Uncertainty quantification for elasto-acoustic nonlinear reduced-order model computational model

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Abstract

The present research concerns the numerical analysis of an uncertain coupled fluid-structure dynamical system, for which the geometrical nonlinearities of the structure induced by the large deformations and the large displacements are taken into account. The structure is coupled with an internal cavity filled with a linear inviscid compressible fluid. The formulation is carried out in terms of displacements and pressure unknowns. The modal basis is made up of the structural modes of the structure and the acoustic modes of the fluid. A nonlinear-reduced order model is then numerically constructed in order to reduce the size of the problem. The uncertainties are then implemented by using the nonparametric probabilistic framework. Note that a peculiar attention that allows the uncertainties on the nonlinear part to be coherently taken into account by preventing the presence of a too large number of random variables used for generating the stochastic model is made. A numerical application is presented.

Keywords: Fluid-structure interaction, Geometric nonlinearities, Uncertainty Quantification

1. Introduction

Nowadays, a major challenge in many industrial areas consists in developing advanced methodologies in order to construct predictive numerical simulation tools, which are representative of the observed dynamical behaviour of the mechanical systems. In particular, it is important to quantify how inherent uncertainties propagate on the considered system. Furthermore, an essential aspect is to pay attention to the various nonlinear effects that can subsequently modify the dynamical response of the response. The present work proposes an extension of the uncertainty quantification for the nonlinear dynamical response of fluid-structure systems, for which the structural part present consequent geometrical nonlinear effects and for which the fluid is assumed to be a linear inviscid compressible fluid. First, we are interested by constructing a nonlinear-reduced-order model [1–4]. A nonlinear mean reduced-order model of the coupled system is constructed by projecting the finite element operators on a chosen projection basis [5]. Then, uncertainties are implemented through the nonparametric probabilistic approach [6, 7], which has been recently improved for the structural part of the nonlinear operators [8]. A numerical example is then presented.

2. Mean nonlinear reduced-order model of the structural-acoustic system

The structural-acoustic system under consideration is made up of a tank structure filled with a linear inviscid compressible fluid. A linear elastic constitutive equation is considered for the structure. It is also assumed that the

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structure undergoes sufficiently large deformations and large displacements in order to consider the geometrical non-linear effects, but also sufficiently moderate so that the fluid behavior remains linear. A total lagrangian formulation around a static equilibrium state taken as a reference configuration is used.

The three-dimensional bounded domain of the physical space $\mathbb{R}^3$ occupied by the structure is denoted by Ω_S with boundary $\partial\Omega_S = \Gamma_{S,0} \cup \Gamma_S \cup \Sigma$. The structure is subjected to a body force field $\mathbf{f}_{\text{vol}}(\mathbf{x}, t)$. It is fixed on $\Gamma_{S,0}$ and subjected to a surface force field $\mathbf{f}_{\text{surf}}(\mathbf{x})$ on Γ_S. The internal fluid occupies a three-dimensional bounded domain Ω_F of $\mathbb{R}^3$ with boundary $\partial\Omega_F = \Gamma_F \cup \Sigma$. It is coupled to the structure through boundary Σ and has a free surface condition on Γ_F. Let $\mathbf{n}_S$ and $\mathbf{n}_F$ be the outward unit normals to $\partial\Omega_S$ and $\partial\Omega_F$. Note that $\mathbf{n}_S = -\mathbf{n}_F$ on Σ. Let $\mathbf{x}$ be the generic point of $\mathbb{R}^3$. The forced response is formulated in the time domain. A formulation in terms of displacements field $\mathbf{u}(\mathbf{x}, t)$ for the structure and in terms of pressure field $p(\mathbf{x}, t)$ for the internal fluid is chosen. The equations related to the mean structural-acoustic system [9] are then written as

$$\rho_S \frac{\partial^2 \mathbf{u}}{\partial t^2} - \text{div}\,(\mathbb{F}\,\mathbb{S}) = \mathbf{f}_{\text{vol}} \quad \text{in} \quad \Omega_S \quad , \tag{1}$$

