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The Numerical Coupling of a Two-Fluid Model with an Homogeneous Relaxation Model

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ABSTRACT. We introduce here some possible way to deal with the unsteady interfacial coupling of two distinct two-phase flow codes. The left code relies on the standard two-fluid approach, and the right code provides approximations of solutions of the Homogeneous Relaxation Model (HRM). The basic idea of the coupling method is to introduce a father model, which corresponds to the two-fluid two-pressure approach, and to define convective fluxes through the interface, once a prolongation of initial conditions on both sides of the coupling interface has been performed. A sketch of the algorithm is provided. More details are given in the paper [HER 06]. A few numerical results show a rather good behaviour of the coupling approach.

KEYWORDS: interfacial coupling, unsteady two-phase flows, finite volumes

1. Introduction

Different two-phase flow models are used by the nuclear community, in order to investigate various situations in the cooling circuit of nuclear power plants. Among these, we should at least mention the standard six-equation two-fluid model (noted STFM afterwards) and the four-equation homogeneous relaxation model (noted HRM in the following). The latter HRM is mainly used for nuclear reactor core applications, but also for standard computations of flow patterns in steam generators, with THYC, FLICA or GENEPI software. The former STFM is actually the keystone of the CATHARE code for instance, which stands for the reference code for French nuclear authorities, especially for simulations of the whole coolant circuit (see [GUE 07] for a more detailed presentation of the whole framework). Some situations may require the interfacial coupling of these codes, either for steady or unsteady numerical experiments. This has motivated the development of new methods and algorithms to cope with these coupled cases. The present work aims at providing a sketch of the whole coupling strategy which has been proposed for such a purpose.

Some recent work on the interfacial coupling of hyperbolic systems of conservation laws has emerged quite recently. Some references ([AMB 08a], etc.) can be found for instance on the website [AMB 03], which basically focus on the interfacial state coupling (see [GOD 96] among others), or alternatively on the flux coupling techniques. Roughly speaking, the former favours the continuity of the conservative variable, whereas the latter aims at ensuring the intrinsic conservation laws. The reader is for instance referred to [GOD 08], which provides an extensive description of recent work on that topic. The method that is proposed in this paper relies on the use of a coupling model, from which both models on each side of the coupling interface may be at least formally recovered. We first briefly present both models to be coupled through the interface in section 2. Then we show which coupling model is used in the present situation, and we provide a sketch of the coupling algorithm in the one-dimensional framework. Since all models involved here are invariant under frame rotation, the extension to the 2D or 3D case is achieved in a straightforward manner by using a projection along the normal to the coupling interface. A few numerical results are eventually presented in a 2D framework. More useful details on the whole approach can be found in [HER 06]. The reference [HER 07a] also provides some recent ongoing work on the interfacial coupling of existing codes. Since the method should apply for pre-existing codes, we do not detail the algorithms that are used in sole codes, and focus on the treatment of boundary conditions through the coupling interface in sections 3 and 4.

2. Models to be coupled

2.1. The left standard two-fluid model: STFM

The derivation of the standard two-fluid approach may be found in [ISH 75] for instance. The governing equations of the STFM are the following:

$$\left\{ \begin{array}{l} \frac{\partial m_k}{\partial t} + \frac{\partial m_k U_k}{\partial x} = 0 ; \\ \frac{\partial m_k U_k}{\partial t} + \frac{\partial m_k U_k^2}{\partial x} + \alpha_k \frac{\partial P}{\partial x} = I_k(W) ; \\ \frac{\partial \alpha_k E_k}{\partial t} + \frac{\partial \alpha_k U_k (E_k + P)}{\partial x} + P_I^* \frac{\partial \alpha_k}{\partial t} = V_I^* I_k(W) ; \end{array} \right. \quad [1]$$

where the state variable is: $W_{STFM}^t = (m_1, m_2, m_1 U_1, m_2 U_2, \alpha_1 E_1, \alpha_2 E_2)$. Here, the void fractions $\alpha_k \in [0, 1]$ comply with $\alpha_1 + \alpha_2 = 1$. Moreover, U_k , ρ_k , $m_k = \alpha_k \rho_k$ respectively stand for the velocity, density and partial mass within phase k . Index 1 refers to the vapour phase, while the water phase is labeled by index 2. The equilibrium pressure P is the same for both phases. A closure law for the interfacial momentum transfer term I_k , which takes drag effects into account, must be given. We thus introduce the classical contribution:

