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COMBINING SHORT AND LONG TERM RESERVOIR OPERATION USING INFINITE HORIZON MODEL PREDICTIVE CONTROL

Luciano Raso¹, Pierre-Olivier Malaterre ²

ABSTRACT

Model Predictive Control (MPC) can be employed for optimal operation of adjustable hydraulic structures. MPC selects the control to apply to the system by solving in real-time an optimal control problem over a finite horizon. The finiteness of the horizon is both the reason of MPC's success and its main limitation. MPC has been in fact successfully employed for short-term reservoir management. Short-term reservoir management deals effectively with fast processes, such as floods, but it is not capable of looking sufficiently ahead to handle long-term issues, such as drought.

We propose an Infinite Horizon MPC solution, tailored for reservoir management, where input signal is structured by use of basis functions. Basis functions reduce the optimization argument to a small number of variables, making the control problem solvable in a reasonable time. We tested this solution on a test case adapted from Manantali Reservoir, on the Senegal River. The long-term horizon offered by IH-MPC is necessary to deal with the strongly seasonal climate of the region for both flood and drought prevention.

Keywords: Model Predictive Control, Reservoir Operation, Infinite Horizon, Manantali

INTRODUCTION

Optimal reservoir operation can be framed as a control problem (Soncini-Sessa et al. 2007), which, for reservoir operation, has been typically solved using methods from the

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23 dynamic programming family. Stochastic Dynamic Programming (SDP) (Stedinger et al.
24 1984; Trezos and Yeh 1987) solve a control problem that is markov. SDP, however, suffers
25 from the so-called “curse of dimensionality,, (Bellman and Dreyfus 1966) and “curse of
26 modelling,, (Tsitsiklis and Van Roy 1996; Bertsekas and Tsitsiklis 1995). The curse of
27 dimensionality limits SDP application to simple systems, made of few variables. Curse of
28 modeling implies the demand of modeling the inflow to the reservoir as a stochastic dynamic
29 system.

30 Model Predictive Control (MPC) is a real-time control technique (Morari et al. 1999;
31 Mayne et al. 2000) suffering neither the curse of dimensionality nor the curse of modelling,
32 as intended for SDP. MPC has been extensively applied on water systems (van Overloop
33 2006), mostly for canals (Malaterre et al. 1998; Malaterre and Rodellar 1997; van Overloop
34 et al. 2014; Horváth et al. 2014), river delta (Dekens et al. 2014; Tian et al. 2015b), also
35 considering quality (Xu et al. 2013; Xu et al. 2010), transport of water and over water (Tian
36 et al. 2013; Tian et al. 2015a) and reservoir operation (Raso et al. 2014b; Schwanenberg
37 et al. 2015; Ficchi et al. 2015; Galelli et al. 2012; Schwanenberg et al. 2014).

38 In reservoir operation, MPC proved to be effective for short-term objectives, such as
39 flood prevention. Short term objectives, however, must be balanced with long-term ones, as
40 drought prevention, among others. MPC, in fact, finds a control action optimal for a finite
41 horizon, but large reservoirs have often a slow dynamic, and effect of control actions are
42 mutually interdependent on a long period. In this case, classic MPC can be employed for
43 short-term optimal control method, but it does not ensure long-term optimality, as effects
44 after the optimization horizon are not included.

45 Methods to integrate the long-term effects within the MPC optimal control problem refer
46 to as Infinite Horizon MPC (Maciejowski 2002). Among these, a suitable approach is input
47 structuring by use of basis function (Wang 2001). We propose here an innovative use of
48 input structuring for Infinite Horizon MPC applied to reservoir operation, and we tested
49 some triangular basis functions. This work extends and generalizes some initial results

50 initially presented in (Raso et al. 2014a). Constraints on inputs can be easily included. We
 51 show an application on a test case adapted from Manantali Reservoir, on the Senegal River.

52 METHOD

53 Consider a water system composed of N_x reservoirs that is operated by N_u discharge
 54 decisions. Discharge decisions are diversions from rivers and releases from reservoirs. A
 55 reservoir may have multiple releases (by different structures or for different users). The
 56 system is influenced by N_d streamflows.

57 We start from framing the reservoir operation problem in control terms. Problem (1)
 58 define the optimal control problem for a reservoir system.

$$\min_{\{\pi_t\}_{t=1}^H} \sum_{t=1}^{H+1} \mathbb{E}_{\mathbf{d}_t} \left[g_t(\mathbf{x}_t, \mathbf{u}_t, \mathbf{d}_t) \right] \quad (1a)$$

Subject to:

$$\mathbf{x}_t = \mathbf{x}_{t-1} + \Delta t \cdot (I \cdot [\mathbf{u}_t, \mathbf{d}_t] - O \cdot [\mathbf{u}_t, \mathbf{d}_t]) \quad (1b)$$

$$0 \leq \mathbf{u}_t \leq \mathbf{u}_{max} \quad (1c)$$

$$\mathbf{x}_{min} \leq \mathbf{x}_t \leq \mathbf{x}_{max} \quad (1d)$$

$$\mathbf{c}_t(\mathbf{x}_t, \mathbf{u}_t, \mathbf{d}_t) \leq 0 \quad (1e)$$

$$\mathbf{d}_t \in \mathbf{D}_t \quad (1f)$$

$$\mathbf{x}_{t=0} \text{ given} \quad (1g)$$

59 In problem (1), vectors $\mathbf{x}_t \in \mathbb{R}^{N_x}$, $\mathbf{u}_t \in \mathbb{R}^{N_u}$, $\mathbf{d}_t \in \mathbb{R}^{N_d}$, represent reservoir volumes,
 60 discharge decisions, stochastic streamflow scenarios at instant t for stocks and in the period
 61 $[t - \Delta t, t)$ for flows; $g_t(\cdot)$ is a \mathbb{R}^N to \mathbb{R} function, representing the system step-cost function
 62 at t , and $N = N_x + N_u + N_d$; $\mathbb{E}[\cdot]$ is the average operator. In some cases different criteria
 63 other than the average may be used, such as the max operator. In Expression (1a), π_t
 64 is the release policy, which gives the optimal release decision in function of to the system