$$\mathbf{u} = \mathbf{0} \quad \text{on} \quad \Gamma_{S,0} \quad , \tag{2}$$

$$(\mathbb{F}\,\mathbb{S}) \cdot \mathbf{n}_S = \mathbf{f}_{\text{surf}} \quad \text{on} \quad \Gamma_S \quad , \tag{3}$$

$$(\mathbb{F}\,\mathbb{S}) \cdot \mathbf{n}_S = p\,\mathbf{n}_S \quad \text{on} \quad \Sigma \quad , \tag{4}$$

$$\frac{1}{\rho_F\,c_F^2} \frac{\partial^2 p}{\partial t^2} - \frac{1}{\rho_F}\Delta p = 0 \quad \text{in} \quad \Omega_F \quad , \tag{5}$$

$$p = 0 \quad \text{on} \quad \Gamma_F \quad , \tag{6}$$

$$\frac{1}{\rho_F} \frac{\partial p}{\partial \mathbf{n}_F} = -\frac{\partial^2 \mathbf{u}}{\partial t^2} \cdot \mathbf{n}_F \quad \text{on} \quad \Sigma \quad , \tag{7}$$

in which the deformation gradient tensor $\mathbb{F}$ is defined by $F_{ij} = u_{i,j} + \delta_{ij}$, in which δ_{ij} is the Kronecker symbol such that $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ otherwise, and where the second Piola-Kirchoff symmetric stress tensor $\mathbb{S}$ is written, for a linear elastic material, as $S_{ij} = a_{ijk\ell} E_{k\ell}$. The fourth-order elasticity tensor $\mathbb{a} = \{a_{ijk\ell}\}_{ijk\ell}$ satisfies the usual symmetry and positive-definiteness properties. The Green strain tensor $\mathbb{E} = \{E_{ij}\}_{ij}$ is then written as the sum of linear and nonlinear terms such that $E_{ij} = \varepsilon_{ij} + \eta_{ij}$, in which $\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$ and $\eta_{ij} = \frac{1}{2}u_{s,i}\,u_{s,j}$. Quantities ρ_S, ρ_F and c_F denote the mass density of the structure, the mass density of the fluid, and the sound velocity respectively.

The structural-acoustic system is then discretized with the finite element method assuming that the finite element meshes of the structure and of the internal fluid are compatible on the coupling interface Σ. A mean nonlinear reduced matrix model of the structural-acoustic system is then constructed. Let $\mathbf{u}$ be the $\mathbb{C}^{n_S}$-vector of the n_S DOF of the structure and let $\mathbf{p}$ be the $\mathbb{C}^{n_F}$-vector corresponding to the finite element discretization of the pressure field of the internal acoustic fluid. The projection basis of the structural-acoustic problem is calculated as follows: (1) The N_S structural modes related to the first N_S positive structural eigenfrequencies of the structure, for which an added mass effect representing the quasi-static effect of the internal fluid of the structure is taken into account [9] , are stored in the $n_S \times N_S$ modal matrix $[\Phi_S]$. (2) The N_F acoustic modes related to the first N_F positive acoustic eigenfrequencies of the internal fluid are stored in the $n_F \times N_F$ modal matrix $[\Phi_F]$. The projection basis allowing the mean reduced matrix model to be constructed is given by

$$\begin{bmatrix} \mathbf{u}(t) \\ \mathbf{p}(t) \end{bmatrix} = \begin{bmatrix} [\Phi_S] & [\mathbb{0}] \\ [\mathbb{0}] & [\Phi_F] \end{bmatrix} \begin{bmatrix} \mathbf{q}_S(t) \\ \mathbf{q}_F(t) \end{bmatrix} \quad , \tag{8}$$

