Bayesian calibration of mechanical parameters of high-speed train suspensions

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Abstract

The objective of the work presented here is a bayesian calibration of parameters describing the mechanical characteristics of high-speed train suspensions for maintenance purposes. This calibration is achieved by comparing simulation results to on-track accelerometric measurements. It requires the estimation on the multidimensionnal admissible set of the parameters of the likelihood function of the train dynamic response. This estimation is achieved thanks to the identification of a kriging metamodel of this likelihood function to reduce the numerical cost. From this metamodel, the posterior probability density function of the parameters is estimated using an MCMC algorithm.

Keywords: statistical inverse problem, railway dynamics, random track irregularities, high-speed train suspensions

1. Introduction

Trains dynamic behavior strongly relies on their suspensions that undergo damage throughout their lifetime. In order to ensure passengers safety and comfort, regular maintenance is performed to guarantee a good state of suspensions. Presently, this maintenance mostly relies on age or mileage criteria. The knowledge of the actual state of suspension characteristics could however allow maintenance rules closer to the real needs to be used. The industrial objective of the work presented here is thus the development of a remote diagnosis method for high-speed train suspensions based on accelerometric on-track measurements. This work is part of a development project conducted by SNCF (the French National Railway Company).

Track geometry (also called track irregularities) constitutes the main excitation source of a rolling train and, consequently, has a major influence on the train dynamic behavior (see [1–4]). Track geometry is also subject to damage caused by railway traffic (see [5, 6]). In order to distinguish suspension damage from track geometry evolution in the accelerometric measurements in the train, railway dynamics simulation is necessary. More precisely, we propose to compare measured accelerations to simulated ones, computed on the track geometry that has been measured together with the accelerations. The experimental data (track geometry and accelerometric measurements) used for this work come from the train *IRIS 320*, a modified TGV specially equiped to perform various measurements at high speed (see [7, 8])

From a scientific point of view, this problem consists in a statistical inverse identification of the train model parameters describing the suspensions mechanical properties. The repetition of this identification on measurements

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performed at different times should allow the time evolution of these parameters to be observed. Appropriate maintenance could then be triggered as soon as they leave the acceptable domain.

2. Description of the analyzed system

2.1. Deterministic mechanical system

In this work, the system considered is a train rolling at variable speed on a track characterized by its design and its geometry.

The track geometry is defined as the geometric irregularities of the rails position with respect to the theoretical track design. The input of the railway dynamical system denoted as $\{\mathbf{x}(t), t \in [0, T]\}$ is the displacement condition imposed to each wheel of the train, in the axis system attached to the train, during the time interval $[0, T]$. It can directly be deduced from the geometry measurements, the train speed record, and the location of the wheelsets along the train.

In the frequency domain, the system output denoted as $\{\mathbf{y}(\omega), \omega \in \Omega\}$ is the train accelerometric response to the track geometry at a certain (possibly varying) speed, in which Ω is the frequency band of interest. For this work, only vertical and lateral accelerations in various points of the train carbodies and bogies are considered. They correspond to the different components of $\mathbf{y}$. The quantity $\mathbf{y}(\omega) \in \mathbb{R}^n$ is the logarithm of the amplitude of the frequency response of the accelerations of various points in the train at frequency ω.

The train is described as a multibody model. It consists of rigid bodies linked together by mechanical joints (mostly stiffnesses and dampers) with nonlinear behavior. Wheel-rail contact law is also nonlinear. The train parameters involved in the identification process are solely mechanical parameters characterizing the train suspensions. They are denoted as $\mathbf{w}$, belonging to the admissible set $C_{\mathbf{w}}$, subset of $\mathbb{R}^q$.

The railway dynamics software used for this work as a black box is *Vampire*. It is represented by the deterministic mapping:

$$\mathbf{h}^{\text{sim}} : (\{\mathbf{x}(t), t \in [0, T]\}, \mathbf{w}) \mapsto \{\mathbf{y}(\omega), \omega \in \Omega\}. \tag{1}$$

It associates the response $\mathbf{y} = \mathbf{h}^{\text{sim}}(\mathbf{x}, \mathbf{w})$ with an excitation $\mathbf{x}$ and vector-valued parameter $\mathbf{w}$.

2.2. Probabilistic modeling of the train response

The analyzed system presents several sources of uncertainty.

By nature, the track geometry is stochastic (see [9]). Consequently, the induced excitation is stochastic as well. This excitation is modeled by $\{\mathbf{X}(t), t \in [0, T]\}$, a real vector-valued stochastic process indexed by the time interval $[0, T]$. Realizations of this process are obtained from the geometry measurements.

