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About the rotation curves of spiral galaxies, a program

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If we were going to make a little trip in the Milky Way

Abstract: Let us take a rotation curve of a spiral galaxy. This program makes the calculus of the inverse of the matrix of forces in a maximal disk, view as a set of stars (or points) with usual symmetry. After, it realizes the calculus of masses and of the surface density of the disk. No halo of dark matter is need. The mass of the twin galaxies, Milky Way and Andromeda, for the same radius from 30 to 38 kpc are similar and in the interval $[2.3 \cdot 10^{11}, 3.5 \cdot 10^{11}]$ times the solar mass.

```
> with(LinearAlgebra):with(CurveFitting):
> k:=160;l:=k/2;Digits:=30:
                                     k := 160
                                     l := 80

> #Tabular: distances and angles
> c:=seq(evalf(cos(Pi*n/l)),n=1..2*l):
d:=seq(evalf((i/(k))^2),i=1..k+1):
> # it is a good choice to simulate a galaxy by point masses
# the number of masses is k^2+1
# we calculate now the forces between the masses
> s:=proc(i,j)
local num,dist,F,n,u;
  u:=0;
  for n to 2*l do
    dist:=evalf(d[i]^2+d[j]^2-2*d[i]*d[j]*c[n]);
    num:=evalf(d[i]-d[j]*c[n]);
    if (n=2*l)and (i=j) then F:=0
    else F:=evalf(num/(dist^(3/2)))
    fi;
    u:=u+F
  od end:
A:=Matrix([seq([seq(s(ii,jj),jj=1..k),evalf(1/d[ii]^2)],ii=1..k),
[seq(2*l,jj=1..k),1]]):
> #with k=160, around 150 seconds
> #The fundamental matrix A (n-body problem) is established.
#So the result (the inverse matrix) is:
> invA:=MatrixInverse(A);#around 12 seconds
```



```

fi:
wmax:=evalf(wmax,25);wminim:=evalf(wminim,25);
wm:=evalf((wmax+wminim)/2,18);
    wmax := 0.999999999999999999935
    wminim := 0.9999999999999999999483
    wm := 1.000000000000000000000000

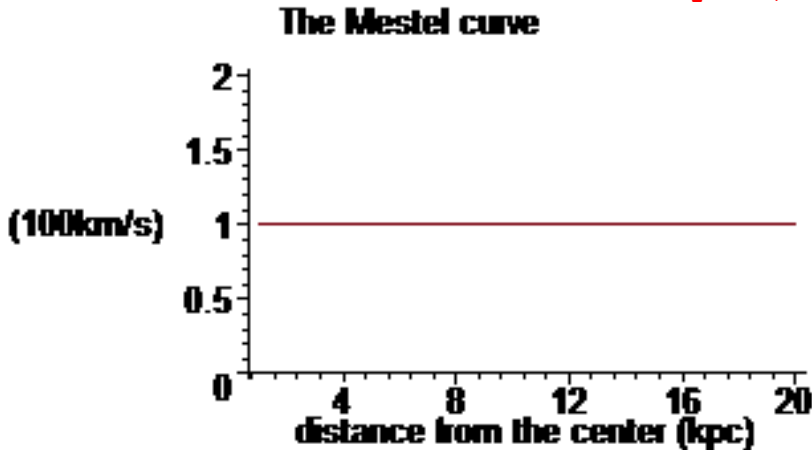
> M:=seq(subs(w=wm,evalf(C[i],18)),i=1..k+1);
M := 0., 0., 0., 0.1000 10-20, 0.1000 10-20, 0.1000 10-20, 0., 0.1000 10-20, 0., 0., 0., 0., 0.,
    0., 0., 0., 0., -0.1000 10-20, 0., 0., 0.1000 10-20, 0., 0., 0., 0., 0., 0., 0., 0.1000 10-20,
    0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
    0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
    0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
    0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
    0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
    0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 1.0000000000000000000100

```

```

> #very well, it's noteworthy!
>
> #For a constant curve (Mestel):
> fv:=proc(x) 1 end;V:=1;R:=20:
> plot([[y,fv(y)]$y=1..R],title=`The Mestel curve`,
labels=[`distance from the center (kpc)``,`(100km/s)`]);

```



```

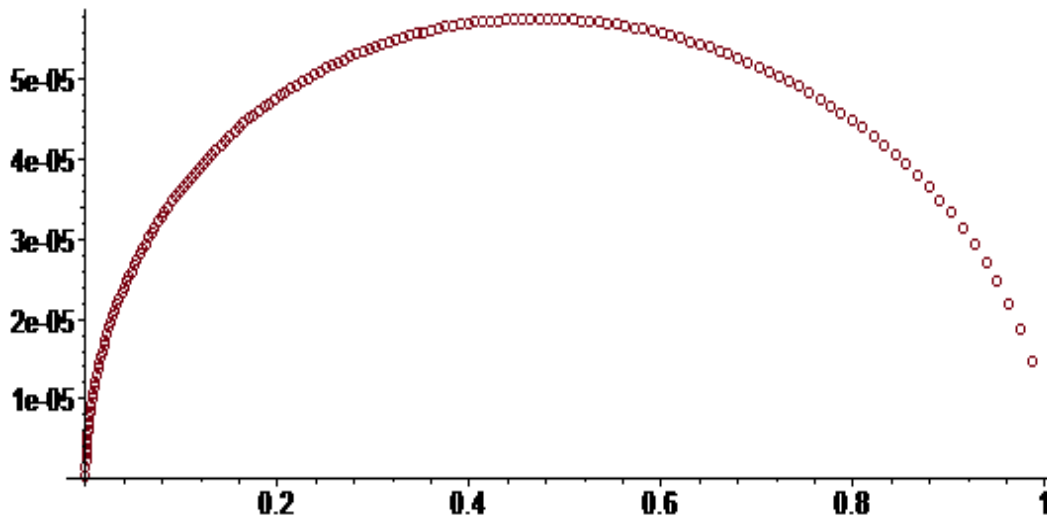
> B:=Vector([seq(w*f(v(d[i]))^2/d[i],i=1..k),1]);
> C:=evalf(MatrixVectorMultiply(invA,B),20):
> wnul:=seq(evalf(solve(C[i]=0,w)),i=1..k+1):
n1:=0;n2:=0:
for j to k+1 do
  N:=seq(subs(w=wnul[j],C[i]),i=1..k+1):truc:=1:
  for i to k+1 do
    if N[i]<-10-15 then truc:=0 fi:
  od;

```

```

if truc=1 then if n1=0 then n1:=j else n2:=j fi fi;
od:
if n1=0 then print(`il_y_a_des_masses_negatives`) else
  if wnul[n1]>wnul[n2] then wmax:=wnul[n1];wminim:=wnul[n2]
    else wmax:=wnul[n2];wminim:=wnul[n1] fi;
  fi:
wmax:=evalf(wmax,25);wminim:=evalf(wminim,25);
wm:=evalf((wmax+wminim)/2,15);
      wmax := 1.55742208213158693494
      wminim := 1.55728972488919912343
      wm := 1.55735590351040
>
M:=seq(subs(w=wm,evalf(C[i],15)),i=1..k):M[1..4];MM:=[seq([d[i],s
ubs(w=wm,evalf(C[i],15))],i=1..k-1)]:
0.23824600929753102 10-6,0.149382278651539238 10-5,
0.2192915484633480012 10-5,0.3055410555907115583 10-5
> plot(MM,style=point,symbol=circle);

```



well, beautiful!

```

> R:=20;V:=1;MasseMestel:=evalf(0.23*10^10*V^2*R/wm,10):
print(`Masse_de_Mestel`,MasseMestel,`en_Masses_Solaires`);
      R := 20

```

V := 1

Masse_de_Mestel, 0.2953724315 10¹¹, en_Masses_Solaires

```

> Mgal:=MasseMestel;#for R=20 kpc
      Mgal := 0.2953724315 1011

```

```

> #mean surface density

```

```

> rho:=[seq(Mgal*evalf(8*1*(M[i]+M[i+1])/Pi/((d[i+1]+d[i+2])^2

```

```

- (d[i]+d[i-1])^2 / (R*10^3)^2 , i=2..(k-1) ] :
> Rho := [seq(ln(rho[i]), i=1..(k-2))] : nops (%) ;

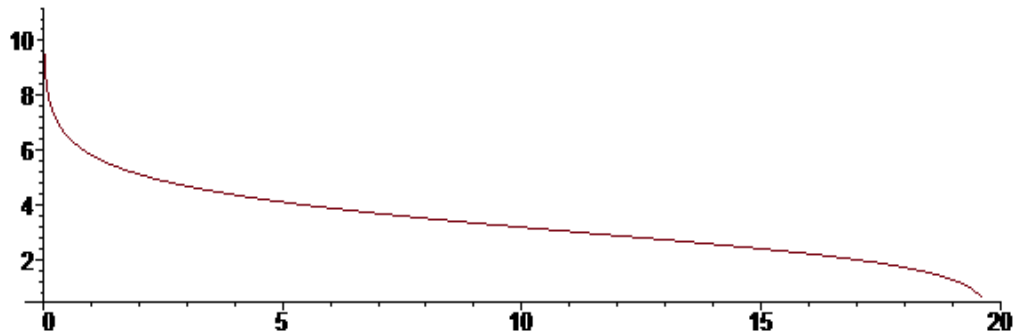
```

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```

> plot([seq([R*(d[i]+d[i+1])/2, Rho[i]], i=1..k-2)],
title=`log-density Mestel curve in Ms/pc^2`);

```



>

```

> #2017, new data for Milky Way curve, coming from Huang (2016)
and Sofu  (2015) :

```

```

> donneesVL := [[0,0], [0.20,233.0], [0.38,268.92], [0.66,250.75],
[1.61,217.83], [2.57,219.58], [3.59,223.11], [4.60,231.24],
[5.08,230.46], [5.58,230.01], [6.10,239.61], [6.57,246.27],
[7.07,243.49], [7.58,242.71], [8.04,243.23], [8.34,239.89],
[8.65,237.26], [9.20,235.30], [9.62,230.99], [10.09,228.41],
[10.58,224.26], [11.09,224.94], [11.58,233.57], [12.07,240.02],
[12.73,242.21], [13.72,261.78], [14.95,259.26], [15.52,268.57],
[16.55,261.17], [17.56,240.66], [18.54,215.31], [19.50,214.99],
[21.25,251.68], [23.78,259.65], [26.22,242.02], [28.71,224.11],
[31.29,211.20], [33.73,217.93], [36.19,219.33], [38.73,213.31],
[41.25,200.05], [43.93,190.15]] :

```

```

> #For Milky Way, with R=20 kpc and we spline the curve of
velocities.

```

```

> donneesVL21 := [[0,0], [0.20,233.0], [0.38,268.92], [0.66,250.75],
[1.61,217.83], [2.57,219.58], [3.59,223.11], [4.60,231.24],
[5.08,230.46], [5.58,230.01], [6.10,239.61], [6.57,246.27],
[7.07,243.49], [7.58,242.71], [8.04,243.23], [8.34,239.89],
[8.65,237.26], [9.20,235.30], [9.62,230.99], [10.09,228.41],
[10.58,224.26], [11.09,224.94], [11.58,233.57], [12.07,240.02],
[12.73,242.21], [13.72,261.78], [14.95,259.26], [15.52,268.57],
[16.55,261.17], [17.56,240.66], [18.54,215.31], [19.50,214.99],
[21.25,251.68]] : nops (%) ;

```

33

```

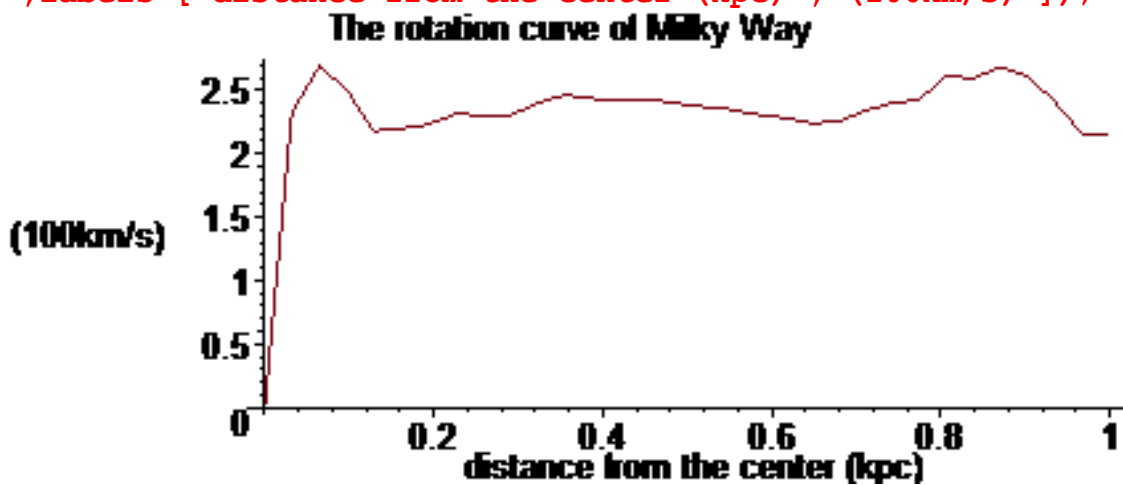
> f := map(u->evalf(op(2,u)/100,5), donneesVL21) ;

```

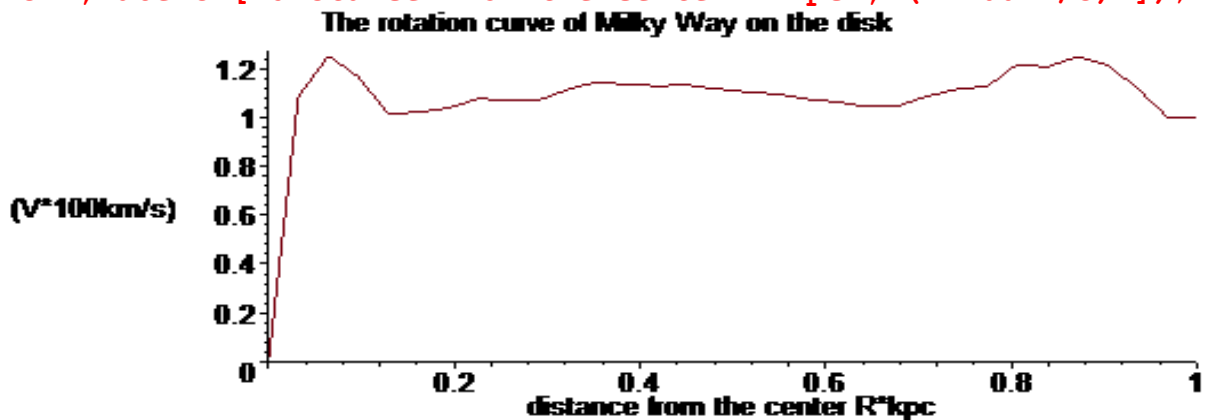
```
f:= [0, 2.3300, 2.6892, 2.5075, 2.1783, 2.1958, 2.2311, 2.3124, 2.3046, 2.3001, 2.3961,
      2.4627, 2.4349, 2.4271, 2.4323, 2.3989, 2.3726, 2.3530, 2.3099, 2.2841, 2.2426,
      2.2494, 2.3357, 2.4002, 2.4221, 2.6178, 2.5926, 2.6857, 2.6117, 2.4066, 2.1531,
      2.1499, 2.5168 ]
```

