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A meta-model to predict the phytoavailability of cadmium in French agricultural soils

André Schneider^{1,2,£}, Zhongbing Lin^{1,2}, Thibault Sterckeman^{3,4}, Christophe Nguyen^{1,2}

- 1: INRA, Biogeochemistry of Trace Elements, UMR 1391 ISPA, 71, avenue Edouard Bourlaux, F-33140 Villenave d'Ornon, France
- 2 : Bordeaux Sciences Agro, UMR 1391 ISPA, F-33170 Gradignan, France
- 3 : INRA, Laboratoire Sols et Environnement, UMR 1120, 2, avenue de la Forêt de Haye, TSA 40602, F54518 Vandoeuvre-lès-Nancy, France
- 4 : Université de Lorraine, Laboratoire Sols et Environnement, UMR 1120, 2, avenue de la Forêt de Haye, TSA 40602, F54518 Vandoeuvre-lès-Nancy, France
- £: andre.schneider@bordeaux.inra.fr

Introduction

To predict the uptake of nutrients by plant, Barber's mechanistic model simulates the behavior of only one chemical species in the soil-plant system. For metals the behavior of three species (M: free ionic M²⁺, ML: complex of M, L: ligand, assumed unique) must be described (see Lin et al. in this session). The understanding of the results of the partial derivative equations (PDE) is difficult. Their numerical integration costs a significant time that can penalize the coupling with a more general model of transfer of metal from the soil to the crop. Steady state analytical solutions (AS) for both models were therefore developed. The outputs of PDE and AS were compared for Cd in agricultural French soils*.

Theory

One species model

The Barber's model was modified to take into account the transport by diffusion and convection, the kinetics of sorption, and the Michaelis-Menten root uptake of Cd²⁺. The PDE were as follows.

$$\begin{cases} \theta \frac{\partial C_{\mathrm{M}}}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(r f \theta D_{\mathrm{M}} \frac{\partial C_{\mathrm{M}}}{\partial r} + r_{0} v_{0} C_{\mathrm{M}} \right) + \left(k_{\mathrm{des}}^{\mathrm{M}} S_{\mathrm{M}} - \theta k_{\mathrm{ads}}^{\mathrm{M}} C_{\mathrm{M}} \right) \\ \frac{\partial S_{\mathrm{M}}}{\partial t} = \theta k_{\mathrm{ads}}^{\mathrm{M}} C_{\mathrm{M}} - k_{\mathrm{des}}^{\mathrm{M}} S_{\mathrm{M}} \end{cases}$$

Three species model

PDE were written for M, ML, and L, including the kinetics of complexation in solution:

$$\begin{cases} \theta \frac{\partial C_{\mathrm{M}}}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(r f \theta D_{\mathrm{M}} \frac{\partial C_{\mathrm{M}}}{\partial r} + r_{0} v_{0} C_{\mathrm{M}} \right) + \theta \left(k_{\mathrm{d}} C_{\mathrm{ML}} - k_{\mathrm{a}}^{\mathrm{cond}} C_{\mathrm{M}} C_{\mathrm{L}} \right) + \left(k_{\mathrm{des}}^{\mathrm{M}} S_{\mathrm{M}} - \theta k_{\mathrm{ads}}^{\mathrm{M}} C_{\mathrm{M}} \right) \\ \theta \frac{\partial C_{\mathrm{L}}}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(r f \theta D_{\mathrm{L}} \frac{\partial C_{\mathrm{L}}}{\partial r} + r_{0} v_{0} C_{\mathrm{L}} \right) + \theta \left(k_{\mathrm{d}} C_{\mathrm{ML}} - k_{\mathrm{a}}^{\mathrm{cond}} C_{\mathrm{M}} C_{\mathrm{L}} \right) + \left(k_{\mathrm{des}}^{\mathrm{L}} S_{\mathrm{L}} - \theta k_{\mathrm{ads}}^{\mathrm{L}} C_{\mathrm{L}} \right) \\ \theta \frac{\partial C_{\mathrm{ML}}}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(r f \theta D_{\mathrm{ML}} \frac{\partial C_{\mathrm{ML}}}{\partial r} + r_{0} v_{0} C_{\mathrm{ML}} \right) + \theta \left(k_{\mathrm{a}}^{\mathrm{cond}} C_{\mathrm{M}} C_{\mathrm{L}} - k_{\mathrm{d}} C_{\mathrm{ML}} \right) + \left(k_{\mathrm{des}}^{\mathrm{ML}} S_{\mathrm{ML}} - \theta k_{\mathrm{ads}}^{\mathrm{ML}} C_{\mathrm{ML}} \right) \\ \frac{\partial S_{\mathrm{M}}}{\partial t} = \theta k_{\mathrm{ads}}^{\mathrm{M}} C_{\mathrm{M}} - k_{\mathrm{des}}^{\mathrm{M}} S_{\mathrm{M}} \\ \frac{\partial S_{\mathrm{L}}}{\partial t} = \theta k_{\mathrm{ads}}^{\mathrm{ML}} C_{\mathrm{L}} - k_{\mathrm{des}}^{\mathrm{ML}} S_{\mathrm{ML}} \\ \frac{\partial S_{\mathrm{ML}}}{\partial t} = \theta k_{\mathrm{ads}}^{\mathrm{ML}} C_{\mathrm{ML}} - k_{\mathrm{des}}^{\mathrm{ML}} S_{\mathrm{ML}} \end{cases}$$