$$I_k = (-1)^k \frac{m_1 m_2}{\tau_U (m_1 + m_2)} (U_1 - U_2) \quad [2]$$

where τ_U is the velocity relaxation time, which can be deduced from classical formulas of drag coefficients. A closure law relates P_I^* to W_{STFM} , we consider here the rough closure: $P_I^* = P$, but other forms such as $P_I^* = P + \beta(W)(m_1 + m_2)(U_2 - U_1)^2$ might be considered. The interface velocity V_I^* is usually taken as the velocity of the dilute phase (which corresponds to the vapour phase in nuclear applications). The total energy within phase k equals $E_k = \rho_k U_k^2 / 2 + \rho_k e_k(P, \rho_k)$. An equation of state (EOS) is of course needed, which prescribes the internal energy $e_k(P, \rho_k)$ within each phase. Hence the system is complete. We recall that the STFM is not hyperbolic, unless $U_1 = U_2$, when using the closure $P_I^* = P$.

2.2. The right homogeneous relaxation model: HRM

The HRM is probably the most widespread model in nuclear two-phase thermohydraulics. The derivation of the model is straightforward when starting from the two-fluid approach ([ISH 75]). If we assume a vanishing relative velocity $U_r = U_2 - U_1 = 0$, the governing equations of the main variable $W_{HRM}^t = (\rho, \rho C, \rho U, E)$ that describes the mixture are the following:

$$\begin{cases} \frac{\partial \rho}{\partial t} + \frac{\partial \rho U}{\partial x} = 0 ; \\ \frac{\partial \rho C}{\partial t} + \frac{\partial \rho U C}{\partial x} = 0 ; \\ \frac{\partial \rho U}{\partial t} + \frac{\partial \rho U^2}{\partial x} + \frac{\partial P}{\partial x} = 0 ; \\ \frac{\partial E}{\partial t} + \frac{\partial U(E + P)}{\partial x} = 0 . \end{cases} \quad [3]$$

Of course, all interfacial transfer terms have vanished when summed up. Once more, we use classical notations, and ρ stands for the density of the mixture (in other words $\rho = m_1 + m_2 = \alpha_1 \rho_1 + \alpha_2 \rho_2$), while U, P, C respectively stand for the velocity of the mixture, the mean pressure and the vapour concentration (that is $U = (m_1 U_1 + m_2 U_2) / (m_1 + m_2)$, $P = \alpha_1 P_1 + \alpha_2 P_2$ and $C = m_1 / (m_1 + m_2)$). The total energy of the mixture is $E = \rho U^2 / 2 + \rho e(\rho, P, C)$, where the closure law for $e(\rho, P, C)$ is given by the right code. Other closure laws for the relative velocity U_r can be found in the literature. If we consider the closure law provided in [GUI 07] for instance, it must be emphasized that the latter does not modify the convective form of the homogeneous model [3]. System [3] is hyperbolic if:

$$(P/\rho - \rho \frac{\partial e}{\partial \rho} |_{\rho, C}) / (\frac{\partial e}{\partial P} |_{\rho, C}) > 0.$$

A classic entropy inequality holds here for the HRM.

3. The coupling model

The coupling model (or father model) enables us to retrieve the left STFM model by enforcing a pressure equilibrium, while it also leads to the right HRM model by

imposing a pressure-velocity-temperature equilibrium. The state variable of the coupling model is: $W^t = (\alpha_1, m_1, m_2, m_1 U_1, m_2 U_2, \alpha_1 E_1, \alpha_2 E_2)$. The void fractions $\alpha_k \in [0, 1]$ still comply with $\alpha_1 + \alpha_2 = 1$. Moreover, $U_k, P_k, \rho_k, m_k = \alpha_k \rho_k$ respectively stand for the velocity, pressure, density and partial mass within phase k . With some abuse of notation, we still note $E_k = \rho_k U_k^2 / 2 + \rho_k e_k(P_k, \rho_k)$ the total energy of phase k . We will thus focus herein on the following two-fluid model (see [BAE 86, KAP 97] for instance):

