Computing hyperbolic two-fluid models with a porous interface

Frédéric Archambeau, Laëtitia Girault, Jean-Marc Hérard, Olivier Hurisse

Computing Two-Phase Flows in Porous Media with a Two-Fluid Hyperbolic Model

Frédéric Archambeau — Laëtitia Girault — Jean-Marc Hérard — Olivier Hurisse

EDF-R&D, Department of Fluid Dynamics, Power Generation and Environment
6 Quai Watier, F-78401 Chatou Cedex

jean-marc.herard@edf.fr

ABSTRACT. *The paper examines the suitability of some finite-volume schemes in order to compute two-fluid models in a porous medium. The hyperbolic two-fluid model is governed by an entropy inequality, and admits unique jump conditions. Closure laws for drag effects and heat exchange are in agreement with standard single pressure two-fluid models. Emphasis is put on the behaviour of finite volume schemes when the computational domain contains a sharp porosity variation. Only two among the three schemes examined herein are shown to preserve a basic porous solution, whatever the mesh size is. Other properties including the preservation of the maximum principle for the void fractions are discussed, and the true behaviour of schemes in a test case representative of the propagation of a rarefaction wave in a pipe with sudden contraction is presented. The behaviour of the third scheme is indeed much better.*

KEYWORDS: *porous medium, two-phase flows, finite volumes, non-conservative schemes*

1. Introduction

Many design and safety studies for pressurised water reactors, steam generators and condensers require the use of the porosity concept in control volumes. So-called component codes in France, such as FLICA (CEA) or THYC (EDF) or system codes such as CATHARE (CEA) rely on this approach. Nonetheless, rather than choosing the homogeneous approach, which corresponds to the basic assumption in the former two codes, it seems promising to consider the water and vapour phases as two different entities. In particular, the latter approach seems reasonable for highly unsteady studies. In that case, there is a need for a two-fluid approach in order to distinguish both phases, such as the one described in [HER 07b] that extends the models [BAE 86, KAP 97, GAL 04]. Beyond this, the growing capacities of computers enable huge computations now, and thus we may benefit from the development of coupling techniques, enabling us to perform simulations in the whole coolant circuit. Some

preliminary works have been achieved quite recently (see [AMB 03] for instance) in that direction, but there is still a need for new techniques in the interfacial coupling of free and porous medium (see [HER 06]). The present work describes some first attempts in that framework. The reader is referred to [HER 07b] for further details on the model.

We first introduce the void fraction $\alpha_k \in [0,1]$ in agreement with $\alpha_1 + \alpha_2 = 1$, the porosity $\epsilon \in [0,1]$, and (for $k = 1, 2$) the mean velocity U_k, the mean pressure P_k, the mean density ρ_k, the internal energy $e_k(P_k, \rho_k)$ in phase k. The state variable is:

$$W_\epsilon^t = (\epsilon m_k, \epsilon m_k U_k, \epsilon \alpha_k E_k) \qquad [1]$$

in $\mathbf{R}^6$, while noting $m_k = \alpha_k \rho_k$ the partial mass in phase k, and $E_k = \rho_k U_k U_k/2 + \rho_k e_k(P_k, \rho_k)$ the total energy of phase k. The equation of state (EOS) is provided through the function $e_k(P_k, \rho_k)$, which may be arbitrary. We will thus focus here on the following two-fluid model:

$$\left\{ \begin{array}{l} \dfrac{\partial \epsilon}{\partial t} = 0 \; ; \\ \dfrac{\partial \alpha_2}{\partial t} + V_I \dfrac{\partial \alpha_2}{\partial x} = \phi_2(\epsilon, \alpha_2, W_\epsilon) \; ; \\ \dfrac{\partial \epsilon m_k}{\partial t} + \dfrac{\partial \epsilon m_k U_k}{\partial x} = 0 \; ; \\ \dfrac{\partial \epsilon m_k U_k}{\partial t} + \dfrac{\partial \epsilon m_k U_k^2}{\partial x} + \epsilon \alpha_k \dfrac{\partial P_k}{\partial x} + \epsilon (P_k - P_I) \dfrac{\partial \alpha_k}{\partial x} = \epsilon I_k(\epsilon, \alpha_2, W_\epsilon) \; ; \\ \dfrac{\partial \epsilon \alpha_k E_k}{\partial t} + \dfrac{\partial \epsilon \alpha_k U_k (E_k + P_k)}{\partial x} + \epsilon P_I \dfrac{\partial \alpha_k}{\partial t} = \epsilon V_I I_k(\epsilon, \alpha_2, W_\epsilon) \; . \end{array} \right. \qquad [2]$$

