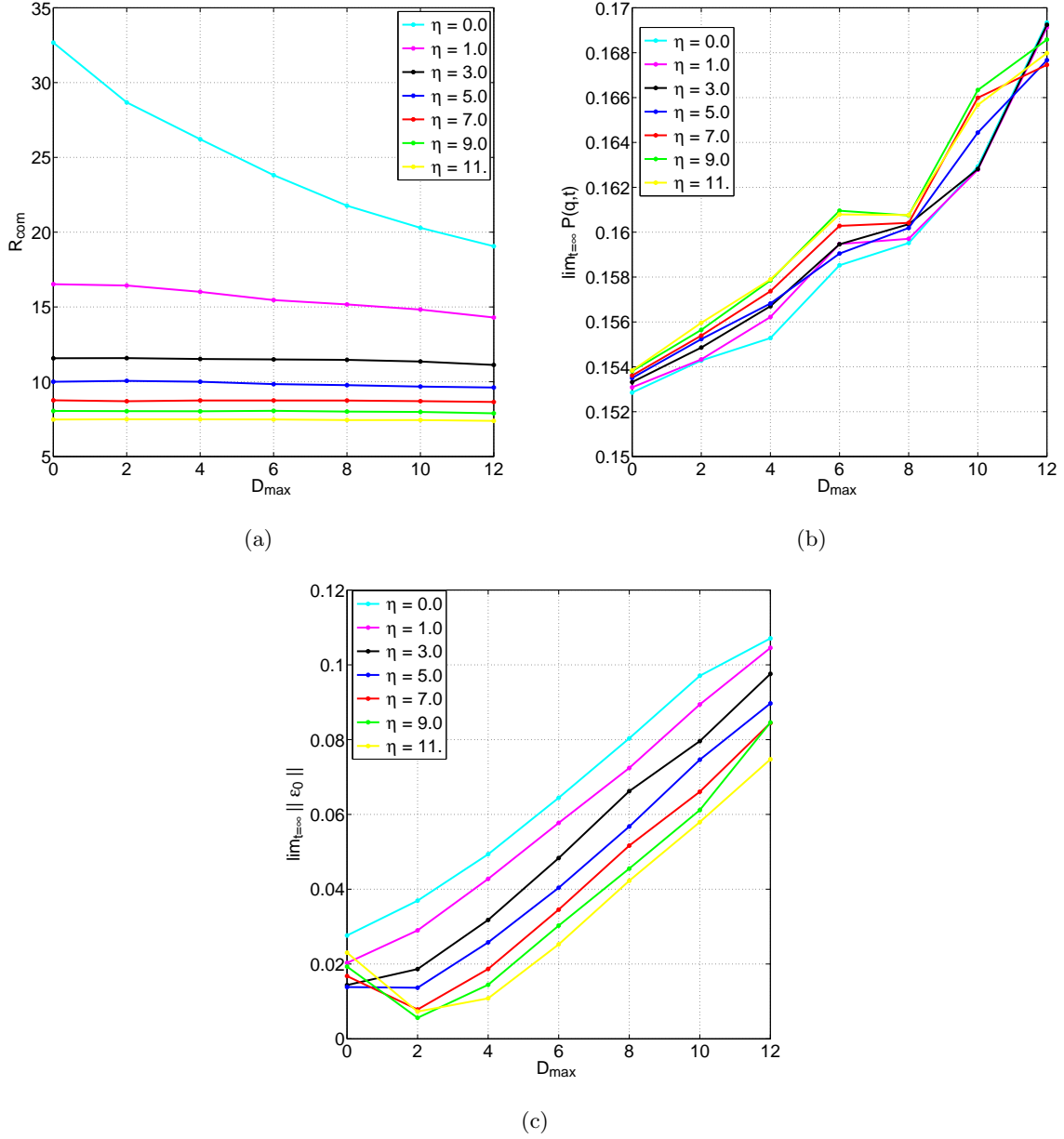


Fig. 6. Evolution of R_{com} , $P(q,t)$ and ε_0 for different values of $D_{\max} \in \{0, 2, 4, 6, 8, 10, 12\}$, $\eta \in \{0, 1, 3, 5, 7, 9, 11\}$ and $\eta_2 = 7.5$. Model (41) and constant estimator (42)-(43) are considerate.

perturbations becomes more important. The parameter η in the CTC 37 can help to reduce R_{com} . Figure 9 (b) and (c) show that the potential energy of the formation $P(q,t)$ and the tracking error ε_0 also increase when the perturbation level increases. Influence of parameter η is also illustrated : Figure 9 (c) shows that increasing η results in make ε_0 decrease when $D_{\max} > 200$. Influence of η on $P(q,t)$ is less clearly detectable than in the case of the DI model.

In Figure 10, it can be observed that R_{com} cannot be reduced below the value of 1: a minimum number of communications is indeed required to converge with the accurate estimator (12).

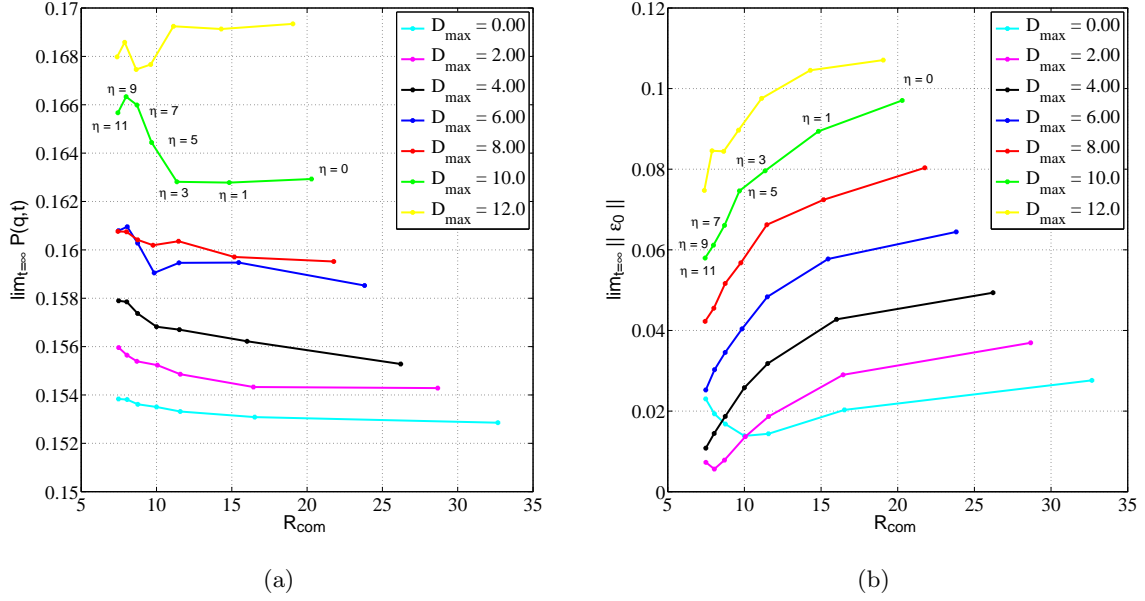


Fig. 7. Evolution of R_{com} , $P(q,t)$ and ε_0 for different values of $D_{max} \in \{0, 2, 4, 6, 8, 10, 12\}$, $\eta \in \{0, 1, 3, 5, 7, 9, 11\}$ and $\eta_2 = 7.5$. Model (41) and constant estimator (42)-(43) are considerate.