65 state, such that $\mathbf{u}_t^* = \pi_t(\mathbf{x}_t)$. Equation (1b) is the continuity equation, represented by the
66 reservoirs mass balance, where Δt is the time-step length, I and O are the input and output
67 matrix, of dimension $N_x \times (N_u + N_d)$, associating at each scenario and discharge decision
68 to its reservoir. $O(i, j)$ and $I(i, j)$ is 1 if the i variable is input or output of reservoir j , 0
69 elsewhere. Hydrological inflow are hydrological scenarios extracted from \mathbf{D}_t , as in Expression
70 (1f), where \mathbf{D}_t is a stochastic variable representing all possible future discharge scenarios.
71 In Inequality (1e), \mathbf{c}_t defines other constraints that apply to the system, such as physical
72 constraints, or other legal or environmental requirements treated as constraints. For example,
73 discharge decision can be limited by water availability within the reservoir. H is the length
74 of simulation, or closed-loop, horizon, on which the system is tested.

75 Solving problem (1) is finding the control strategy π_t , be either a function mapping ob-
76 served state to optimal control, or a tree of decisions according to the observed discharge
77 (Shapiro and Andrzej 2003). Different methodologies try to tackle the optimal release policy
78 identification problem for reservoir operation. Simulation-based methods (Sulis and Sechi
79 2013), known in the operational research community as policy function approximations (Pow-
80 ell and Meisel 2015), are often used by analysts having their main expertise in hydrology,
81 where the class of functions, or set of rules, is defined a priori, and some parameters are
82 adjusted according to simulation results. Apart from simulation-based rules, methods from
83 the dynamic programming family, typically Stochastic Dynamic Programming (SDP) (Ste-
84 dinger et al. 1984), has been extensively employed to solve Problem (1) by taking advantage
85 of its markov structure. SDP, however, suffers from the curse of dimensionality and the curse
86 of modelling: the SDP functional optimization is particularly complex to solve numerically,
87 therefore application are limited to systems made of few variables, and state transitions must
88 be defined explicitly, requiring a stochastic representation of the inflow process. Stochas-
89 tic Dual Dynamic Programming (Pereira and Pinto 1991; Tilmant et al. 2008) attenuates
90 the curse of dimensionality (Shapiro 2011), and Sampling Stochastic dynamic Program-
91 ming (Kelman et al. 1990; Faber and Stedinger 2001) tackles the curse of modelling, but

92 no methods from the dynamic programming family overcomes effectively both limitations.
 93 Evolutionary algorithms for reservoir operation are methods for non-linear optimization used
 94 to optimise some parameters that define the release policies (Nicklow et al. 2009; Reed et al.
 95 2013), but their application to large systems has been little tested.

96 Model Predictive Control (MPC) is an alternative control method to tackle Problem (1).
 97 In MPC, at each control instant t , the control actions are obtained by solving on-line, i.e.
 98 at each control time-step, the following optimal control problem.

$$\min_U \left[\sum_{k=1}^h g_k(\mathbf{x}_k, \mathbf{u}_k, \mathbf{d}_k) + g_{h+1}(\mathbf{x}_h) \right] \quad (2a)$$

Subject to:

$$\mathbf{x}_k = \mathbf{x}_{k-1} + \Delta t \cdot \left(I \cdot [\mathbf{u}_k, \mathbf{d}_k] - O \cdot [\mathbf{u}_k, \mathbf{d}_k] \right) \quad (2b)$$

$$0 \leq \mathbf{u}_k \leq \mathbf{u}_{\max} \quad (2c)$$

$$\mathbf{x}_{\min} \leq \mathbf{x}_k \leq \mathbf{x}_{\max} \quad (2d)$$

$$\mathbf{c}_k(\mathbf{x}_k, \mathbf{u}_k, \mathbf{d}_k) \leq 0 \quad (2e)$$

$$\mathbf{x}_{k=0} = \mathbf{x}_t, \{\mathbf{d}_k\}_{k=1}^h \text{ given} \quad (2f)$$

99 In Equations (2), $U = \{\mathbf{u}_k\}_{k=1}^h$, where k is the time index, going from 1 to the final
 100 time-step of the optimization, or open-loop, horizon, $h \ll H$; g_h the final penalty that sums
 101 up all the future costs beyond the control horizon.

102 MPC uses the system model in Equations (2b-2e) to predict the system behavior in
 103 response to the control actions over a finite future horizon. The model takes the current state
 104 of the system as initial state, and a deterministic forecasts of disturbances as uncontrolled
 105 input, as in Equations (2f). Once system model, cost function, initial state and forecasted
 106 disturbance are given, MPC solves problem (2) and finds the optimal control trajectory for
 107 the future prediction horizon. At each time-step, only the first value of the optimal control

108 trajectory is applied to the real system, i.e. $\mathbf{u}_t^* = \mathbf{u}_{k=1}$; then the horizon is shifted ahead
109 and the procedure is repeated at the next controlling instant using the latest up-to-date
110 information.