Obviously, we must specify closure laws for the source terms (ϕ_2, I_k), which agree with: $I_1 + I_2 = 0$. We assume here that the latter closures are in agreement with the global entropy inequality, choosing $\phi_2 = \alpha_1 \alpha_2 (P_2 - P_1)/\tau_P$ and $I_k = (-1)^k m_1 m_2 (U_1 - U_2)/\tau_U/(m_1 + m_2)$ (see [COQ 02, GAL 04, HER 07a, HER 07b]). Moreover, the couple (V_I, P_I) is assumed to be one among the two couples (U_k, P_{3-k}), with $k \in 1, 2$. Hence, we are ensured that: (i) system [2] is hyperbolic; (ii) unique jump conditions hold within each single field; (iii) smooth solutions are governed by a physically relevant entropy inequality. Therefore, since the non-conservative products are only active in a linearly degenerate field, we know that the converged approximation (w.r.t. the mesh size) obtained when computing a Riemann problem associated with [2] will not depend on the scheme, as may happen for other choices of V_I (see [GUI 07] for instance).

Since we aim at developing schemes and methods which allow the interfacial coupling of codes based on formulation [2], and its counterpart where $\epsilon = 1$, on the whole computational domain, we will focus in this paper on the definition and the ability of some finite volume schemes ([EYM 00]) to cope with coupled situations including a sudden change in the porosity distribution. The first scheme corresponds

to the classical Rusanov scheme, the second scheme being a slight modification of the latter. The third scheme is quite different. It relies on former propositions by Greenberg and Leroux (see [GRE 96]) revisited by Kroner and Thanh (see [KRO 06], and [BOU 04] too). The latter scheme does not require solving an exact Riemann problem around each cell interface (see [GOD 59]), and thus is much simpler than the original well-balanced scheme.

2. Two basic exact solutions

We define two basic solutions of system [2], whatever the EOS is.

– Basic solution S_1:
Solution S_1 corresponds to the following generalized unsteady contact solution:

$$\left\{ \begin{array}{l} \epsilon(x) = \epsilon_0 \\ P_1(x,t) = P_2(x,t) = P_0 \\ U_1(x,t) = U_2(x,t) = U_0 \end{array} \right. \quad [3]$$

while both ρ_k and α_2 comply with the governing equation $\dfrac{\partial f}{\partial t} + U_0 \dfrac{\partial f}{\partial x} = 0$.

– Basic solution S_2:
Distribution $\epsilon(x)$ is arbitrary. Solution S_2 corresponds to the steady solution:

$$\left\{ \begin{array}{l} P_1(x,t) = P_2(x,t) = P_0 \\ U_1(x,t) = U_2(x,t) = 0 \end{array} \right. \quad [4]$$

while both $m_k(x,t) = m_k(x,0)$ and $\alpha_2(x,t) = \alpha_2(x,0)$.