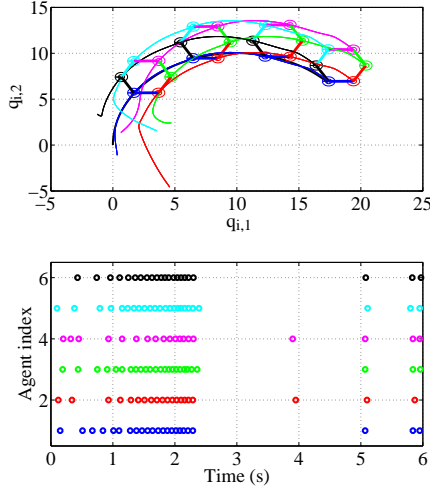
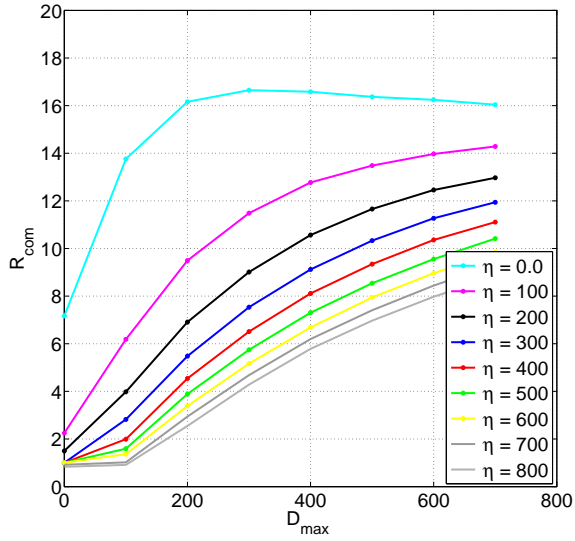


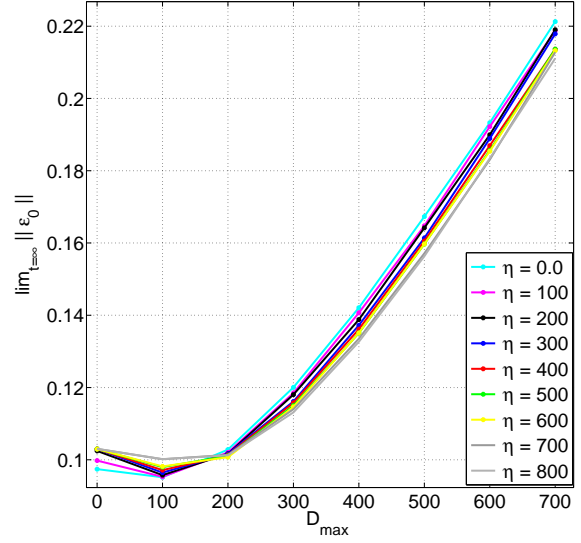
Fig. 8. Hexagonal formation and tracking problem with $D_{max} = 50$, $\eta = 50$, and $\eta_2 = 7.5$. Circles represents agents (top figure) and communication events (bottom figure). $R_{com} = 5\%$, $P(q,T) = 0.001$ and $\|\varepsilon_0\| = 0.1$. $T = 6$ s.

8 Conclusion

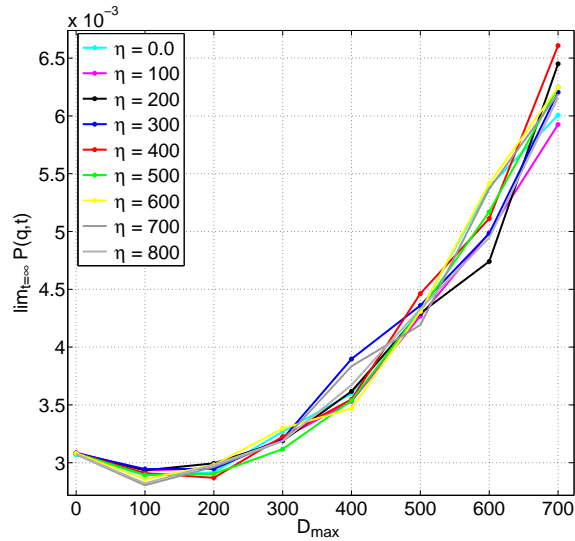
This paper presents an adaptive control and event-triggered communication strategy to reach a target formation for multi-agent systems with perturbed Euler-Lagrange dynamics. From estimate information of agents dynamics, an estimator has been proposed to provide the missing information required by the control. Convergence to a desired formation and influence of state perturbations on the convergence and on the amount of required communications have been studied. Tracking control to follow an desired trajectory has been considerate and added to the formation control. A distributed event-triggered condition to converge to a desired formation and follow the reference trajectory while reduce the number of communications have been studied. Simulations have shown the effectiveness of the proposed method in presence of state perturbations when their level remains moderate. The time interval between consecutive communications has been shown to be strictly positive.



(a)



(b)



(c)

Fig. 9. Evolution of R_{com} , $P(q, t)$ and ε_0 for different values of $D_{\text{max}} \in \{0, 100, 200, \dots, 700\}$, $\eta \in \{0, 100, 200, \dots, 800\}$ and $\eta_2 = 7.5$. The SS model (44) and accurate estimator (12) are considered.

In future work, the considered problem will be extended to communication delay and package drop.

Acknowledgments

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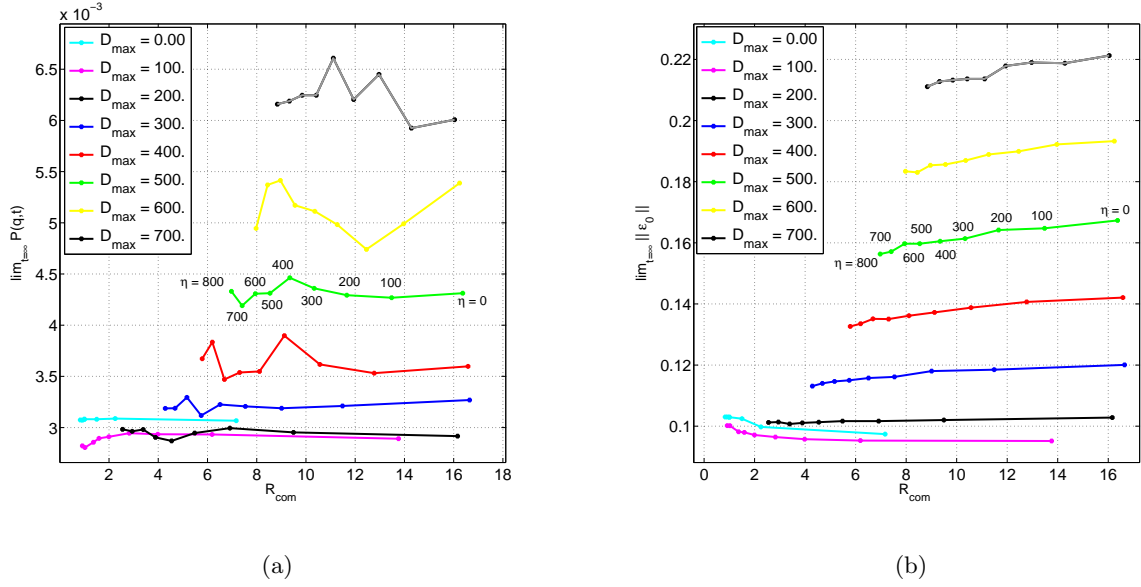


Fig. 10. Evolution of R_{com} , $P(q,t)$ and ϵ_0 for different values of $D_{max} \in \{0, 100, 200, \dots, 700\}$, $\eta \in \{0, 100, 200, \dots, 800\}$ and $\eta_2 = 7.5$. The SS model (44) and accurate estimator (12) are considered.

9 Appendix

9.1 Proof of Theorem 4

Consider a given value of D_{max} and η , one shows first that the MAS is input-to-state practically stable. One then evaluates the influence of D_{max} and η on the behavior of the MAS.

9.1.1 Proof of the input-to-state practical stability of the MAS

Consider the continuous positive-definite candidate Lyapunov function

$$V = \frac{1}{2} \sum_{i=1}^N (s_i^T M_i s_i + \Delta \theta_i^T \Gamma_i^{-1} \Delta \theta_i) + \frac{k_g}{2} \left[\frac{1}{2} P(q, t) + \sum_{i=1}^N k_0 \|q_i - q_i^*\|^2 \right] \quad (45)$$

where $\Delta \theta_i = \bar{\theta}_i - \theta_i$. The time derivative of V is

$$\dot{V} = \sum_{i=1}^N \left[\frac{1}{2} s_i^T \dot{M}_i s_i + s_i^T M_i \dot{s}_i + \Delta \theta_i^T \Gamma_i^{-1} \dot{\Delta \theta}_i \right] + \frac{k_g}{2} \frac{d}{dt} \left[\frac{1}{2} P(q, t) + \sum_{i=1}^N k_0 \|q_i - q_i^*\|^2 \right] \quad (46)$$

where, from (27), one has $\dot{s}_i = \ddot{q}_i - \ddot{q}_i^* + k_p \dot{q}_i$. Injecting (11) in (46) one obtains

$$\begin{aligned} \dot{V} = & \sum_{i=1}^N \left[\frac{1}{2} s_i^T \dot{M}_i s_i + s_i^T M_i \dot{s}_i + \Delta \theta_i^T Y_i(q_i, \dot{q}_i, \bar{p}_i, \bar{p}_i) \bar{s}_i \right] \\ & + \frac{k_g}{2} \frac{d}{dt} \left[\frac{1}{2} P(q, t) + \sum_{i=1}^N k_0 \|q_i - q_i^*\|^2 \right]. \end{aligned} \quad (47)$$