111 MPC is an on-line, or “real-time”, technique, meaning that Problem (2) is solved contem-
112 poraneously to the system operation. At every control instant, MPC uses the most up-to-date
113 system state and disturbance forecast. In MPC, a control policy $\mathbf{u}_t^* = \pi(\mathbf{x}_t, \{\mathbf{d}_k\}_{k=1}^h)$ is found
114 on-line by solving a deterministic optimization problem as defined in Equations (2), which is
115 much easier to solve than its stochastic equivalent. MPC is not affected by the limitations of
116 SDP, and it can be applied to much larger systems, using discharge forecasts not influenced
117 by release decisions as input to MPC.

118 Robustness to uncertainty is a key question in MPC research literature (Morari et al.
119 1999). At each decision instant, MPC uses the most up-to-date information. This feedback
120 mechanism due to the continuous system update gives to MPC a form of ‘inherent robust-
121 ness’ (Mayne et al. 2000), which may be sufficient to produce satisfactory results in the face
122 of the present uncertainty. If this is not the case, synthetic robust MPC (Bemporad and
123 Morari 1999) methods can augment the system robustness to the desired level, generally at
124 the cost of additional computational complexity (Muñoz de la Peña et al. 2005).

125 In MPC, the cost-to-go function g_h should theoretically sum up all the costs, from instant
126 h to infinite, for having left the system in x_h at the end of the control horizon. In practice,
127 however, this function is difficult to obtain. If g_t is a Lyapunov function, and the control
128 horizon is sufficiently long, MPC ensures stability (Maciejowski 2002), even without g_h . An
129 example of Lyapunov function widely used in MPC for trajectory following problems is a
130 quadratic penalty on the state deviance from the optimal trajectory (van Overloop 2006; Xu
131 et al. 2010; Negenborn et al. 2009; van Overloop et al. 2008). This property is extensively
132 used in MPC applications for canal control, where the objective is trajectory tracking, but
133 reservoir objectives are rarely well represented by Lyapunov functions.

134 An alternative way to guarantee stability is adding a constraint on the final state (De Nicolao

135 et al. 1996). However, this solution requires the identification of a desired final state, which
136 can be unknown. This is often the case in reservoir operation, where MPC applications use
137 historical final penalties (Ficchi et al. 2015) which does not guarantee optimality. Moreover,
138 if the horizon is too short, this MPC configuration runs the risk of having an infeasible
139 problem.

140 Infinite horizon MPC is a family of solutions dealing with the finiteness of the optimiza-
141 tion horizon. Within this family, input structuring (Wang 2001) seems to be particularly
142 suited for reservoir operation. In input structuring, the control are not optimized directly,
143 but they are arranged according to a convenient form. Among the different forms of input
144 structuring we selected basis functions. Heuberger et al. (2005) offers a clear and accurate
145 description of basis functions and their use for system identification.

146 Equation (3) shows input structuring using basis function.

$$147 \quad \mathbf{u}_k = \sum_{i=1}^N \boldsymbol{\lambda}_i \cdot f_i(k) \quad (3)$$

148 where $f_i(k)$ are fixed time-variant functions and $\boldsymbol{\lambda}_i \in \mathbb{R}^{N_u}$ are their weights, selected with
149 an optimization procedure.

150 Basis functions can represent a smooth signal using few parameters λ_i , being therefore
151 a potential appropriate approach for input structuring in reservoir operation. Reservoirs,
152 in fact, filter out the high frequency variability of inflow. Consequently, the control signal
153 (i.e. the releases) varies slowly too. Moreover, periodic basis functions can follow the yearly
154 periodicity of natural systems.

155 If input structuring is to be used in optimization, optimizing $\boldsymbol{\lambda}_i$ instead of \mathbf{u}_k reduces
156 the degrees of freedom from $h \times N_u$ to $N \times N_u$. In a rolling horizon optimization problem,
157 reducing the degrees of freedom allows the use of a much larger h , i.e. extending the control
158 horizon without having an explosive growth in computational complexity. Input structuring
159 reduces the computational complexity related to the horizon length, not affecting or being
160 affected by other sources of complexity related to the system size or the number of objectives.

161 For this reason the proposed methodology is applicable to large systems in the same way as
 162 MPC when applied for short-term operation.

163 Using input structuring in MPC, it is particularly important that the optimal control
 164 sequence is well represented in proximity of the first control value, which will be eventually
 165 applied to the system. Therefore, basis functions must be selected such that the control
 166 signal at the initial part of the horizon is regulated by a larger degree of freedom, i.e. a
 167 larger number of bases. Control values far ahead in the horizon have less influence on the
 168 first control value and can be represented by relatively less basis. Influences of control at
 169 instant k on first control value shades as k get larger with no clear boundary. Selection of
 170 basis functions shapes must follow this regression.

