

A CLOSING LEMMA FOR POLYNOMIAL AUTOMORPHISMS OF \mathbb{C}^2

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ABSTRACT. We prove that for a polynomial diffeomorphism of \mathbb{C}^2 , the support of any invariant measure, apart from a few obvious cases, is contained in the closure of the set of saddle periodic points.

1. INTRODUCTION AND RESULTS

Let f be a polynomial diffeomorphism of \mathbb{C}^2 with non-trivial dynamics. This hypothesis can be expressed in a variety of ways, for instance it is equivalent to the positivity of topological entropy. The dynamics of such transformations has attracted a lot of attention in the past few decades (the reader can consult e.g. [B] for basic facts and references).

In this paper we make the standing assumption that f is dissipative, i.e. that the (constant) Jacobian of f satisfies $|\text{Jac}(f)| < 1$.

We classically denote by J^+ the forward Julia set, which can be characterized as usual in terms of normal families, or by saying that $J^+ = \partial K^+$, where K^+ is the set of points with bounded forward orbits. Reasoning analogously for backward iteration gives the backward Julia set $J^- = \partial K^-$. Thus the 2-sided Julia set is naturally defined by $J = J^+ \cap J^-$. Another interesting dynamically defined subset is the closure J^* of the set of saddle periodic points (which is also the support of the unique entropy maximizing measure [BLS]).

The inclusion $J^* \subset J$ is obvious. It is a major open question in this area of research whether the converse inclusion holds. Partial answers have been given in [BS1, BS3, D, LP, GP].

Let ν be an ergodic f -invariant probability measure. If ν is hyperbolic, that is, its two Lyapunov exponents¹ are non-zero and of opposite sign, then the so-called Katok closing lemma [K] implies that $\text{Supp}(\nu) \subset J^*$. It may also be the case that ν is supported in the Fatou set: then from the classification of recurrent Fatou components in [BS2], this happens if and only if ν is supported on an attracting or semi-Siegel periodic orbit, or is the Haar measure on a cycle of k circles along which f^k is conjugate to an irrational rotation (recall that f is assumed dissipative). Here by semi-Siegel periodic orbit, we mean a linearizable periodic orbit with one attracting and one irrationally indifferent multipliers.

The following “ergodic closing lemma” is the main result of this note:

Theorem 1.1. *Let f be a dissipative polynomial diffeomorphism of \mathbb{C}^2 with non-trivial dynamics, and ν be any invariant measure supported on J . Then $\text{Supp}(\nu)$ is contained in J^* .*

A consequence is that if $J \setminus J^*$ happens to be non-empty, then the dynamics on $J \setminus J^*$ is “transient” in a measure-theoretic sense. Indeed, if $x \in J$, we can form an invariant

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¹Recall that in holomorphic dynamics, Lyapunov exponents always have even multiplicity.

probability measure by taking a cluster limit of $\frac{1}{n} \sum_{k=0}^n \delta_{f^k(x)}$ and the theorem says that any such invariant measure will be concentrated on J^* . More generally the same argument implies:

Corollary 1.2. *Under the assumptions of the theorem, if $x \in J^+$, then $\omega(x) \cap J^* \neq \emptyset$.*

Here as usual $\omega(x)$ denotes the ω -limit set of x . Note that for $x \in J^+$ then it is obvious that $\omega(x) \subset J$. It would be interesting to know whether the conclusion of the corollary can be replaced by the sharper one: $\omega(x) \subset J^*$.

Theorem 1.1 can be formulated slightly more precisely as follows.

Theorem 1.3. *Let f be a dissipative polynomial diffeomorphism of \mathbb{C}^2 with non-trivial dynamics, and ν be any ergodic invariant probability measure. Then one of the following situations holds:*

- (i) *either ν is atomic and supported on an attracting or semi-Siegel cycle;*
- (ii) *or ν is the Haar measure on an invariant cycle of circles contained in a periodic rotation domain;*
- (iii) *or $\text{Supp}(\nu) \subset J^*$.*

Note that the additional ergodicity assumption on ν is harmless since any invariant measure is an integral of ergodic ones. The only new ingredient with respect to Theorem 1.1 is the fact that measures supported on periodic orbits that do not fall in case (i), that is, are either semi-parabolic or semi-Cremer, are supported on J^* . For semi-parabolic points this is certainly known to the experts although apparently not available in print. For semi-Cremer points this follows from the hedgehog construction of Firsova, Lyubich, Radu and Tanase (see [LRT]). For completeness we give complete proofs below.