```
> R:=20; a:=nops(f)-2; V:=f[a+1];
      R:=20
      a:=31
      V:=2.1499
```

```
> plot([y/31, f[y+1]]$y=0..31, title='The rotation curve of Milky
Way', labels=['distance from the center (kpc)', '(100km/s)']);
```

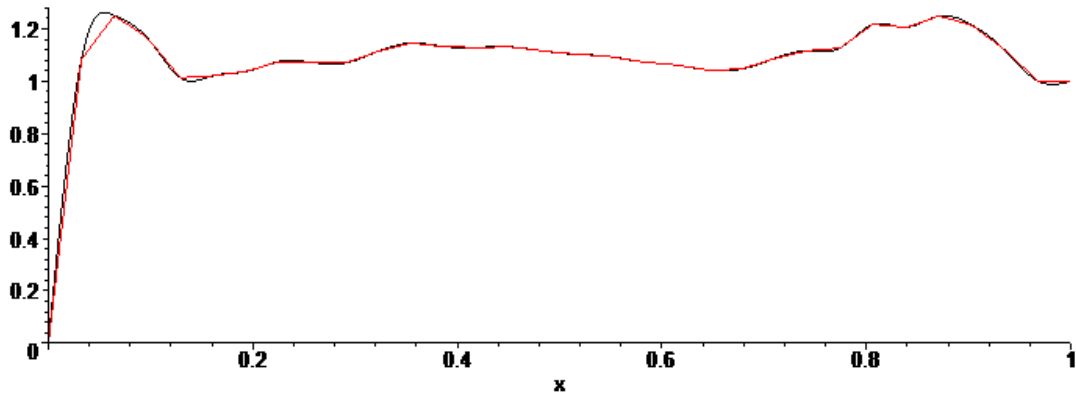


```
> fv:=proc(x)
      (f[trunc(a*x)+1]+(a*x-trunc(a*x))*(f[trunc(a*x)+2]-f[trunc(a*x)
+1]))/V
end:
> plot(fv(x), x=0..1, title='The rotation curve of Milky Way on the
disk', labels=['distance from the center R*kpc', '(V*100km/s)']);
```



```
> F:= [seq([(i-1)/31, f[i]/V], i=1..32)]:
>
> g:=x->Spline(F, x):
```

```
> plot({g(x),fv(x)},x=0..1,color=[red,black]);
```



```
>
```

```
> #The second member of the linear system, coming from observed velocities.
```

```
BB:=Vector([seq(w*g(d[i])^2/d[i],i=1..k),1]):
```

```
#w as the meaning of the inverse of the mass M of the galaxy
```

```
> #calculus of mass as function of w
```

```
C:=evalf(MatrixVectorMultiply(invA,BB),30):
```

```
> #search of w=wmin and w=wmax such that all the mass are >=0
```

```
wnul:=seq(evalf(solve(C[i]=0,w)),i=1..k+1):
```

```
n1:=0:n2:=0:
```

```
for j to k+1 do
```

```
  N:=seq(subs(w=wnul[j],C[i]),i=1..k+1):truc:=1:
```

```
  for i to k+1 do
```

```
    if N[i]<-10^(-5) then truc:=0 fi:
```

```
  od;
```

```
  if truc=1 then if n1=0 then n1:=j else n2:=j fi fi;
```

```
od:
```

```
if n1=0 then print(`il y a des masses negatives`) else
```

```
  if wnul[n1]>wnul[n2] then wmax:=wnul[n1];wminim:=wnul[n2]
```

```
  else wmax:=wnul[n2];wminim:=wnul[n1] fi;
```

```
  fi:
```

```
wmax:=evalf(wmax,15);wminim:=evalf(wminim,15);
```

```
wm:=evalf((wmax+wminim)/2,15);wmax-wminim;
```

```
  wmax := 1.29664337753167
```

```
  wminim := 1.29664335097775
```

```
  wm := 1.29664336425471
```

```
  0.2655392 10-7
```

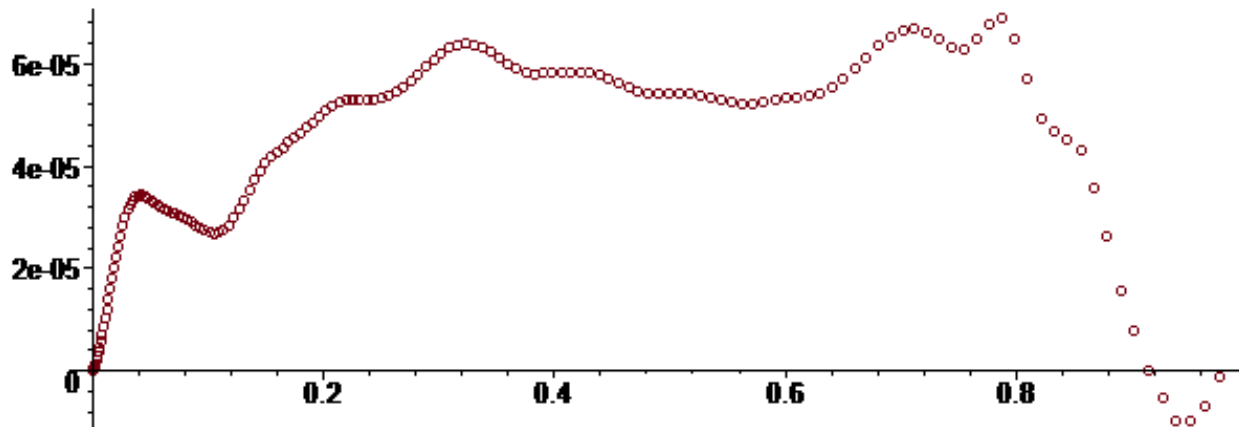
```
>
```

```
M:=seq(subs(w=wm,evalf(C[i],15)),i=1..k):M[1..3];MM:=seq([d[i],subs(w=wm,evalf(C[i],15))],i=1..k-2):
```

```
  0.5740575241300 10-10,0.469177057255007 10-8,0.30394016978535919 10-7
```



```
> plot(MM, style=point, symbol=circle);
```



```
> MasseGalaxie:=evalf(0.23*10^10*v^2*R/wm,10):
print(`Masse_de_la_Galaxie`,MasseGalaxie,`en_Masses_Solaires`);
Masse_de_la_Galaxie,0.1639735538 10^12,en_Masses_Solaires
```

```
>
evalf(0.23*10^10*v^2*R/wmax,10),evalf(0.23*10^10*v^2*R/wminim,10)
;
0.1639735521 10^12,0.1639735555 10^12
```

```
> #we have a problem at the end of the curve, but
```

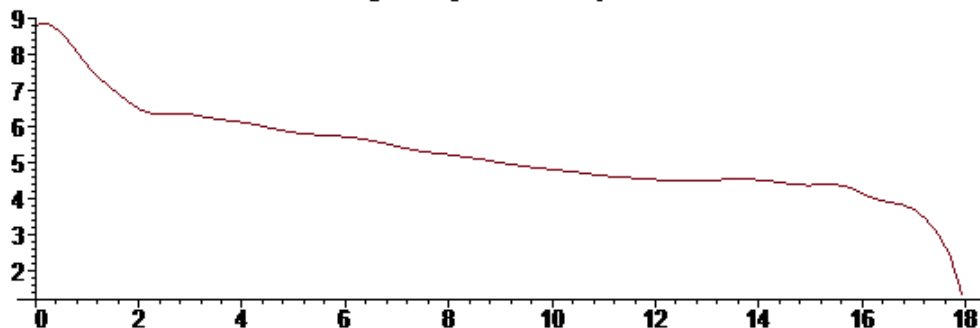
```
> Mgal:=MasseGalaxie;#for R=20 kpc
Mgal := 0.1639735538 10^12
```

```
> #mean surface density
```

```
> rho:=[seq(Mgal*evalf(8*1*(M[i]+M[i+1])/Pi/((d[i+1]+d[i+2])^2
-(d[i]+d[i-1])^2)/(R*10^3)^2),i=2..(k-1))]:nops(%);
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```

```
> Rho:=[seq(ln(rho[i]),i=1..(k-9))]:nops(%);
151
```

```
> plot([seq([R*(d[i]+d[i+1])/2,Rho[i]],i=1..k-9)],
title=`log-density curve in Ms/pc^2`);
log-density curve in Ms/pc^2
```



```
>
> #Now for an extended Milky Way, (always without spherical halo)
```

```

>
> #For Milky Way, with R=25 kpc and we spline the curve of
velocities.
> donneesVL26:=[[0, 0], [.20, 233.0], [.38, 268.92], [.66,
250.75], [1.61, 217.83], [2.57, 219.58], [3.59, 223.11], [4.60,
231.24], [5.08, 230.46], [5.58, 230.01], [6.10, 239.61], [6.57,
246.27], [7.07, 243.49], [7.58, 242.71], [8.04, 243.23], [8.34,
239.89], [8.65, 237.26], [9.20, 235.30], [9.62, 230.99], [10.09,
228.41], [10.58, 224.26], [11.09, 224.94], [11.58, 233.57],
[12.07, 240.02], [12.73, 242.21], [13.72, 261.78], [14.95,
259.26], [15.52, 268.57], [16.55, 261.17], [17.56, 240.66],
[18.54, 215.31], [19.50, 214.99], [21.25, 251.68], [23.78,
259.65], [26.22, 242.02]]:nops(%);

```

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```

> f:=map(u->evalf(op(2,u)/100,5), donneesVL26);
f:=[0, 2.3300, 2.6892, 2.5075, 2.1783, 2.1958, 2.2311, 2.3124, 2.3046, 2.3001, 2.3961,
2.4627, 2.4349, 2.4271, 2.4323, 2.3989, 2.3726, 2.3530, 2.3099, 2.2841, 2.2426,
2.2494, 2.3357, 2.4002, 2.4221, 2.6178, 2.5926, 2.6857, 2.6117, 2.4066, 2.1531,
2.1499, 2.5168, 2.5965, 2.4202 ]

```

```

> R:=25;a:=nops(f)-2;V:=(f[a+1]+f[a+2])/2;
R:=25

```

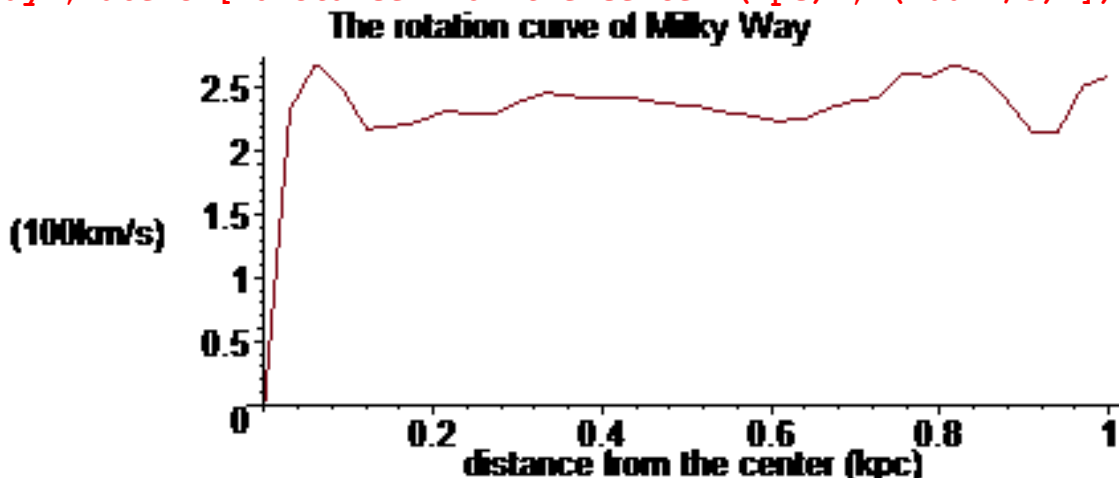
a:=33

V:=2.50835000000000000000

```

> plot([y/33,f[y+1]]$y=0..33),title=`The rotation curve of Milky
Way`,labels=[`distance from the center (kpc)`,`(100km/s)`];

```



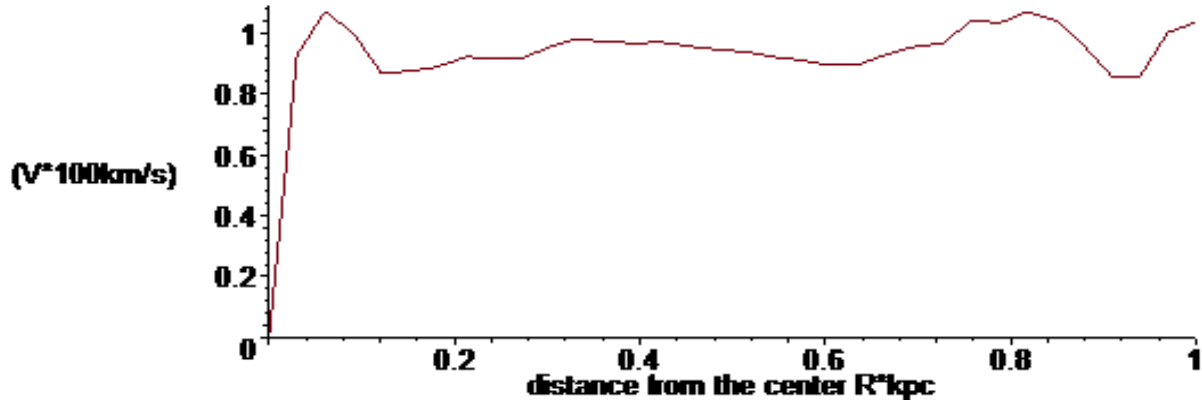
```

