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ABSTRACT. *We present in this paper a two-fluid hyperbolic model in order to compute gas-particle flows that are highly loaded with particles. The two-phase flow model is an extension of the classical Baer-Nunziatto model. All closure laws for drag effects, mass and energy transfer terms are in agreement with the entropy inequality. The pressure relaxation terms slightly differ from their counterpart in the dilute case. A fractional step method which complies with the entropy inequality enables to perform computations of both the equilibrium model and the relaxed model, while accounting for the granular pressure. Special emphasis is given here on the pressure relaxation step, which guarantees that the volume fraction remains within its bounds. A few numerical results are presented on coarse and fine meshes. Other results are available in [CGH 07, Nus 07].*

KEYWORDS: *Granular Pressure, Two-phase flows, Finite Volumes, Relaxation schemes*

1. Introduction

We propose herein a two-phase flow model relying on the two-fluid approach for the prediction of gas-particle flows highly loaded with particles. The convective part of the model is hyperbolic with natural assumptions on the physical states. Thus the initial-value problem mimicking unsteady flows seems to be meaningful. Furthermore, the structure of waves is such that jump conditions through any travelling discontinuity are uniquely defined, though the system contains non-conservative terms. Actually, the relaxation process leads to distinct pressures within each phase, in order to account for the granular pressure $R = p_2 - p_1$. When $b_2 = 0$, the model is the well-known Baer-Nunziatto model ([BaN 86]).

We will present the model first, and recall its basic properties. Afterwards, we will give the main ingredients that are used to compute pressure relaxation effects. We will consider both cases where the pressure relaxation time τ_P is null or not. We nonetheless recall that the space of hyperbolic states is drastically reduced when $\tau_P = 0$. Therefore, we may expect that computations in the latter case will blow up when the mesh size tends to 0.

2. The two-phase flow model

The solid (respectively the gas) phase subscript is 2 (resp. 1). Within each phase $k = 1, 2$, we note ρ_k, u_k, e_k, P_k the partial density ρ_k, the phase velocity u_k, the internal energy e_k and the mean pressure P_k. $E_k = \rho_k e_k + \rho_k u_k^2/2$ denotes the total energy within phase k. The equation of state provides e_k as a given function of (P_k, ρ_k). Void fractions α_k comply with: $\alpha_1 + \alpha_2 = 1$. We also introduce $Z = (\alpha_2, \rho_k, u_k, P_k)$ in $\mathbf{R}^7$. The governing set of equations for Z is:

$$
\begin{aligned}
&\alpha_{2,t} + v_I \alpha_{2,x} = \epsilon_2 P,\\
&m_{k,t} + (m_k u_k)_x = \epsilon_k M,\\
&(m_k u_k)_t + (m_k u_k^2 + \alpha_k p_k)_x - p_I \alpha_{k,x} = \epsilon_k (Q_0 + v_I M),\\
&(m_k E_k)_t + ((m_k E_k + \alpha_k p_k) u_k)_x + p_I \alpha_{k,t} = \epsilon_k (S_0 + v_I Q_0 + (v_i^2/2 + \mu_2) M),
\end{aligned}
\tag{1}
$$

where $\epsilon_k = (-1)^{k+1}$. We will use here: $p_I = p_1$ and $v_I = u_2$, which is quite natural in the present situation (see [BaN 86], and [CGH 02] for other choices). Closure laws for source terms M, Q_0, P, S_0 will be detailed below. For that purpose, we need to introduce entropies s_k for each phase, that agree with the constraint formally written as:

$$
T_k ds_k = de_k - \frac{p_k}{\rho_k^2} d\rho_k + b_k d\alpha_k = T_k \left(\frac{\partial s_k}{\partial e_k}\right) de_k + T_k \left(\frac{\partial s_k}{\partial \rho_k}\right) d\rho_k + T_k \left(\frac{\partial s_k}{\partial \alpha_k}\right) d\alpha_k
\tag{2}
$$