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2. PROOFS

In this section we prove Theorem 1.3 by dealing separately with the atomic and the non-atomic case. Theorem 1.1 follows immediately. Recall that f denotes a dissipative polynomial diffeomorphism with non trivial dynamics and ν an f -invariant ergodic probability measure.

2.1. Preliminaries. Using the theory of laminar currents, it was shown in [BLS] that any saddle periodic point belongs to J^* . More generally, if p and q are saddle points, then $J^* = \overline{W^u(p) \cap W^u(q)}$ (see Theorems 9.6 and 9.9 in [BLS]). This result was generalized in [DL] as follows. If p is any saddle point and $X \subset W^u(p)$, we respectively denote by $\text{Int}_i X$, $\text{cl}_i X$, $\partial_i X$ the interior, closure and boundary of X relative to the intrinsic topology of $W^u(p)$, that is the topology induced by the biholomorphism $W^u(p) \simeq \mathbb{C}$.

Lemma 2.1 ([DL, Lemma 5.1]). *Let p be a saddle periodic point. Relative to the intrinsic topology in $W^u(p)$, $\partial_i(W^u(p) \cap K^+)$ is contained in the closure of the set of transverse homoclinic intersections. In particular $\partial_i(W^u(p) \cap K^+) \subset J^*$.*

Here is another statement along the same lines, which can easily be extracted from [BLS].

Lemma 2.2. *Let $\psi : \mathbb{C} \rightarrow \mathbb{C}^2$ be an entire curve such that $\psi(\mathbb{C}) \subset K^+$. Then for any saddle point p , $\psi(\mathbb{C})$ admits transverse intersections with $W^u(p)$.*

Proof. This is identical to the first half of the proof of [DL, Lemma 5.4]. □

We will repeatedly use the following alternative which follows from the combination of the two previous lemmas. Recall that a Fatou disk is a holomorphic disk along which the iterates $(f^n)_{n \geq 0}$ form a normal family.

Lemma 2.3. *Let \mathcal{E} be an entire curve contained in K^+ , p be any saddle point, and t be a transverse intersection point between \mathcal{E} and $W^u(p)$. Then either $t \in J^*$ or there is a Fatou disk $\Delta \subset W^u(p)$ containing t .*

Proof. Indeed, either $t \in \partial_i(W^u(p) \cap K^+)$ so by Lemma 2.1, $t \in J^*$, or $t \in \text{Int}_i(W^u(p) \cap K^+)$. In the latter case, pick any open disk $\Delta \subset \text{Int}_i(W^u(p) \cap K^+)$ containing t . Since Δ is contained in K^+ , its forward iterates remain bounded so it is a Fatou disk. □

2.2. The atomic case. Here we prove Theorem 1.3 when ν is atomic. By ergodicity, this implies that ν is concentrated on a single periodic orbit. Replacing f by an iterate we may assume that it is concentrated on a fixed point. Since f is dissipative there must be an attracting eigenvalue. A first possibility is that this fixed point is attracting or semi-Siegel. Then we are in case (i) and there is nothing to say. Otherwise p is semi-parabolic or semi-Cremer and we must show that $p \in J^*$. In both cases, p admits a strong stable manifold $W^{ss}(p)$ associated to the contracting eigenvalue, which is biholomorphic to \mathbb{C} by a theorem of Poincaré. Let q be a saddle periodic point and t be a point of transverse intersection between $W^{ss}(p)$ and $W^u(q)$. If $t \in J^*$, then since $f^n(t)$ converges to p as $n \rightarrow \infty$ we are done. Otherwise there is a non-trivial Fatou disk Δ transverse to $W^{ss}(p)$ at t . Let us show that this is contradictory.