3. Three distinct finite volume schemes

We introduce standard notations for finite volume schemes. Within each finite volume of size $h_i = x_{i+1/2} - x_{i-1/2}$, the mean value of W at time t^n in cell i is:

$$W_i^n = (\int_{[x_{i-1/2}, x_{i+1/2}]} W(x,t^n)dx)/h_i \quad [5]$$

The time step Δt^n will comply with a standard CFL condition. Moreover, we define: $\overline{a}_{i+1/2} = (a_i + a_{i+1})/2$, whatever a is. We define the flux f_ϵ in $\mathbf{R}^6$:

$$f_\epsilon(W_\epsilon, \alpha_2, \epsilon)^t = (\epsilon m_k U_k, \epsilon m_k U_k^2, \epsilon \alpha_k U_k (E_k + P_k)) \quad [6]$$

The computation of the whole set [2] is achieved with a fractional step method which is in agreement with the overall entropy inequality. The homogeneous problem associated with the left-hand side of [2] is computed first. Source terms are then accounted for using an implicit scheme, which is exactly the one described in [GAL 04]. We thus only describe the first evolution step here.

3.1. *Classical Rusanov scheme* LF

The cell scheme which is used to compute the evolution step simply reads:

$$\begin{array}{ll} h_i((\alpha_2)_i^{n+1} - (\alpha_2)_i^n) & +\Delta t^n (V_I)_i^n ((\overline{\alpha}_2)_{i+1/2}^n - (\overline{\alpha}_2)_{i-1/2}^n) \\ & +\Delta t^n (c_{i+1/2}^n - c_{i-1/2}^n) = 0 \end{array} \quad [7]$$

where $c_{i+1/2}^n = -r_{i+1/2}^n ((\alpha_2)_{i+1}^n - (\alpha_2)_i^n)/2$, while:

$$\begin{array}{ll} h_i((W_\epsilon)_i^{n+1} - (W_\epsilon)_i^n) & +\Delta t^n (F_{i+1/2}^{LF}((W_\epsilon)_l^n, (\alpha_2)_l^n, \epsilon_l) - F_{i-1/2}^{LF}((W_\epsilon)_l^n, (\alpha_2)_l^n, \epsilon_l)) \\ & +\Delta t^n (NCT_\epsilon)_i^n = 0 \end{array} \quad [8]$$

where the numerical flux is defined by:

$$\begin{array}{ll} F_{i+1/2}^{LF}((W_\epsilon)_l^n, (\alpha_2)_l^n, \epsilon_l) = & (f_\epsilon((W_\epsilon)_i^n, (\alpha_2)_i^n, \epsilon_i) + f_\epsilon((W_\epsilon)_{i+1}^n, (\alpha_2)_{i+1}^n, \epsilon_{i+1}) \\ & -r_{i+1/2}^n ((W_\epsilon)_{i+1}^n - (W_\epsilon)_i^n))/2 \end{array} \quad [9]$$

The scalar $r_{i+1/2}^n$ represents the maximal value of the spectral radius of the Jacobian matrices $A((W_\epsilon)_l^n, (\alpha_2)_l^n, \epsilon_l)$ for $l = i, i+1$. The contribution connected with the first-order non-conservative terms $(NCT_\epsilon)_i^n$ is approximated by:

$$(NCT_\epsilon)_i^n = \left\{ \begin{array}{ll} 0 & \\ \epsilon_i & (((P_k)_i^n - (P_I)_i^n)((\overline{\alpha}_k)_{i+1/2}^n - (\overline{\alpha}_k)_{i-1/2}^n) \\ & +(\alpha_k)_i^n ((\overline{P}_k)_{i+1/2}^n - (\overline{P}_k)_{i-1/2}^n)) \\ -\epsilon_i & (P_I)_i^n (V_I)_i^n ((\overline{\alpha}_k)_{i+1/2}^n - (\overline{\alpha}_k)_{i-1/2}^n) \end{array} \right. \quad [10]$$

3.2. *A modified Rusanov scheme* MLF

This scheme is similar to the previous one. The update for the void fractions is:

$$\begin{array}{ll} h_i((\alpha_2)_i^{n+1} - (\alpha_2)_i^n) & +\Delta t^n (V_I)_i^n ((\overline{\alpha}_2)_{i+1/2}^n - (\overline{\alpha}_2)_{i-1/2}^n) \\ & +\Delta t^n (d_{i+1/2,-}^n - d_{i-1/2,+}^n) = 0 \end{array} \quad [11]$$

where:

$$\begin{array}{l} d_{i+1/2,-}^n = -\frac{(\hat{\epsilon})_{i+1/2}}{2\epsilon_i} r_{i+1/2}^n ((\alpha_2)_{i+1}^n - (\alpha_2)_i^n) \\ d_{i-1/2,+}^n = -\frac{(\hat{\epsilon})_{i-1/2}}{2\epsilon_i} r_{i-1/2}^n ((\alpha_2)_i^n - (\alpha_2)_{i-1}^n) \end{array} \quad [12]$$