Analytical solutions

They rely on the following assumptions: constant free ligand concentration

> steady state conditions

 \triangleright no inter-root competition ($\delta < r_1$)

cylindrical geometry

The diffusion layer thickness for a totally inert (δ_{IN}) or a totally labile (δ_{LAB}) complex is deduced from a simpler model (infinite sink, planar geometry, no convection). It increases with time:

$$\delta_{\mathrm{IN}} \approx 0.44 \sqrt{\frac{4 f \theta D_{\mathrm{M}} t}{b_{\mathrm{M}}} + \frac{f D_{\mathrm{M}}}{k_{\mathrm{ads,M}}} \left(1 - \frac{\theta}{b_{\mathrm{M}}} \right)} \qquad \delta_{\mathrm{LAB}} = \delta_{\mathrm{IN}} \sqrt{\frac{1 + \varepsilon K^{*}}{1 + \pi K^{*}}} \qquad t > \frac{1}{4 k_{\mathrm{des,M}}}$$

The analytical solutions of the uptake (U) for both cases are Best's equations (van Leeuwen, 1999) differing only by the values of their *a*- and *b*-parameters:

$$U = \frac{(1+a+b)}{2b} \left(1 - \left(1 - \frac{4b}{(1+a+b)^2} \right)^{1/2} \right) I_{\text{max}} t$$

One species model:

$$a_{\rm IN} = \frac{K_{\rm m}}{c_{\rm M}^*} \left(\frac{r_0}{r_0 + \delta_{\rm IN}}\right)^{Pe}$$

$$a_{\text{IN}} = \frac{K_{\text{m}}}{c_{\text{M}}^*} \left(\frac{r_0}{r_0 + \delta_{\text{IN}}} \right)^{Pe}$$

$$b_{\text{IN}} = \frac{I_{\text{max}}}{v_0 c_{\text{M}}^*} \left(1 - \left(\frac{r_0}{r_0 + \delta_{\text{IN}}} \right)^{Pe} \right)$$

$$Pe = \frac{v_0 r_0}{f \theta D_{\text{M}}}$$

Totally labile complex:

$$a_{\text{LAB}} = \frac{K_{\text{m}}}{c_{\text{M}}^*} \left(\frac{r_0}{r_0 + \delta_{\text{LAB}}}\right)^{\frac{1+K}{1+\varepsilon K^*}Pe} \qquad b_{\text{LAB}} = \frac{I_{\text{max}}}{v_0 (1+K^*)c_{\text{M}}^*} \left(1 - \left(\frac{r_0}{r_0 + \delta_{\text{LAB}}}\right)^{\frac{1+K}{1+\varepsilon K^*}Pe}\right)$$