> fv:=proc(x)
(f[trunc(a*x)+1]+(a*x-trunc(a*x))*(f[trunc(a*x)+2]-f[trunc(a*x)
+1]))/V
end:
> plot(fv(x),x=0..1,title=`The rotation curve of Milky Way on the

```

```
disk`,labels=[`distance from the center R*kpc`,`(V*100km/s)`]);
```

The rotation curve of Milky Way on the disk

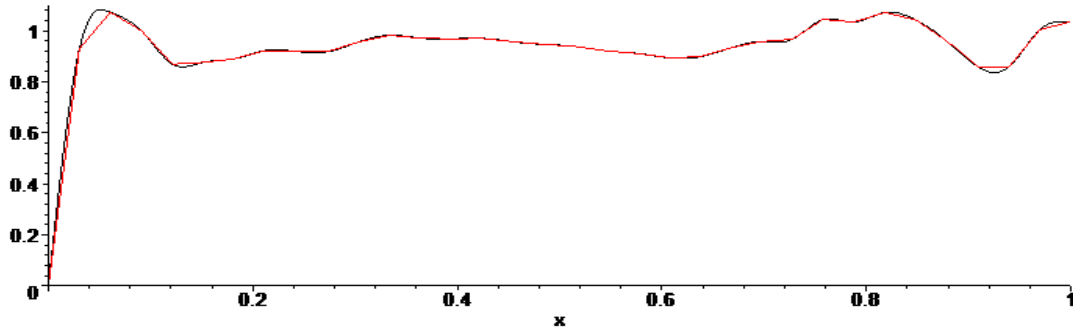


```
>
```

```
> F:=seq([(i-1)/33,f[i]/V],i=1..34):
```

```
> g:=x->Spline(F,x):
```

```
> plot({g(x),fv(x)},x=0..1,color=[red,black]);
```



```
> #The second member of the linear system, coming from observed velocities.
```

```
> BB:=Vector([seq(w*g(d[i])^2/d[i],i=1..k),1]):
```

```
#w as the meaning of the inverse of the mass M of the galaxy
```

```
> C:=evalf(MatrixVectorMultiply(invA,BB),30):
```

```
> #search of w=wmin and w=wmax such that all the mass are >=0
```

```
> wnul:=seq(evalf(solve(C[i]=0,w)),i=1..k+1):
```

```
n1:=0:n2:=0:
```

```
for j to k+1 do
```

```
  N:=seq(subs(w=wnul[j],C[i]),i=1..k+1):truc:=1:
```

```
  for i to k+1 do
```

```
    if N[i]<-10^(-6) then truc:=0 fi:
```

```
  od;
```

```
  if truc=1 then if n1=0 then n1:=j else n2:=j fi fi;
```

```
od:
```

```
if n1=0 then print(`il_y_a_des_masses_negatives`) else
```

```
  if wnul[n1]>wnul[n2] then wmax:=wnul[n1];wminim:=wnul[n2]
```

```
  else wmax:=wnul[n2];wminim:=wnul[n1] fi;
```

```
fi:
```

```
wmax:=evalf(wmax,15);wminim:=evalf(wminim,15);
wm:=evalf((wmax+wminim)/2,15);wmax-wminim;
wmax := 1.63914534386586
```

```
wminim := 1.63914531059209
```

```
wm := 1.63914532722898
```

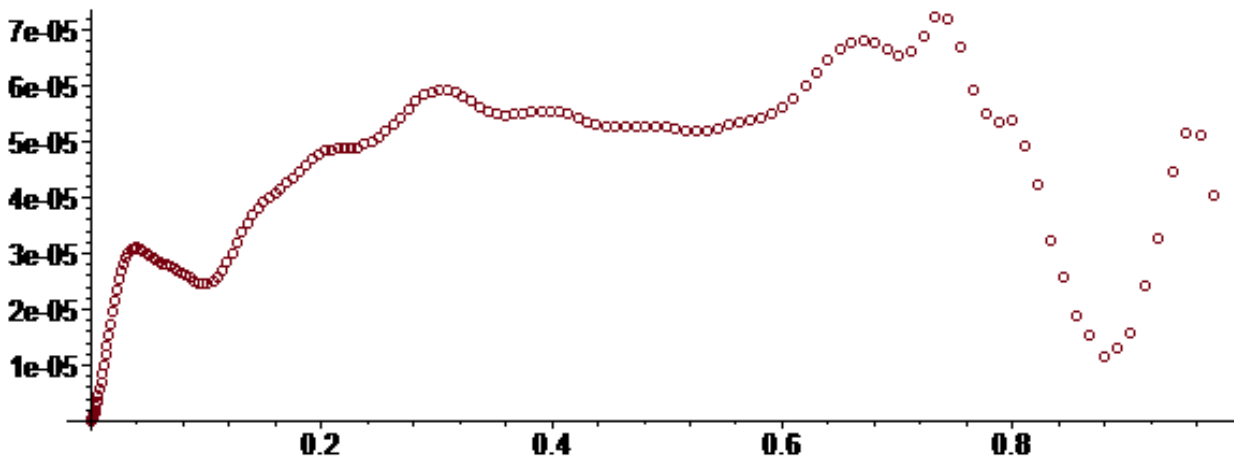
```
0.3327377 10-7
```

```
>
```

```
M:=seq(subs(w=wm,evalf(C[i],15)),i=1..k):M[1..3];MM:=seq([d[i],subs(w=wm,evalf(C[i],15))],i=1..k-2):
```

```
0.5690259176508 10-10,0.465064286684057 10-8,0.30127473812319455 10-7
```

```
> plot(MM,style=point,symbol=circle);
```



```
> MasseGalaxie:=evalf(0.23*1010*V2*R/wm,5):
```

```
print(`Masse_de_la_Galaxie`,MasseGalaxie,`en_Masses_Solaires`);
Masse_de_la_Galaxie,0.22072 1012,en_Masses_Solaires
```

```
>evalf(0.23*1010*V2*R/wmax,8),evalf(0.23*1010*V2*R/wminim,8);
0.22071236 1012,0.22071236 1012
```

```
> #no problem at the end of the curve
```

```
> Mgal:=MasseGalaxie;#for R=25 kpc
```

```
Mgal := 0.22072 1012
```

```
> #mean surface density
```

```
rho:=seq(Mgal*evalf(8*1*(M[i]+M[i+1])/Pi/((d[i+1]+d[i+2])2-
(d[i]+d[i-1])2)/(R*103)2),i=2..(k-1)):
```

```
> #log-density curve
```

```
i:='i':Rho:=seq(ln(rho[i]),i=1..(k-2)):
```

```
i:='i':courbelog:=plot([R*(d[i]+d[i+1])/2,Rho[i]]$i=2..k-2,
title=`log-density curve in Ms/pc2`):
```

```
> with(stats):Digits:=5:s:=trunc(k/6);
```

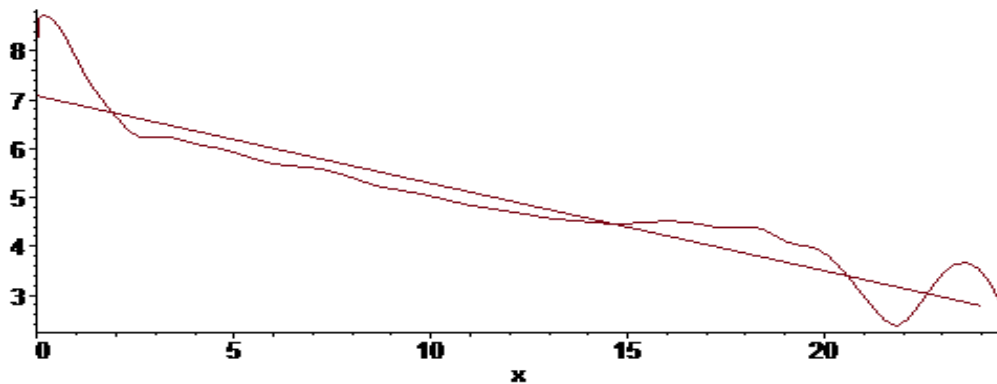
```

liste:=[ [seq(R*(d[i]+d[i+1])/2,i=s..(k-5))], [seq(Rho[i+1],i=s-1..
(k-6))] ]:
eqfit:=fit[leastsquare[[x,y]]](liste);
eqfonction:=unapply(rhs(eqfit),x):
courberegr:=plot(eqfonction(x),x=0..R-1):
with(plots):
display({courberegr,courbelog});Digits:=20:
s:=26

```

$$eqfit := y = -0.17884 x + 7.0770$$

log-density curve in Ms/pc²



```

>
> #Now for Milky Way, with R=30 kpc
> donneesVL31:=[[0, 0], [.20, 233.0], [.38, 268.92], [.66,
250.75], [1.61, 217.83], [2.57, 219.58], [3.59, 223.11], [4.60,
231.24], [5.08, 230.46], [5.58, 230.01], [6.10, 239.61], [6.57,
246.27], [7.07, 243.49], [7.58, 242.71], [8.04, 243.23], [8.34,
239.89], [8.65, 237.26], [9.20, 235.30], [9.62, 230.99], [10.09,
228.41], [10.58, 224.26], [11.09, 224.94], [11.58, 233.57],
[12.07, 240.02], [12.73, 242.21], [13.72, 261.78], [14.95,
259.26], [15.52, 268.57], [16.55, 261.17], [17.56, 240.66],
[18.54, 215.31], [19.50, 214.99], [21.25, 251.68], [23.78,
259.65], [26.22, 242.02], [28.71, 224.11], [31.29,
211.20]]:nops(%);

```

37

```

> f:=map(u->evalf(op(2,u)/100,5), donneesVL31);
f:=[0., 2.3300, 2.6892, 2.5075, 2.1783, 2.1958, 2.2311, 2.3124, 2.3046, 2.3001, 2.3961,
2.4627, 2.4349, 2.4271, 2.4323, 2.3989, 2.3726, 2.3530, 2.3099, 2.2841, 2.2426,
2.2494, 2.3357, 2.4002, 2.4221, 2.6178, 2.5926, 2.6857, 2.6117, 2.4066, 2.1531,
2.1499, 2.5168, 2.5965, 2.4202, 2.2411, 2.1120]

```

>

```

R:=30;a:=nops(f)-2;v:=evalf((f[a+1]+f[a+2])/2,7);

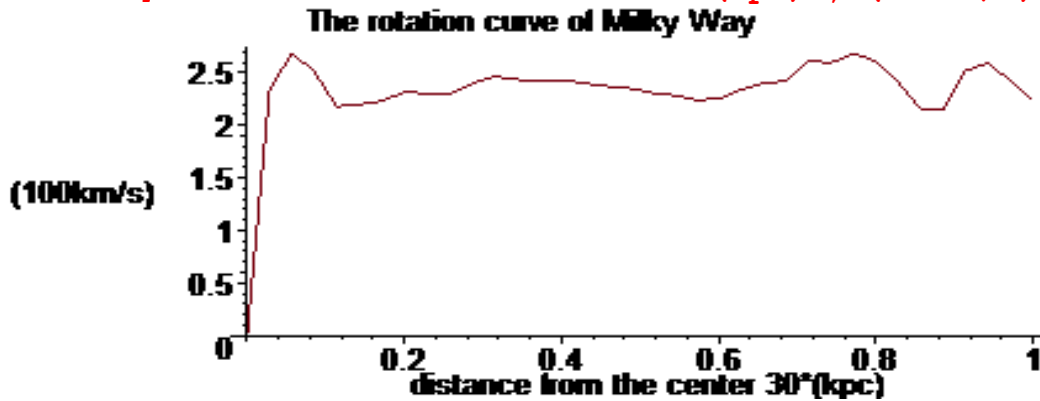
```

R:=30

a:=35

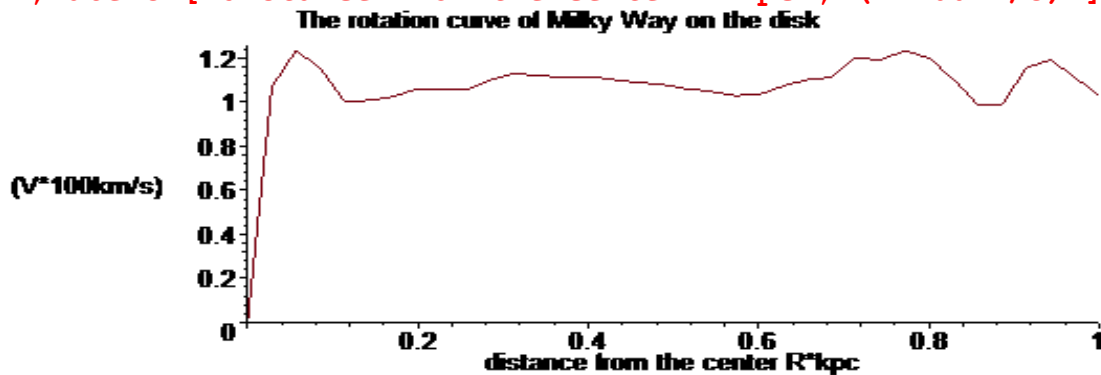
$V := 2.176550$

```
> plot([y/35, f[y+1]]$y=0..35], title=`The rotation curve of Milky Way`, labels=[`distance from the center 30*(kpc)`, `(100km/s)`]);
```



```
> fv:=proc(x)
(f[trunc(a*x)+1]+(a*x-trunc(a*x))*(f[trunc(a*x)+2]-f[trunc(a*x)+1]))/V end;
```

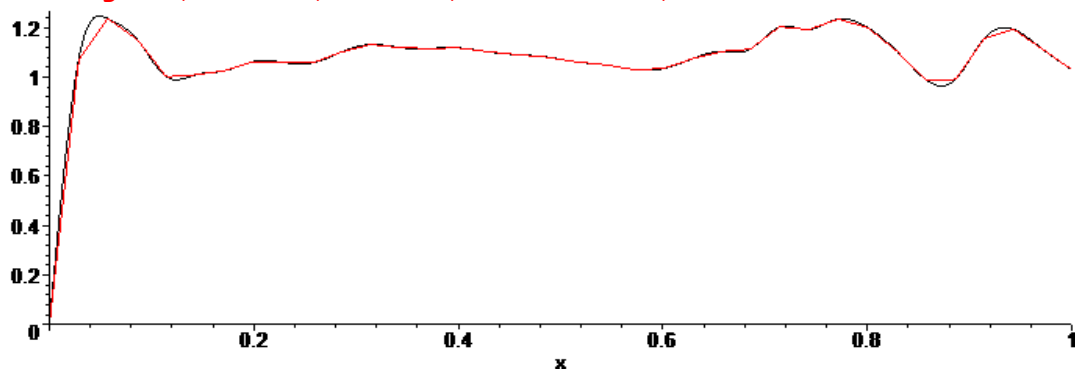
```
> plot(fv(x), x=0..1, title=`The rotation curve of Milky Way on the disk`, labels=[`distance from the center R*kpc`, `(V*100km/s)`]);
```