The numerical flux in [8] is replaced by:

$$\begin{array}{ll} F_{i+1/2}^{MLF}((W_\epsilon)_l^n, (\alpha_2)_l^n, \epsilon_l) = & (f_\epsilon((W_\epsilon)_i^n, (\alpha_2)_i^n, \epsilon_i) + f_\epsilon((W_\epsilon)_{i+1}^n, (\alpha_2)_{i+1}^n, \epsilon_{i+1}) \\ & -r_{i+1/2}^n \quad (\hat{\epsilon})_{i+1/2} \quad (\frac{(W_\epsilon)_{i+1}^n}{\epsilon_{i+1}} - \frac{(W_\epsilon)_i^n}{\epsilon_i}))/2 \end{array} \quad [13]$$

where $(\hat{\epsilon})_{i+1/2} = (\overline{\epsilon})_{i+1/2}$ or : $(\hat{\epsilon})_{i+1/2} = (2\epsilon_i \epsilon_{i+1})/(\epsilon_i + \epsilon_{i+1})$.

3.3. *A simplified well-balanced scheme* $SWBLF$

The basic idea is the following. First, the update for α_2 is still achieved following [7]. Then, we introduce $W \in \mathbf{R}^6$ and $f(W, \alpha_2) \in \mathbf{R}^6$:

$$\begin{array}{l} W^t = (m_k, m_k U_k, m_k E_k) \\ f(W, \alpha_2)^t = (m_k U_k, m_k U_k^2 + \alpha_k P_k, \alpha_k U_k (E_k + P_k)) \end{array} \quad [14]$$

ϵ is assumed to be constant within each cell, the cell scheme now reads:

$$\begin{array}{ll} h_i(W_i^{n+1} - W_i^n) & +\Delta t^n (F_{i+1/2,-}^{SWBLF}(W_l^n, (\alpha_2)_l^n, \epsilon_l) - F_{i-1/2,+}^{SWBLF}(W_l^n, (\alpha_2)_l^n, \epsilon_l)) \\ & +\Delta t^n (NCT)_i^n = 0 \end{array} \quad [15]$$

where the numerical fluxes and the contribution NCT are defined by:

$$\begin{array}{ll} F_{i+1/2,-}^{SWBLF}(W_l^n, (\alpha_2)_l^n, \epsilon_l) = & (f((W)_i^n, (\alpha_2)_i^n) + f((W_{i+1/2,-}^n, (\alpha_2)_{i+1/2,-}^n) \\ & -r_{i+1/2}^n (W_{i+1/2,-}^n - W_i^n)))/2 \\ F_{i-1/2,+}^{SWBLF}(W_l^n, (\alpha_2)_l^n, \epsilon_l) = & (f((W)_i^n, (\alpha_2)_i^n) + f((W_{i-1/2,+}^n, (\alpha_2)_{i-1/2,+}^n) \\ & -r_{i-1/2}^n (W_i^n - W_{i-1/2,+}^n)))/2 \end{array} \quad [16]$$

$$NCT_i^n = -(0, (P_I)_i^n ((\overline{\alpha}_k)_{i+1/2}^n - (\overline{\alpha}_k)_{i-1/2}^n), (P_I)_i^n (V_I)_i^n ((\overline{\alpha}_k)_{i+1/2}^n - (\overline{\alpha}_k)_{i-1/2}^n)) \quad [17]$$