in which $\mathbf{q}_S$ and $\mathbf{q}_F$ are the $\mathbb{C}^{N_S}$-vector and the $\mathbb{C}^{N_F}$-vector of the generalized coordinates related to the structure and to the internal fluid. From such projection basis, the linear reduced operators $[\mathcal{M}_S]$, $[\mathcal{M}_F]$, $[\mathcal{K}_S]$ and $[\mathcal{K}_F]$ and the coupling reduced operator $[\mathcal{C}]$ are constructed and the nonlinear quadratic and cubic reduced operators $\mathcal{K}^{(2)}_{S,\alpha\beta\gamma}$ and $\mathcal{K}^{(3)}_{S,\alpha\beta\gamma\delta}$ are explicitly constructed according [5] . The generalized coordinates are then solution of the matrix equation

$$\begin{bmatrix} [\mathcal{M}_S] & 0 \\ -[\mathcal{C}]^T & [\mathcal{M}_F] \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}_S(t) \\ \ddot{\mathbf{q}}_F(t) \end{bmatrix} + \begin{bmatrix} [\mathcal{D}_S] & 0 \\ 0 & [\mathcal{D}_F] \end{bmatrix} \begin{bmatrix} \dot{\mathbf{q}}_S(t) \\ \dot{\mathbf{q}}_F(t) \end{bmatrix} + \begin{bmatrix} [\mathcal{K}_S] & [\mathcal{C}] \\ 0 & [\mathcal{K}_F] \end{bmatrix} \begin{bmatrix} \mathbf{q}_S(t) \\ \mathbf{q}_F(t) \end{bmatrix} + \begin{bmatrix} \mathcal{F}^{NL}(\mathbf{q}_S(t)) \\ 0 \end{bmatrix} = \begin{bmatrix} \mathcal{F}_S(t) \\ 0 \end{bmatrix} \quad , \tag{9}$$

in which the nonlinear reduced force $\mathcal{F}^{NL}(\mathbf{q}_S)$ is defined from the nonlinear quadratic and cubic stiffness operators such that

$$\mathcal{F}_\alpha^{NL}(\mathbf{q}_S) = \mathcal{K}_{S,\alpha\beta\gamma}^{(2)} \, q_{S,\beta} \, q_{S,\gamma} + \mathcal{K}_{S,\alpha\beta\gamma\delta}^{(3)} \, q_{S,\beta} \, q_{S,\gamma} \, q_{S,\delta} \quad . \tag{10}$$

For the application presented in this paper, the damping model is written as $[\mathcal{D}_S] = b_S [\mathcal{K}_S]$ and $[\mathcal{D}_F] = b_F [\mathcal{K}_F]$. Furthermore, the vector $\mathcal{F}_S(t)$ is the external reduced load vector of the structure, assuming that there is no external source for the fluid. The presence of the geometric nonlinearities yields the equations of the mean nonlinear reduced-order model to be considered in the time domain. The forced response of the structural-acoustic problem is then investigated in the time-domain by considering an equivalent time-evolution problem with zero initial conditions over a finite time interval, which includes almost all of the signal energy of the excitation. The strategy is to simultaneously and uniformly excite all the frequencies of a chosen frequency band of excitation so that only one computation of the nonlinear dynamical problem is required.

3. Stochastic nonlinear reduced-order model of the structural-acoustic system

The uncertainty propagation of the nonlinear dynamical response of the structural-acoustic system is then investigated by implementing the uncertainties through the nonparametric probabilistic approach whose review can be found in [6].

- Let $[\mathcal{A}]$ be one of the matrices $[\mathcal{M}_S]$, $[\mathcal{M}_F]$, $[\mathcal{D}_S]$, $[\mathcal{D}_F]$ or $[\mathcal{K}_F]$ which are symmetric positive-definite matrices and let $N \in \{N_S, N_F\}$ be its order. In such case, the corresponding random matrix $[\mathbf{A}]$ is then written as

$$[\mathbf{A}] = [L_A]^{\mathrm{T}} [\mathbf{G}_A(\delta_A)] [L_A] \quad , \tag{11}$$