The last term in (47) may be written as

$$\begin{aligned}
& \frac{1}{2} \frac{d}{dt} \left[\frac{1}{2} P(q, t) + \sum_{i=1}^N k_0 \|q_i - q_i^*\|^2 \right] \\
&= \frac{1}{4} \frac{d}{dt} \sum_{i=1}^N \sum_{j=1}^N k_{ij} \|r_{ij} - r_{ij}^*\|^2 + \frac{1}{2} \frac{d}{dt} \sum_{i=1}^N k_0 \|q_i - q_i^*\|^2 \\
&= \sum_{i=1}^N \left[\frac{1}{2} \sum_{j=1}^N k_{ij} (\dot{r}_{ij} - \dot{r}_{ij}^*)^T (r_{ij} - r_{ij}^*) + k_0 (\dot{q}_i - \dot{q}_i^*)^T (q_i - q_i^*) \right] \\
&= \sum_{i=1}^N \left[\frac{1}{2} \sum_{j=1}^N k_{ij} [(\dot{q}_i - \dot{q}_i^*)^T (r_{ij} - r_{ij}^*) - (\dot{q}_j - \dot{q}_j^*)^T (r_{ij} - r_{ij}^*)] \right. \\
&\quad \left. + k_0 (\dot{q}_i - \dot{q}_i^*)^T (q_i - q_i^*) \right] \\
&= \sum_{i=1}^N \left[\frac{1}{2} \sum_{j=1}^N k_{ij} [(\dot{q}_i - \dot{q}_i^*)^T (r_{ij} - r_{ij}^*) - (\dot{q}_i - \dot{q}_i^*)^T (r_{ji} - r_{ji}^*)] \right. \\
&\quad \left. + k_0 (\dot{q}_i - \dot{q}_i^*)^T \varepsilon_i \right]. \tag{48}
\end{aligned}$$

Since $r_{ji} = -r_{ij}$, one gets

$$\begin{aligned}
\frac{1}{2} \frac{d}{dt} \left[\frac{1}{2} P(q, t) + \sum_{i=1}^N k_0 \|q_i - q_i^*\|^2 \right] &= \sum_{i=1}^N (\dot{q}_i - \dot{q}_i^*)^T \left[\sum_{j=1}^N k_{ij} (r_{ij} - r_{ij}^*) + k_0 \varepsilon_i \right] \\
&= \sum_{i=1}^N (\dot{q}_i - \dot{q}_i^*)^T g_i. \tag{49}
\end{aligned}$$

Combining (47) and (49), one obtains

$$\dot{V} = \sum_{i=1}^N \left[\frac{1}{2} s_i^T M_i \dot{s}_i + s_i^T M_i \dot{s}_i + \Delta \theta_i^T Y_i(q_i, \dot{q}_i, \dot{p}_i, \bar{p}_i) \bar{s}_i + k_g (\dot{q}_i - \dot{q}_i^*)^T g_i \right]. \tag{50}$$

One focuses now on the term $M_i \dot{s}_i$. Using again (27), one may write

$$M_i \dot{s}_i + C_i s_i = M_i (\ddot{q}_i - \ddot{q}_i^* + k_p \dot{q}_i) + C_i (\dot{q}_i - \dot{q}_i^* + k_p g_i) \tag{51}$$

Using (1), one gets

$$M_i \dot{s}_i + C_i s_i = \tau_i + d_i - G + M_i (k_p \dot{g}_i - \ddot{q}_i^*) + C_i (k_p g_i - \dot{q}_i^*), \tag{52}$$

where one used (1). Now, introducing (30), one gets

$$\begin{aligned}
M_i \dot{s}_i + C_i s_i &= -k_s \bar{s}_i - k_g \bar{g}_i - Y_i(q_i, \dot{q}_i, k_p \dot{g}_i - \ddot{q}_i^*, k_p \bar{g}_i - \dot{q}_i^*) \bar{\theta}_i \\
&\quad + M_i (k_p \dot{g}_i - \ddot{q}_i^*) + C_i (k_p g_i - \dot{q}_i^*) + d_i \tag{53}
\end{aligned}$$

In what follows, one uses Y_i in place of $Y_i(q_i, \dot{q}_i, k_p \dot{g}_i - \ddot{q}_i^*, k_p \bar{g}_i - \dot{q}_i^*)$ to lighten notations. Since $\Delta \theta_i = \bar{\theta}_i - \theta_i$, one obtains

$$\begin{aligned}
s_i^T M_i \dot{s}_i &= -k_s s_i^T \bar{s}_i - k_g s_i^T \bar{g}_i - s_i^T C_i s_i + s_i^T (M_i (k_p \dot{g}_i - \ddot{q}_i^*) + C_i (k_p g_i - \dot{q}_i^*)) \\
&\quad - s_i^T Y_i(\theta_i + \Delta \theta_i) + s_i^T d_i. \tag{54}
\end{aligned}$$

Using (2) in (54) leads to

$$-s_i^T Y_i (\theta_i + \Delta\theta_i) = -s_i^T Y_i \Delta\theta_i - s_i^T (M_i (k_p \dot{g}_i - \ddot{q}_i^*) + C_i (k_p \bar{g}_i - \dot{q}_i^*)). \quad (55)$$

Considering (2) and (54) in (50), one gets

$$\begin{aligned} \dot{V} = & \sum_{i=1}^N \left[\frac{1}{2} s_i^T \dot{M}_i s_i - k_s s_i^T \bar{s}_i - k_g s_i^T \bar{g}_i - s_i^T C_i s_i + s_i^T (M_i (k_p \dot{g}_i - \ddot{q}_i^*) + C_i (k_p \bar{g}_i - \dot{q}_i^*)) \right. \\ & - s_i^T (M_i (k_p \dot{g}_i - \ddot{q}_i^*) + C_i (k_p \bar{g}_i - \dot{q}_i^*)) - s_i^T Y_i \Delta\theta_i + \bar{s}_i^T Y_i \Delta\theta_i \\ & \left. + k_g (\dot{q}_i - \dot{q}_i^*)^T g_i + s_i^T d_i \right]. \end{aligned} \quad (56)$$

Now, introduce (24) in (27) to get

$$s_i = \dot{q}_i - \dot{q}_i^* + k_p \left[\sum_{j=1}^N k_{ij} (q_i - q_j - r_{ij}^*) + k_0 \varepsilon_i \right]. \quad (57)$$

Since $e_j^i = \hat{q}_j^i - q_j$, one gets

$$\begin{aligned} s_i &= \dot{q}_i - \dot{q}_i^* + k_p \left[\sum_{j=1}^N k_{ij} (q_i - \hat{q}_j^i + e_j^i - r_{ij}^*) + k_0 \varepsilon_i \right] \\ &= \dot{q}_i - \dot{q}_i^* + k_p \left[\sum_{j=1}^N k_{ij} (\bar{r}_{ij} - r_{ij}^*) + k_0 \varepsilon_i \right] + k_p \sum_{\substack{j=1 \\ j \neq i}}^N k_{ij} e_j^i \\ &= \bar{s}_i + k_p E_j^i, \end{aligned} \quad (58)$$

with since $k_{ii} = 0$

$$E_j^i = \sum_{i=1}^N k_{ij} e_j^i. \quad (59)$$

Using similar derivations, one may show that

$$g_i = \bar{g}_i + E_j^i. \quad (60)$$

Replacing (58) and (60) in (56), one gets

$$\begin{aligned} \dot{V} = & \sum_{i=1}^N \left[s_i^T \left[\frac{1}{2} \dot{M}_i - C_i \right] s_i - k_s s_i^T \bar{s}_i - k_g (\dot{q}_i - \dot{q}_i^* + k_p g_i)^T \bar{g}_i \right. \\ & \left. + k_p s_i^T (M_i \dot{E}_j^i + C_i E_j^i) + k_p E_j^{iT} Y_i \Delta\theta_i + k_g (\dot{q}_i - \dot{q}_i^*)^T g_i + s_i^T d_i \right]. \end{aligned} \quad (61)$$