The values $W_{i-1/2,+}^n$ and $W_{i+1/2,-}^n$ are obtained by solving the non-linear equations:

$$\begin{array}{l} I_m^0(W_{i-1/2,+}^n, (\alpha_2)_{i-1/2,+}^n, \epsilon_i) = I_m^0(W_{i-1}^n, (\alpha_2)_{i-1}^n, \epsilon_{i-1}) \\ I_m^0(W_{i+1/2,-}^n, (\alpha_2)_{i+1/2,-}^n, \epsilon_i) = I_m^0(W_{i+1}^n, (\alpha_2)_{i+1}^n, \epsilon_{i+1}) \end{array} \quad [18]$$

where $I_0^0(W, \alpha_2, \epsilon) = \alpha_2$, $I_{3k-2}^0(W, \alpha_2, \epsilon) = \epsilon m_k U_k$, $I_{3k-1}^0(W, \alpha_2, \epsilon) = s_k(P_k, \rho_k)$, $I_{3k}^0(W, \alpha_2, \epsilon) = e_k(P_k, \rho_k) + P_k/\rho_k + U_k^2/2$ for $k = 1, 2$, where s_k denotes the specific entropy for phase k (see [HER 07b]). In practice, this requires solving two uncoupled non-linear scalar equations at each cell interface $i + 1/2$, the solution of which is trivial when $\epsilon_i = \epsilon_{i+1}$, or when $(U_k)_i^n (U_k)_{i+1}^n = 0$.

3.4. *Properties*

– *Property 1:* We will now assume that the equation of state takes the form: $\rho_k e_k(P_k, \rho_k) = a_{k,0}\rho_k + g_k(P_k)$ in each phase k. The three schemes LF, MLF and $SWBLF$ described above preserve the discrete form of the basic solution S_1, whatever the mesh size is, since $(U_1)_i^n = (U_2)_i^n = U_0$ and $(P_1)_i^n = (P_2)_i^n = P_0$ imply that $(U_1)_i^{n+1} = (U_2)_i^{n+1} = U_0$ and $(P_1)_i^{n+1} = (P_2)_i^{n+1} = P_0$, if $\epsilon_i = \epsilon_0$.

– *Property 2:* We will still assume that the equation of state takes the form: $\rho_k e_k(P_k, \rho_k) = a_{k,0}\rho_k + g_k(P_k)$ in each phase k. Both schemes MLF and $SWBLF$ preserve the discrete form of the basic solution S_2 on any mesh, since

$(U_1)_i^n = (U_2)_i^n = 0$ and $(P_1)_i^n = (P_2)_i^n = P_0$ imply that $(U_1)_i^{n+1} = (U_2)_i^{n+1} = 0$ and also $(P_1)_i^{n+1} = (P_2)_i^{n+1} = P_0$, with any ϵ_i. The standard LF scheme does not.

– *Property 3:* The maximum principle for void fractions holds, and positive partial masses are ensured when applying any scheme among LF, MLF and $SWBLF$, provided that a standard CFL-like condition holds.

Note: We must emphasize that the specific EOS which maintain S_1, S_2 on any mesh size include perfect gas EOS and also stiffened gas EOS. The reader is referred to [GAL 02] which examines similar problems for single-phase flows.