in which $[L_A]$ is the $(N \times N)$ upper triangular matrix issued from the Cholesky factorization of $[\mathcal{A}]$, and where $[\mathbf{G}_A]$ is a full random matrix with values in the set of all the positive-definite symmetric $(N \times N)$ matrices.
- The random coupling reduced operator $[\mathcal{C}]$, which is a $(N_S \times N_F)$ rectangular matrix is written as

$$[\mathcal{C}] = [T_C] [L_C]^{\mathrm{T}} [\mathbf{G}_C(\delta_C)] [L_C] \quad , \tag{12}$$

in which the matrices $[T_C]$ and $[L_C]$ are the $(N \times N)$ matrices defined by $[L_C] = [S]^{-1/2} [V_C]^{\mathrm{T}}$ and $[T_C] = [\mathcal{C}] [V_C] [S] [V_C]^{\mathrm{T}}$, in which the columns of the $(N_F \times N)$ full matrix $[V_C]$ contains the with $N < N_S$ right-singular vectors of matrix $[\mathcal{C}]$ related to the corresponding largest singular value stored by decreasing order in the $(P \times P)$ diagonal matrix $[S]$ and where $[\mathbf{G}_C]$ is a full random matrix with values in the set of all the positive-definite symmetric $(N \times N)$ matrices.
- Concerning the implementation of the uncertainties on the linear and the nonlinear terms issued from the linear elasticity constitutive equation, the methodology is briefly described below. The main idea introduced in [7] consist in globally introducing the uncertainties on all linear and nonlinear stiffness terms through a stiffness operator self-containing the linear, quadratic and cubic stiffness terms. Such reduced operator is represented by the $(P \times P)$ real matrix $[\mathcal{K}]$ with $P = N_S(N_S + 1)$. It has been shown in [7] that matrix $[\mathcal{K}]$ is a symmetric and positive-definite matrix. Consequently, the nonparametric probabilistic approach can be easily extended to the geometrically nonlinear context. The random matrix $[\mathcal{K}]$ can be written as $[\mathcal{K}] = [L_K]^T [\mathbf{G}_K(\delta_K)] [L_K]$, in which the $(P \times P)$ matrix $[L_K]$ is issued from the Choleski factorization of matrix $[\mathcal{K}]$ and where $[\mathbf{G}_K(\delta_K)]$ is a full random matrix with values in the set of all the positive-definite symmetric $(P \times P)$ matrices. The main drawbacks of this method is that the dimension P of the random germ $[\mathbf{G}_K(\delta_K)]$ drastically increases with N and that a scale effect prevents to compare the influence of the uncertainties on the nonlinear stiffness terms with respect to the linear stiffness terms in the nonlinear dynamical analysis [10]. The main idea proposed in [8] is to use another factorization of matrix $[\mathcal{K}]$, which involves a rectangular matrix of dimension $(N \times P)$ with $N \ll P$. This is achieved by performing a reduction of matrix $[\mathcal{K}]$. Let us then consider the eigenvalue problem

$$[\mathcal{K}] \psi_\alpha = \lambda_\alpha \psi_\alpha \quad . \tag{13}$$

Matrix $[\mathcal{K}]$ can then be approximated by the $(P \times P)$ matrix $[\widetilde{\mathcal{K}}]$ such that $[\widetilde{\mathcal{K}}] = [\widetilde{L}_K]^T [\widetilde{L}_K]$, in which $[\widetilde{L}_K]$ is the full $(N \times P)$ matrix defined by $[\widetilde{L}_K] = [\Lambda^N]^{\frac{1}{2}} [\Psi^N]^T$, where $[\Lambda^N]$ is the $(N \times N)$ diagonal matrix such that