In the semi-parabolic case, this is classical. A short argument goes as follows (compare [U, Prop. 7.2]). Replace f by an iterate so that the neutral eigenvalue is equal to 1. Since f has no curve of fixed points there are local coordinates (x, y) near p in which $p = (0, 0)$, $W_{\text{loc}}^{ss}(p)$ is the y -axis $\{x = 0\}$ and f takes the form

$$(x, y) \mapsto (x + x^{k+1} + h.o.t., by + h.o.t.) ,$$

with $|b| < 1$ (see [U, §6]). Then f^n is of the form

$$(x, y) \mapsto (x + nx^{k+1} + h.o.t., b^n y + h.o.t.) ,$$

so we see that f^n cannot be normal along any disk transverse to the y axis and we are done.

In the semi-Cremer case we rely on the hedgehog theory of [FLRT, LRT]. Let $\phi : \mathbb{D} \rightarrow \Delta$ be any parameterization, and fix local coordinates (x, y) as before in which $p = (0, 0)$, $W_{\text{loc}}^{ss}(p)$ is the y -axis and f takes the form

$$(x, y) \mapsto (e^{i2\pi\theta}x, by) + h.o.t.$$

Let B be a small neighborhood of the origin in which the hedgehog is well-defined. Reducing Δ and iterating a few times if necessary, we can assume that for all $k \geq 0$, $f^k(\Delta) \subset B$ and ϕ is of the form $s \mapsto (s, \phi_2(s))$. Then the first coordinate of $f^n \circ \phi$ is of the form $s \mapsto e^{i2n\pi\theta}s + h.o.t.$. If $(n_j)_{j \geq 0}$ is a subsequence such that $f^{n_j} \circ \phi$ converges to some $\psi = (\psi_1, \psi_2)$, we get that $\psi_1(s) = \alpha s + h.o.t.$, where $|\alpha| = 1$. Thus $\psi(\mathbb{D}) = \lim f^{n_j}(\Delta)$ is a non-trivial holomorphic disk Γ through 0 that is smooth at the origin.

For every $k \in \mathbb{Z}$ we have that $f^k(\Gamma) = \lim f^{n_j+k}(\Delta) \subset B$. Therefore by the local uniqueness of hedgehogs (see [LRT, Thm 2.2]) Γ is contained in \mathcal{H} . It follows that \mathcal{H} has non-empty relative interior in any local center manifold of p and from [LRT, Cor. D.1] we infer that p is semi-Siegel, which is the desired contradiction.

2.3. The non-atomic case. Assume now that ν is non-atomic. If ν gives positive mass to the Fatou set, then by ergodicity it must give full mass to a cycle of recurrent Fatou components. These were classified in [BS2, §5]: they are either attracting basins or rotation domains. Since ν is non-atomic we must be in the second situation. Replacing f by f^k we may assume that we are in a fixed Fatou component Ω . Then Ω retracts onto some Riemann surface S which is biholomorphic to a disk or an annulus and on which the dynamics is that of an irrational rotation. Furthermore all orbits in Ω converge to S . Thus ν must give full mass to S , and since S is foliated by invariant circles, by ergodicity ν gives full mass to a single circle. Finally the unique ergodicity of irrational rotations implies that ν is the Haar measure.

Therefore we are left with the case where $\text{Supp}(\nu) \subset J$, that is, we must prove Theorem 1.1. Let us start by recalling some facts on the Oseledets-Pesin theory of our mappings. Since ν is ergodic by the Oseledets theorem there exists $1 \leq k \leq 2$, a set \mathcal{R} of full measure and for $x \in \mathcal{R}$ a measurable splitting of $T_x\mathbb{C}^2$, $T_x\mathbb{C}^2 = \bigoplus_{i=1}^k E_i(x)$ such that for $v \in E_i(x)$, $\lim_{n \rightarrow \infty} \frac{1}{n} \log \|df_x^n(v)\| = \chi_i$. Moreover, $\sum \chi_i = \log |\text{Jac}(f)| < 0$, and since ν is non-atomic both χ_i cannot be both negative (this is already part of Pesin's theory, see [BLS, Prop. 2.3]). Thus $k = 2$ and the exponents satisfy $\chi_1 < 0$ and $\chi_2 \geq 0$ (up to relabelling). Without loss of generality, we may further assume that points in \mathcal{R} satisfy the conclusion of the Birkhoff ergodic theorem for ν .