Three species model

$$\begin{cases} \mu \leq \delta_{\text{IN}} \Rightarrow \mu = \sqrt{\frac{fD_{\text{M}}}{k_{\text{a}}^*}} \\ \mu > \delta_{\text{IN}} \Rightarrow \mu = \delta_{\text{IN}} \\ U = (U_{\text{IN}} - U_{\text{LAB}}) \frac{\mu}{\delta_{\text{IN}}} + U_{\text{LAB}} \end{cases}$$
 μ : reaction layer thickness

Results

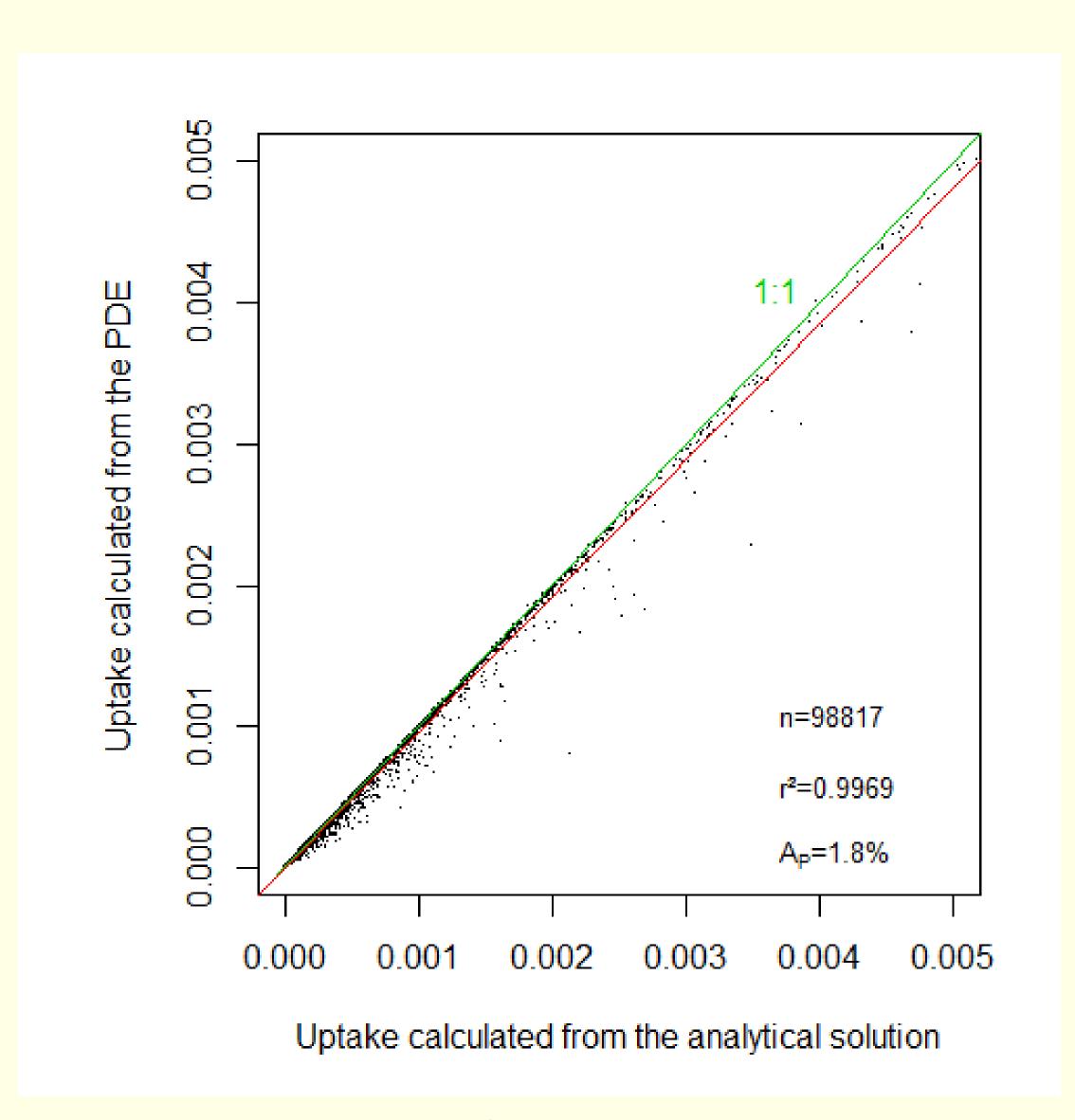


Fig. 1. One species model. Comparison between PDE and AS outputs (µmol cm⁻² root during one month).