```
> F:=[seq([(i-1)/35, f[i]/V], i=1..36)]:
```

```
> g:=x->Spline(F, x):
```

```
> plot({g(x), fv(x)}, x=0..1, color=[red, black]);
```



```
> #The second member of the linear system, coming from observed velocities.
```

```
> BB:=Vector([seq(w*g(d[i])^2/d[i], i=1..k), 1]):
```

```

#w as the meaning of the inverse of the mass M of the galaxy
> C:=evalf(MatrixVectorMultiply(invA,BB),30):
> #search of w=wmin and w=wmax such that all the mass are >=0
> wnul:=seq(evalf(solve(C[i]=0,w)),i=1..k+1):
n1:=0:n2:=0:
for j to k+1 do
  N:=seq(subs(w=wnul[j],C[i]),i=1..k+1):truc:=1:
  for i to k+1 do
    if N[i]<-10^(-5) then truc:=0 fi:
  od;
  if truc=1 then if n1=0 then n1:=j else n2:=j fi fi;
od:
if n1=0 then print(`il_y_a_des_masses_negatives`) else
  if wnul[n1]>wnul[n2] then wmax:=wnul[n1];wminim:=wnul[n2]
  else wmax:=wnul[n2];wminim:=wnul[n1] fi;
fi:
wmax:=evalf(wmax,15);wminim:=evalf(wminim,15);
wm:=evalf((wmax+wminim)/2,15);wmax-wminim;
      wmax := 1.29626222698922
      wminim := 1.29626219763040
      wm := 1.29626221230981
      0.2935882 10-7

```

```

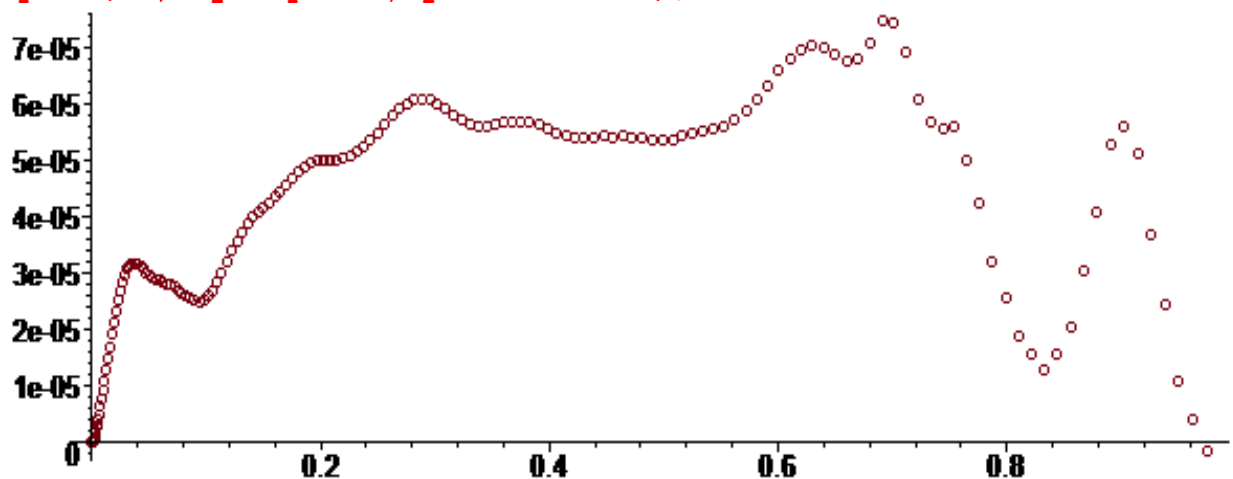
>
M:=seq(subs(w=wm,evalf(C[i],15)),i=1..k):M[1..3];MM:=seq([d[i],s
subs(w=wm,evalf(C[i],15))],i=1..k-2):
0.6348821720099 10-10,0.518888344801622 10-8,0.33614133380241097 10-7

```

```

> plot(MM,style=point,symbol=circle);

```



```

> MasseGalaxie:=evalf(0.23*10^10*V^2*R/wm,8):
print(`Masse_de_la_Galaxie`,MasseGalaxie,`en_Masses_Solaires`);
      Masse_de_la_Galaxie,0.25217007 1012,en_Masses_Solaires

```

```
>evalf(0.23*10^10*V^2*R/wmax,8),evalf(0.23*10^10*V^2*R/wminim,8);
0.25217007 1012,0.25217007 1012
```

```
> #a very little problem at the end.
```

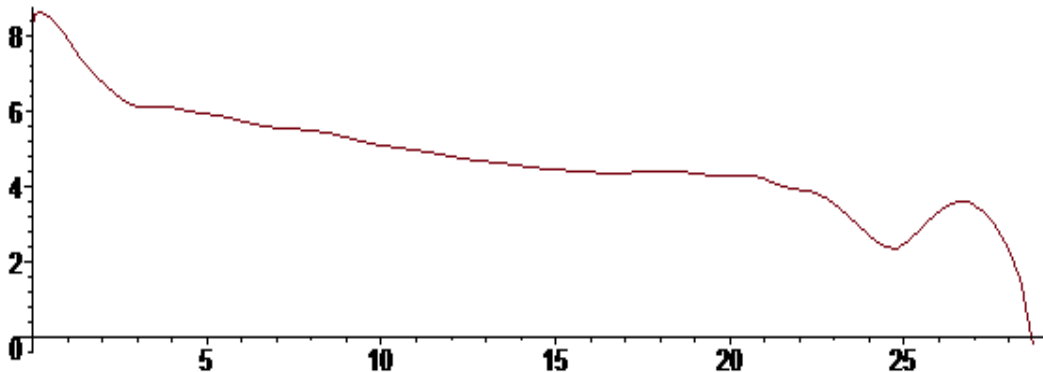
```
> Mgal:=MasseGalaxie;#for R=30 kpc
Mgal :=0.25217007 1012
```

```
> #mean surface density
```

```
> rho:=[seq(Mgal*evalf(8*1*(M[i]+M[i+1])/Pi/((d[i+1]+d[i+2])^2
-(d[i]+d[i-1])^2)/(R*10^3)^2),i=2..(k-1)):nops(%);
158
```

```
> Rho:=[seq(ln(rho[i]),i=1..(k-3)):nops(%);
157
```

```
> plot([seq([R*(d[i]+d[i+1])/2,Rho[i]],i=1..k-4)],
title=`log-density curve in Ms/pc2`);
log-density curve in Ms/pc2
```



```
> densitéVL_30:=[seq([R*(d[i]+d[i+1])/2,Rho[i]],i=1..k-4)]:
> #And for Milky Way, with R=40 kpc ? see Fosué and also Huang
> donneesVL41:=[[0,0],[0.20,233.0],[0.38,268.92],[0.66,250.75],
[1.61,217.83],[2.57,219.58],[3.59,223.11],[4.60,231.24],
[5.08,230.46],[5.58,230.01],[6.10,239.61],[6.57,246.27],
[7.07,243.49],[7.58,242.71],[8.04,243.23],[8.34,239.89],
[8.65,237.26],[9.20,235.30],[9.62,230.99],[10.09,228.41],
[10.58,224.26],[11.09,224.94],[11.58,233.57],[12.07,240.02],
[12.73,242.21],[13.72,261.78],[14.95,259.26],[15.52,268.57],
[16.55,261.17],[17.56,240.66],[18.54,215.31],[19.50,214.99],
[21.25,251.68],[23.78,259.65],[26.22,242.02],[28.71,224.11],
[31.29,211.20],[33.73,217.93],[36.19,219.33],[38.73,213.31],
[41.25,200.05]]:nops(%);
41
```

```
> f:=map(u->evalf(op(2,u)/100,5),donneesVL41);
```



```
f:= [0, 2.3300, 2.6892, 2.5075, 2.1783, 2.1958, 2.2311, 2.3124, 2.3046, 2.3001, 2.3961,
      2.4627, 2.4349, 2.4271, 2.4323, 2.3989, 2.3726, 2.3530, 2.3099, 2.2841, 2.2426,
      2.2494, 2.3357, 2.4002, 2.4221, 2.6178, 2.5926, 2.6857, 2.6117, 2.4066, 2.1531,
      2.1499, 2.5168, 2.5965, 2.4202, 2.2411, 2.1120, 2.1793, 2.1933, 2.1331, 2.0005]
```

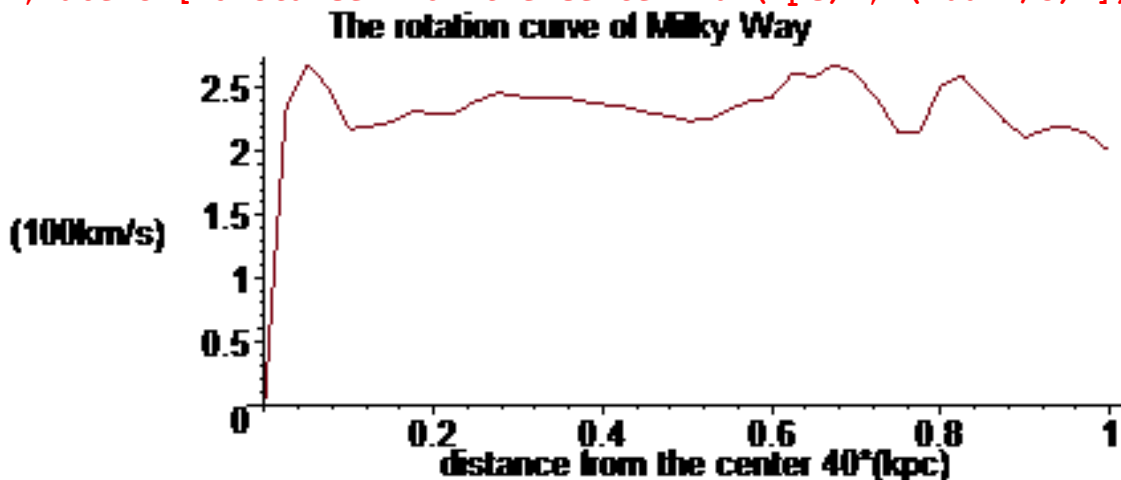
```
R:=40;a:=nops(f)-2;V:=evalf((f[a+1]+f[a+2])/2,7);
```

```
R:=40
```

```
a:=39
```

```
V:=2.066800
```

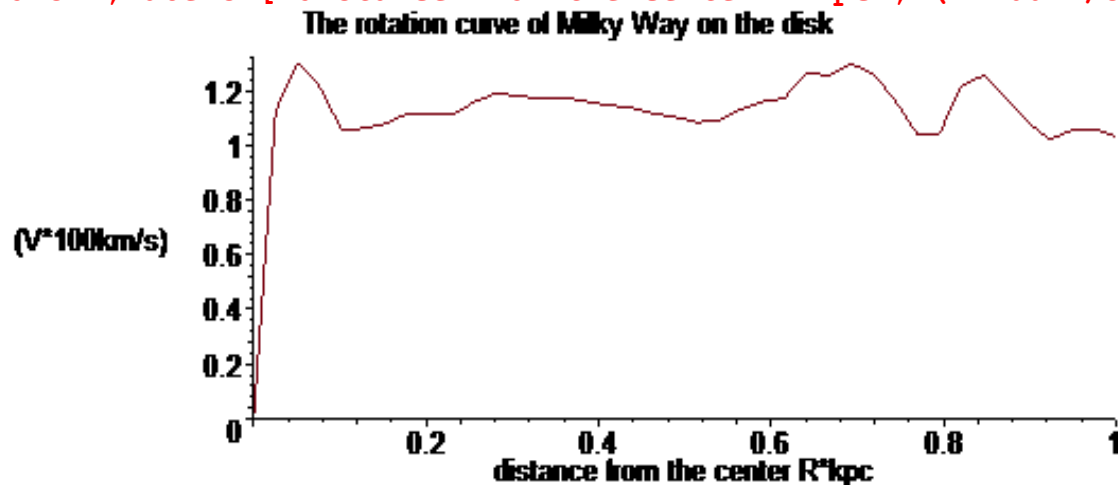
```
> plot([y/40,f[y+1]]$y=0..40),title=`The rotation curve of Milky
Way`,labels=[`distance from the center 40*(kpc)`,`(100km/s)`];
```



```
> fv:=proc(x)
```

```
(f[trunc(a*x)+1]+(a*x-trunc(a*x))*(f[trunc(a*x)+2]-f[trunc(a*x)
+1]))/V end;
```

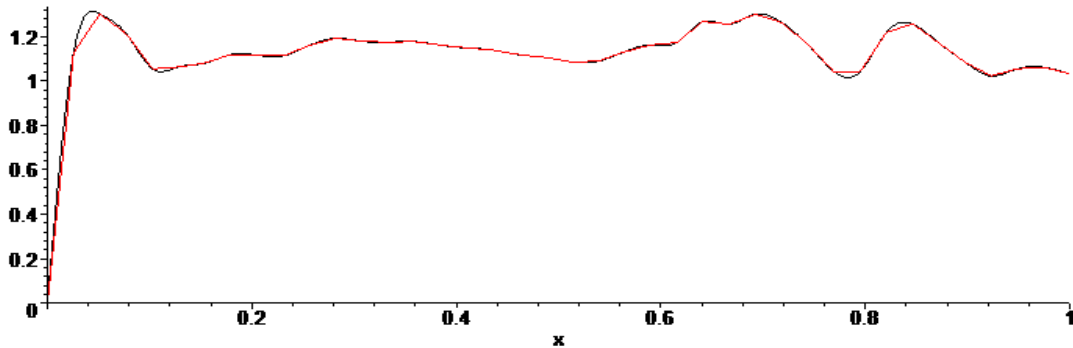
```
> plot(fv(x),x=0..1,title=`The rotation curve of Milky Way on the
disk`,labels=[`distance from the center R*kpc`,`(V*100km/s)`]);
```



```
> F:=[seq([(i-1)/39,f[i]/V],i=1..40)]:
```

```
> g:=x->Spline(F,x):
```

```
> plot({g(x),fv(x)},x=0..1,color=[red,black]);
```



```
> #The second member of the linear system, coming from observed velocities.
```