As observed in the introduction, the ergodic closing lemma is well-known when $\chi_2 > 0$ so we might only consider the case $\chi_2 = 0$ (our proof actually treats both cases simultaneously).

To ease notation, let us denote by $E^s(x)$ the stable Oseledets subspace and by χ^s the corresponding Lyapunov exponent ($\chi^s < 0$). The Pesin stable manifold theorem (see e.g. [FHY] for details) asserts that there exists a measurable set $\mathcal{R}' \subset \mathcal{R}$ of full measure, and a family of holomorphic disks $W_{\text{loc}}^s(x)$, tangent to $E^s(x)$ at x for $x \in \mathcal{R}'$, and such that $f(W_{\text{loc}}^s(x)) \subset W_{\text{loc}}^s(f(x))$. In addition for every $\varepsilon > 0$ there exists a set \mathcal{R}'_ε of measure $\nu(\mathcal{R}'_\varepsilon) \geq 1 - \varepsilon$ and constants r_ε and C_ε such that for $x \in \mathcal{R}'_\varepsilon$, $W_{\text{loc}}^s(x)$ contains a graph of slope at most 1 over a ball of radius r_ε in $E^s(x)$ and for $y \in W_{\text{loc}}^s(x)$, $d(f^n(y), f^n(x)) \leq C_\varepsilon \exp((\chi^s + \varepsilon)n)$ for every $n \geq 0$. Furthermore, local stable manifolds vary continuously on \mathcal{R}'_ε .

From this we can form global stable manifolds by declaring² that $W^s(x)$ is the increasing union of $f^{-n}(W_{\text{loc}}^s(f^n(x)))$. Then it is a well-known fact that $W^s(x)$ is a.s. biholomorphically equivalent to \mathbb{C} (see e.g. [BLS, Prop 2.6]). Indeed, almost every point visits \mathcal{R}'_ε infinitely many times, and from this we can view $W^s(x)$ as an increasing union of disks D_j such that the modulus of the annuli $D_{j+1} \setminus D_j$ is uniformly bounded from below. Discarding a set of zero measure if necessary, it is no loss of generality to assume that $\bigcup_{\varepsilon>0} \mathcal{R}'_\varepsilon = \mathcal{R}'$ and that for every $x \in \mathcal{R}'$, $W^s(x) \simeq \mathbb{C}$.

To prove the theorem we show that for every $\varepsilon > 0$, $\mathcal{R}'_\varepsilon \subset J^*$. Fix $x \in \mathcal{R}'_\varepsilon$ and a saddle point p . By Lemma 2.2 there is a transverse intersection t between $W^s(x)$ and $W^u(p)$. Since

²If ν has a zero exponent, this may not be the stable manifold of x in the usual sense, that is, there might exist points outside $W^s(x)$ whose orbit approach that of x .

x is recurrent and $d(f^n(x), f^n(t)) \rightarrow 0$, to prove that $x \in J^*$ it is enough to show that $t \in J^*$. We argue by contradiction so assume that this is not the case. Then by Lemma 2.3 there is a Fatou disk Δ through t inside $W^u(p)$. Reducing Δ a little if necessary we may assume that f^n is a normal family in some neighborhood of $\bar{\Delta}$ in $W^u(p)$.