171 Basis functions have been extensively used for system identification (Van Den Hof et al.
 172 1995; Van den Hof and Ninness 2005; Heuberger et al. 1995). However, in MPC, constraints
 173 on \mathbf{u}_k imply constraint on $\boldsymbol{\lambda}_i$. In Equations (4) we define the infinite horizon MPC problem
 174 with input structuring by basis functions.

$$\min_{\Lambda} \sum_{k=1}^{h-1} e^{-r \cdot k} \left[\cdot g_k(\mathbf{x}_k, \mathbf{u}_k, \mathbf{d}_k) + c_k(\mathbf{x}_k) \right] \quad (4a)$$

Subject to:

$$\mathbf{x}_k = \mathbf{x}_{k-1} + \Delta t \cdot (I \cdot [\mathbf{u}_k, \mathbf{d}_k] - O \cdot [\mathbf{u}_k, \mathbf{d}_k]) \quad (4b)$$

$$U = M \cdot \Lambda \quad (4c)$$

$$0 \leq M \cdot \Lambda \leq \mathbf{u}_{\max} \quad (4d)$$

$$\mathbf{x}_{k=0} = \mathbf{x}_t, \{\mathbf{d}_k\}_{t=1}^h \text{ given} \quad (4e)$$

175 In Equation (4), $\Lambda = [\boldsymbol{\lambda}_1], \dots, \boldsymbol{\lambda}_N]$, r is the discount rate, M is a $N \times h$ vector defined
 176 by the basis functions, such that $M(k, i) = f_i(k)$. Note that Equation (4c) is a linear
 177 transformation, implying that the problem stays linear in Λ , no matter whether the basis

178 functions are linear or not.

179 State constraints, as in inequalities (1d) are integrated as soft constraints, such that

$$180 \quad c_k(\mathbf{x}_k) = \max\{0, w_x \cdot (\mathbf{x}_k - \mathbf{x}_{\max}), w_x \cdot (\mathbf{x}_{\min} - \mathbf{x}_k)\} \quad (5)$$

181 where $w_x \gg 0$.

182 Extending the long term far beyond the horizon where forecast are reliable requires the
 183 inclusion of climatic information and add large uncertainty (Zhao et al. 2011). Uncertainty
 184 that jeopardizes MPC robustness can be dealt with method for synthetic robust methods,
 185 such as Multiple MPC (van Overloop et al. 2008), Tree-Based MPC (Raso et al. 2014b;
 186 Maestre et al. 2012a; Maestre et al. 2012b) or others (Bemporad and Morari 1999; Muñoz
 187 de la Peña et al. 2005; Muñoz de la Peña 2005).

188 **Triangular basis function and triangles selection**

189 We use triangular basis function because of their simplicity to be communicated and to
 190 be defined from few parameters. Equations (6) define a generic triangular basis function i .

$$191 \quad f_i(k) = \begin{cases} 1 - \frac{T_i+k}{L_i} & \text{if } T_i + L_i < k \leq T_i \\ 1 + \frac{T_i+k}{R_i} & \text{if } T_i < k \leq T_i + R_i \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

192 Each triangle i is determined by its peak instant, T_i , its left base, L_i , and its right base
 193 R_i . Figure 1 shows a graphical visualization of the triangles and their parameters. An
 194 alternative family of basis function could be a combination of exponential functions with
 195 different decay rate, and a sum of sines and cosines with difference frequency.

196 Basis function accuracy must be progressive going ahead on time, this progression de-
 197 pending on the system characteristics. We give here some general indications for triangular
 198 functions, highlighting the advantages of some specific shapes.

199 We suggest selecting progressive triangles, i.e. $L_i < R_i, L_i + 1 > L_i$, in the early stage of the

200 horizon. In MPC in fact, only the first control value will be applied to the systems. The
201 first control is more sensitive to controls that are closer in time; therefore it is better to have
202 a higher degree of freedom in the initial part of the horizon. The first triangle should have
203 its peak T at the initial time-step.

204 Sufficiently far from present condition, periodicity becomes dominating. For $t > P$, where
205 P is the system periodicity, triangles having L_i and R_i equal to $P/2$ are able to follow the
206 periodic trend. In this part of the horizon, T should be equal to $P \times j$ and multiple of
207 $P \times (j + 1/2)$, where j is an integer going from zero to the number of years contained in the
208 control horizon. Selection of independent triangles, such that $L_{i+1} = R_i$, and $T_{i+1} = T_i + L_i$,
209 makes constraints independent.

210 TEST CASE

211 The method is tested on a system adapted from Manantali reservoir case. Manantali
212 is located in Mali, on the Senegal River, presently used mainly for electricity production.
213 Plans for agro-business on the Senegal River valley could change the management in the
214 short future (Fraval et al. 2002). In this case, the objective of energy production must be
215 balanced with flood and drought prevention. The hydrology on the Senegal River is strongly
216 seasonal, influenced by the tropical rainy season in the upper basin.

217 The reservoir is modeled by the continuity equation as in Equation (4b). The system
218 disturbance is the uncontrolled inflow, d_t , which is the observed discharge at Soukoutali.
219 The system controls are the release through the turbines, u_t^r , and the release through the
220 spillages, u_t^s . Controls are constrained between zero and maximum release through turbines,
221 u_{\max}^r , and maximum release through spillages, u_{\max}^s . The operational volume is constrained
222 between x_{\min} and x_{\max} . In this experiment, the operational volume is reduced to increase
223 the difficulty of the reservoir operation. Evaporation from the reservoir and other losses are
224 neglected.