```
> BB:=Vector([seq(w*g(d[i])^2/d[i],i=1..k),1]):
```

```
#w as the meaning of the inverse of the mass M of the galaxy
```

```
> C:=evalf(MatrixVectorMultiply(invA,BB),30):
```

```
> #search of w=wmin and w=wmax such that all the mass are >=0
```

```
> wnul:=seq(evalf(solve(C[i]=0,w)),i=1..k+1):
```

```
n1:=0:n2:=0:
```

```
for j to k+1 do
```

```
  N:=seq(subs(w=wnul[j],C[i]),i=1..k+1):truc:=1:
```

```
  for i to k+1 do
```

```
    if N[i]<-10(-6) then truc:=0 fi:
```

```
  od;
```

```
  if truc=1 then if n1=0 then n1:=j else n2:=j fi fi;
```

```
od:
```

```
if n1=0 then print(`il y a des masses negatives`) else
```

```
  if wnul[n1]>wnul[n2] then wmax:=wnul[n1];wminim:=wnul[n2]
```

```
  else wmax:=wnul[n2];wminim:=wnul[n1] fi;
```

```
fi:
```

```
wmax:=evalf(wmax,15);wminim:=evalf(wminim,15);
```

```
wm:=evalf((wmax+wminim)/2,15);wmax-wminim;
```

```
wmax := 1.25770737549743
```

```
wminim := 1.25770734123387
```

```
wm := 1.25770735836565
```

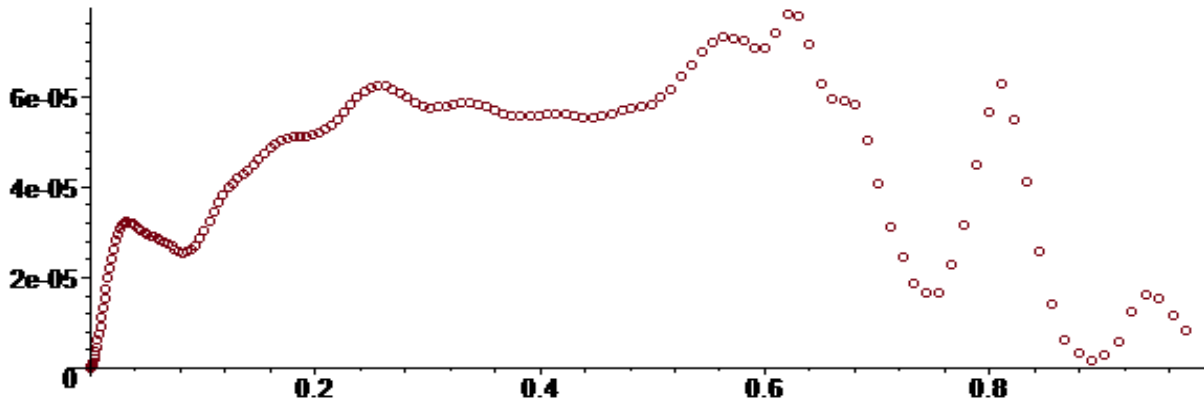
```
0.3426356 10-7
```

```
>M:=seq(subs(w=wm,evalf(C[i],15)),i=1..k):M[1..3];MM:=[seq([d[i],
```

```
subs(w=wm,evalf(C[i],15))],i=1..k-2)]:
```

```
0.763660545193 10-10,0.62413761483094 10-8,0.4043194233463329 10-7
```

```
> plot(MM,style=point,symbol=circle);
```



```

> MasseGalaxie:=evalf(0.23*10^10*v^2*R/wm,8):
print(`Masse_de_la_Galaxie`,MasseGalaxie,`en_Masses_Solaires`);
      Masse_de_la_Galaxie,0.31246770 1012,en_Masses_Solaires

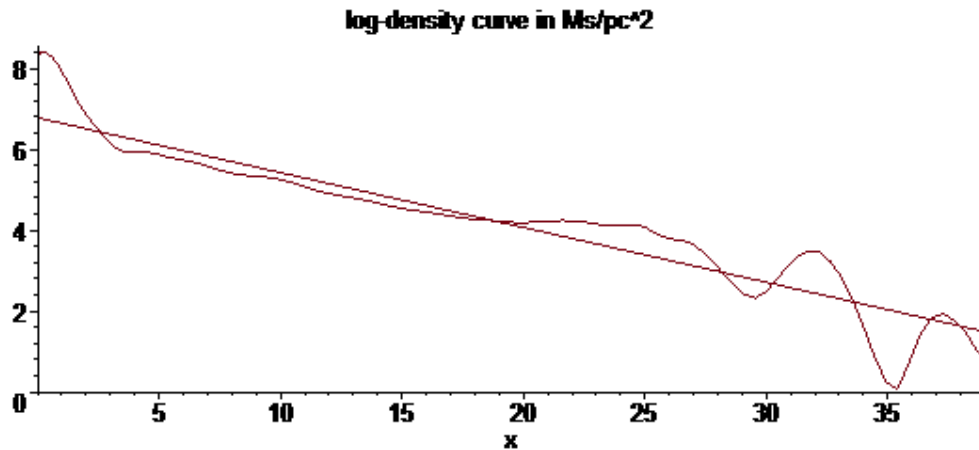
>evalf(0.23*10^10*v^2*R/wmax,8),evalf(0.23*10^10*v^2*R/wminim,8);
      0.31246770 1012,0.31246772 1012

> #no problem at the end.
> Mgal:=MasseGalaxie;#for R=40 kpc
      Mgal :=0.31246770 1012

> #mean surface density
rho:=[seq(Mgal*evalf(8*1*(M[i]+M[i+1])/Pi/((d[i+1]+d[i+2])^2
-(d[i]+d[i-1])^2)/(R*10^3)^2),i=2..(k-2))]:

> #log-density curve
i:='i':Rho:=[seq(ln(rho[i]),i=1..(k-3))]:
i:='i':courbelog:=plot([[R*(d[i]+d[i+1])/2,Rho[i]]$i=2..k-3],
title=`log-density curve in Ms/pc^2`):
> with(stats):Digits:=5:s:=trunc(k/6);
liste:=[[seq(R*(d[i]+d[i+1])/2,i=s..(k-5))],[seq(Rho[i+1],i=s-1..
(k-6))]]:
eqfit:=fit[leastsquare][[x,y]](liste);
eqfonction:=unapply(rhs(eqfit),x):
courberegr:=plot(eqfonction(x),x=0..R-1):
with(plots):
display({courberegr,courbelog});Digits:=25:
      s := 26

```



> #For Fosué and also Huang, the Keplerian mass of the Milky Way is around $0.5 \cdot 10^{12}$ solar mass. So the radius of the disk of the Milky Way seems to be around 40 kpc, (no need of a hypothetical halo).

>

> #Thus : no need of a spherical halo to explain the flatness problem. Moreover the plane of dwarf galaxy satellites could be understood.

> #This program is robust and fast! If the galaxy is now in a universe, it is not difficult to modify this program (a correction from 1% to 3% for the mass which is less than the uncertainties coming from velocities).

> #with $k=160$, total time is less than 200 seconds.



Is this the end of the trip?

> #Now let us go to Andromeda.

```

> #For Andromeda NE
R:=30;V:=2.35;
f:=[0,2.,2.1,2.25,2.3,2.4,2.3,2.35,2.35,2.4,2.45,2.5,2.5,2.55
,2.5,2.45,2.4,2.35,2.35,2.35,2.4,2.35,2.3,2.35,2.4,2.4,2.35,2.35,
2.35,2.35]:
a:=nops(f)-2;
fv:=proc(x)
(f[trunc(a*x)+1]+(a*x-trunc(a*x))*(f[trunc(a*x)+2]-f[trunc(a*x)
+1]))/V
end:

```

$R := 30$

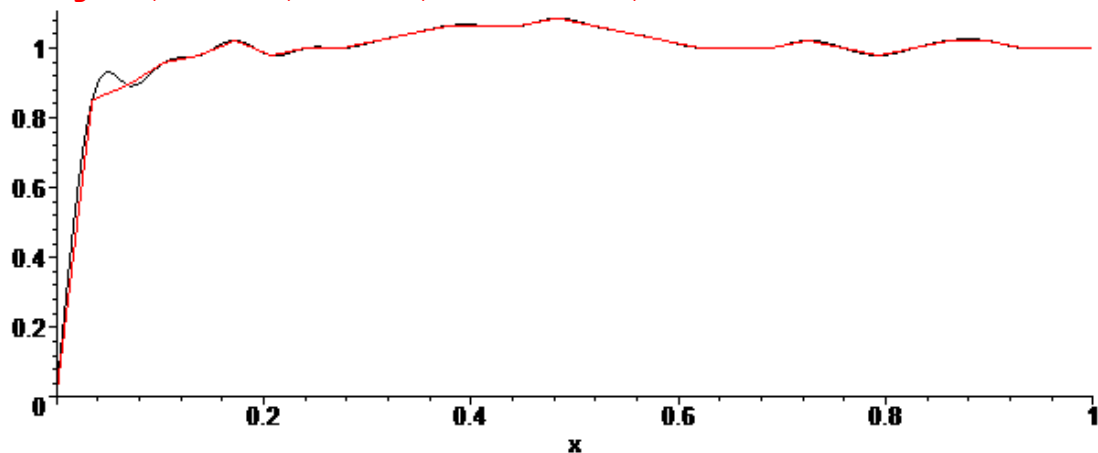
$V := 2.35$

$a := 29$

```

> F:= [seq([(i-1)/29, f[i]/V], i=1..30)]:
> g:=x->Spline(F,x):
> plot({g(x), fv(x)}, x=0..1, color=[red, black]); #well

```



```

> #The second member of the linear system, coming from observed
velocities.

```

```

BB:=Vector([seq(w*g(d[i])^2/d[i], i=1..k), 1]):

```

```

> #calculus of mass as function of w

```

```

C:=evalf(MatrixVectorMultiply(invA, BB), 30):

```

```

> #search of w=wmin and w=wmax such that all the mass are >=0

```

```

wnul:=seq(evalf(solve(C[i]=0, w)), i=1..k+1):

```

```

n1:=0:n2:=0:

```

```

for j to k+1 do

```

```

  N:=seq(subs(w=wnul[j], C[i]), i=1..k+1): truc:=1:

```

```

  for i to k+1 do

```

```

    if N[i]<-10^(-5) then truc:=0 fi:

```

```

  od;

```

```

  if truc=1 then if n1=0 then n1:=j else n2:=j fi fi;

```

```

od:

```

```

if n1=0 then print(`il_y_a_des_masses_negatives`) else

```

```

if wnul[n1]>wnul[n2] then wmax:=wnul[n1];wminim:=wnul[n2]
else wmax:=wnul[n2];wminim:=wnul[n1] fi;
fi:
wmax:=evalf(wmax,15);wminim:=evalf(wminim,15);
wm:=evalf((wmax+wminim)/2,15);wmax-wminim;
wmax := 1.52637658960144

wminim := 1.52637656731640

wm := 1.52637657845892

0.2228504 10-7

```

```

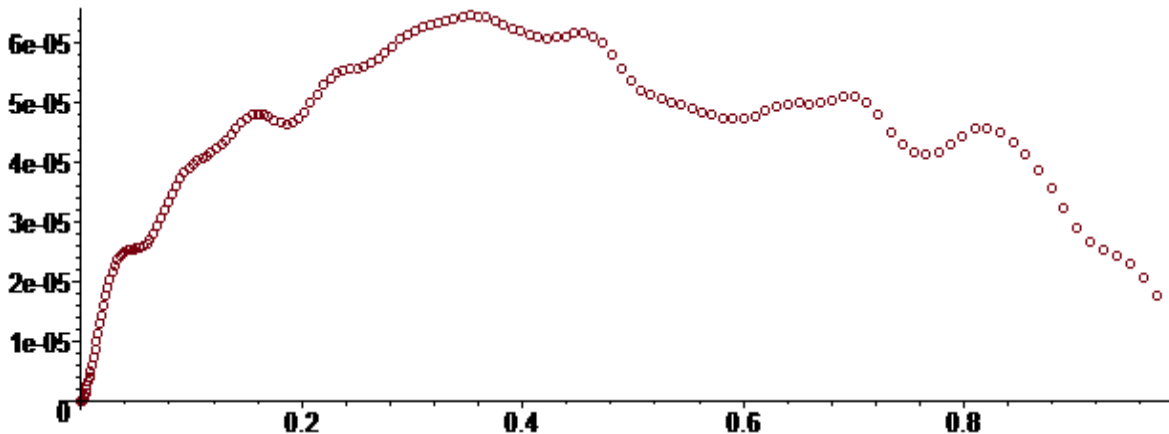
>
M:=seq(subs(w=wm,evalf(C[i],15)),i=1..k):M[1..3];MM:=seq([d[i],s
ubs(w=wm,evalf(C[i],15))],i=1..k-2):
0.4092599432665 10-10,0.334487997012523 10-8,0.21668667076291949 10-7

```

```

> plot(MM,style=point,symbol=circle);

```



```

> MasseGalaxie:=evalf(0.23*1010*V2*R/wm,5):
print(`Masse_d'Andromède NE`,MasseGalaxie,`en_Masses_Solaires`);
Masse_d'Andromède NE,0.24964 1012,en_Masses_Solaires

```

```

>
evalf(0.23*1010*V2*R/wmax,8),evalf(0.23*1010*V2*R/wminim,8);
0.24964515 1012,0.24964515 1012

```

```

> Mgal:=MasseGalaxie:
> #mean surfacic density
> rho:=[seq(Mgal*evalf(8*1*(M[i]+M[i+1])/Pi/((d[i+1]+d[i+2])2
-(d[i]+d[i-1])2)/(R*103)2),i=2..(k-1))]:
> Rho:=[seq(ln(rho[i]),i=1..(k-2))]:nops(%);

```

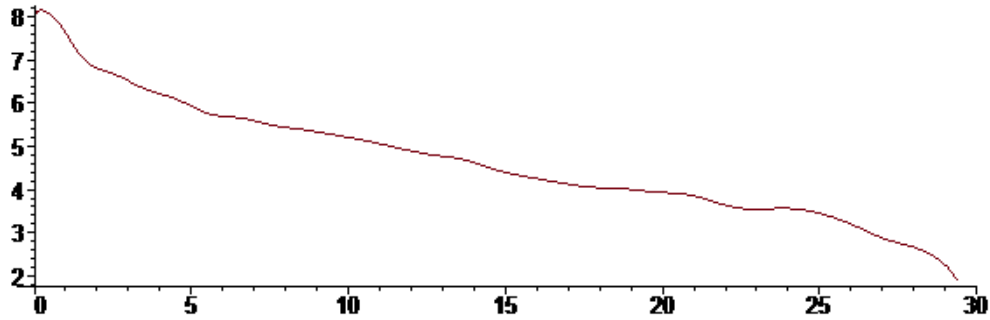
158