$[\Lambda^N]_{\alpha\alpha} = \lambda_\alpha$, where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_N$ and where the columns of the $(P \times N)$ matrix $[\Psi^N]$ are the eigenvector ψ_α, $\alpha \in \{1, \ldots, N\}$ related to eigenvalues λ_α. Random matrix $[\mathcal{K}]$ is then replaced by the random matrix $[\widetilde{\mathcal{K}}]$ such that

$$[\widetilde{\mathcal{K}}] = [\widetilde{L}_K]^T [\widetilde{\mathbf{G}}_K(\delta_K)] [\widetilde{L}_K] + ([\widetilde{\mathcal{K}}] - [\mathcal{K}]) \quad , \tag{14}$$

in which $[\widetilde{\mathbf{G}}_K(\delta_K)]$ is a $(N \times N)$ random matrix with values in the set of the symmetric positive-definite matrices. Note that Eq. (14) ensures that stiffness operator $[\widetilde{\mathcal{K}}]$ is almost surely positive definite.

It can be proved that the probability distribution and the random generator of each random $(N \times N)$ matrix $[\mathbf{G}_A(\delta_A)]$ is detailed in [6, 11]. The dispersion of each random matrix $[\mathbf{G}_A]$ is controlled by the hyperparameter δ_A belonging an admissible set $\mathbf{\Delta}_A$. Consequently, the uncertainty level of the coupled fluid-structure system is entirely controlled by the $\mathbb{R}^7$-valued hyperparameter $\mathbb{\delta} = (\delta_{M_S}, \delta_{D_S}, \delta_C, \delta_{M_F}, \delta_{D_F}, \delta_{K_F}, \delta_K)$, belonging to the admissible set $\mathbb{\Delta}^7$.

Concerning the numerical procedure for computing the response of the stochastic nonlinear reduced-order model in the time domain, the Monte Carlo numerical simulation is used with an implicit and unconditionally stable integration scheme (Newmark method with the averaging acceleration scheme) combined with either the fixed point method or with an adapted efficient algorithm based on the arc-length method [10, 12, 13], depending on the nonlinearity rate. A posterior nonlinear dynamical analysis is then performed in the frequency domain by using Fast Fourier Transform.

4. Numerical application

The fluid-structure coupled dynamical system under consideration is a cylindrical tank partially filled with a linear inviscid acoustic fluid that is described in a global cartesian coordinate system $(O, \mathbf{e}_1, \mathbf{e}_1, \mathbf{e}_3)$, where O is the center of the cylinder basis and where the cylinder axis is defined along $\mathbf{e}_3$. Its geometry is characterized by the mean radius $r_m = 0.93\,m$, thickness $e = 0.14\,m$, $h = 2\,m$, and bottom thickness $h_b = 0.5\,m$. The bottom of the cylindrical shell is clamped. The material is a linear isotropic homogeneous elastic material for which the Young modulus $E = 2.1 \times 10^{11}\,N \times m^{-2}$, the Poisson ratio $\nu = 0.3$, and the mass density $\rho_S = 7860\,Kg \times m^{-3}$. The internal fluid occupies a cylindrical volume with radius $r_f = 0.86\,m$ and height $h_F = 0.75\,m$. It has a free surface. The fluid has mass density $\rho_F = 1000\,Kg.m^{-3}$ and sound velocity $c_F = 1480\,m \times s^{-1}$. The finite element model of the system has 1024 three-dimensional solid finite elements with 8 nodes with $n_S = 3750$ Dofs. The finite element model of the fluid has 768 three-dimensional fluid finite elements with 8 nodes with $n_F = 819$ Dofs and is represented by Fig. 1. The structure is subjected to a transverse load such that all the nodes located at the top of the cylindrical shell are uniformly excited along $\mathbf{e}_2$. The damping model is characterized by $b_S = 1.5 \times 10^{-5}$ and $b_F = 10^{-5}$.
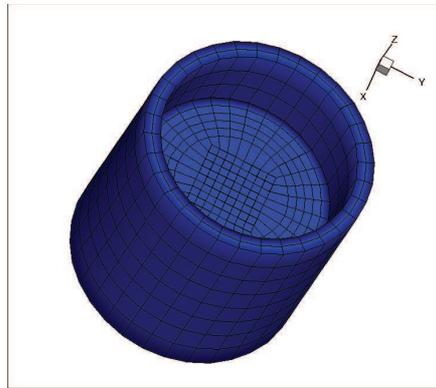

Figure 1: Finite element model of the mean fluid-structure system.