Let

$$\dot{V}_1 = \sum_{i=1}^N 2k_p s_i^T (M_i \dot{E}_j^i + C_i E_j^i)$$

and

$$\dot{V}_2 = 2k_p \sum_{i=1}^N E_j^{iT} Y_i \Delta \theta_i.$$

Since $\frac{1}{2}\dot{M}_i - C_i$ is skew symmetric or definite negative, $s_i^T \left[\frac{1}{2}\dot{M}_i - C_i \right] s_i \leq 0$. For all $b > 0$ and all vectors x and y of similar size, one has

$$x^T y \leq \frac{1}{2} \left(b x^T x + \frac{1}{b} y^T y \right). \quad (62)$$

Using (62) with $b = 1$, one deduces that $d_i^T s_i \leq \frac{1}{2} (D_{\max}^2 + s_i^T s_i)$ and that

$$\begin{aligned} \dot{V} \leq & \sum_{i=1}^N \left[-k_s s_i^T \bar{s}_i - k_g k_p g_i^T \bar{g}_i + \frac{1}{2} s_i^T s_i + \frac{1}{2} D_{\max}^2 \right. \\ & \left. + k_g (\dot{q}_i - \dot{q}_i^*)^T (g_i - \bar{g}_i) \right] + \frac{1}{2} (\dot{V}_1 + \dot{V}_2) \end{aligned} \quad (63)$$

One notices that $r_{ij} = q_i - q_j = q_i - \hat{q}_j^i + e_j^i = \bar{r}_{ij} + e_j^i$, thus

$$\begin{aligned} \|s_i - \bar{s}_i\|^2 &= s_i^T s_i - 2s_i^T \bar{s}_i + \bar{s}_i^T \bar{s}_i \\ \|k_p E_j^i\|^2 &= s_i^T s_i - 2s_i^T \bar{s}_i + \bar{s}_i^T \bar{s}_i \\ s_i^T \bar{s}_i &= -\frac{1}{2} \|k_p E_j^i\|^2 + \frac{1}{2} s_i^T s_i + \frac{1}{2} \bar{s}_i^T \bar{s}_i \end{aligned} \quad (64)$$

In the same way, from (64), one shows that

$$g_i^T \bar{g}_i = -\frac{1}{2} \|E_j^i\|^2 + \frac{1}{2} g_i^T g_i + \frac{1}{2} \bar{g}_i^T \bar{g}_i. \quad (65)$$

Injecting (65) in (63),

$$\begin{aligned} \dot{V} \leq & \sum_{i=1}^N \left[\frac{k_s}{2} \left(k_p^2 \|E_j^i\|^2 - s_i^T s_i - \bar{s}_i^T \bar{s}_i \right) + k_p k_g \frac{1}{2} \left(\|E_j^i\|^2 - g_i^T g_i - \bar{g}_i^T \bar{g}_i \right) + \frac{1}{2} s_i^T s_i + \frac{1}{2} D_{\max}^2 \right. \\ & \left. + k_g (\dot{q}_i - \dot{q}_i^*)^T (g_i - \bar{g}_i) \right] + \frac{1}{2} (\dot{V}_1 + \dot{V}_2) \\ \leq & \sum_{i=1}^N \left[-\frac{(k_s - 1)}{2} s_i^T s_i - \frac{k_s}{2} \bar{s}_i^T \bar{s}_i + \frac{k_s k_p^2 + k_g k_p}{2} \|E_j^i\|^2 - \frac{1}{2} k_p k_g (g_i^T g_i + \bar{g}_i^T \bar{g}_i) + \frac{1}{2} D_{\max}^2 \right. \\ & \left. + k_g (\dot{q}_i - \dot{q}_i^*)^T (g_i - \bar{g}_i) \right] + \frac{1}{2} (\dot{V}_1 + \dot{V}_2). \end{aligned} \quad (66)$$

Using (62) with $b = b_i > 0$, one shows that $2\dot{q}_i^T (g_i - \bar{g}_i) \leq \left(b_i \|\dot{q}_i\|^2 + \frac{1}{b_i} \|E_j^i\|^2 \right)$. Using this result in (66), one gets

$$\begin{aligned} \dot{V} \leq & \frac{1}{2} \sum_{i=1}^N \left[-(k_s - 1) s_i^T s_i - k_s \bar{s}_i^T \bar{s}_i + \left(k_s k_p^2 + k_g k_p + \frac{k_g}{b_i} \right) \|E_j^i\|^2 + b_i k_g \|\dot{q}_i - \dot{q}_i^*\|^2 \right. \\ & \left. - k_p k_g (g_i^T g_i + \bar{g}_i^T \bar{g}_i) + D_{\max}^2 \right] + \frac{1}{2} (\dot{V}_1 + \dot{V}_2). \end{aligned} \quad (67)$$

Consider now \dot{V}_1 . Using (62) with $b = 1$, the fact that M_i is symmetric positive definite, and that $x^T M_i x < k_M x^T x$,

one obtains

$$\begin{aligned}
\sum_{i=1}^N 2k_p s_i^T (M_i \dot{E}_j^i + C_i E_j^i) &\leq \sum_{i=1}^N k_p \left(s_i^T M_i s_i + s_i^T s_i + \left[\dot{E}_j^{iT} M_i \dot{E}_j^i + E_j^{iT} C_i^T C_i E_j^i \right] \right) \\
&\leq \sum_{i=1}^N k_p \left((k_M + 1) s_i^T s_i + \left[k_M \dot{E}_j^{iT} \dot{E}_j^i + E_j^{iT} C_i^T C_i E_j^i \right] \right)
\end{aligned} \tag{68}$$

Focus now on the terms $E_j^{iT} C_i^T C_i E_j^i$

$$\begin{aligned}
\sum_{i=1}^N E_j^{iT} C_i^T C_i E_j^i &= \sum_{i=1}^N \left(\sum_{j=1}^N k_{ij} e_j^i \right)^T C_i^T C_i \left(\sum_{\ell=1}^N k_{i\ell} e_\ell^i \right) \\
&\leq \sum_{i=1}^N \sum_{j=1}^N \sum_{\ell=1}^N k_{i\ell} k_{ij} \|C_i\|^2 e_j^{iT} e_\ell^i.
\end{aligned} \tag{69}$$

Using (62) with $b = 1$, one gets

$$\begin{aligned}
\sum_{i=1}^N E_j^{iT} C_i^T C_i E_j^i &\leq \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N \sum_{\ell=1}^N k_{i\ell} k_{ij} \|C_i\|^2 (e_j^{iT} e_j^i + e_\ell^{iT} e_\ell^i) \\
&\leq \sum_{i=1}^N \sum_{j=1}^N \sum_{\ell=1}^N k_{i\ell} k_{ij} \|C_i\|^2 (e_j^{iT} e_j^i) \\
&\leq \sum_{i=1}^N \alpha_i \sum_{j=1}^N k_{ij} \|C_i\|^2 (e_j^{iT} e_j^i).
\end{aligned} \tag{70}$$

Since one has assumed that (40) and (39) are satisfied, one has $\hat{q}_j^i = \hat{q}_j^j$, $e_j^i = e_j^j$. As a consequence,

$$\sum_{i=1}^N \sum_{j=1}^N k_{ij} \|e_j^i\|^2 = \sum_{i=1}^N \sum_{j=1}^N k_{ij} \|e_j^j\|^2 = \sum_{i=1}^N \sum_{j=1}^N k_{ji} \|e_i^i\|^2. \tag{71}$$

and since $k_{ij} = k_{ji}$,

$$\begin{aligned}
\sum_{i=1}^N E_j^{iT} C_i^T C_i E_j^i &\leq \sum_{i=1}^N \left(\alpha_M \sum_{j=1}^N \left[k_{ij} \|e_i^i\|^2 \|C_j\|^2 \right] \right) \\
&\leq \sum_{i=1}^N \left(\alpha_M \sum_{j=1}^N \left[k_{ij} \|e_i^i\|^2 k_C^2 \|\hat{q}_j\|^2 \right] \right).
\end{aligned} \tag{72}$$

Then, the second CTC (38) leads to

$$\sum_{i=1}^N E_j^{iT} C_i^T C_i E_j^i \leq \sum_{i=1}^N \left(\alpha_M k_C^2 \|e_i^i\|^2 \sum_{j=1}^N k_{ij} \left(\|\hat{q}_j^i\| + \eta_2 \right)^2 \right). \tag{73}$$