$$\left\{ \begin{array}{l} \frac{\partial \alpha_1}{\partial t} + V_I \frac{\partial \alpha_1}{\partial x} = \phi_1(W) ; \\ \frac{\partial m_k}{\partial t} + \frac{\partial m_k U_k}{\partial x} = 0 ; \\ \frac{\partial m_k U_k}{\partial t} + \frac{\partial m_k U_k^2}{\partial x} + \alpha_k \frac{\partial P_k}{\partial x} + (P_k - P_I) \frac{\partial \alpha_k}{\partial x} = I_k(W) ; \\ \frac{\partial \alpha_k E_k}{\partial t} + \frac{\partial \alpha_k U_k (E_k + P_k)}{\partial x} + P_I \frac{\partial \alpha_k}{\partial t} = V_I I_k(W) . \end{array} \right. \quad [4]$$

A closure law for the source terms ϕ_k must be added, and these should comply with: $\phi_1 + \phi_2 = 0$. We will prescribe: $\phi_k = (-1)^k \alpha_1 \alpha_2 (P_2 - P_1) / (\tau_P \Pi_0)$, where τ_P stands for the pressure relaxation time, while Π_0 refers to some pressure constant given by user. In practice, nuclear applications in pressurised water reactors obviously lead to the choice $(V_I, P_I) = (U_1, P_2)$, where the index 1 refers to the dilute (vapour) phase, while the index 2 refers to the water phase. With this definition of (V_I, P_I) , the father model [4] is hyperbolic; moreover, shock solutions are uniquely defined though the system contains some non-conservative contributions, and a meaningful entropy inequality governs the motion of smooth solutions (see [COQ 02, GAL 04]). In the following, we will note $Z = (\alpha_1, \rho_1, U_1, P_1, \rho_2, U_2, P_2)$. The entropy inequality which governs regular solutions of the seven-equation model is the exact counterpart of the entropy inequality associated with the standard two-fluid model. If we define the specific entropy $s_k(P_k, \rho_k)$ within each phase, which compels with: $c_k^2 \frac{\partial s_k}{\partial P_k} |_{\rho_k} + \frac{\partial s_k}{\partial \rho_k} |_{P_k} = 0$, where: $c_k^2 = (\frac{\partial e_k}{\partial P_k} |_{\rho_k})^{-1} (P_k / \rho_k^2 - \frac{\partial e_k}{\partial \rho_k} |_{P_k})$, we can introduce the entropy $\eta = m_1 s_1 + m_2 s_2$, and the entropy flux $f_\eta = m_1 U_1 s_1 + m_2 U_2 s_2$; thus, regular solutions of both STFM and coupling model obey the inequality:

$$\frac{\partial \eta}{\partial t} + \frac{\partial f_\eta}{\partial x} \geq 0.$$

4. Main ideas of the coupling algorithm

Boundary conditions are imposed on each side of the coupling interface that separates the STFM and the HRM finite volume codes, by solving the 1D Riemann problem associated with the LHS of [4] together with initial conditions Z_L, Z_R as defined below. Eventually, the interface state $Z^* = Z^{Riemann}(Z_L, Z_R, (x - x_{int}) / \Delta t = 0)$ is accounted for by both codes as a boundary condition through a flux prescription.

1) *Defining initial conditions on both sides of the coupling interface*

- Starting from the STFM-cell value on the left side of the interface:

$$Z_{STFM}^- = (\alpha_1^-, \rho_1^-, U_1^-, \rho_2^-, U_2^-, P^-)$$

we define Z_L on the left side of the coupling interface as follows:

$$Z_L = (\alpha_1^-, \rho_1^-, U_1^-, P_1^- = P^-, \rho_2^-, U_2^-, P_2^- = P^-)$$