```

> plot([seq([R*(d[i]+d[i+1])/2,Rho[i]],i=1..k-2)],
title=`log-density curve in Ms/pc2`);

```

log-density curve in Ms/pc²



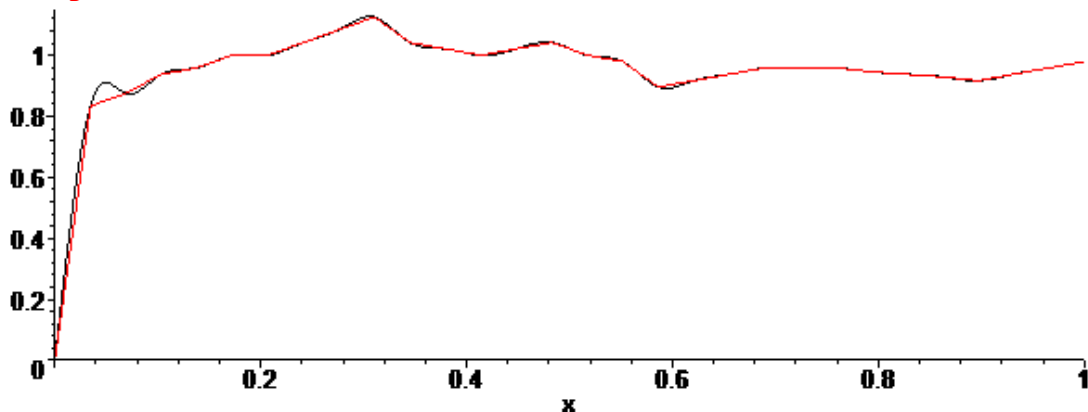
```
>densitéAndromède_NE:=[seq([R*(d[i]+d[i+1])/2,Rho[i]],i=1..k-2)]:
> #For Andromede SW :
#The observed rotation curve for Andromeda SW
R:=30;V:=2.4;
f:=[0,2.,2.1,2.25,2.3,2.4,2.4,2.5,2.6,2.7,2.5,2.45,2.4,2.45,2.5,2
.4,2.35,
2.15,2.2,2.25,2.3,2.3,2.3,2.27,2.25,2.23,2.2,2.25,2.3,2.35,2.4]:
a:=nops(f)-2;
fv:=proc(x)
(f[trunc(a*x)+1]+(a*x-trunc(a*x))*(f[trunc(a*x)+2]-f[trunc(a*x)
+1]))/V
end:
```

$R := 30$

$V := 2.4$

$a := 29$

```
> F:=[seq([(i-1)/29,f[i]/V],i=1..30)]:
> g:=x->Spline(F,x):
> plot({g(x),fv(x)},x=0..1,color=[red,black]);#well
```



```
> #The second member of the linear system, coming from observed
velocities.
```

```
BB:=Vector([seq(w*g(d[i])^2/d[i],i=1..k),1]):
```

```
#w as the meaning of the inverse of the mass M of the galaxy
```

```
> #calculus of mass as fonction of w
```

```
C:=evalf(MatrixVectorMultiply(invA,BB),30):
```

```

> #search of w=wmin and w=wmax such that all the mass are >=0
wnul:=seq(evalf(solve(C[i]=0,w)),i=1..k+1):
n1:=0:n2:=0:
for j to k+1 do
  N:=seq(subs(w=wnul[j],C[i]),i=1..k+1):truc:=1:
  for i to k+1 do
    if N[i]<-10^(-5) then truc:=0 fi:
  od:
  if truc=1 then if n1=0 then n1:=j else n2:=j fi fi:
od:
if n1=0 then print(`il_y_a_des_masses_negatives`) else
  if wnul[n1]>wnul[n2] then wmax:=wnul[n1];wminim:=wnul[n2]
  else wmax:=wnul[n2];wminim:=wnul[n1] fi:
fi:
wmax:=evalf(wmax,15);wminim:=evalf(wminim,15);
wm:=evalf((wmax+wminim)/2,15);wmax-wminim;
      wmax := 1.67975395286903
      wminim := 1.67975392689909
      wm := 1.67975393988406
      0.2596994 10-7

```

```

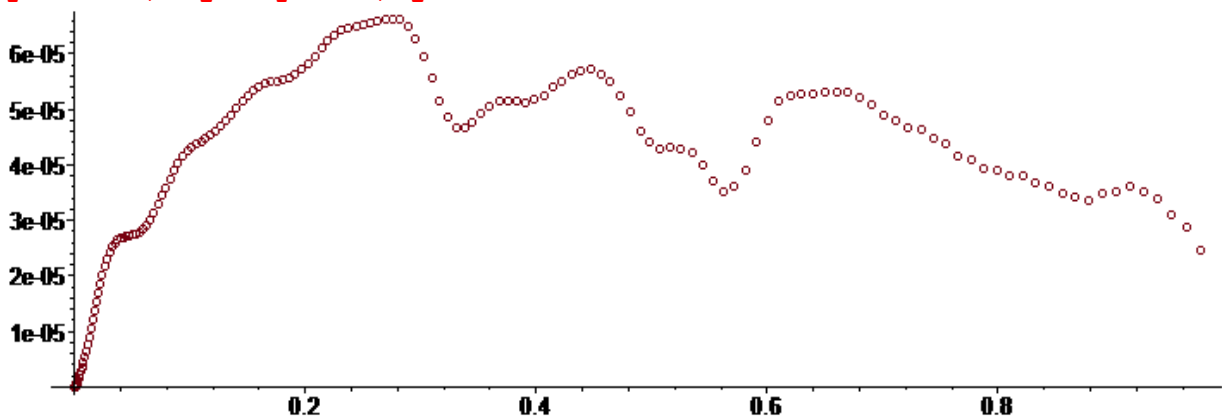
>M:=seq(subs(w=wm,evalf(C[i],15)),i=1..k):M[1..3];MM:=[seq([d[i],
subs(w=wm,evalf(C[i],15))],i=1..k-2)]:
0.4333838421490 10-10,0.354204531074112 10-8,0.22945938137795726 10-7

```

```

> plot(MM,style=point,symbol=circle);

```



```

> MasseGalaxie:=evalf(0.23*10^10*V^2*R/wm,5):
print(`Masse_d'Andromède SW`,MasseGalaxie,`en_Masses_Solaires`);
      Masse_d'Andromède SW,0.23660 1012,en_Masses_Solaires

```

```

>evalf(0.23*10^10*V^2*R/wmax,8),evalf(0.23*10^10*V^2*R/wminim,8);
0.23660608 1012,0.23660608 1012

```

```

> Mgal:=MasseGalaxie:

```



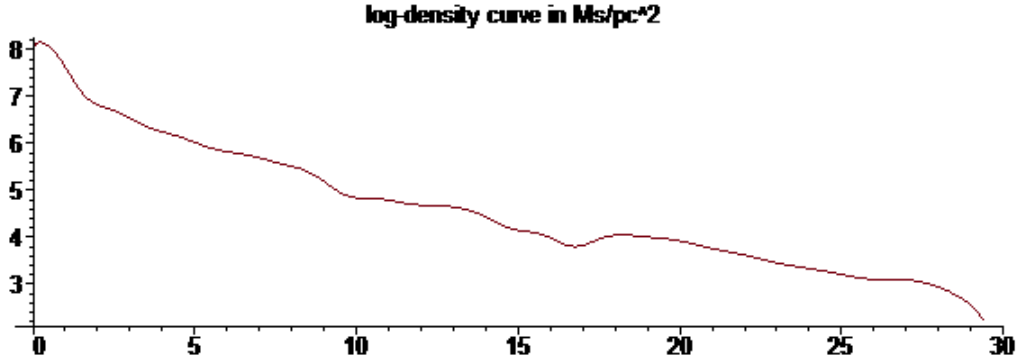
```

> #mean surface density
> rho:=[seq(Mgal*evalf(8*1*(M[i]+M[i+1])/Pi/((d[i+1]+d[i+2])^2
- (d[i]+d[i-1])^2)/(R*10^3)^2),i=2..(k-1)):nops(%);
158

> Rho:=[seq(ln(rho[i]),i=1..(k-2))]:nops(%);
158

> plot([seq([R*(d[i]+d[i+1])/2,Rho[i]],i=1..k-2)],
title=`log-density curve in Ms/pc^2`);

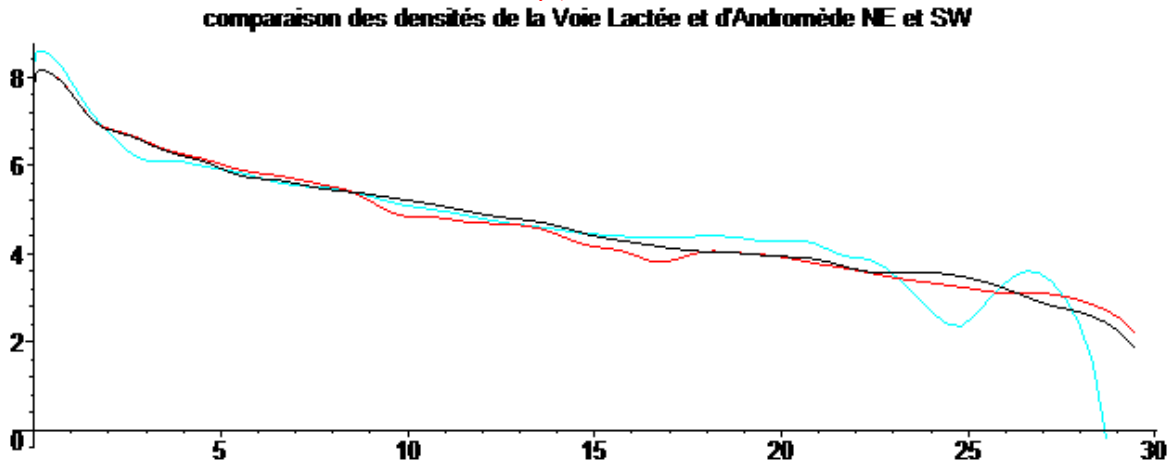
```



```

> densitéAndromède_SW:=[seq([R*(d[i]+d[i+1])/2,Rho[i]],i=1..k-2)]:
>
> plot([densitéAndromède_NE,densitéAndromède_SW,densitéVL_30],colo
r=[black,red,cyan], title=`comparaison des densités de la Voie
Lactée et d'Andromède NE et SW`);

```



```

> #the surface densities are similar up to 24 kpc from the
center; the color and mass are respectively for Andro_NE,
Andro_SW, Milky Way, [black,red,cyan],
[.24964e12,.23660e12,.25217007e12]; so Milky Way and Andromeda
are twin galaxies. This fact is new since 2014.

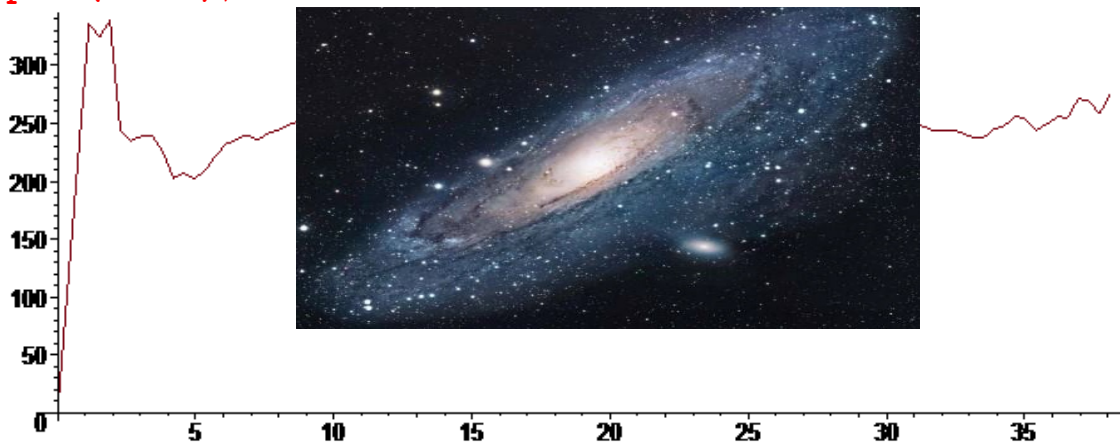
```

Is this the end of the journey?

> #But the data for Andromède are old (1996) so let us now take the recent data (Chemin, L., Carignan, C., & Foster, T. 2009, ApJ, 705, 1395) valid up to 38 kpc for the entire galaxy.