Similarly, one shows that

$$\sum_{i=1}^N E_j^{iT} E_j^i \leq \sum_{i=1}^N \alpha_M^2 \|e_i^i\|^2$$

and

$$\sum_{i=1}^N \dot{E}_j^{iT} \dot{E}_j^i \leq \sum_{i=1}^N \alpha_M^2 \|e_i^i\|^2.$$

Consider now \dot{V}_2

$$\begin{aligned} \dot{V}_2 &= 2k_p \sum_{i=1}^N E_j^{iT} Y_i \Delta \theta_i \\ &= 2k_p \sum_{i=1}^N \left(\sum_{j=1}^N k_{ij} e_j^i \right)^T Y_i \Delta \theta_i. \end{aligned} \quad (74)$$

Since $e_j^i = e_j^j$, one gets

$$\begin{aligned} \dot{V}_2 &= 2k_p \sum_{i=1}^N \left(\sum_{j=1}^N k_{ij} e_j^j \right)^T Y_i \Delta \theta_i \\ &= 2k_p \sum_{j=1}^N \sum_{i=1}^N (k_{ji} e_i^i)^T Y_j \Delta \theta_j \\ &= 2k_p \sum_{i=1}^N e_i^{iT} \sum_{j=1}^N k_{ji} Y_j \Delta \theta_j. \end{aligned} \quad (75)$$

Let $0_n = [0, \dots, 0]^T \in \mathbb{R}^n$ be the all-zero vector. If $e_i^i = 0_n$, one has $2k_p e_i^{iT} \sum_{j=1}^N k_{ji} Y_j \Delta \theta_j = 0$. Considering now the case $e_i^i \neq 0_n$. Using (62) with $b = b_{i2} > 0$, one obtains

$$\dot{V}_2 = 2k_p \sum_{i=1}^N E_j^{iT} Y_i \Delta \theta_i \quad (76)$$

$$\leq k_p \sum_{i=1}^N \left(b_{i2} E_j^{iT} E_j^i + \frac{1}{b_{i2}} \|Y_i \Delta \theta_i\|^2 \right). \quad (77)$$

Since $\sum_{i=1}^N E_j^{iT} E_j^i \leq \sum_{i=1}^N \alpha_M^2 \|e_i^i\|^2$, one gets

$$\begin{aligned} \dot{V}_2 &\leq \sum_{i=1}^N k_p \left(\alpha_M^2 b_{i2} \|e_i^i\|^2 + \frac{1}{b_{i2}} \|Y_i\| \|\Delta \theta_i\|^2 \right) \\ &\leq \sum_{i=1}^N k_p \left(\alpha_M^2 b_{i2} \|e_i^i\|^2 + \frac{1}{b_{i2}} \|Y_i\| \|\Delta \theta_{i,\max}\|^2 \right), \end{aligned} \quad (78)$$

where $\Delta \theta_{i,\max}$ is given by (34).

Since $e_i^i \neq 0_n$, choosing $b_{i2} = \frac{1 + \|Y_i\| \Delta\theta_{i,\max}}{\|e_i^i\|}$, one obtains $\dot{V}_2 \leq \dot{V}_3$ with

$$\begin{aligned} \dot{V}_3 &= \sum_{i=1}^N k_p \left[\alpha_M^2 \left(\frac{1 + \|Y_i\| \Delta\theta_{i,\max}}{\|e_i^i\|} \right) \|e_i^i\|^2 + \frac{\|e_i^i\| \|Y_i\| \Delta\theta_{i,\max}}{(1 + \|Y_i\| \Delta\theta_{i,\max})} \right] \\ &= \sum_{i=1}^N k_p \|e_i^i\| \left[\alpha_M^2 (1 + \|Y_i\| \Delta\theta_{i,\max}) + \frac{\|Y_i\| \Delta\theta_{i,\max}}{(1 + \|Y_i\| \Delta\theta_{i,\max})} \right]. \end{aligned} \quad (79)$$

Injecting (68), (73), and (79) in (67), one gets

$$\begin{aligned} \dot{V} &\leq \frac{1}{2} \sum_{i=1}^N \left[-(k_s - 1 - k_p(k_M + 1)) s_i^T s_i - k_s \bar{s}_i^T \bar{s}_i + D_{\max}^2 \right. \\ &\quad - k_p k_g g_i^T g_i - k_p k_g \bar{g}_i^T \bar{g}_i + k_g b_i \|\dot{q}_i - \dot{q}_i^*\|^2 + k_p k_M \alpha_M^2 \|e_i^i\|^2 \\ &\quad \left. + \alpha_M^2 \left(k_s k_p^2 + k_g k_p + \frac{k_g}{b_i} \right) \|e_i^i\|^2 + \alpha_M k_p k_C^2 \|e_i^i\|^2 \sum_{j=1}^N k_{ij} \left[\|\dot{q}_j^i\| + \eta_2 \right]^2 \right] + \frac{1}{2} \dot{V}_3. \end{aligned} \quad (80)$$

The CTC (37) leads to

$$\begin{aligned} \dot{V} &\leq \frac{1}{2} \sum_{i=1}^N \left[-(k_s - 1 - k_p(k_M + 1)) s_i^T s_i - k_g k_p g_i^T g_i + D_{\max}^2 + \eta \right] \\ \dot{V} &\leq \frac{1}{2} \sum_{i=1}^N \left[-k_1 s_i^T s_i - k_g k_p g_i^T g_i + D_{\max}^2 + \eta \right] \end{aligned} \quad (81)$$

with $k_1 = k_s - 1 - k_p(k_M + 1)$.

Following the steps given in Appendix 9.3.1 from (106) to (110), one shows that

$$\dot{V} \leq -c_3 V + \frac{N}{2} [D_{\max}^2 + \eta] + \frac{c_3}{2} \sum_{i=1}^N (\Delta\theta_i^T \Gamma_i^{-1} \Delta\theta_i), \quad (82)$$

where $c_3 > 0$ is a positive constant. Introducing $\Delta_{\max} = \max_{i=1:N} (\sup_{t>0} (\Delta\theta_i^T \Gamma_i^{-1} \Delta\theta_i))$, one has

$$\dot{V} \leq -c_3 V + \frac{N}{2} [c_3 \Delta_{\max} + D_{\max}^2 + \eta]. \quad (83)$$

Define the function W such that $W(0) = V(0)$ and

$$\dot{W} = -c_3 W + \frac{N}{2} [D_{\max}^2 + \eta + c_3 \Delta_{\max}]. \quad (84)$$

Using the initial condition $W(0) = V(0)$, the solution of (84) is

$$W(t) = \exp(-c_3 t) V(0) + (1 - \exp(-c_3 t)) \frac{N}{2c_3} [D_{\max}^2 + \eta + c_3 \Delta_{\max}]. \quad (85)$$

Then, using the Lemma 3.4 in [1] (Comparison lemma), one has $V(t) \leq W(t)$ and so

$$V(t) \leq \exp(-c_3 t) V(0) + (1 - \exp(-c_3 t)) \frac{N}{2c_3} [D_{\max}^2 + \eta + c_3 \Delta_{\max}] \quad (86)$$

Since M_i and Γ_i are symmetric, there exists matrices S_{M_i} and S_{Γ_i} such that $M_i = S_{M_i}^T S_{M_i}$ and $\Gamma_i = S_{\Gamma_i}^T S_{\Gamma_i}$. Introduce now

$$y_M = \left[(S_{M_1} s_1)^T \dots (S_{M_i} s_i)^T \dots (S_{M_N} s_N)^T \right]^T \quad (87)$$

$$y_\Gamma = \left[(S_{\Gamma_1}^{-1} \Delta \theta_1)^T \dots (S_{\Gamma_i}^{-1} \Delta \theta_i)^T \dots (S_{\Gamma_N}^{-1} \Delta \theta_N)^T \right]^T \quad (88)$$

$$y_q = \left[(q_1 - q_1^*)^T \dots (q_i - q_i^*)^T \dots (q_N - q_N^*)^T \right]^T \quad (89)$$

$$z = \left[y_M^T \ y_\Gamma^T \ \sqrt{k_g k_0} y_q^T \ \sqrt{\frac{k_g}{2}} P(x, t) \right]^T \quad (90)$$