4. Behaviour of schemes in specific test-cases

The first test case corresponds to a rough representation of a loss of coolant accident, focusing on the propagation of the rarefaction wave that hits the free/porous interface separating the pipes and the steam generator. The computational domain includes a free region ($\epsilon = 1$) on the left side of an interface located at $x = 0.35$, and a porous region (steam generator, $\epsilon = 0.6$) on the right side of the latter interface. The initial void fraction profile is $\alpha_2(x < 0.3, 0) = 0.05$ and $\alpha_2(x > 0.3, 0) = 0.95$. Other initial conditions are chosen as follows: $P_1(x, 0) = P_2(x, 0) = P_L$ for $x < 0.3$, $P_1(x, 0) = P_2(x, 0) = P_R$ for $x > 0.3$, and $U_1(x, 0) = U_2(x, 0) = 0$, where $P_L = 1.10^5$ and $P_R = 20.10^5$. Figures 1 and 2 show the behaviour of the Riemann invariants I_3^0, I_6^0, s_1 and s_2 when the rarefaction has passed the interface. The mesh contains 1000 regular cells, and the CFL number is $1/2$. Subscript 2 refers to water. Results are clearly in favour of $SWBLF$. The second test case corresponds to a classical one-dimensional Riemann problem, where the left and right states of the porosity are $\epsilon_L = 1$, and $\epsilon_R = 0.6$ respectively. Figure 3 provides the measure of the L^1 norm of the error when focusing on the SWBLF scheme. The EOS for the vapour and the liquid are perfect gas EOS ($\gamma_1 = 1.1$ and $\gamma_2 = 1.4$). The solution contains two contact waves associated with $\lambda = 0$ and $\lambda = U_2$, and one shock wave in the vapour phase corresponding to $\lambda = U_2 + c_2$. Subscript 2 now refers to the gas phase, and $(V_I, P_I) = (U_2, P_1)$. The coarser and finer meshes contain 10^2 and 2.10^5 cells.

5. References

[AMB 03] AMBROSO A., CHALONS C. , COQUEL F., GODLEWSKI E. , LAGOUTIÈRE F., RAVIART P. A., SEGUIN N., Working group on the interfacial coupling of models, http://www.ann-jussieu.fr/groupes/cea, 2003.

[BAE 86] BAER M.R., NUNZIATO J.W., "A two-phase mixture theory for the deflagration to detonation transition (DDT) in reactive granular materials", *Int. J. Multiphase Flow*, vol. 12, n°6, 1986, pp. 861–889.

[BOU 04] BOUCHUT F., *"Nonlinear stability of finite volume methods for hyperbolic conservation laws, and well-balanced schemes for sources"*, Frontiers in Mathematics series, Birkhauser, 2004.

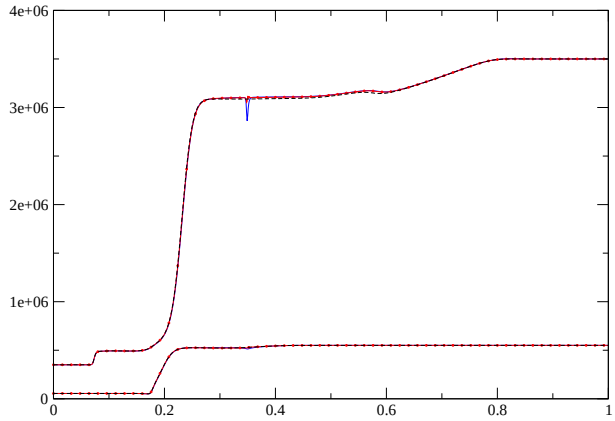

Figure 1. *Riemann invariants I_3^0 (top) and I_6^0 (bottom). LF scheme (blue line), MLF scheme (red line with circles), SWBLF scheme (dark dashed line).* $\gamma_1 = 1.4, \gamma_2 = 1.1$

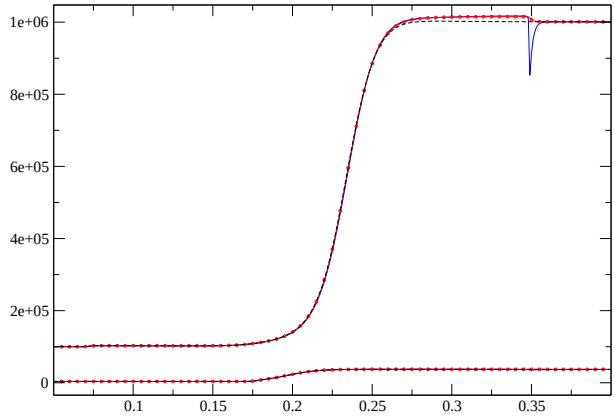

Figure 2. *Specific entropies s_1 (bottom) and s_2 (top). LF scheme (blue line), MLF scheme (red line with circles), SWBLF scheme (dark dashed line). Zoom on* $[0, 0.4]$

[COQ 02] COQUEL F., GALLOUËT T., HÉRARD J.M., SEGUIN N., "Closure laws for a two-fluid two-pressure model", *C. R. Acad. Sci. Paris*, vol. I-332, 2002, pp. 927–932.