```
>Andro:=[[0,0],[1.14,336.],[1.52,324.5],[1.90,339.],[2.28,243.5],
[2.66,235.],[3.04,239.],[3.43,239.],[3.81,226.],[4.19,203.],
[4.57,207.],[4.95,202.5],[5.33,209.],[5.71,221.5],[6.09,232.],
[6.85,240.],[7.23,235.5],[7.61,241.5],[7.99,244.5],[8.37,249],
[8.75,252],[9.13,253.],[9.51,259.],[9.90,259.],[9.51,259.],
[10.28,262.],[10.66,269.],[11.04,267.],[11.42,266.],
[11.80,264.5],[12.18,264.7],[12.56,265.3],[12.94,265.2],
[13.32,262.],[13.32,262.],[13.70,261.],[14.08,259.],[14.46,258.],
[14.84,258.5],[15.23,259.2],[15.61,262.7],[15.99,266.],
[16.37,270.],[16.75,270.],[17.13,269.],[17.51,268.5],
[17.89,263.],[18.27,257.],[18.65,254.],[19.03,252.],
[19.41,249.5],[19.79,245.7],[20.18,243.7],[20.56,247.9],
[20.94,242.3],[21.32,239.2],[21.70,239.5],[22.08,236.1],
[22.46,233.8],[22.84,233.1],[23.22,230.1],[23.60,232.1],
[23.98,228.7],[24.36,229.1],[24.75,227.9],[25.13,226.9],
[25.51,225.1],[25.89,225.4],[26.27,230.3],[26.65,229.],
[27.03,229.9],[27.41,230.1],[27.79,229.8],[28.17,230.4],
[28.56,230.9],[28.94,229.8],[29.32,228.8],[29.70,228.3],
[30.08,243.6],[30.46,247.3],[30.84,247.8],[31.22,248.4],
[31.61,244.5],[31.99,244.5],[32.37,244.4],[32.75,241.7],
[33.13,237.7],[33.51,237.6],[33.89,244.9],[34.27,247.9],
[34.66,256.3],[35.04,253.5],[35.42,244.3],[35.80,249.3],
[36.18,255.7],[36.56,255.0],[36.94,271.1],[37.32,269.8],
[37.71,258.2],[38.09,275.1]]]:
```

> plot(Andro);



> Comparison between Andromeda, with these new data, and the Milky Way for 3 radius R=30, 35 and 38 kpc. we take now k:=240.

>For R=30 kpc

```
>donneesVL30:=[[0,0],[0.20,233.0],[0.38,268.92],[0.66,250.75],
[1.61,217.83],[2.57,219.58],[3.59,223.11],[4.60,231.24],
[5.08,230.46],[5.58,230.01],[6.10,239.61],[6.57,246.27],
```

```
[7.07,243.49],[7.58,242.71],[8.04,243.23],[8.34,239.89],
[8.65,237.26],[9.20,235.30],[9.62,230.99],[10.09,228.41],
[10.58,224.26],[11.09,224.94],[11.58,233.57],[12.07,240.02],
[12.73,242.21],[13.72,261.78],[14.95,259.26],[15.52,268.57],
[16.55,261.17],[17.56,240.66],[18.54,215.31],[19.50,214.99],
[21.25,251.68],[23.78,259.65],[26.22,242.02],[28.71,224.11],
[31.29,211.20]]:nops(%);
```

37

```
> f:=map(u->evalf(op(2,u)/100,5),donneesVL30);
f:=[0.,2.3300,2.6892,2.5075,2.1783,2.1958,2.2311,2.3124,2.3046,2.3001,2.3961,
2.4627,2.4349,2.4271,2.4323,2.3989,2.3726,2.3530,2.3099,2.2841,2.2426,
2.2494,2.3357,2.4002,2.4221,2.6178,2.5926,2.6857,2.6117,2.4066,2.1531,
2.1499,2.5168,2.5965,2.4202,2.2411,2.1120]
```

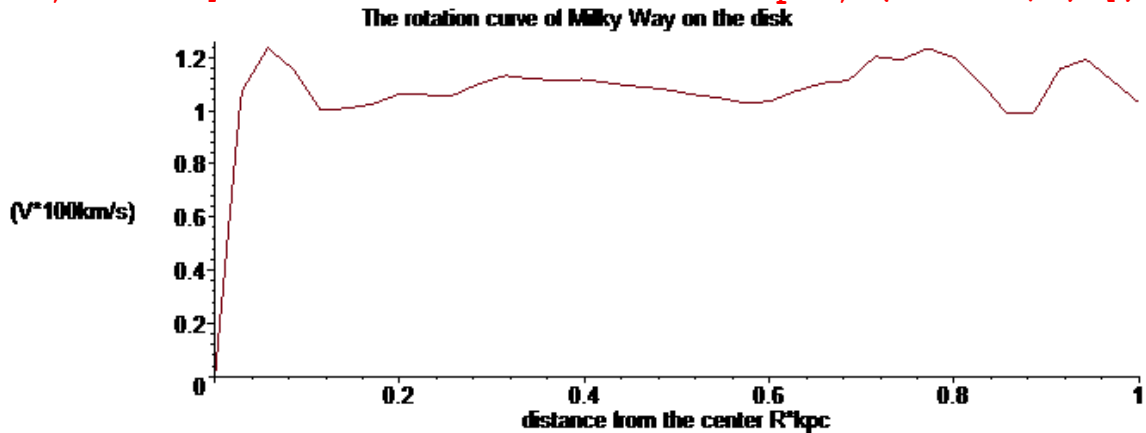
```
R:=30;a:=nops(f)-2;V:=evalf((f[a+1]+f[a+2])/2,7);
```

R := 30

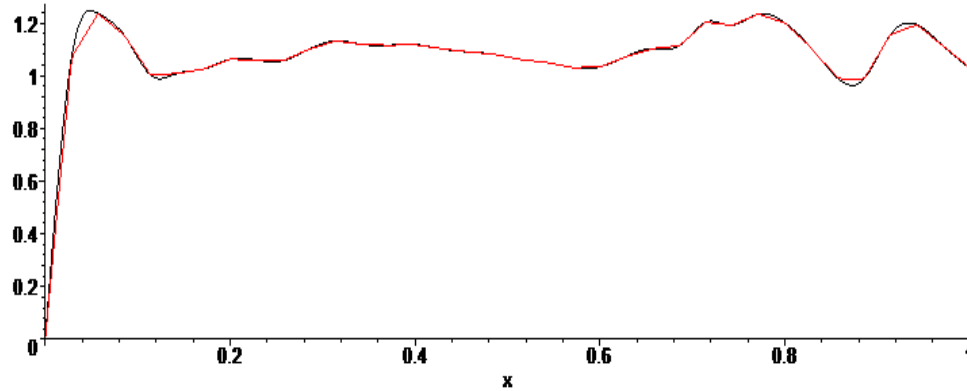
a := 35

V := 2.176550

```
> fv:=proc(x)
(f[trunc(a*x)+1]+(a*x-trunc(a*x))*(f[trunc(a*x)+2]-f[trunc(a*x)
+1]))/V end;
> plot(fv(x),x=0..1,title=`The rotation curve of Milky Way on the
disk`,labels=[`distance from the center R*kpc`,`(V*100km/s)`]);
```



```
> F:=[seq([(i-1)/35,f[i]/V],i=1..37)];
> g:=x->Spline(F,x);
> plot({g(x),fv(x)},x=0..1,color=[red,black]);
```



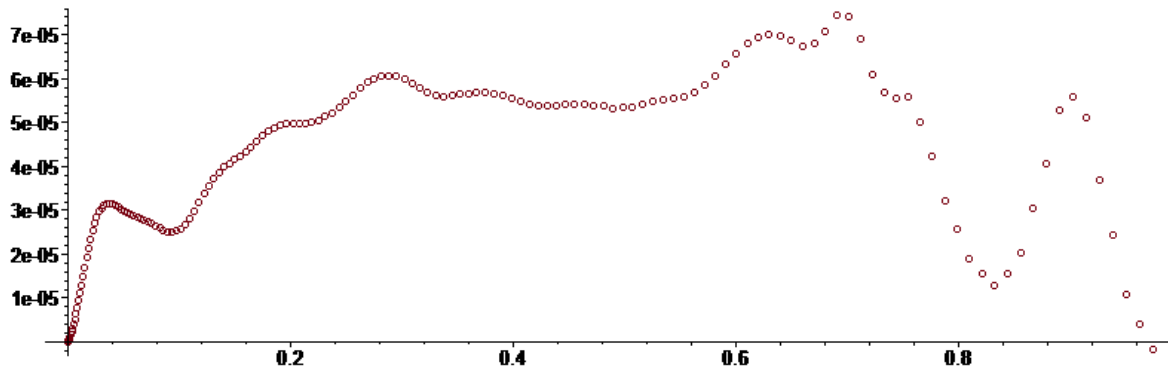
```

> BB:=Vector([seq(w*g(d[i])^2/d[i],i=1..k),1]):
#w as the meaning of the inverse of the mass M of the galaxy
> C:=evalf(MatrixVectorMultiply(invA,BB),30):
> #search of w=wmin and w=wmax such that all the mass are >=0
> wnul:=seq(evalf(solve(C[i]=0,w)),i=1..k+1):
n1:=0:n2:=0:
for j to k+1 do
  N:=seq(subs(w=wnul[j],C[i]),i=1..k+1):truc:=1:
  for i to k+1 do
    if N[i]<-10^(-5) then truc:=0 fi:
  od;
  if truc=1 then if n1=0 then n1:=j else n2:=j fi fi;
od:
if n1=0 then print(`il_y_a_des_masses_negatives`) else
  if wnul[n1]>wnul[n2] then wmax:=wnul[n1];wminim:=wnul[n2]
  else wmax:=wnul[n2];wminim:=wnul[n1] fi;
fi:
wmax:=evalf(wmax,15);wminim:=evalf(wminim,15);
wm:=evalf((wmax+wminim)/2,15);wmax-wminim;
      wmax := 1.29609974554824
      wminim := 1.29609971619675
      wm := 1.29609973087250
      0.2935149 10-7

>M:=seq(subs(w=wm,evalf(C[i],15)),i=1..k):M[1..3];MM:=seq([d[i],
subs(w=wm,evalf(C[i],15))],i=1..k-2]:
0.6348035233179 10-10,0.518823756785924 10-8,0.33609949360446503 10-7

> plot(MM,style=point,symbol=circle);

```



```
> MasseGalaxie:=evalf(0.23*10^10*V^2*R/wm,8):
print(`Masse_de_la_Galaxie`,MasseGalaxie,`en_Masses_Solaires`);
      Masse_de_la_Galaxie,0.25220169 1012,en_Masses_Solaires

>evalf(0.23*10^10*V^2*R/wmax,8),evalf(0.23*10^10*V^2*R/wminim,8);
      0.25220169 1012,0.25220169 1012
```

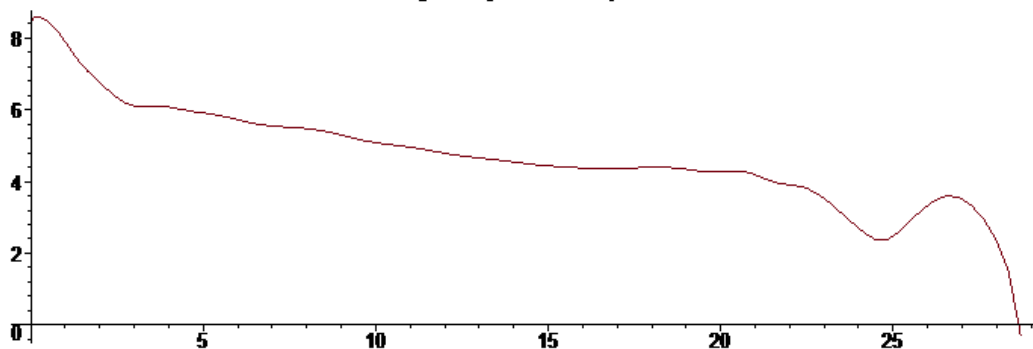
```
> Mgal:=MasseGalaxie;
      Mgal := 0.25220169 1012
```

```
> MgalVL30:=.25220e12;
      MgalVL30 := 0.25220 1012
```

```
> #mean surface density
rho:=[seq(Mgal*evalf(8*1*(M[i]+M[i+1])/Pi/((d[i+1]+d[i+2])^2
-(d[i]+d[i-1])^2)/(R*10^3)^2),i=2..(k-2))]:
> rho[150..157];
[36.5501577683586838323 ,35.3001252716176763077 ,28.3864158784650650320 ,
 19.4033352341991819725 ,10.9967957362445686830 ,
 4.61118506733003064041 ,0.732748184283954685502 ,
 -1.04006947883587880957 ]
```

#log-density curve

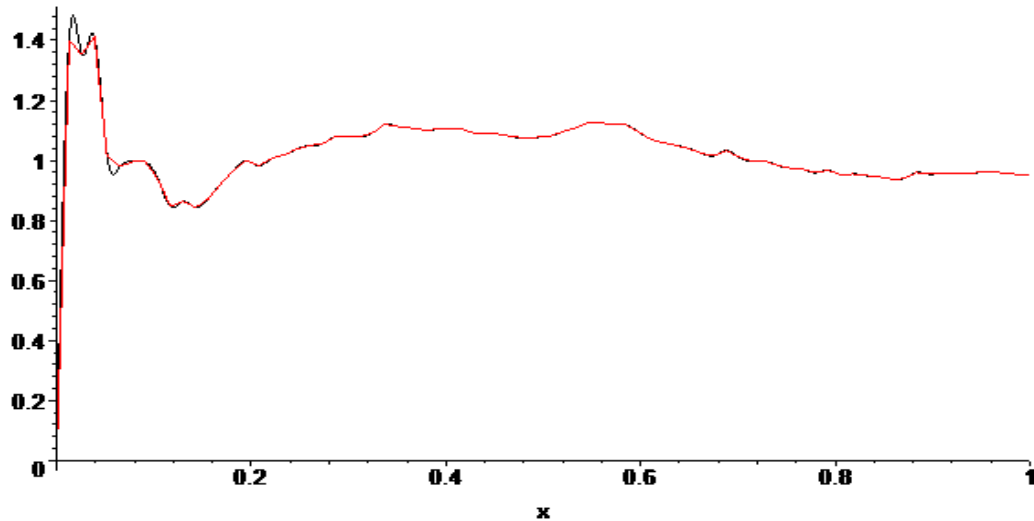
```
Rho:=[seq(ln(rho[i]),i=1..(k-4))]:
> plot([seq([R*(d[i]+d[i+1])/2,Rho[i]],i=1..k-4)],
title=`log-density curve in Ms/pc^2`);
      log-density curve in Ms/pc2
```



```

> densitéVL30:= [seq([R*(d[i]+d[i+1])/2,Rho[i]],i=1..k-4)]:
>
> Andro30:= [[0,0],[1.14,336.],[1.52,324.5],[1.90,339.],
[2.28,243.5],[2.66,235.],[3.04,239.],[3.43,239.],[3.81,226.],
[4.19,203.],[4.57,207.],[4.95,202.5],[5.33,209.],[5.71,221.5],
[6.09,232.],[6.85,240.],[7.23,235.5],[7.61,241.5],[7.99,244.5],
[8.37,249],[8.75,252],[9.13,253.],[9.51,259.],[9.90,259.],
[9.51,259.],[10.28,262.],[10.66,269.],[11.04,267.],[11.42,266.],
[11.80,264.5],[12.18,264.7],[12.56,265.3],[12.94,265.2],
[13.32,262.],[13.32,262.],[13.70,261.],[14.08,259.],[14.46,258.],
[14.84,258.5],[15.23,259.2],[15.61,262.7],[15.99,266.],
[16.37,270.],[16.75,270.],[17.13,269.],[17.51,268.5],
[17.89,263.],[18.27,257.],[18.65,254.],[19.03,252.],
[19.41,249.5],[19.79,245.7],[20.18,243.7],[20.56,247.9],
[20.94,242.3],[21.32,239.2],[21.70,239.5],[22.08,236.1],
[22.46,233.8],[22.84,233.1],[23.22,230.1],[23.60,232.1],
[23.98,228.7],[24.36,229.1],[24.75,227.9],[25.13,226.9],
[25.51,225.1],[25.89,225.4],[26.27,230.3],[26.65,229.],
[27.03,229.9],[27.41,230.1],[27.79,229.8],[28.17,230.4],
[28.56,230.9],[28.94,229.8],[29.32,228.8],[29.70,228.3],
[30.08,243.6]]]:
> #For Andromeda30
R:=30;V:=2.40;
f:=map(u->op(2,u)/100,Andro30):
a:=nops(f)-2;
fv:=proc(x)
(f[trunc(a*x)+1]+(a*x-trunc(a*x))*(f[trunc(a*x)+2]-f[trunc(a*x)
+1]))/V
end:
R := 30
V := 2.40
a := 77
> F:= [seq([(i-1)/77,f[i]/V],i=1..78)]:
> g:=x->Spline(F,x):
> plot({g(x),fv(x)},x=0..1,color=[red,black]);