225 The hydrological input d_t uses both real-time forecast and climatic information, gliding
226 from the real-time information into the climatic one going ahead on time: d_t is the Bayesian

227 Model Averaging (Raftery et al. 2005) of the forecasted inflow, d^{fr} , and the climatic one,
228 d^{cl} , weighted by their reliability.

$$229 \quad d_k = B_k \cdot d_{t+k} + (1 - B_k) \cdot D \quad (7)$$

230 Where B_t , representing the forecast reliability, is the product of the inflow autocorre-
231 lation lag 1 ϕ_τ , from t to $t + k$, such that $B_k = \prod_t^{t+k} \phi_\tau$. This is equivalent to use a
232 Periodic Autoregressive lag 1 model (Bartolini et al. 1988) as forecast model. Using of an
233 average climatic year as climatic disturbance would filter out extremes; we use instead an ob-
234 served inflow at each control time-step, randomly extracted from the observed inflow data,
235 $\{d^{cl}\}_{k=1}^h \in D^{cl} = \{d^{obs}\}_{\tau=k}^{k+h}$. When the reservoir residence time is large enough, its slow
236 dynamic will serve as low pass filter, which will average out the effects of different inflow
237 scenarios used at each time-step. This is expected to have little effect on the reservoir vol-
238 ume signal. In this experiment, we consider reservoir management having three objectives:
239 flood and drought prevention, and energy production. Flood and drought prevention are
240 represented by the cost function, gt^{tg} , in Equation (8b): keeping the total discharge as close
241 as possible to the target flow, $q_{tg} = 200m^3/s$, attains both flood and drought prevention.
242 The electricity production is proportional to the product of hydraulic head into discharge
243 through the turbines, $\propto \Delta h_t \cdot u_t^r$. This Equation is a convex function that must be maxi-
244 mized. We cannot use this function directly as objective within the optimization problem
245 because we use a convex optimization method, namely the interior-point method, which does
246 not guarantee, in this case, the convergence to the global optimum (Boyd and Vandenberghe
247 2004). The Objective function for energy production will be, instead, Equation (8a), which
248 is the function for energy production linearized as in (Raso et al. 2015). In Equation (8a),
249 $\Delta h_0 = 52.5 m$ is the nominal hydraulic head, $u_0^r = 500 m^3/s$ the nominal release trough
250 turbines, and $A_0 = 4.6e8 m^2$ the nominal reservoir surface. The negative sign means that
251 its value must be maximized.

$$g_t^e = -(\Delta h_0 \cdot u_t^r + u_0^r/A_0 \cdot v_t) \quad (8a)$$

$$g_t^{tg} = (u_t^r + u_t^s - q_{tg})^2 \quad (8b)$$

$$g_t = w_e \cdot g_t^e + w_{tg} \cdot g_t^{tg} \quad (8c)$$

252 The aggregated objective function g_t , in Equation (8c), is the weighted sum of g_t^e and
253 g_t^{tg} . Flood and drought prevention objectives have higher priority on energy production,
254 therefore w_{tg} is larger than w_e , being 0.8 and 0.2, respectively; the decaying factor r in
255 Equation (4a) is set to 0.973, selected to be close to zero at the end of the 3 year horizon.
256 The reservoir average residence time is in fact about one year. The system state, at the end
257 of the 3 year optimization horizon, contains a negligible trace of the initial system state.
258 The final state, having little influence on the first release decision, can be weighted much
259 less in the optimization. Other values may be tested to analyse the results sensitivity to
260 this parameter. We use 10 independent triangles, defined by T_i , S_i , and L_i as in Table
261 1. Triangles are selected so that the resulting composition has a higher degree of freedom,
262 therefore a higher accuracy, at beginning of the control horizon. In this case we selected
263 five asymmetric triangles with increasing left and right base length as the peak time T gets
264 larger. Other symmetric triangles with a larger base length are used to catch the system
265 periodicity.

266 Results

267 To evaluate the proposed method, we analyze both the role of input structuring and that
268 of uncertainty, isolating their effects in departing from the optimal solution. We consider
269 three solutions: i) Infinite Horizon MPC using triangular input structuring and realistic fore-
270 cast, ii) Infinite Horizon MPC using triangular input structuring and perfect forecast, iii)
271 Infinite Horizon MPC with no input structuring and perfect forecast. Comparing first and
272 second case shows the loss due to uncertainty; comparing second and third shows the loss

273 due to input structuring. In the third case, solving the optimal control problem requires a
274 large computation time, and it is not applicable in reality. This experiment serves, however,
275 as upper boundary of system performance. We use some indicators to measure performance:
276 i) Average yearly energy production, for electricity production, ii) days per year when flow is
277 lower than $100 \text{ m}^3/\text{s}$, for drought prevention, iii) days per year when flow is larger than 800
278 m^3/s , for flood prevention, and iv) the quadratic distance from the target discharge, as used
279 within the objective function, for both drought and flood prevention. The first indicator is
280 to be maximized, the others to be reduced. We run a four-year simulation, from the 1st
281 January 2005 to the 31st December 2008.

282
283 Table 2 presents a summary of simulation results for the three cases under evaluation, for
284 the considered indicators. This table shows that both uncertainty and input structuring leads
285 to a reduction of system performance. Performance loss is relatively small if compared to
286 the loss due to the presence of a relevant uncertainty for energy production, and comparable
287 for flood and drought prevention indicator. Simulation using input structuring and realistic
288 forecast, if compared to simulation using input structuring and perfect forecast, shows a
289 small deterioration on drought prevention, which is a slow, predictable process. On the
290 other hand energy production, which is a combination of short-long term goals decreases
291 moderately. Flood prevention, being the effect of a faster and less predictable process, shows
292 a major worsening. Results from simulation using structured and un-structured inputs, both
293 using perfect forecast, are nearly equivalent, suggesting that input structuring can be applied
294 with little effect on results. The performance loss can be reduced by increasing the number
295 of basis function (i.e. triangles), even if this will lead to an increase of computational time.
296 Using the interior point method in a Matlab[®] optimizer, on a processor 2,9 GHz Intel Core
297 i7, the computation time required to find a solution was 12-20 seconds for the case using
298 input structuring, and about 4 hours for the case without input structuring. The latter is
299 patently unacceptable for practical application.