We also define the chemical potential μ_k by :

$$\mu_k = e_k + \frac{p_k}{\rho_k} - T_k s_k \tag{3}$$

Closure laws for source terms M, Q_0, S_0, P will comply with:

$$\begin{aligned} M &= m_1 m_2 (\mu_2 - \mu_1 + (u_1 - u_2)^2/2)/(\tau_M (m_1 \mu_1 + m_2 \mu_2)) \\ Q_0 &= m_1 m_2 (u_2 - u_1)/(\tau_U (m_1 + m_2)) \\ S_0 &= m_1 m_2 (T_2 - T_1)/(\tau_T (m_1 + m_2)) \\ P &= \alpha_1 \alpha_2 (p_1 - (p_2 + m_2 b_2))/(\tau_P ((p_1 + p_2 + m_2 b_2))) \end{aligned} \tag{4}$$

assuming of course positive time scales $\tau_P, \tau_U, \tau_M, \tau_T$. Q_0, S_0 correspond to classic closure laws in the two-phase flow literature ; the interfacial mass transfer term M slightly differs from its counterpart for homogeneous flows, where the corrective term $(u_1 - u_2)^2/2$ is -implicitely- null. The Baer-Nunziatto model corresponds to $b_2 = 0$. In our case, b_2 is a given function of the unknowns P_2, ρ_2, α_2, and $b_1 = 0$.

3. Main properties

3.1. *Hyperbolicity*

We focus on the LHS of the governing equations (1). Introducing the unknown

$$Y = (\alpha_1, \rho_1, u_1, s_1, \rho_2, u_2, s_2)^T. \tag{5}$$

the system (1) may be rewritten

$$Y_t + B(Y) Y_x = 0, \tag{6}$$

with

$$B(Y) = \begin{bmatrix} u_2 & & & & & & & \\ \frac{\rho_1 (u_1 - u_2)}{\alpha_1} & u_1 & \rho_1 & & & & & \\ & \frac{c_1^2}{\rho_1} & u_1 & \frac{p_{1,s_1}}{\rho_1} & & & & \\ & & & u_1 & & & & \\ & & & & u_2 & \rho_2 & & \\ \frac{p_1 - p_2}{m_2} & & & & \frac{c_2^2}{\rho_2} & u_2 & \frac{p_{2,s_2}}{\rho_2} & \\ & & & & & & u_2 & \end{bmatrix} \tag{7}$$

where, for $k = 1, 2$, c_k is a given function of P_k, ρ_k, thanks to the EOS. Indeed, considering P_k as a given function of s_k, ρ_k, we have: $c_k = \frac{\partial p_k(\rho_k, s_k)}{\partial \rho_k}$.

Property : System (6) *is hyperbolic (all eigenvalues of* $B(Y)$ *are real). Its eigenvalues are :* $\lambda_{1,2} = u_2$, $\lambda_3 = u_1$, $\lambda_{4,5} = u_1 \pm c_1$, $\lambda_{6,7} = u_2 \pm c_2$. *It is not resonant unless when* $|u_1 - u_2| = c_k$. *In the latter case, the set of right eigenvectors no longer spans the whole space* $\mathbf{R}^7$.

3.2. *Jump conditions*

Property: The $1, 2, 3$ *fields are Linearly Degenerated. Other fields are Genuinely Non Linear, unless resonance occurs. Moreover, discontinuous solutions of system (1) are uniquely defined in single waves.*

Actually, non conservative terms $p_I \alpha_{k,x}$ and $p_I \alpha_{k,t}$ are null in GNL fields. The reader is refered to [CGH 02, GHS 04, CGH 07] for more details.