Then, $V(t)$ can be rewritten as

$$V(z) = \frac{1}{2} z^T z. \quad (91)$$

Using (91) in (86), one has $\forall t \geq 0$

$$\begin{aligned} \|z(t)\|^2 &\leq \exp(-c_3 t) \|z(0)\|^2 + (1 - \exp(-c_3 t)) \frac{N}{c_3} [D_{\max}^2 + \eta + c_3 \Delta_{\max}] \\ \|z(t)\| &\leq \sqrt{\exp(-c_3 t) \|z(0)\|^2 + (1 - \exp(-c_3 t)) \frac{N}{c_3} [D_{\max}^2 + \eta + c_3 \Delta_{\max}]} \\ \|z(t)\| &\leq \sqrt{\exp(-c_3 t) \|z(0)\|^2} + \sqrt{(1 - \exp(-c_3 t)) \frac{N}{c_3} [D_{\max}^2 + \eta + c_3 \Delta_{\max}]} \\ \|z(t)\| &\leq \exp\left(-\frac{c_3}{2} t\right) \|z(0)\| + \sqrt{\frac{N}{c_3} [D_{\max}^2 + \eta + c_3 \Delta_{\max}]} \end{aligned} \quad (92)$$

and so

$$\|z(t)\| \leq \beta(\|z(0)\|, t) + \rho \quad (93)$$

with $\rho = \sqrt{\frac{N}{c_3} [D_{\max}^2 + \eta + c_3 \Delta_{\max}]}$, $\beta(\|z(0)\|, t) = \exp\left(-\frac{c_3}{2} t\right) \|z(0)\|$, and $\beta \in \mathcal{KL}$. Using Definition 2.1 from [16], (93) implies that the MAS is input-to-state practically stable.

9.1.2 Convergence of V

From (93), we know the system is ISpS. Moreover, from (82), one has

$$\dot{V} \leq -c_3 V + \frac{N}{2} [D_{\max}^2 + \eta] + \frac{c_3}{2} \sum_{i=1}^N (\Delta \theta_i^T \Gamma_i^{-1} \Delta \theta_i) \quad (94)$$

Then, if initially

$$-c_3 V(0) + \frac{N}{2} [D_{\max}^2 + \eta] + \frac{c_3}{2} \sum_{i=1}^N (\Delta \theta_i^T \Gamma_i^{-1} \Delta \theta_i) < 0 \quad (95)$$

one has $\dot{V} \leq 0$ and V is decreasing. Then, one has from (86)

$$\begin{aligned}
\lim_{t \rightarrow \infty} \frac{1}{2} \sum_{i=1}^N (s_i^T M_i s_i + \Delta \theta_i^T \Gamma_i^{-1} \Delta \theta_i) + \frac{k_g}{2} \left(\sum_{i=1}^N k_0 \|\varepsilon_i\|^2 + \frac{1}{2} P(q, t) \right) &\leq \frac{N}{2c_3} [D_{\max}^2 + \eta + c_3 \Delta_{\max}] \\
\lim_{t \rightarrow \infty} \frac{k_g}{2} \left(\sum_{i=1}^N k_0 \|\varepsilon_i\|^2 + \frac{1}{2} P(q, t) \right) &\leq \frac{N}{2c_3} [D_{\max}^2 + \eta + c_3 \Delta_{\max}] \\
&\quad - \lim_{t \rightarrow \infty} \frac{1}{2} \sum_{i=1}^N (s_i^T M_i s_i + \Delta \theta_i^T \Gamma_i^{-1} \Delta \theta_i) \\
\lim_{t \rightarrow \infty} \sum_{i=1}^N k_0 \|\varepsilon_i\|^2 + \frac{1}{2} P(q, t) &\leq \frac{N}{k_g c_3} [D_{\max}^2 + \eta + c_3 \Delta_{\max}]. \tag{96}
\end{aligned}$$

Asymptotically, the formation and tracking error are bounded.

9.2 Proof of $t_{i,k+1} - t_{i,k} > 0$

From the CTC (37), a communication is triggered at $t = t_{i,k}^-$ when

$$\begin{aligned}
k_s \bar{s}_i^T \bar{s}_i + k_p k_g \bar{g}_i^T \bar{g}_i + \eta &= \alpha_M^2 \left(k_e \|e_i^i\|^2 + k_p k_M \|\dot{e}_i^i\|^2 \right) \\
&\quad + \alpha_M k_C^2 k_p \|e_i^i\|^2 \sum_{j=1}^N k_{ji} \left[\|\dot{q}_j^i\| + \eta_2 \right]^2 + k_g b_i \|\dot{q}_i - \dot{q}_i^*\|^2 \\
&\quad + k_p \|e_i^i\| \left[\alpha_M^2 \left(1 + \|Y_i\| \Delta \theta_{i,\max} \right)^2 + \frac{\|Y_i\| \Delta \theta_{i,\max} \|^2}{\left(1 + \|Y_i\| \Delta \theta_{i,\max} \right)^2} \right] \tag{97}
\end{aligned}$$

with $k_e = \left(k_s k_p^2 + k_g k_p + \frac{k_g}{b_i} \right)$. Then, the estimation errors e_i^i and \dot{e}_i^i are reset and one has $e_i^i(t_{i,k}^+) = 0$ and $\dot{e}_i^i(t_{i,k}^+) = 0$. As a consequence, the CTC (37) in Theorem 4 is not satisfied at $t = t_{i,k}^+$ iff

$$k_s \bar{s}_i^T \bar{s}_i + k_p k_g \bar{g}_i^T \bar{g}_i + \eta > k_g b_i \|\dot{q}_i - \dot{q}_i^*\|^2. \tag{98}$$

To prove the absence of Zeno behavior, *i.e.*, that $t_{i,k+1} > t_{i,k}$, one has to show that (98) is satisfied.

Using the property $x^T y \geq -\frac{1}{2} \left(b_{i2} x^T x + \frac{1}{b_{i2}} y^T y \right)$ for some $b_{i2} > 0$, one deduces that

$$\begin{aligned}
\bar{s}_i^T \bar{s}_i &= k_p^2 \bar{g}_i^T \bar{g}_i + \|\dot{q}_i - \dot{q}_i^*\|^2 + 2k_p \bar{g}_i^T (\dot{q}_i - \dot{q}_i^*) \\
&\geq (k_p^2 - k_p b_{i2}) \bar{g}_i^T \bar{g}_i + \left(1 - \frac{k_p}{b_{i2}} \right) \|\dot{q}_i - \dot{q}_i^*\|^2. \tag{99}
\end{aligned}$$

Using (99), a sufficient condition for (98) to be satisfied is

$$\begin{aligned}
k_s (k_p^2 - k_p b_{i2}) \bar{g}_i^T \bar{g}_i + k_s \left(1 - \frac{k_p}{b_{i2}}\right) \|\dot{q}_i - \dot{q}_i^*\|^2 + k_p k_g \bar{g}_i^T \bar{g}_i + \eta &> k_g b_i \|\dot{q}_i - \dot{q}_i^*\|^2 \\
k_s \left(1 - \frac{k_p}{b_{i2}}\right) \|\dot{q}_i - \dot{q}_i^*\|^2 + [k_p k_g + k_s (k_p^2 - k_p b_{i2})] \bar{g}_i^T \bar{g}_i + \eta &> k_g b_i \|\dot{q}_i - \dot{q}_i^*\|^2 \\
k_1 \bar{g}_i^T \bar{g}_i + \eta &> k_2 \|\dot{q}_i - \dot{q}_i^*\|^2
\end{aligned} \tag{100}$$

where $k_1 = [k_p k_g + k_s (k_p^2 - k_p b_{i2})]$ and $k_2 = [k_g b_i - k_s (1 - \frac{k_p}{b_{i2}})]$. To ensure that the inequality (100) is satisfied independently of the values of \bar{g}_i and \dot{q}_i , it is sufficient to find b_i and b_{i2} such that $k_1 > 0$ and $k_2 < 0$. Consider first k_1 .

$$\begin{aligned}
k_p k_g + k_s (k_p^2 - k_p b_{i2}) &> 0 \\
\frac{k_g}{k_s} &> (-k_p + b_{i2}) \\
\frac{k_s k_p + k_g}{k_s} &> b_{i2}.
\end{aligned} \tag{101}$$

Focus now on k_2

$$\begin{aligned}
k_g b_i - k_s \left(1 - \frac{k_p}{b_{i2}}\right) &< 0 \\
\frac{k_g b_i}{k_s} &< 1 - \frac{k_p}{b_{i2}} \\
\frac{1}{b_{i2}} &< \frac{1}{k_p} \left(1 - \frac{k_g b_i}{k_s}\right).
\end{aligned} \tag{102}$$

Since $b_{i2} > 0$, one has $\frac{k_g b_i}{k_s} < 1$ and so $b_i < \frac{k_s}{k_g}$. Then

$$\frac{k_s k_p}{k_s - k_g b_i} < b_{i2}. \tag{103}$$

Finally, one has to find a condition on b_i such that (101) and (102) can be satisfied simultaneously

$$\frac{k_s k_p + k_g}{k_s} > b_{i2} > \frac{k_s k_p}{k_s - k_g b_i}. \tag{104}$$

One may find b_{i2} if

$$\begin{aligned}
k_s - k_g b_i &> \frac{k_s^2 k_p}{k_s k_p + k_g} \\
\frac{1}{k_g} \left(k_s - \frac{k_s^2 k_p}{k_s k_p + k_g}\right) &> b_i \\
b_i &< \frac{k_s}{k_s k_p + k_g}.
\end{aligned} \tag{105}$$

which also ensures that $b_i < \frac{k_s}{k_g}$. Thus, once $b_i < \frac{k_s}{k_s k_p + k_g}$, there exists some b_{i2} such that (104) is satisfied. As a consequence $t_{i,k+1} - t_{i,k} > 0$.