The multibody modeling contains inaccuracies and simplifications compared to the real system. Numerical solving is also a source of errors. To perform a robust identification, a train model uncertainty has to be introduced. Moreover, the accelerometric measurements contain noise and uncertainties. This two types of uncertainties (model and measurements) are globally taken into account thanks to an output predictive error $\mathbf{B}$ added to the simulated response. This $\mathbb{R}^n$-valued stochastic process $\{\mathbf{B}(\omega), \omega \in \Omega\}$ indexed by frequency band Ω is taken as a gaussian process.

Despite vector-valued parameter $\mathbf{w}$ being deterministic for a given train at a given date, the identification procedure does not allow this parameter to be exactly known. The uncertainty on parameter $\mathbf{w}$ must be quantified. Consequently, the train parameter $\mathbf{w}$ is modeled by a random vector $\mathbf{W}$, with values in the admissible set $C_{\mathbf{w}}$.

The system output is thus modeled by the $\mathbb{R}^n$-valued stochastic process $\{\mathbf{Y}^{\text{mod}}(\omega), \omega \in \Omega\}$ indexed by the frequency band Ω according to the following equation:

$$\mathbf{Y}^{\text{mod}} = \mathbf{h}^{\text{sim}}(\mathbf{X}, \mathbf{W}) + \mathbf{B}. \tag{2}$$

3. Bayesian calibration of the train parameters

3.1. Method

Our objective is to estimate the posterior probability density function (pdf) of random vector $\mathbf{W}$, written as $p_{\mathbf{W}}^{\text{post}}$, conditionned by process $\mathbf{Y}^{\text{mod}}$ for which a set of ν independent realizations $\{\mathbf{y}^{\text{exp},i}\}_{1\leq i\leq\nu}$ is available. This set corresponds to ν measurements of the train response performed on ν track stretches at the same date. The conditionnal pdf of $\mathbf{Y}^{\text{mod}}$ given $\mathbf{W}=\mathbf{w}$ is written as $\mathbf{y}\mapsto p_{\mathbf{Y}^{\text{mod}}\,|\,\mathbf{W}}(\mathbf{y}\,|\,\mathbf{w})$. According to the Bayes formula, at point $\mathbf{w}\in\mathcal{C}_{\mathbf{w}}$, we have:

$$p_{\mathbf{W}}^{\text{post}}(\mathbf{w}) \propto p_{\mathbf{Y}^{\text{mod}}\,|\,\mathbf{W}}(\{\mathbf{y}^{\text{exp},i}\}_i\,|\,\mathbf{w})\times p_{\mathbf{W}}^{\text{prior}}(\mathbf{w})\,, \tag{3}$$

in which the prior pdf $p_{\mathbf{W}}^{\text{prior}}$ is chosen as a uniform pdf on admissible set $\mathcal{C}_{\mathbf{w}}$. Measurements of the track geometry are available for μ stretches, from which a set $\{\mathbf{x}^{\text{exp},j}\}_{1\leq j\leq\mu}$ of μ realizations of $\mathbf{X}$ is obtained. The likekihood function (see [10]) $p_{\mathbf{Y}^{\text{mod}}\,|\,\mathbf{W}}(\{\mathbf{y}^{\text{exp},i}\}_i\,|\,\mathbf{w})$, written as $\mathcal{L}(\mathbf{w})$, can then be estimated according to Eq. (4):

$$\begin{aligned}\mathcal{L}(\mathbf{w}) &\approx \prod_{i=1}^{\nu}\frac{1}{\mu}\sum_{j=1}^{\mu} p_{\mathbf{Y}^{\text{mod}}\,|\,\mathbf{W},\mathbf{X}}(\mathbf{y}^{\text{exp},i}\,|\,\mathbf{w}\,,\mathbf{x}^{\text{exp},j})\\ &\approx \prod_{i=1}^{\nu}\frac{1}{\mu}\sum_{j=1}^{\mu} p_{\mathbf{B}}(\mathbf{y}^{\text{exp},i}-\mathbf{h}^{\text{sim}}(\mathbf{x}^{\text{exp},j}\,,\mathbf{w}))\,.\end{aligned} \tag{4}$$

Process $\mathbf{B}$ being gaussian, the latter density can explicitly be computed.