3.3. *Entropy inequality*

Property: Smooth solutions of system (1) satisfy the following equation:

$$\begin{aligned}&(\sum m_k s_k)_t + (\sum m_k u_k s_k)_x = \frac{P}{T_2}\left(p_1 - (p_2 + m_2 b_2)\right) + \\ &\frac{M}{T_1}\left(\frac{(u_1 - u_2)^2}{2} + \mu_2 - \mu_1\right) + \frac{Q_0}{T_1}(u_2 - u_1) + \frac{S_0}{T_1 T_2}(T_2 - T_1) \geq 0.\end{aligned} \tag{8}$$

The sign of the RHS of (8) is highly connected with the form of closure laws (4).

4. An entropy-consistent fractional step approach

4.1. *Overall algorithm*

The numerical method enables to simulate both the original seven-equation model (1), and its relaxed form wrt pressure (that is using the algebric law: $p_2 + m_2 b_2 - p_1 = 0$, instead of the first equation of (1)), which is here the standard Gough model [Gou 79]. To achieve this, we use a three-step splitting algorithm, which can summerized as:

– Step (I): First evolve the homogeneous seven-equation model (LHS of (1)), while getting rid of source terms ;

– Step (II): While freezing partial masses and momentum within each phase,

- (IIa) compute the pressure equilibrium that takes the granular stress into account ($p_2 + m_2 b_2 - p_1 = 0$), and update void fractions at the same time,

- *or*:

- (IIb) compute both pressures P_1 and P_2 and the void fraction α_2 by accounting for pressure relaxation terms when $\tau_P \neq 0$,

– Step (III): Compute other source terms including drag effects, interfacial mass and energy transfer terms Q_0, M, S_0.

The evolution step (I) is solved using Rusanov scheme or any approximate Godunov scheme (including VFRoe-ncv scheme, see [GHS 04] for instance). Many algorithms

may be proposed to compute step (III), and we refer to [HeH 06] for instance for more details. We detail below the computation of step (II). Step (IIa) will be retained when computing the Gough model, step (IIb) otherwise.

4.2. *Pressure relaxation step II*

The pressure relaxation step requires solving the following set of PDE:

$$\begin{aligned} &\alpha_{k,t} = (-1)^{k+1} P, \\ &m_{k,t} = Q_{k,t} = 0, \\ &(m_k e_k)_t + p_1 \alpha_{k,t} = 0. \end{aligned} \tag{9}$$

– Instantaneous pressure relaxation (IIa)
The volume fraction is updated in such a way that we recover the relation $p_2 + m_2 b_2 = p_1$ at the end of the time step. Using superscript 0 for physical values in a given cell at the end of the evolution step, and taking into account the fact that both the partial masses and the velocities are frozen in the step, we need to compute (α_1, p_1, p_2) solution of:

$$\begin{aligned} &m_k = m_k^0, \quad Q_k = Q_k^0, \\ &p_2 + m_2 b_2 - p_1 = 0, \\ &m_1 e_1 + m_2 e_2 = m_1^0 e_1^0 + m_2^0 e_2^0, \\ &(m_1 e_1 - m_1^0 e_1^0) + p_1(\alpha_1 - \alpha_1^0) = 0. \end{aligned} \tag{10}$$

In order to go further on, we will focus on the stiffened gas EOS recalled below:

$$(\gamma_k - 1)\rho_k e_k = p_k + \gamma_k \pi_k \tag{11}$$

where $\pi_k > 0$ and $\gamma_k > 1$ are given constants for $k = 1, 2$. We choose $\pi_2 > \pi_1$, which is in agreement with our framework . We also assume that $b_2 = -\rho_2^{\gamma_2 - 1}\theta(\alpha_2)$ (see [CGH 07]). Hence, we need to find (α_1, p_1, p_2) in each cell solution of:

$$\begin{aligned} &p_2 - \alpha_2 \rho_2^{\gamma_2} \theta(\alpha_2) - p_1 = 0, \\ &\alpha_2 \frac{p_2 + \pi_2}{\gamma_2 - 1} - \alpha_2^0 \frac{p_2^0 + \pi_2}{\gamma_2 - 1} + (p_1 + \pi_2)(\alpha_2 - \alpha_2^0) = 0, \\ &\alpha_1 \frac{p_1 + \pi_1}{\gamma_1 - 1} - \alpha_1^0 \frac{p_1^0 + \pi_1}{\gamma_1 - 1} + (p_1 + \pi_1)(\alpha_1 - \alpha_1^0) = 0. \end{aligned} \tag{12}$$