Since ν is non-atomic and stable manifolds vary continuously for the C^1 topology on \mathcal{R}'_ε , there is a set A of positive measure such that if $y \in A$, $W^s(y)$ admits a transverse intersection with Δ . The iterates $f^n(\Delta)$ form a normal family and $f^n(\Delta)$ is exponentially close to $f^n(A)$. Let (n_j) be some subsequence such that $f^{n_j}|_\Delta$ converges. Then the limit map has either generic rank 0 or 1, that is if $\phi : \mathbb{D} \rightarrow \Delta$ is a parameterization, $f^{n_j} \circ \phi$ converges uniformly on \mathbb{D} to some limit map ψ , which is either constant or has generic rank 1. Set $\Gamma = \psi(\mathbb{D})$. Let ν' be a cluster value of the sequence of measures $(f^{n_j})_*(\nu|_A)$. Then ν' is a measure of mass $\nu(A)$, supported on $\bar{\Gamma}$ and $\nu' \leq \nu$. Since ν gives no mass to points, the rank 0 case is excluded so Γ is a (possibly singular) curve. Notice also that if z is an interior point of Δ (i.e. $z = \phi(\zeta)$ for some $\zeta \in \mathbb{D}$), then $\lim f^{n_j}(z) = \psi(\zeta)$ is an interior point of Γ . This shows that ν' gives full mass to Γ (i.e. it is not concentrated on its boundary). Then the proof of Theorem 1.1 is concluded by the following result of independent interest.

Proposition 2.4. *Let f be a dissipative polynomial diffeomorphism of \mathbb{C}^2 with non-trivial dynamics, and ν be an ergodic non-atomic invariant measure, giving positive measure to a subvariety. Then ν is the Haar measure on an invariant cycle of circles contained in a periodic rotation domain.*

In particular a non-atomic invariant measure supported on J gives no mass to subvarieties.

Proof. Let f and ν be as in the statement of the proposition, and Γ_0 be a subvariety such that $\nu(\Gamma_0) > 0$. Since ν gives no mass to the singular points of Γ_0 , by reducing Γ_0 a bit we may assume that Γ_0 is smooth. If M is an integer such that $1/M < \nu(\Gamma_0)$, by the pigeonhole principle there exists $0 \leq k \leq l \leq M$ such that $\nu(f^k(\Gamma_0) \cap f^l(\Gamma_0)) > 0$, so $f^k(\Gamma_0)$ and $f^l(\Gamma_0)$ intersect along a relatively open set. Thus replacing f by some iterate f^N (which does not change the Julia set) we can assume that $\Gamma_0 \cap f(\Gamma_0)$ is relatively open in Γ_0 and $f(\Gamma_0)$. Let now $\Gamma = \bigcup_{k \in \mathbb{Z}} f^k(\Gamma_0)$. This is an invariant, injectively immersed Riemann surface with $\nu(\Gamma) > 0$. Notice that replacing f by f^N may corrupt the ergodicity of ν so if needed we replace ν by a component of its ergodic decomposition (under f^N) giving positive (hence full) mass to Γ .

We claim that Γ is biholomorphic to a domain of the form $\{z \in \mathbb{C}, r < |z| < R\}$ for some $0 \leq r < R \leq \infty$, that $f|_{\Gamma_0}$ is conjugate to an irrational rotation, and ν is the Haar measure on an invariant circle. This is *a priori* not enough to conclude the proof since at this stage nothing prevents such an invariant “annulus” to be contained in J .

To prove the claim, note first that since Γ is non-compact, it is either biholomorphic to \mathbb{C} or \mathbb{C}^* , or it is a hyperbolic Riemann surface³. In addition Γ possesses an automorphism f with a non-atomic ergodic invariant measure. In the case of \mathbb{C} and \mathbb{C}^* all automorphisms are affine and the only possibility is that f is an irrational rotation. In the case of a hyperbolic Riemann surface, the list of possible dynamical systems is also well-known (see e.g. [M, Thm 5.2]) and again the only possibility is that f is conjugate to an irrational rotation on a disk or an annulus. The fact that ν is a Haar measure follows as before.

³In the situation of Theorem 1.1 we further know that $\Gamma \subset K$ so the first two cases are excluded.

Let γ be the circle supporting ν , and $\tilde{\Gamma} \subset \Gamma$ be a relatively compact invariant annulus containing Γ in its interior. To conclude the proof we must show that γ is contained in the Fatou set. This will result from the following lemma, which will be proven afterwards.