The estimation of the posterior pdf of parameter $\mathbf{W}$ on the whole admissible set $\mathcal{C}_{\mathbf{w}}$ requires the evaluation of $\mathcal{L}(\mathbf{w})$ on numerous points $\mathbf{w}$ in $\mathcal{C}_{\mathbf{w}}$. Each one of these evaluations requires to run μ calculations of the train response $\{\mathbf{h}^{\text{sim}}(\mathbf{x}^{\text{exp},j}\,,\mathbf{w})\}_{1\leq j\leq\mu}$, which is highly time-consuming. In order to perform this estimation in a reasonable amount of time, a kriging metamodel $\widetilde{\mathcal{L}}$ of $\mathcal{L}$ is identified on admissible set $\mathcal{C}_{\mathbf{w}}$ (see [11, 12]), thanks to a preliminary training of $\mathcal{C}_{\mathbf{w}}$. The estimation $E\{\widetilde{\mathcal{L}}(\mathbf{w})\}$, where $E\{.\}$ denotes the mathematical expectation, can be quickly computed and is then used in place of $\mathcal{L}(\mathbf{w})$.

For estimating the posterior pdf with accuracy, the metamodel needs to be as precise as possible around the points in which the likelihood function is maximum. The preliminary exploration of set $\mathcal{C}_{\mathbf{w}}$, consisting in introducing training points in $\mathcal{C}_{\mathbf{w}}$, aims at characterizing the general trends of likelihood function $\mathcal{L}$ on the whole set. The set of training points must therefore be space-filling. The metamodel should then be refined around the points in which the likelihood function is maximum. The EGO algorithm (standing for Efficient Global Optimization, see [13]) is an efficient way to do so. The principle is to perform new runs of the simulation on a few points in $\mathcal{C}_{\mathbf{w}}$ selected according to an expected improvement criterium. The metamodel can then be updated with the value of the likelihood function obtained on these new points.

Using the metamodel $\widetilde{\mathcal{L}}$, the posterior pdf of $\mathbf{W}$ is estimated with the MCMC (Monte Carlo Markov Chain) method. The classical Metropolis-Hastings algorithm (see [14]) is used, with a gaussian transition probability.

To sum up, the identification method consists of four steps :

- the preliminary training of admissible set $\mathcal{C}_{\mathbf{w}}$, defining the training points;
- the identification of the likelihood function metamodel, $\widetilde{\mathcal{L}}$, from the values of the likelihood function computed at the the training points;
- using the EGO algorithm to add points to the training points in order to refine the metamodel around the point in which the likelihood function is maximum;
- the identification of the posterior pdf of parameter $\mathbf{W}$ using the MCMC method with the likelihood function evaluated with metamodel $\widetilde{\mathcal{L}}$.

3.2. Results

The method presented in Section 3.1 has been applied to a test set $\{\mathbf{y}^{\text{exp},i}\}_{1\leq i\leq\nu}$ generated by simulation with fixed parameter $\mathbf{w}^{\text{ref}}$, vector of $q=7$ components.

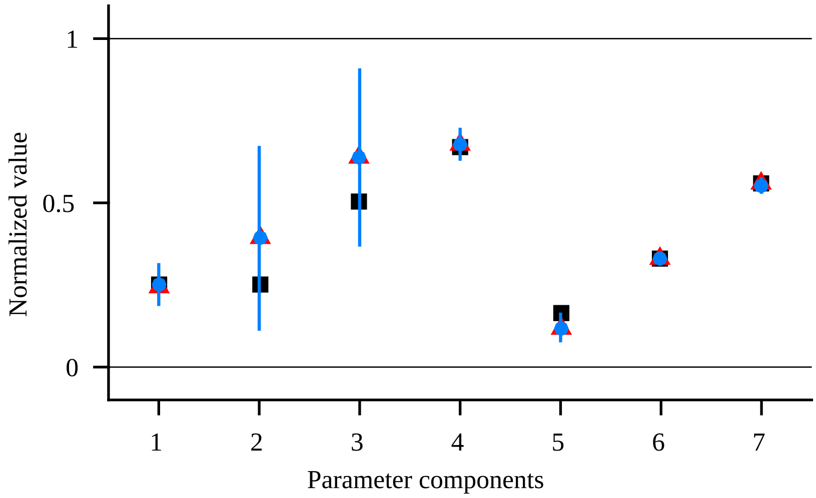

Figure 1: Comparison of the results obtained with bayesian calibration with the reference, using a normalized scale: for $1 \leq k \leq 7$ (parameter components), optimal value w_k^{joint} (red triangles), optimal value w_k^{marg} (blue circles), with 95% confidence intervals (blue lines), and reference value w_k^{ref} (black squares).