300 Figure 3 shows discharge decisions and reservoir volume for the first year of closed-loop
301 simulation. Discharge decision on simulation using real forecast is noisy: decision is influ-
302 enced by the random extraction of a future discharge scenario. Discharge increases in early
303 august, as precautionary measure, in anticipation to an high flow which eventually does not
304 occurs. The reservoir filter out the high frequency variability of release decisions and inflow.
305 Reservoir volume on simulation using real forecast is, on the rising part, lower than volume
306 on simulation using perfect forecast, using less efficiently the reservoir capacity. Figure 3
307 shows the presence of few small violations on volume constraint, due to the implementation
308 of volume constraints as soft ones. In this system, in fact, constraints on the reservoir volume
309 represent a legal, rather than a physical condition, therefore small violations are acceptable.

310 Figure 4 shows open-loop optimization results at a specific decision instant: plot (a)
311 shows the input and output discharges, and plot (b) the resulting reservoir volume. For
312 both plots we show the nominal inflow, for which the release decisions are optimised, and
313 the observed inflow, that will actually happen, for the first year of open-loop simulation.
314 The controller tries to balance the hydrological variability to keep the total discharge as
315 close as possible to the target discharge. In the dry season the outflow is higher than the
316 inflow, and the reservoir is drawn down, keeping a low water volume until the rising part of
317 the hydrograph, in preparation of the peak. The reservoir is eventually filled, and spillages
318 are minimized. The plot below shows state constraints violation, at around $t = 150$ and
319 $t = 220$. These constraints violation are small, being about 1% of the reservoir volume, and
320 sufficiently far in time from the initial release decision. Their influence on the latter is very
321 likely to be minimal, and therefore they do not affect the control quality. If this is not the
322 case, weight w_x in Expression (5) can be increased. For $w_x \rightarrow \infty$, in fact, the soft constraint
323 $c_k(\mathbf{x}_k)$ “approaches, the behavior of a hard constraint.

324 Effects of input structuring are evident in plot (a), where a single triangle take into
325 account the entire high flow period. Plot (b) shows a large divergence between the effects on
326 reservoir volume of observed and nominal discharge, which adds evidence that robustness to

327 uncertainty is a relevant issue for the proposed method.

328 **CONCLUSIONS**

329 This paper presented an Infinite Horizon Model Predictive Control method specifically
330 designed for reservoir operations. Input structuring can be employed thanks to smoothness
331 of the control signal. The control smoothness is related to the slow dynamic of reservoir
332 systems: the reservoir filter out the high variability of inflow, therefore the control signal
333 (i.e. the releases) varies slowly too. Basis functions, often employed in system identification,
334 were used here for control. Input structuring reduces the computational complexity related
335 to the horizon length, and not to other sources of complexity, such as the system size or
336 the number of objectives. For this reason the proposed methodology is applicable to large
337 systems as MPC when applied for short-term operation.

338 We selected triangular basis function for their simplicity to be communicated and defined.
339 Triangular basis function can handle hypercube constraints on inputs, and we gave some
340 indication on how to select these triangles. Alternative families of basis function that could
341 have been employed are, among others, a combination of sines and cosines with different
342 frequencies, or a combination of exponential functions with different decay rates. We leave
343 to further research the exploration of effective basis functions.

344 We suggested selecting progressive independent triangles in the early stage, and periodic
345 ahead on time. In water systems, in fact, both water demand and hydrological processes
346 are periodic. The proposed method largely reduces the number of variables to be optimized,
347 reducing the optimization problem complexity. We tested the proposed method for the
348 operational management of Manantali reservoir, on the Senegal River, with the objective of
349 flood and drought prevention, and energy production. Analysis shows that input structuring
350 may have a negative effect on the system performance, mostly related to fast, uncertain
351 processes. The extent of performance loss depends on which indicator is considered, being
352 small or, for some indicator, equivalent to the performance loss due to the presence of inflow
353 uncertainty.

354 Selecting the proper number of basis functions is the result of a trade-off between system
355 performance and computation time. A larger number of triangles would increase both the
356 computation time and the performance. The latter, however, will saturate. Further research
357 could explore how performance and computation time change in function of number of basis
358 functions.

359 The question on how to deal with forecast uncertainty is still open in Infinite Horizon
360 MPC using input structuring. We suggest using the proposed method in combination with
361 a compatible synthetic robust MPC algorithm, selected from the vast control literature on
362 the topic. Notwithstanding this limitation, the method we propose can potentially handle
363 large systems, made of multiple reservoirs or routing downstream of the reservoir, offering
364 an optimal compromise between short and long term objectives.