- Starting from the HRM-cell value on the right side of the interface:

$$Z_{HRM}^+ = (\rho^+, \rho^+ C^+, \rho^+ U^+, E^+)$$

we define Z_R on the right side of the coupling interface as:

$$Z_R = (\alpha_1^+, \rho_1^+, U_1^+, P_1^+ = P^+, \rho_2^+, U_2^+, P_2^+ = P^+)$$

This is achieved by enforcing a pressure, velocity and temperature equilibrium between phases. This implies $U_1^+ = U_2^+ = U^+$, noting: $1/T_k = (\frac{\partial s_k}{\partial P_k}|_{\rho_k})/(\frac{\partial e_k}{\partial P_k}|_{\rho_k})$:

$$\begin{cases} P_1^+ = P_2^+ = P^+ \\ T_1(P_1^+, \rho_1^+) = T_2(P_2^+, \rho_2^+) = T^+ \\ \alpha_1^+ \rho_1^+ = \rho^+ C^+ \\ (1 - \alpha_1^+) \rho_2^+ = \rho^+ (1 - C^+) \\ \alpha_1^+ \rho_1^+ e_1(P_1^+, \rho_1^+) + (1 - \alpha_1^+) \rho_2^+ e_2(P_2^+, \rho_2^+) = (E - \rho U^2/2)^+ \end{cases} \quad [5]$$

If EOS are chosen as stiffened gas EOS within each phase: $(\gamma_k - 1)\rho_k e_k = P_k + \gamma_k P_{\infty,k}$, an explicit formula may be obtained. In that case, a direct elimination leads to a non-linear equation wrt P^+ , which admits a unique solution (see [HER 06]) in agreement with $0 \leq P^+ \leq P_{\infty,\phi}$. Hence, we obtain physically relevant values:

$$\begin{cases} T^+ = \frac{\alpha_1^+(P^+ + P_{\infty,1})}{\rho^+ C^+ C_{V,1}(\gamma_1 - 1)} \\ \alpha_1^+ = \frac{1}{1 + \psi(P^+)} \\ \psi(P^+) = \frac{(1 - C^+)(C_{V,2})(\gamma_2 - 1)(P^+ + P_{\infty,1})}{C^+(C_{V,1})(\gamma_1 - 1)(P^+ + P_{\infty,2})}. \end{cases} \quad [6]$$

The specific heat with constant volume has been noted $C_{V,k}$ within phase k .

2) *Solving the Riemann problem at the coupling interface*

The Riemann problem at the coupling interface associated with [4], Z_L and Z_R is solved using either the Rusanov scheme or an approximate Godunov scheme such as the one described in [GAL 04]. Once the intermediate state Z^* defined above is known, we may calculate the flux on the right side of the interface (HRM side) by summing up both flux components associated with the total mass and momentum, that is:

$$F_\rho^+ = (\alpha_1 \rho_1 U_1 + \alpha_2 \rho_2 U_2)^*$$

$$F_{\rho U}^+ = (\alpha_1(\rho_1 U_1^2 + P_1) + \alpha_2(\rho_2 U_2^2 + P_2))^*$$

and the counterpart corresponding to the total energy flux:

$$F_E^+ = (\alpha_1 U_1 (E_1 + P_1) + \alpha_2 U_2 (E_2 + P_2))^*.$$

The mass flow rate for the vapour is $F_{\rho C}^+ = (\alpha_1 \rho_1 U_1)^*$. The implementation is straightforward on the left (STFM) side of the coupling interface, since we only need to skip the first component corresponding to the void fraction in [4].

5. Numerical results

In this first academic test case, the one-dimensional computational domain contains 500 regular cells and the CFL number is $CFL = 0.5$. The coupling interface between the STFM domain (left side) and the HRM domain (right side) is located at $x = 0$. A Rusanov scheme has been used here to calculate convective fluxes through the coupling interface. The initial condition is such that: $P(x < x_0, t = 0) = 150 \times 10^5$, $P(x > x_0, t = 0) = 155 \times 10^5$, where $x_0 = -0.15$. The flow is at rest at the beginning of the computation ($U_1(x, t = 0) = U_2(x, t = 0) = 0$), and the void fraction is uniform ($\alpha_1(x, t = 0) = 0.995$). The pressure disequilibrium inside the STFM region generates a rarefaction wave which moves towards the right side. The grey-green line in Figure 1 indicates the result obtained with the coupled simulation, whereas the black-red line corresponds to the sole calculation using the STFM model on both sides of the coupling interface $x = 0$. We have plotted here the state variable of the HRM model (liquid mass fraction C , density ρ , velocity U , and pressure P) in order to obtain a relevant comparison. Obviously, we can notice a difference in the mass fraction profile, which is essentially due to the fact that no drift velocity is accounted for in the right HRM domain. Many other experiments can be found in [HER 06].