The frequency band of excitation is $\mathbb{B}_{exc} = [200, 1\,400]\,Hz$ such that $s = 2/3$ and $\Delta\nu = 1\,200\,Hz$ according to Eq.(14). The stochastic reduced-order model is constructed with $N_S = 50$ and $N_F = 40$, for which a convergence analysis concerning the displacements of the structure and the pressure in the fluid has been made. The chosen observations are the displacements U_x, U_y, U_z along directions $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ of the point of coordinates $(-1, 0, 2)$ located at the top of the structure and the pressure P of the point of coordinated $(-0.64, 0, 1)$, located in the fluid. Figures 2 and 3 compare the confidence region of the nonlinear stochastic response of the fluid-structure system at these observation points when uncertainty is located on the stiffness structure only $\delta_K = 0.2$ or on the stiffness fluid only $\delta_{K_S} = 0.2$.
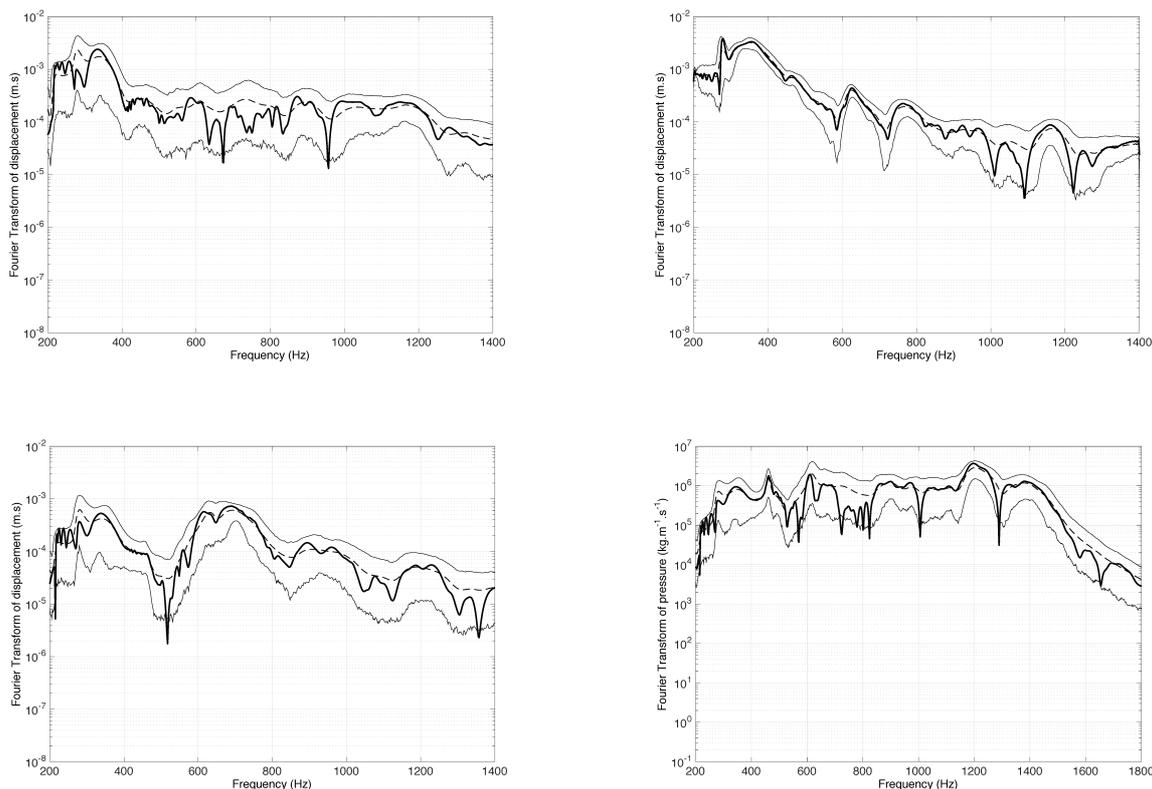

Figure 2: For $\delta_{K_S} = 0.2$, graph of the deterministic response (thick line), the mean response (thin dashed line) and of the confidence region related to (a) displacement $\widehat{U}_x(\nu)$, (b) displacement $\widehat{U}_y(\nu)$, (c) displacement $\widehat{U}_z(\nu)$, (d) pressure $\widehat{P}(\nu)$.