– Pressure relaxation (IIb)
A convenient way to compute this step is described in detail in [GHS 04]. Other methods are of course possible (see for instance [Gui 07]).

Property : Assume that a stiffened gas law holds for each phase. The existence and uniqueness of the volume fraction in $[0, 1]$ *is guaranteed by step (IIa) when discretized following* (10)*. A similar result holds for step (IIb) when using the method described in [GHS 04]. In the latter case, more general EOS may be used.*

5. Numerical application

We consider here a simple 1D test case without granular stress (taken from [CEG 97]) in order to demonstrate the non-hyperbolic behavior of the model when the relaxation time tends to zero. It is a simple Riemann problem in the interval $[-1/2, 1/2]$. The two phases are perfect gases with $\gamma_1 = 1.0924$ and $\gamma_2 = 1.0182$. We plot the solution at time $t = 0.0008$. The CFL number is fixed to 0.9. The initial data are

$$\begin{array}{cccc} & (L) & (R) \\ \rho_1 & 76.45430093 & 57.34072568 \\ u_1 & 0 & 0 \\ p_1 & 200 \times 10^5 & 150 \times 10^5 \\ \rho_2 & 836.1239718 & 358.8982226 \\ u_2 & 0 & 0 \\ p_2 & 200 \times 10^5 & 150 \times 10^5 \\ \alpha_1 & 0.25 & 0.25 \end{array} \tag{13}$$

When using a coarse mesh with 50 cells (typical in an industrial context), the solution seems to be smooth. The volume fraction α_1, the velocities and pressures are plotted on Figure 1. Then , we perform the same computation with 1000, 10000 and 100000 cells and plot the void fraction on Figure 2.

We observe that spurious instabilities arise, due to the non-hyperbolic behavior of the equilibrium model. Thus a modeling with a non-vanishing relaxation time is highly recommended. The introduction of the granular stress generally enlarges the stability domain. More numerical experiments with granular stress and industrial applications are detailed in [Nus 07] and [CGH 07].

6. Conclusion

The introduction of granular pressure effects in the relaxed (that is $\tau_P = 0$) Baer Nunziatto model enables to extend the hyperbolicity domain. Nonetheless, it does not guarantee that, for any initial conditions lying in the hyperbolic domain, the solution will remain in the hyperbolic domain when time increases. In practice, this model (corresponding to step (IIa)) enables to compute real cases at least on *coarse meshes*. However, refining the mesh usually leads to large oscillations even in standard cases (see preprint [CGH 07]).

On the other hand, the full seven-equation model corresponding to step (IIb) does not suffer from this drawback, and thus should be prefered. Moreover, many numerical experiments seem to confirm that the comparison of models (IIa) and (IIb) on industrial meshes provides very similar results.