9.3 Complementary proof elements

9.3.1 Differential equation satisfied by V

From (81), one gets

$$\dot{V} \leq \frac{1}{2} \sum_{i=1}^N [-k_m (s_i^T s_i - k_g g_i^T g_i) + D_{\max}^2 + \eta] \quad (106)$$

where $k_m = \min \{k_1, k_p\}$. Using (117), one may write

$$\begin{aligned} \sum_{i=1}^N g_i^T g_i &\geq \sum_{i=1}^N k_0^2 \|\varepsilon_i\|^2 + \left(2k_0 + \frac{\alpha_{\min} k_{\min}}{k_{\max}}\right) P(q, t) \\ &\geq k_2 \left(\sum_{i=1}^N k_0^2 \|\varepsilon_i\|^2 + \frac{1}{2} P(q, t) \right) \end{aligned} \quad (107)$$

where

$$k_2 = \begin{cases} 2 \left(2k_0 + \frac{\alpha_{\min} k_{\min}}{k_{\max}}\right) & \text{if } \left(2k_0 + \frac{\alpha_{\min} k_{\min}}{k_{\max}}\right) < \frac{1}{2} \\ 1 & \text{else.} \end{cases}$$

Then

$$\begin{aligned} \sum_{i=1}^N g_i^T g_i &\geq k_2 \left(\sum_{i=1}^N k_0^2 \|\varepsilon_i\|^2 + \frac{1}{2} P(q, t) \right) \\ &\geq k_3 \left(\sum_{i=1}^N k_0 \|\varepsilon_i\|^2 + \frac{1}{2} P(q, t) \right) \end{aligned} \quad (108)$$

where $k_3 = k_2 k_0$ if $k_0 < 1$, $k_3 = 1$ else. Then

$$\begin{aligned} \dot{V} &\leq -\frac{1}{2} \sum_{i=1}^N (k_m s_i^T s_i) - \frac{k_3 k_g}{2} \left(\sum_{i=1}^N k_0 \|\varepsilon_i\|^2 + \frac{1}{2} P(q, t) \right) + \frac{N}{2} (D_{\max}^2 + \eta) \\ &\leq -\frac{1}{k_M^*} \left[\frac{1}{2} \sum_{i=1}^N k_m (k_M s_i^T s_i) + \frac{k_3 k_g}{2} \left(\sum_{i=1}^N k_0 \|\varepsilon_i\|^2 + \frac{1}{2} P(q, t) \right) \right] + \frac{N}{2} (D_{\max}^2 + \eta) \\ &\leq -\frac{k_4}{k_M^*} \left[\frac{1}{2} \sum_{i=1}^N (k_M s_i^T s_i) + \frac{k_g}{2} \left(\sum_{i=1}^N k_0 \|\varepsilon_i\|^2 + \frac{1}{2} P(q, t) \right) \right] + \frac{N}{2} (D_{\max}^2 + \eta) \end{aligned} \quad (109)$$

with $k_M^* = 1$ if $k_M < 1$ and $k_M^* = k_M$ else, and $k_4 = \min(k_m, k_3)$. Let $c_3 = \frac{k_4}{k_M^*}$ and one gets

$$\begin{aligned} \dot{V} &\leq -c_3 \left[\frac{1}{2} \sum_{i=1}^N [s_i^T M_i s_i + \Delta \theta_i^T \Gamma_i^{-1} \Delta \theta_i] + \frac{k_g}{2} \left(\sum_{i=1}^N k_0 \|\varepsilon_i\|^2 + \frac{1}{2} P(q, t) \right) \right] \\ &\quad + \frac{N}{2} [D_{\max}^2 + \eta] + \frac{c_3}{2} \sum_{i=1}^N \Delta \theta_i^T \Gamma_i^{-1} \Delta \theta_i \\ \dot{V} &\leq -c_3 V + \frac{N}{2} [D_{\max}^2 + \eta] + \frac{c_3}{2} \sum_{i=1}^N (\Delta \theta_i^T \Gamma_i^{-1} \Delta \theta_i). \end{aligned} \quad (110)$$

The evaluation of c_3 is described in Appendix 9.3.4.

9.3.2 Upper-bound on $\sum_{i=1}^N g_i^T g_i$

From (24), one may write

$$\begin{aligned} \sum_{i=1}^N g_i^T g_i &= \sum_{i=1}^N \left[\sum_{j=1}^N k_{ij} (r_{ij} - r_{ij}^*) + k_0 \varepsilon_i \right]^T \left[\sum_{j=1}^N k_{ij} (r_{ij} - r_{ij}^*) + k_0 \varepsilon_i \right] \\ &= \sum_{i=1}^N \left[\left\| \sum_{j=1}^N k_{ij} (r_{ij} - r_{ij}^*) \right\|^2 + \|k_0 \varepsilon_i\|^2 + 2 (k_0 \varepsilon_i)^T \left(\sum_{j=1}^N k_{ij} (r_{ij} - r_{ij}^*) \right) \right]. \end{aligned} \quad (111)$$

Let

$$P_1 = \sum_{i=1}^N \varepsilon_i^T \left(\sum_{j=1}^N k_{ij} (r_{ij} - r_{ij}^*) \right). \quad (112)$$

Since $r_{ij} - r_{ij}^* = q_i - q_j - (q_i^* - q_j^*) = \varepsilon_i - \varepsilon_j$,

$$\begin{aligned} P_1 &= \sum_{i=1}^N \sum_{j=1}^N k_{ij} \varepsilon_i^T (\varepsilon_i - \varepsilon_j) \\ &= \sum_{i=1}^N \sum_{j=1}^N k_{ij} (\varepsilon_i^T \varepsilon_i - \varepsilon_i^T \varepsilon_j). \end{aligned} \quad (113)$$

Using the fact that $2a^T b = a^T a + b^T b - (a - b)^T (a - b)$, one gets

$$P_1 = \sum_{i=1}^N \sum_{j=1}^N k_{ij} \left(\|\varepsilon_i\|^2 - \frac{1}{2} (\|\varepsilon_i\|^2 + \|\varepsilon_j\|^2 - \|\varepsilon_i - \varepsilon_j\|^2) \right). \quad (114)$$

Since $k_{ij} = k_{ji}$ and $\varepsilon_i - \varepsilon_j = r_{ij} - r_{ij}^*$

$$\begin{aligned} P_1 &= \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N k_{ij} \|\varepsilon_i\|^2 - \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N k_{ji} \|\varepsilon_j\|^2 + \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N k_{ij} \|r_{ij} - r_{ij}^*\|^2 \\ &= \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N k_{ij} \|\varepsilon_i\|^2 - \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N k_{ij} \|\varepsilon_i\|^2 + P(q, t) \\ &= P(q, t). \end{aligned} \quad (115)$$

Injecting P_1 in (111), one gets

$$\sum_{i=1}^N g_i^T g_i = \sum_{i=1}^N \left[\left\| \sum_{j=1}^N k_{ij} (r_{ij} - r_{ij}^*) \right\|^2 + \|k_0 \varepsilon_i\|^2 \right] + 2k_0 P(q, t) \quad (116)$$

and using (125), one gets

$$\sum_{i=1}^N g_i^T g_i \geq \sum_{i=1}^N k_0^2 \|\varepsilon_i\|^2 + \left(2k_0 + \frac{\alpha_{\min} k_{\min}}{k_{\max}}\right) P(q, t) \quad (117)$$