The second industrial test case corresponds to a rough representation of a transient flow in the core of the reactor, after a sudden increase of the released power through the fuel assembly. The central part of the core is governed by the HRM model, while the flow in the coolant circuit in the three inlet and outlet pipes is represented by the STFM model. The three coupling interfaces have been located inside the pipes, in order to minimize transverse perturbations (see [HER 07b]). Though the interfacial coupling procedure appears to be stable, the global analysis is difficult, and suffers from the absence of any reference solution. More details can be found in [HER 07a].

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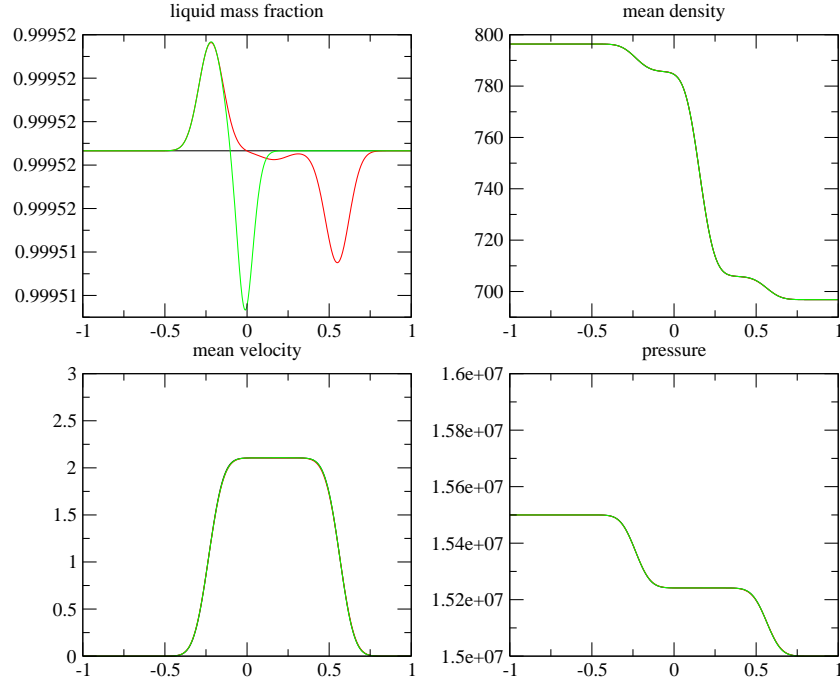


Figure 1. Simulation of waves passing through the coupling interface (at $x = 0$). The left (resp. right) code corresponds to the two-fluid (resp. HRM) model. The green line corresponds to the coupled simulation, the red one to the sole STFM computation.

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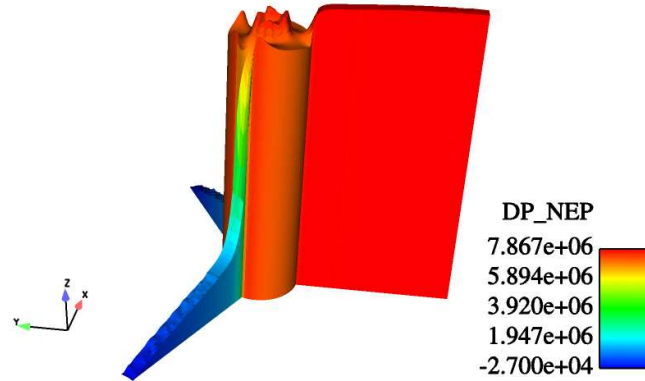


Figure 2. Pressure distribution (wrt a pressure reference $P_0 = 150.10^5 Pa$)

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