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506 **List of Tables**

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TABLE 1. Triangular function specification: Peak time (T), Left base (L), and right side (R) defining the 10 triangles.

i	1	2	3	4	5	6	7	8	9	10
T	1	7	18	41	87	178	269	360	543	726
L	0	6	11	23	46	91	91	91	182	182
R	6	11	23	46	91	91	91	182	182	182

TABLE 2. Results for the analyzed configurations

Indicator \uparrow max / \downarrow min [<i>Unit</i>]	Electricity production \uparrow [$\times 10e5 MWh/year$]	Drought prevention \downarrow [<i>d/year</i>]	Flood prevention \downarrow [<i>d/year</i>]	Quadratic cost (Flood and Drought) \downarrow [$(m^3/s)^2$] $\times 10e7$
Basis functions, real forecast	9.1	65	7	6.3
Basis function, perfect forecast	9.5	75	0	5.5
No structuring, perfect forecast	9.6	71	0	4.3

510 **List of Figures**

511 1 **Example of basis triangular functions. For triangle 3, the peak time**
512 **T , the left base L and the right base R are highlighted. 27**
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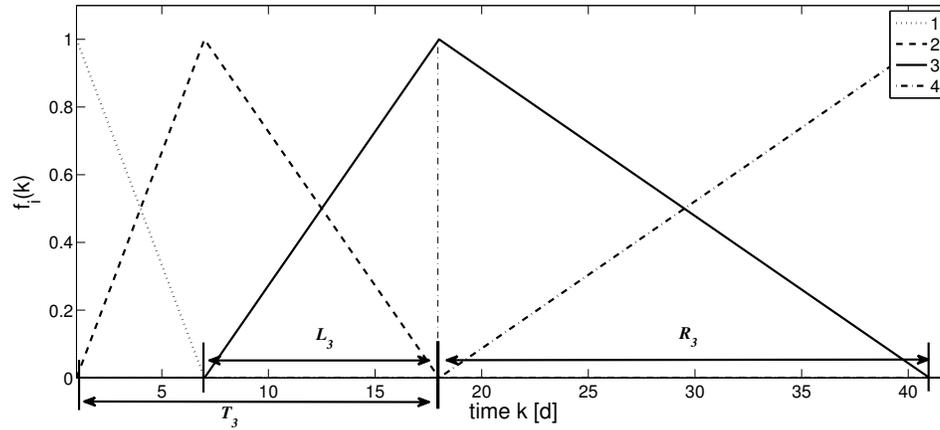


FIG. 1. Example of basis triangular functions. For triangle 3, the peak time T , the left base L and the right base R are highlighted.

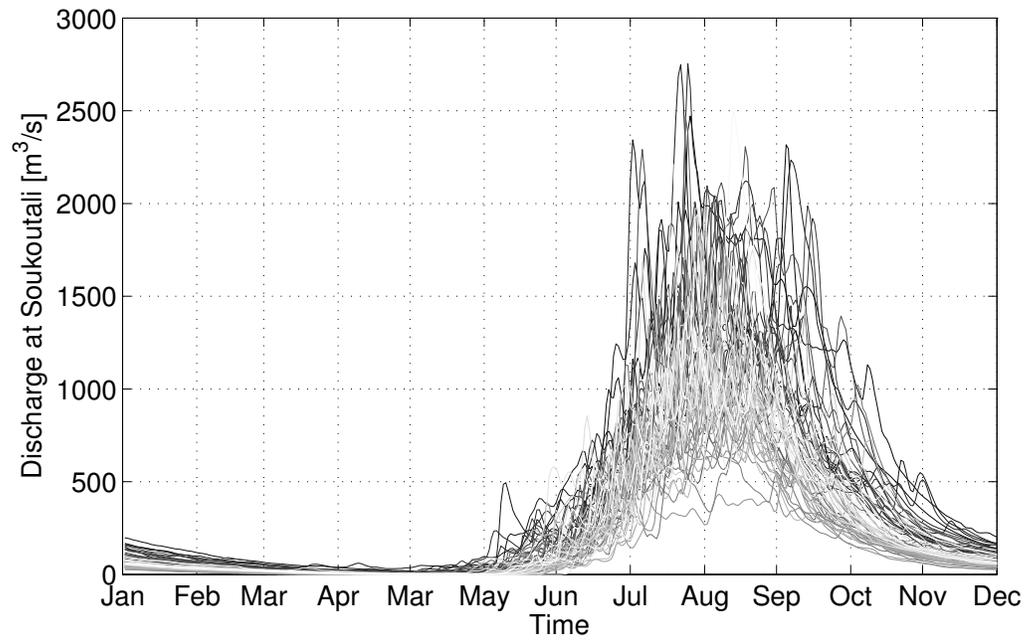
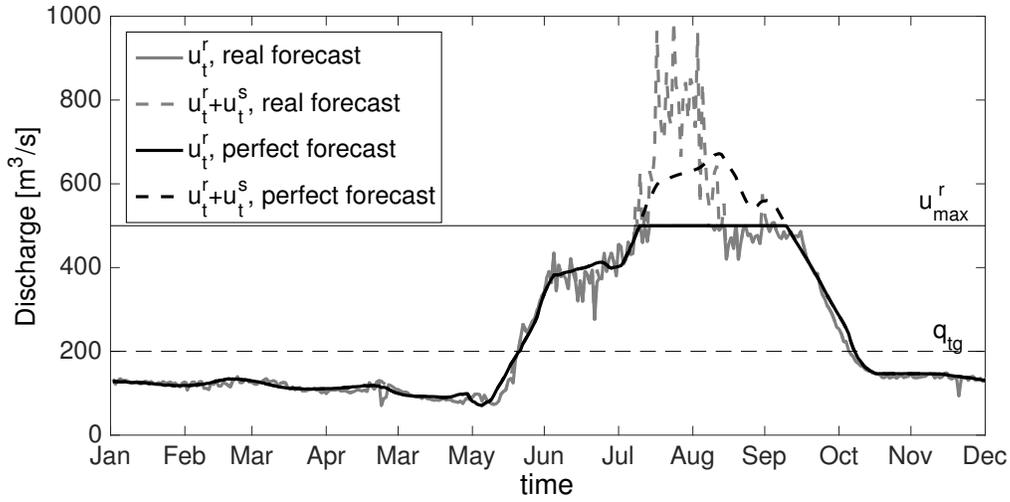
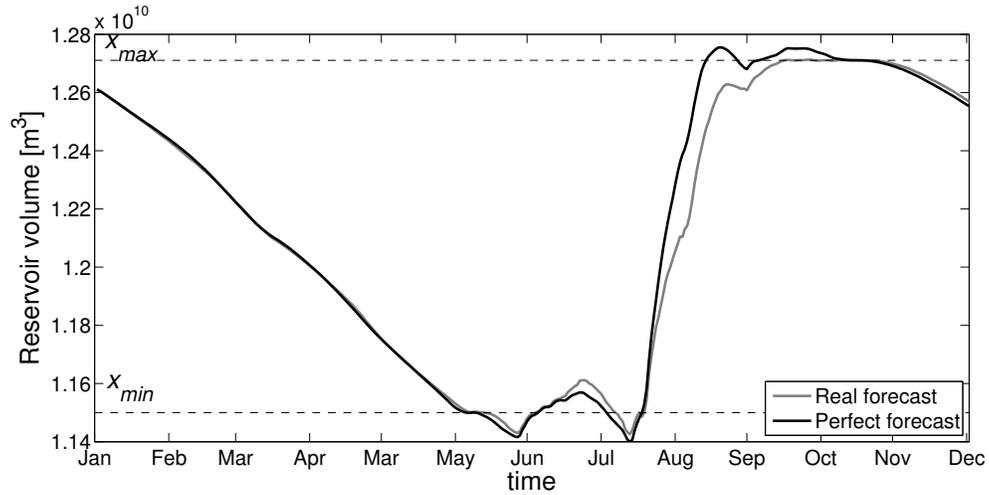