7. References

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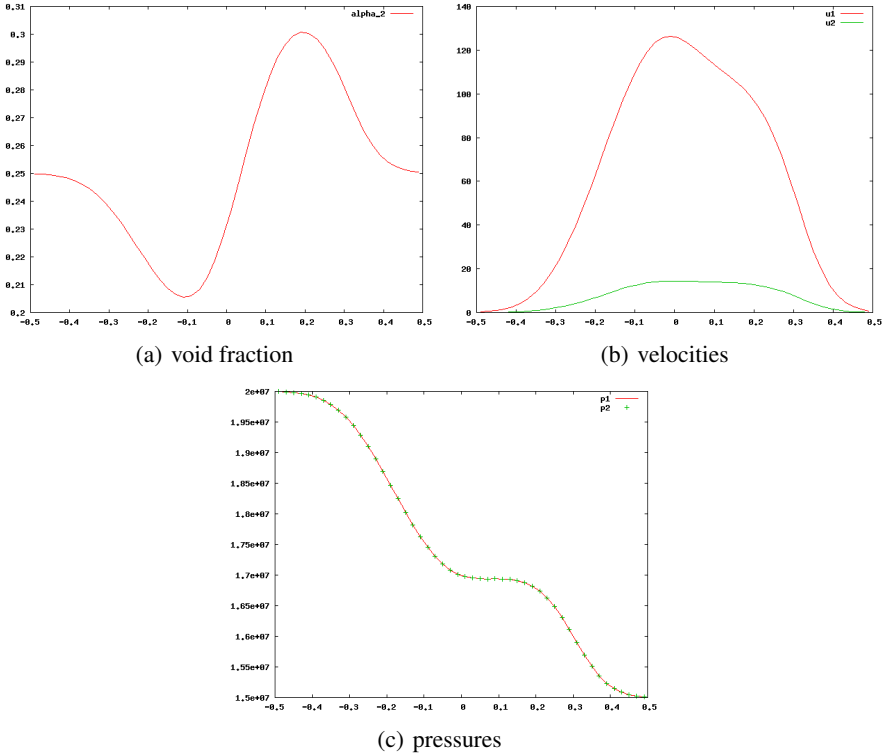
Figure 1. *Riemann problem, 50 cells computation .*



(a) void fraction (b) velocities

(c) pressures

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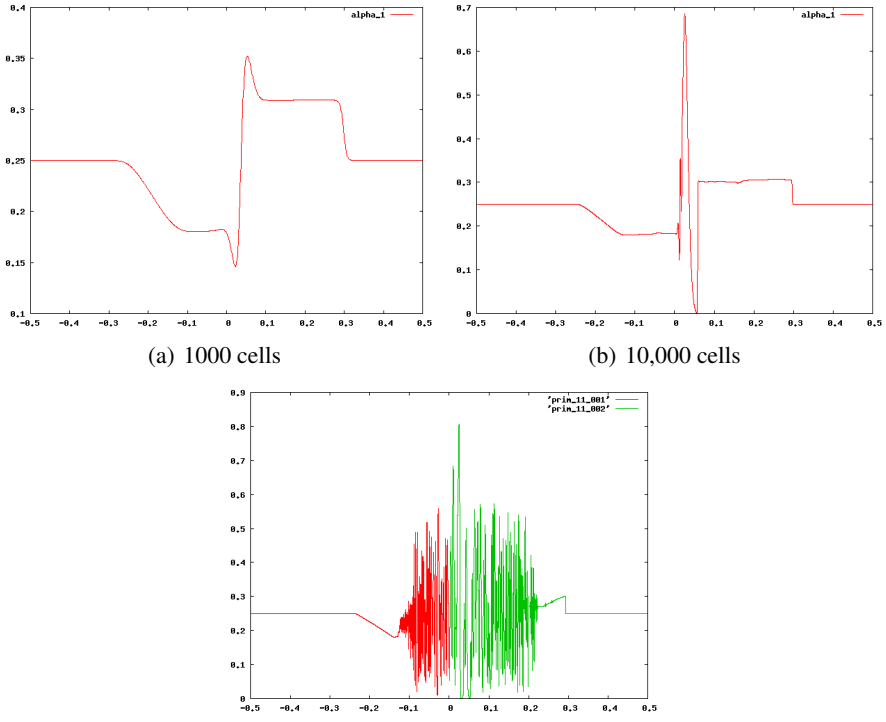
Figure 2. *Riemann problem, 1000 to 100,000 cells computations, void fraction.*



(a) 1000 cells (b) 10,000 cells

(c) 100,000 cells