5. Conclusion

A numerical method for computing the nonlinear dynamical behaviour of an uncertain coupled fluid-structure system has been presented. A numerical application demonstrates the feasability of the proposed method.

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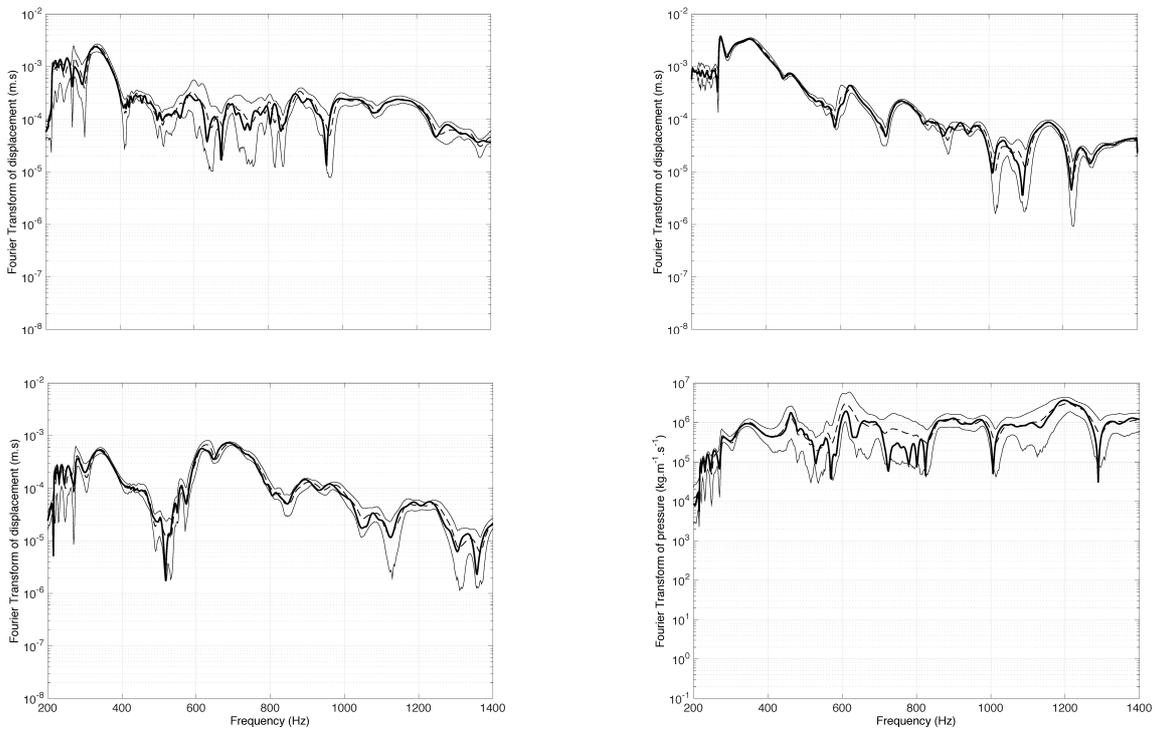
Figure 3: For $\delta_{K_F} = 0.2$, graph of the deterministic response (thick line), the mean response (thin dashed line) and of the confidence region related to (a) displacement $\widehat{U}_x(\nu)$, (b) displacement $\widehat{U}_y(\nu)$, (c) displacement $\widehat{U}_z(\nu)$, (d) pressure $\widehat{P}(\nu)$.

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