FIG. 2. Inflow at Soukoutali, from 1 January 1950 to 31 December 2013, daily time-step.



(a) Release



(b) Volume

FIG. 3. Simulation results, from 1 January 2005 to 31 December 2005

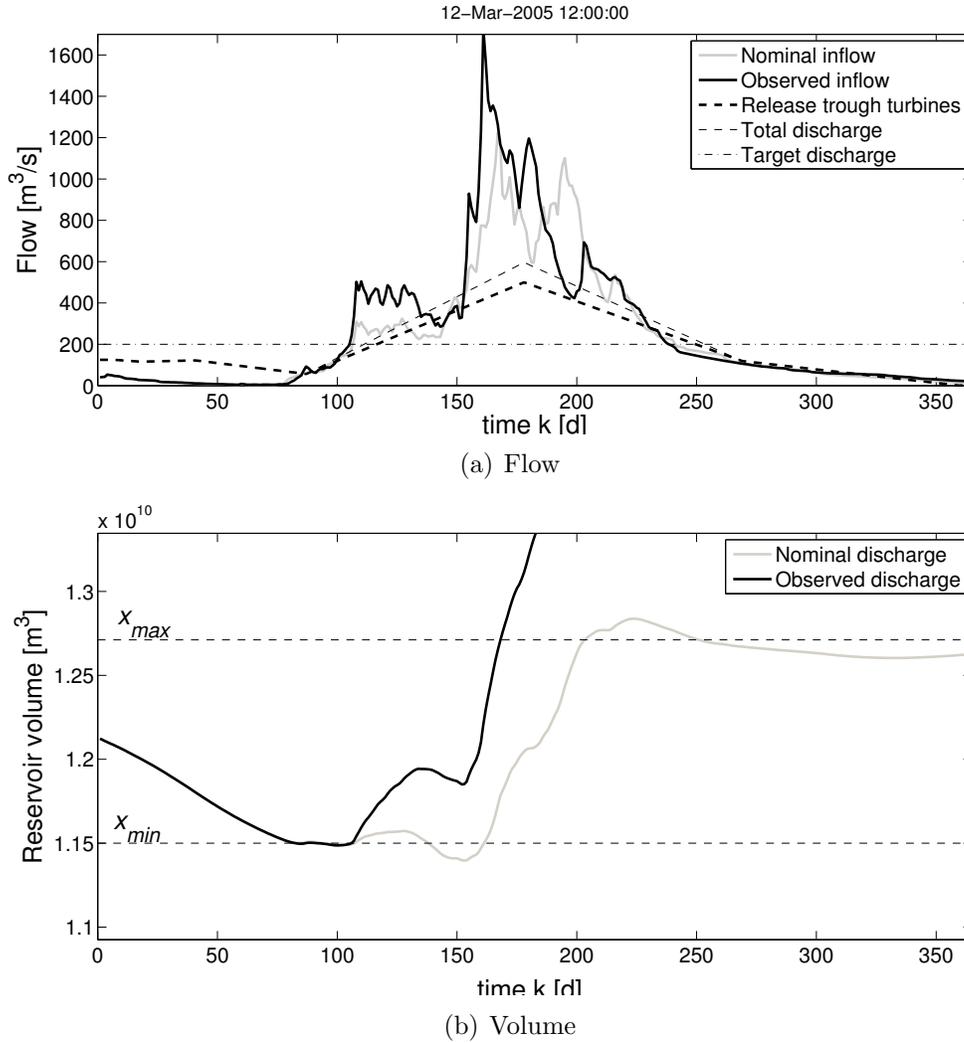


FIG. 4. Open loop results at $t = 12$ March 2005, for k from 1 to 365