For the seven components (indexed by k) of $\mathbf{w}$, Fig. 1 shows:

- the optimal value $\mathbf{w}^{\text{joint}}$ that maximizes the posterior pdf of $\mathbf{W}$ estimated by MCMC:

$$\mathbf{w}^{\text{joint}} = \arg\max_{\mathbf{w} \in C_{\mathbf{w}}} p_{\mathbf{W}}^{\text{post}}(\mathbf{w})\,; \tag{5}$$

- for $1 \leq k \leq 7$, the optimal value w_k^{marg} that maximizes the marginal pdf of the component W_k of $\mathbf{W}$:

$$w_k^{\text{marg}} = \arg\max_{w_k \in C_{w_k}} p_{W_k}^{\text{post}}(w_k)\,; \tag{6}$$

- the 95% confidence interval around w_k^{marg}, defined as the interval $[w_k^{\text{marg}} - \delta_k \,, w_k^{\text{marg}} + \delta_k]$ where component w_k has a probability of 0.95 to be located;
- the reference value $\mathbf{w}^{\text{ref}}$.

In Fig. 1, all the components of $\mathbf{w}$ are normalized such that the variation interval of each component is shrinked to $[0\,,1]$. Fig. 2 shows the marginal posterior pdf $w_k \mapsto p_{W_k}^{\text{post}}(w_k)$ for the first, the second, and the fifth components of $\mathbf{w}$. On each graph, the reference value w_k^{ref} and the boundaries of the 95% confidence interval are indicated as well.

In Fig. 1, one can observe that w_k^{joint} and w_k^{marg} are close for each component k. The statistical dependencies between the components of $\mathbf{W}$ do not influence the location of the maximum of the posterior pdf. The correspondence between those values and the reference values is satisfying for five components: 1, 4, 5, 6, and 7. However, the confidence intervals show that the accuracy of the identification is not the same for the different parameter components. For example, the accuracy of the identified value for the first component is much lower than for the sixth one.

The accuracy of the idenfication for components 2 and 3 is not so good. However, for those two components, we can also observe that the confidence interval is much larger than for all the other components. An explanation for this poor identification is that the train response is less sensitive to these two parameters than to the other ones. These results highlight the interest of the bayesian calibration: the width of the confidence interval allows us to assess the quality of the identification.

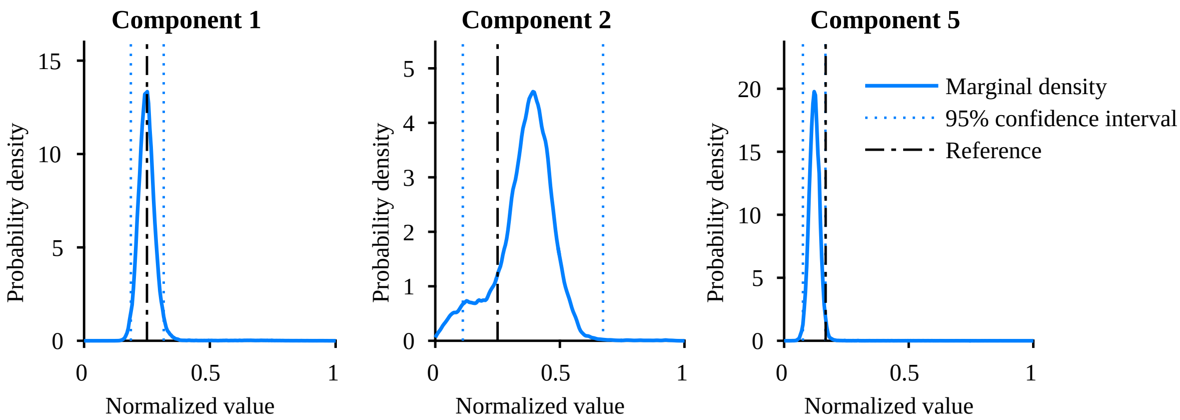

Figure 2: Marginal posterior pdf (solid blue line) for the first (left graph), the second (center graph), and fifth (right graph) parameter components, with 95% confidence interval boundaries (dotted blue lines), compared to the reference values w_k^{ref} (dash-dot black line).

4. Conclusion and perspective

In this paper, we have presented a bayesian calibration method that allows for performing the inverse identification of mechanical parameters of high-speed train suspensions as well as the assesment of the quality of this identification. The method requires the computation of the likelihood function for measurements of the train dynamic response. In order to reduce the numerical cost, a kriging metamodel of the likelihood function is built. Using this metamodel, the posterior pdf of the vector-valued random parameter is estimated with an MCMC algorithm.

The method has been applied to a numerical experiment, with promising results. Focus should be put on the refining of the metamodel that may involve some difficulties for precisely locating the maximum of the likelihood function. The validation of this approach using actual measurements is in progress.

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