9.3.3 Upper-bound on $\sum_{i=1}^N \left\| \sum_{j=1}^N k_{ij} (r_{ij} - r_{ij}^*) \right\|^2$

One may write

$$\begin{aligned} \sum_{i=1}^N \left\| \sum_{j=1}^N k_{ij} (r_{ij} - r_{ij}^*) \right\|^2 &= \sum_{i=1}^N \left(\sum_{j=1}^N k_{ij} (r_{ij} - r_{ij}^*) \right)^T \left(\sum_{\ell=1}^N k_{i\ell} (r_{i\ell} - r_{i\ell}^*) \right) \\ &= \sum_{i=1}^N \sum_{\ell=1}^N \sum_{j=1}^N k_{i\ell} k_{ij} (r_{ij} - r_{ij}^*)^T (r_{i\ell} - r_{i\ell}^*) \end{aligned} \quad (118)$$

Using the fact $2a^T b = a^T a + b^T b - (a - b)^T (a - b)$

$$\sum_{i=1}^N \left\| \sum_{j=1}^N k_{ij} (r_{ij} - r_{ij}^*) \right\|^2 = \sum_{i=1}^N \left[\frac{1}{2} \sum_{\ell=1}^N \sum_{j=1}^N k_{i\ell} k_{ij} \left[\|r_{ij} - r_{ij}^*\|^2 + \|r_{i\ell} - r_{i\ell}^*\|^2 - \|r_{ij} - r_{ij}^* - (r_{i\ell} - r_{i\ell}^*)\|^2 \right] \right] \quad (119)$$

One has

$$(r_{ij} - r_{ij}^*) - (r_{i\ell} - r_{i\ell}^*) = (r_{ij} - r_{i\ell}) - (r_{ij}^* - r_{i\ell}^*) \quad (120)$$

$$= r_{\ell j} - r_{\ell j}^* \quad (121)$$

Injecting this result in (119) leads to

$$\sum_{i=1}^N \left\| \sum_{j=1}^N k_{ij} (r_{ij} - r_{ij}^*) \right\|^2 = \sum_{i=1}^N \left[\frac{1}{2} \sum_{\ell=1}^N \sum_{j=1}^N k_{i\ell} k_{ij} \left[\|r_{ij} - r_{ij}^*\|^2 + \|r_{i\ell} - r_{i\ell}^*\|^2 - \|r_{\ell j} - r_{\ell j}^*\|^2 \right] \right] \quad (122)$$

with $k_{\max} = \max_{\substack{\ell = 1 \dots N \\ j = 1 \dots N}} (k_{\ell j})$

$$\begin{aligned} k_{\max} \sum_{i=1}^N \left\| \sum_{j=1}^N k_{ij} (r_{ij} - r_{ij}^*) \right\|^2 &\geq \sum_{i=1}^N \left[\frac{1}{2} \sum_{\ell=1}^N \sum_{j=1}^N k_{i\ell} k_{ij} k_{\ell j} \left[\|r_{ij} - r_{ij}^*\|^2 + \|r_{i\ell} - r_{i\ell}^*\|^2 - \|r_{\ell j} - r_{\ell j}^*\|^2 \right] \right] \\ k_{\max} \sum_{i=1}^N \left\| \sum_{j=1}^N k_{ij} (r_{ij} - r_{ij}^*) \right\|^2 &\geq \frac{1}{2} \sum_{i=1}^N \sum_{\ell=1}^N \sum_{j=1}^N k_{i\ell} k_{ij} k_{\ell j} \|r_{ij} - r_{ij}^*\|^2 + \frac{1}{2} \sum_{i=1}^N \sum_{\ell=1}^N \sum_{j=1}^N k_{i\ell} k_{ij} k_{\ell j} \|r_{i\ell} - r_{i\ell}^*\|^2 \\ &\quad - \frac{1}{2} \sum_{i=1}^N \sum_{\ell=1}^N \sum_{j=1}^N k_{i\ell} k_{ij} k_{\ell j} \|r_{\ell j} - r_{\ell j}^*\|^2 \\ k_{\max} \sum_{i=1}^N \left\| \sum_{j=1}^N k_{ij} (r_{ij} - r_{ij}^*) \right\|^2 &\geq \frac{1}{2} \sum_{i=1}^N \sum_{\ell=1}^N \sum_{j=1}^N k_{i\ell} k_{ij} k_{\ell j} \|r_{ij} - r_{ij}^*\|^2 + \frac{1}{2} \sum_{i=1}^N \sum_{\ell=1}^N \sum_{j=1}^N k_{i\ell} k_{ij} k_{\ell j} \|r_{ij} - r_{ij}^*\|^2 \\ &\quad - \frac{1}{2} \sum_{i=1}^N \sum_{\ell=1}^N \sum_{j=1}^N k_{i\ell} k_{ij} k_{\ell j} \|r_{ij} - r_{ij}^*\|^2 \\ k_{\max} \sum_{i=1}^N \left\| \sum_{j=1}^N k_{ij} (r_{ij} - r_{ij}^*) \right\|^2 &\geq \frac{1}{2} \sum_{i=1}^N \sum_{\ell=1}^N \sum_{j=1}^N k_{i\ell} k_{ij} k_{\ell j} \|r_{ij} - r_{ij}^*\|^2. \end{aligned} \quad (123)$$

Let $k_{\min} = \min_{\ell=1 \dots N} (k_{\ell j} \neq 0)$ and $\alpha_{\min} = \min_{i=1, \dots, N} \alpha_i$. One may write

$$\begin{aligned}
\sum_{i=1}^N \sum_{\ell=1}^N \sum_{j=1}^N k_{i\ell} k_{ij} k_{\ell j} \|r_{ij} - r_{ij}^*\|^2 &= \sum_{i=1}^N \sum_{\ell=1}^N k_{i\ell} \sum_{j=1}^N k_{ij} k_{\ell j} \|r_{ij} - r_{ij}^*\|^2 \\
&\geq \sum_{i=1}^N \sum_{\ell=1}^N k_{i\ell} k_{\min} \sum_{j=1}^N k_{ij} \|r_{ij} - r_{ij}^*\|^2 \\
&\geq \sum_{i=1}^N \alpha_i k_{\min} \sum_{j=1}^N k_{ij} \|r_{ij} - r_{ij}^*\|^2 \\
&\geq \alpha_{\min} k_{\min} \sum_{i=1}^N \sum_{j=1}^N k_{ij} \|r_{ij} - r_{ij}^*\|^2 \\
&\geq 2\alpha_{\min} k_{\min} P(q, t)
\end{aligned} \tag{124}$$

Injecting (124) in (123) one gets

$$\begin{aligned}
k_{\max} \sum_{i=1}^N \left\| \sum_{j=1}^N k_{ij} (r_{ij} - r_{ij}^*) \right\|^2 &\geq \alpha_{\min} k_{\min} P(q, t) \\
\sum_{i=1}^N \left\| \sum_{j=1}^N k_{ij} (r_{ij} - r_{ij}^*) \right\|^2 &\geq \frac{\alpha_{\min} k_{\min}}{k_{\max}} P(q, t).
\end{aligned} \tag{125}$$

9.3.4 Evaluation of c_3

One has

$$\begin{aligned}
c_3 &= \frac{k_4}{k_M^*} \\
&= \frac{\min(k_m, k_3)}{\max\{1, k_M\}} \\
&= \frac{\min\{\min\{k_1, k_p\}, \min\{k_2 k_0, 1\}\}}{\max\{1, k_M\}} \\
&= \frac{\min\{k_1, k_p, 1, k_2 k_0\}}{\max\{1, k_M\}} \\
&= \frac{\min\left\{k_1, k_p, 1, k_0 \min\left\{2\left(2k_0 + \frac{\alpha_{\min} k_{\min}}{k_{\max}}\right), 1\right\}\right\}}{\max\{1, k_M\}} \\
&= \frac{\min\left\{k_1, k_p, 1, k_0, 2k_0\left(2k_0 + \frac{\alpha_{\min} k_{\min}}{k_{\max}}\right)\right\}}{\max\{1, k_M\}}
\end{aligned} \tag{126}$$

where $k_1 = k_s - 1 - k_p (k_M + 1)$, $\alpha_{\min} = \min_{i=1, \dots, N} \alpha_i$, $k_{\max} = \max_{\ell=1 \dots N} (k_{\ell j})$ and $k_{\min} = \min_{\ell=1 \dots N} (k_{\ell j} \neq 0)$

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