Lemma 2.5. *f admits a dominated splitting along $\tilde{\Gamma}$.*

See [S] for generalities on the notion of dominated splitting. In our setting, since Γ is an invariant complex submanifold and f is dissipative, the dominated splitting actually implies a normal hyperbolicity property. Indeed, observe first that $f|_{\tilde{\Gamma}}$ is an isometry for the Poincaré metric Poin_{Γ} of Γ , which is equivalent to the induced Riemannian metric on $\tilde{\Gamma}$. In particular $C^{-1} \leq \|df^n|_{T\tilde{\Gamma}}\| \leq C$ for some $C > 0$ independent of n . Therefore a dominated splitting for $f|_{\tilde{\Gamma}}$ means that there is a continuous splitting of $T\mathbb{C}^2$ along $\tilde{\Gamma}$, $T_x\mathbb{C}^2 = T_x\Gamma \oplus V_x$, and for every $x \in \tilde{\Gamma}$ and $n \geq 0$ we have $\|df_x^n|_{V_x}\| \leq C'\lambda^n$ for some $C' > 0$ and $\lambda < 1$. In other words, f is normally contracting along $\tilde{\Gamma}$. Thus in a neighborhood of γ , all orbits converge to Γ . This completes the proof of Proposition 2.4. \square

Proof of Lemma 2.5. By the cone criterion for dominated splitting (see [N, Thm 1.2] or [S, Prop. 3.2]) it is enough to prove that for every $x \in \Gamma$ there exists a cone \mathcal{C}_x about $T_x\Gamma$ in $T_x\mathbb{C}^2$ such that the field of cones $(\mathcal{C}_x)_{x \in \tilde{\Gamma}}$ is strictly contracted by the dynamics. For $x \in \Gamma$, choose a vector $e_x \in T_x\Gamma$ of unit norm relative to the Poincaré metric Poin_{Γ} and pick f_x orthogonal to e_x in $T_x\mathbb{C}^2$ and such that $\det(e_x, f_x) = 1$. Since $\text{Poin}_{\Gamma}|_{\tilde{\Gamma}}$ is equivalent to the metric induced by the ambient Riemannian metric, there exists a constant C such that for all $x \in \tilde{\Gamma}$, $C^{-1} \leq \|e_x\| \leq C$. Thus, the basis (e_x, f_x) differs from an orthonormal basis by bounded multiplicative constants, i.e. there exists $C^{-1} \leq \alpha(x) \leq C$ such that $(\alpha(x)e_x, \alpha^{-1}(x)f_x)$ is orthonormal.

Let us work in the frame $\{(e_x, f_x), x \in \Gamma\}$. Since $df|_{\Gamma}$ is an isometry for the Poincaré metric and $f(\Gamma) = \Gamma$, the matrix expression of df_x in this frame is of the form

$$\begin{pmatrix} e^{i\theta(x)} & a(x) \\ 0 & e^{-i\theta(x)}J \end{pmatrix},$$

where J is the (constant) Jacobian. Fix λ such that $|J| < \lambda < 1$, and for $\varepsilon > 0$, let $\mathcal{C}_x^\varepsilon \subset T_x\mathbb{C}^2$ be the cone defined by

$$\mathcal{C}_x^\varepsilon = \{ue_x + vf_x, |v| \leq \varepsilon|u|\}.$$

Let also $A = \sup_{x \in \tilde{\Gamma}} |a(x)|$. Working in coordinates, if $(u, v) \in \mathcal{C}_x^\varepsilon$ then

$$df_x(u, v) =: (u_1, v_1) = (e^{i\theta(x)}u + a(x)v, e^{-i\theta(x)}Jv),$$

hence

$$|u_1| \geq |u| - A|v| \geq |u|(1 - A\varepsilon) \text{ and } |v_1| = |Jv| \leq \varepsilon|J||u|$$

We see that if ε is so small that $|J| < \lambda(1 - A\varepsilon)$, then for every $x \in \tilde{\Gamma}$ we have that $|v_1| \leq \lambda\varepsilon|u_1|$, that is, $df_x(\mathcal{C}_x^\varepsilon) \subset \mathcal{C}_{f(x)}^{\lambda\varepsilon}$. The proof is complete. \square

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