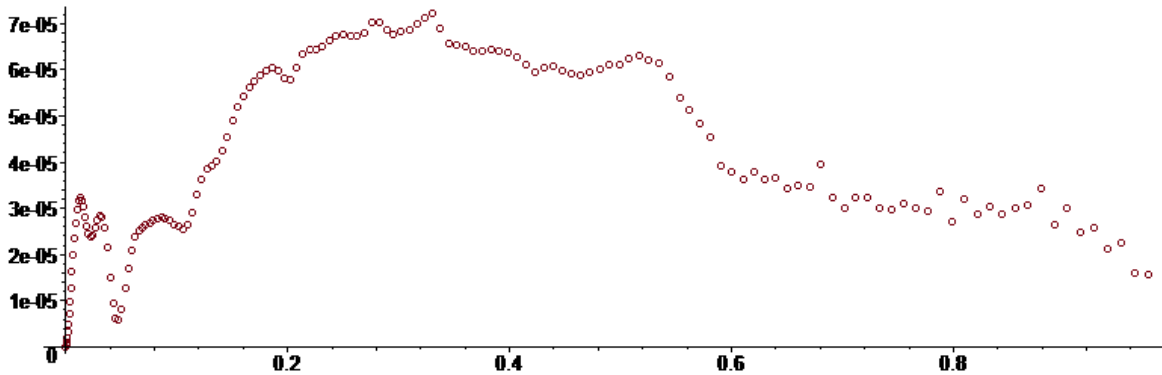
```



```

> BB:=Vector([seq(w*g(d[i])^2/d[i],i=1..k),1]):
> C:=evalf(MatrixVectorMultiply(invA,BB),30):
> wnul:=seq(evalf(solve(C[i]=0,w)),i=1..k+1):
n1:=0:n2:=0:
for j to k+1 do
  N:=seq(subs(w=wnul[j],C[i]),i=1..k+1):truc:=1:
  for i to k+1 do
    if N[i]<-10^(-5) then truc:=0 fi:
  od:
  if truc=1 then if n1=0 then n1:=j else n2:=j fi fi:
od:
if n1=0 then print(`il y a des masses negatives`) else
  if wnul[n1]>wnul[n2] then wmax:=wnul[n1];wminim:=wnul[n2]
  else wmax:=wnul[n2];wminim:=wnul[n1] fi:
fi:
wmax:=evalf(wmax,15);wminim:=evalf(wminim,15);
wm:=evalf((wmax+wminim)/2,15);wmax-wminim;
      wmax := 1.58684496047333
      wminim := 1.58684479838473
      wm := 1.58684487942903
      0.16208860 10-6
>M:=seq(subs(w=wm,evalf(C[i],15)),i=1..k):M[1..3];MM:=seq([d[i],
subs(w=wm,evalf(C[i],15))],i=1..k-2]):
0.28632900277784 10-9,0.2340073285137875 10-7,0.151561437343734699 10-6
> plot(MM,style=point,symbol=circle);

```



```
> MasseGalaxie:=evalf(0.23*10^10*v^2*R/wm,5):
print(`Masse_d'Andromède`,MasseGalaxie,`en_Masses_Solaires`);
      Masse_d'Andromède,0.25047 1012,en_Masses_Solaires
```

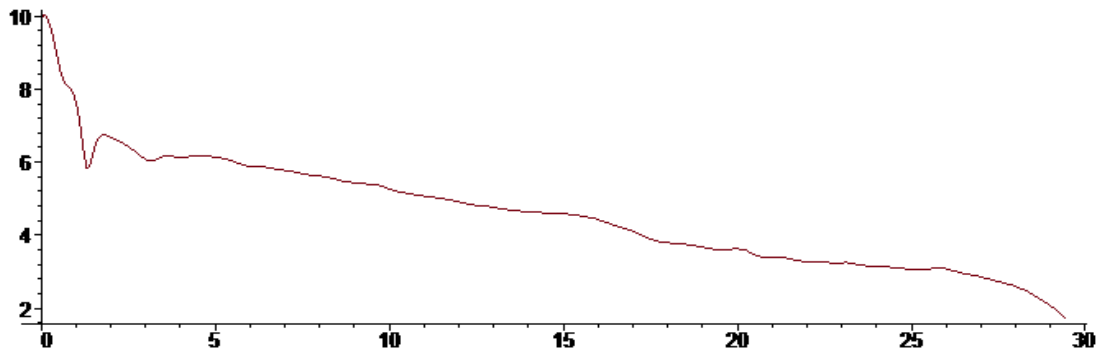
```
> MgalAndro30:=.25047e12;
      MgalAndro30 := 0.25047 1012
```

```
> rho:=[seq(MgalAndro30*evalf(8*1*(M[i]+M[i+1])/Pi/
((d[i+1]+d[i+2])^2
-(d[i]+d[i-1])^2)/(R*10^3)^2),i=2..(k-1)):nops(%);
      158
```

```
> rho[150..158];
[18.8202629941841191441 ,17.9042367161536068983 ,16.2306684089766064085 ,
 14.8298265462072842882 ,13.5229207371455547674 ,
 11.6634467504719020297 ,9.42836959439959923588 ,
 7.52745871867826191984 ,5.53210191320532546773 ]
```

```
> Rho:=[seq(ln(rho[i]),i=1..(k-2)):nops(%);
      158
```

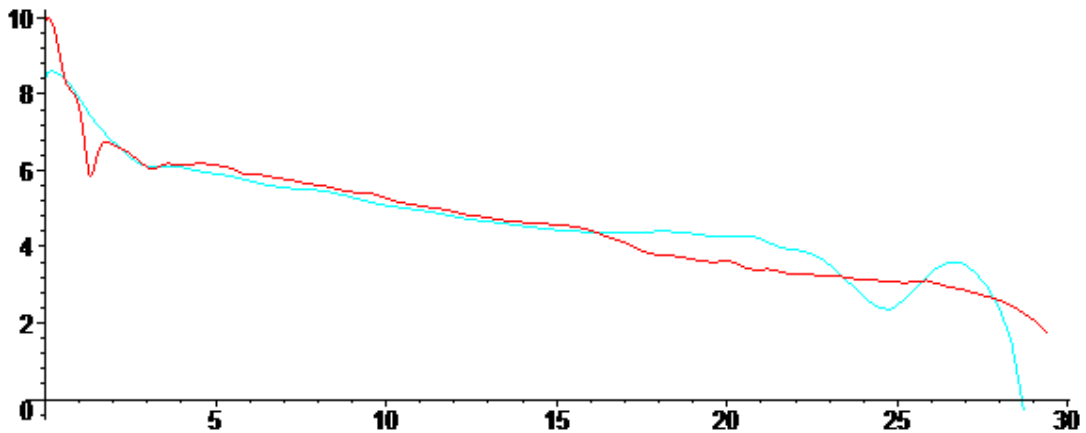
```
> plot([seq([R*(d[i]+d[i+1])/2,Rho[i]],i=1..k-2)],
title=`log-density curve in Ms/pc^2`);
      log-density curve in Ms/pc2
```



```
> densitéAndromède30:=[seq([R*(d[i]+d[i+1])/2,Rho[i]],i=1..k-2)];
> plot([densitéAndromède30,densitéVL30],color=[red,cyan],
title=`comparaison des densités de la Voie Lactée et
```


d'Andromède`);

comparaison des densités de la Voie Lactée et d'Andromède



>

> MgalAndro30;MgalVL30;

$$0.25047 \cdot 10^{12}$$

$$0.25220 \cdot 10^{12}$$

> #recall the mass of Andromeda with the old data, for the same radius:

> `Masse_d'Andromède NE`= .24964e12;`Masse_d'Andromède SW`= .23660e12;

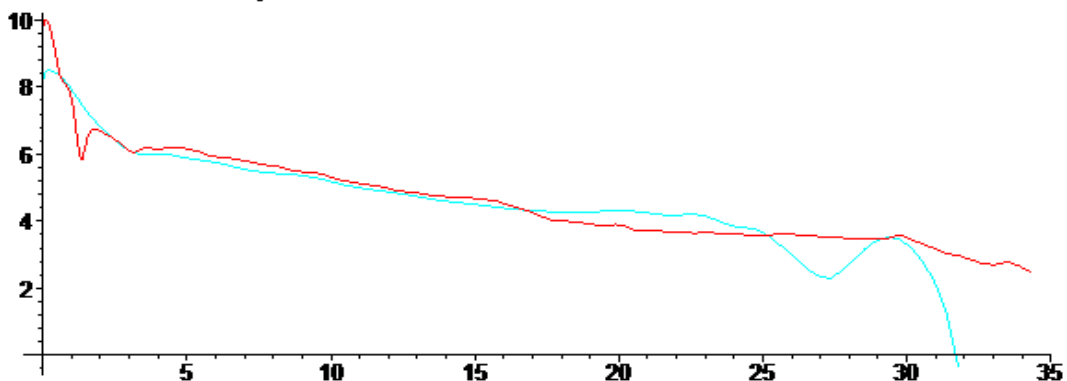
$$\text{Masse_d'Andromède NE} = 0.24964 \cdot 10^{12}$$

$$\text{Masse_d'Andromède SW} = 0.23660 \cdot 10^{12}$$

> #Now if the two galaxies have a bigger radius, R=35 or R=38 kpc

> # For the radius 35 kpc the final results are :

comparaison des densités de la Voie Lactée et d'Andromède



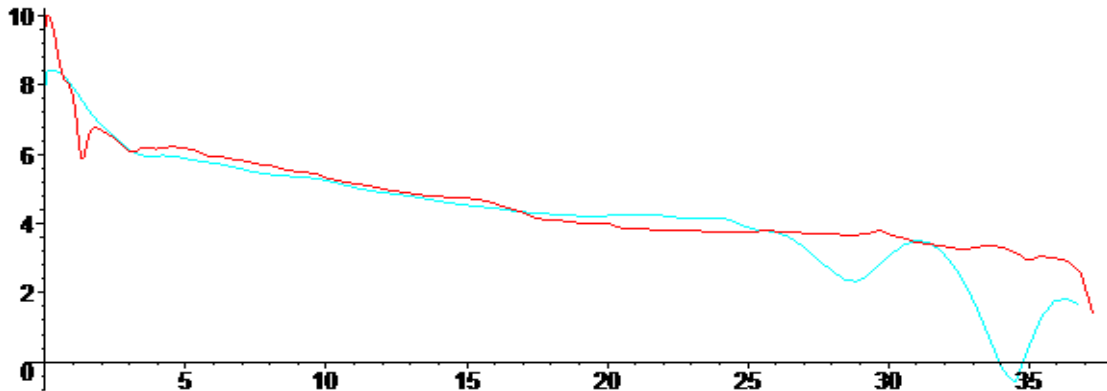
> MgalAndro35=.30873e12;MgalVL35=.28104e12;

$$\text{MgalAndro35} = 0.30873 \cdot 10^{12}$$

$$\text{MgalVL35} = 0.28104 \cdot 10^{12}$$

> #And for the radius 38 kpc the final results are :

comparaison des densités de la Voie Lactée et d'Andromède



> $M_{galAndro38} = .34705e12$; $M_{galVL38} = .302285e12$;

$$M_{galAndro38} = 0.34705 \cdot 10^{12}$$

$$M_{galVL38} = 0.302285 \cdot 10^{12}$$

Some recent studies show that the Kepler radius of these two galaxies, with a halo of dark matter, would be of the order of 40 to 50 kpc with a Kepler mass of the order of $0.5 \cdot 10^{12}$ solar masses. Our results confirm these facts **but without halo of dark matter.**

Why?

It is a long story of mathematical errors in the use of the theory of integration. Even if for about 6 years, one of the errors is corrected by the use of the functions of Bessel, there are still others. A disk of matter is not a sphere!

Until now we worked with the Newton theory of the gravitation. But it is probably that relativistic theories could provide similar results. Indeed the Einstein theory and also the conformal theory have been tested by many scientists around the world. A first problem, the speed of light is finish. Another problem, we worked with a spiral galaxy in an empty universe. The first problem has an aftermath, not for the relativist gravitation but for the trip of Gaston with his gaffophone because the radius of the Milky Way is greater than 50 000 light years. Probably this journey is a dream.



The second problem, with a spiral galaxy in an empty universe, is difficult to overcome. Indeed the equality between the inertial mass m_I and the gravitational mass m_G is a consequence of the Newton's Laws but in Einstein gravitational theory it is a basic

principle. How to explain that m_I is not null in an empty universe? In a non-empty universe and endowed with the Mach's principle, it is possible. But we have a small aftermath: the Kepler's law, so the Kepler's radius, don't exist. Nevertheless we have another radius, the radius of attraction of the galaxy which was defined by J. M. Souriau (cf. Mizony 2005). I made a calculus for the Milky Way in a de Sitter universe 20 years ago, the relativist corrections are very small, for example the mass is around 1% heavier. Since, many scientists made also alternative proofs (cf. Mannheim, Marmet, Cooperstock, ... , and these last years a Chinese group and an Australian). All, they find, using different forms for the disk and often Einstein equations, similar results: the Bessel's functions, similar mass, ..., with no need of a dark matter halo. So no "core-cusp" problem for the "vast rotating disk of dwarf galaxies surrounding the Andromeda galaxy". As a **conclusion** we have to note that the mass of the twin galaxies for the same radius from 30 to 38 kpc are similar and in the interval **[2.3 10¹¹, 3.5 10¹¹] times the solar mass.**

The creation of the "dark matter" fiction is similar to the "gaffophone" story.

References :

- Y. Huang, X.-W. Liu, H.-B. Yuan, M.-S. Xiang, H.-W. Zhang, B.-Q. Chen, J.-J. Ren, C. Wang, Y. Zhang, Y.-H. Hou, Y.-F. Wang, Z.-H. Cao, *The Milky Way's rotation curve out to 100 kpc and its constraint on the Galactic mass distribution*, Monthly Notices of the Royal Astronomical Society, Volume 463, Issue 3, p.2623-2639, 12/2016; [arXiv:1604.01216v2](https://arxiv.org/abs/1604.01216v2)
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