[EYM 00] EYMARD R., GALLOUËT T., HERBIN R., "*Finite Volume Methods*", P.G. Ciarlet and P.L. Lions editors, North Holland, 2000.

[GAL 02] GALLOUËT T., HÉRARD J.-M., SEGUIN N., "A hybrid scheme to compute contact discontinuities in one dimensional Euler systems", *Math. Model. and Numer. Anal.*, vol. 36, 2002, pp. 1133-1159.

[GAL 04] GALLOUËT T., HÉRARD J.-M., SEGUIN N., "Numerical modelling of two phase

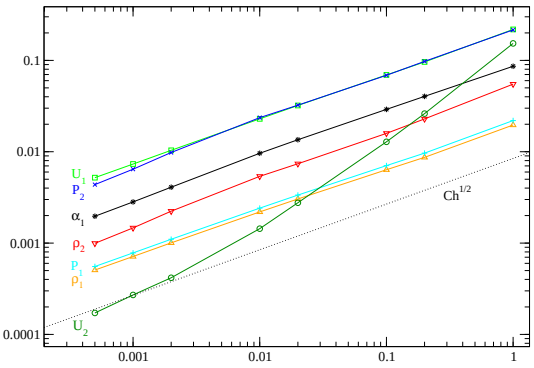

Figure 3. *SWBLF: L^1 norm of the error for variables ρ_k, U_k, P_k and α_2.*

flows using the two-fluid two-pressure approach", *Math. Mod. Meth. in Appl. Sci.*, vol. 14, n°5, 2004, pp. 663-700.

[GOD 59] GODUNOV S.K., "Finite difference method for numerical computation of discontinuous solutions of the equations of fluid dynamics", *Mat. Sb.*, vol. 47, 1959, pp. 271-300.

[GRE 96] GREENBERG J.M., LEROUX A.Y., "A well balanced scheme for the numerical processing of source terms in hyperbolic equations", *SIAM J. Num. Anal.*, vol. 33, n°1, 1996, pp. 1–16.

[GUI 07] GUILLEMAUD V., "Modélisation et simulation numérique des écoulements diphasiques par une approche bifluide à deux pressions", PhD thesis, Université Aix Marseille I, 27th March, 2007.

[HER 06] HÉRARD J.M. , "A rough scheme to couple free and porous media", *Int. J. Finite Volumes (electronic)*, http://www.latp.univ-mrs.fr/IJFV/, vol. 3, n°2, 2006, pp. 1-28.

[HER 07a] HÉRARD J.-M., "A three-phase flow model", *Mathematical Computer Modelling*, vol. 45, 2007, pp. 432-455.

[HER 07b] HÉRARD J.-M., "Un modèle hyperbolique diphasique bi-fluide en milieu poreux", *EDF report H-I81-2007-02622-FR, unpublished*, 2007.

[KAP 97] KAPILA A.K., SON S.F., BDZIL J.B., MENIKOFF R., STEWART D.S., "Two phase modeling of a DDT: structure of the velocity relaxation zone", *Phys. of Fluids*, vol. 9, n°12, 1997, pp. 3885–3897.

[KRO 06] KRONER D. , THANH M.D., "Numerical solution to compressible flows in a nozzle with variable cross-section", *SIAM J. Numer. Anal.* , vol. 43, n°2, 2006, pp. 796–824.

Acknowledgments:
This work has been achieved in the framework of the NEPTUNE project, with financial support from CEA (Commissariat à l'Énergie Atomique), EDF (Électricité de France), IRSN (Institut de Radioprotection et de Sûreté Nucléaire) and AREVA-NP. Part of the financial support of the second author is provided by ANRT (Association Nationale de la Recherche Technique, Ministère chargé de la Recherche) through a EDF/CIFRE contract, and also by FSE (Fonds Social Européen).