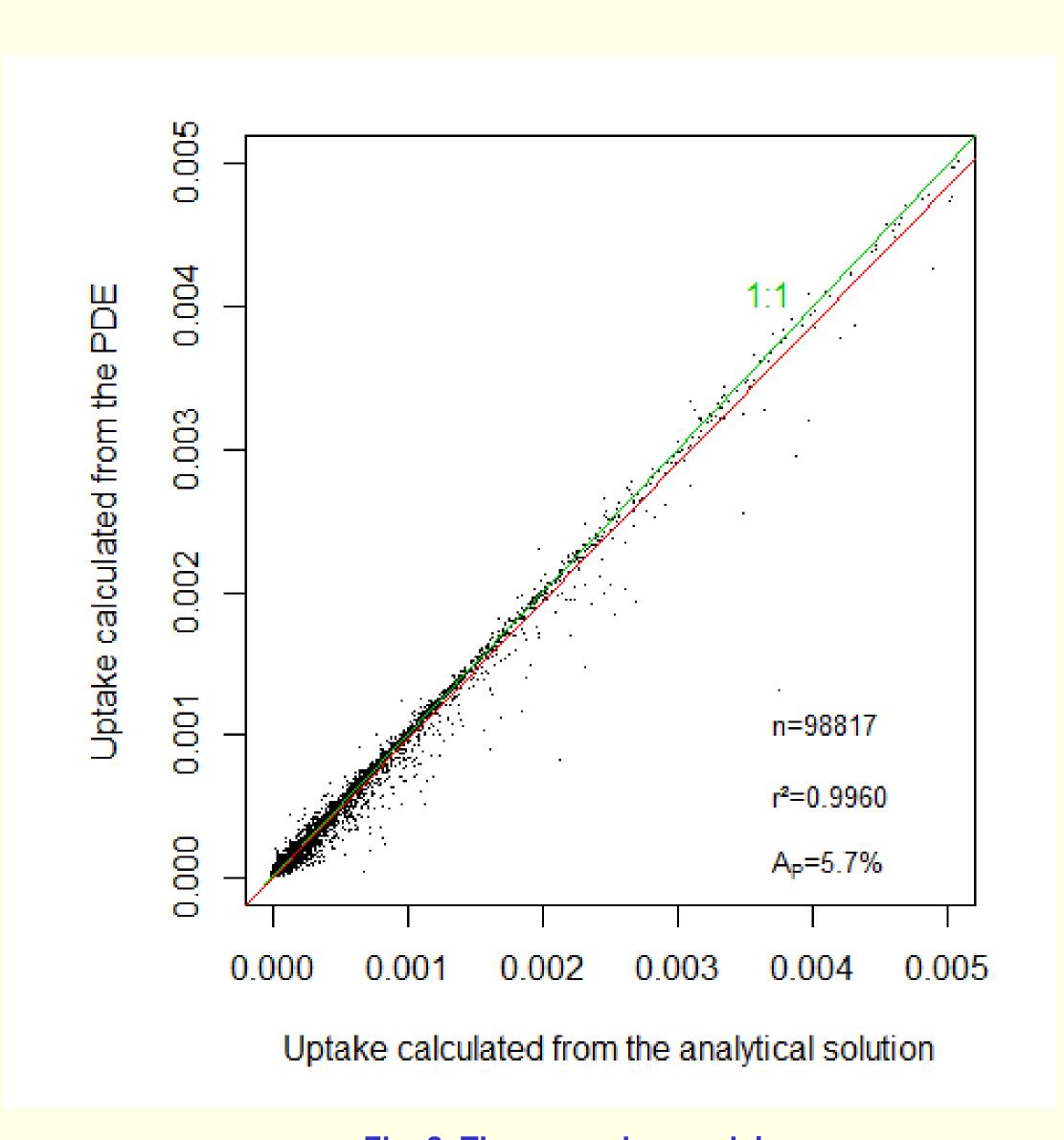


Fig. 2. Three species model. Comparison between PDE and AS outputs (µmol cm⁻² root during one month).

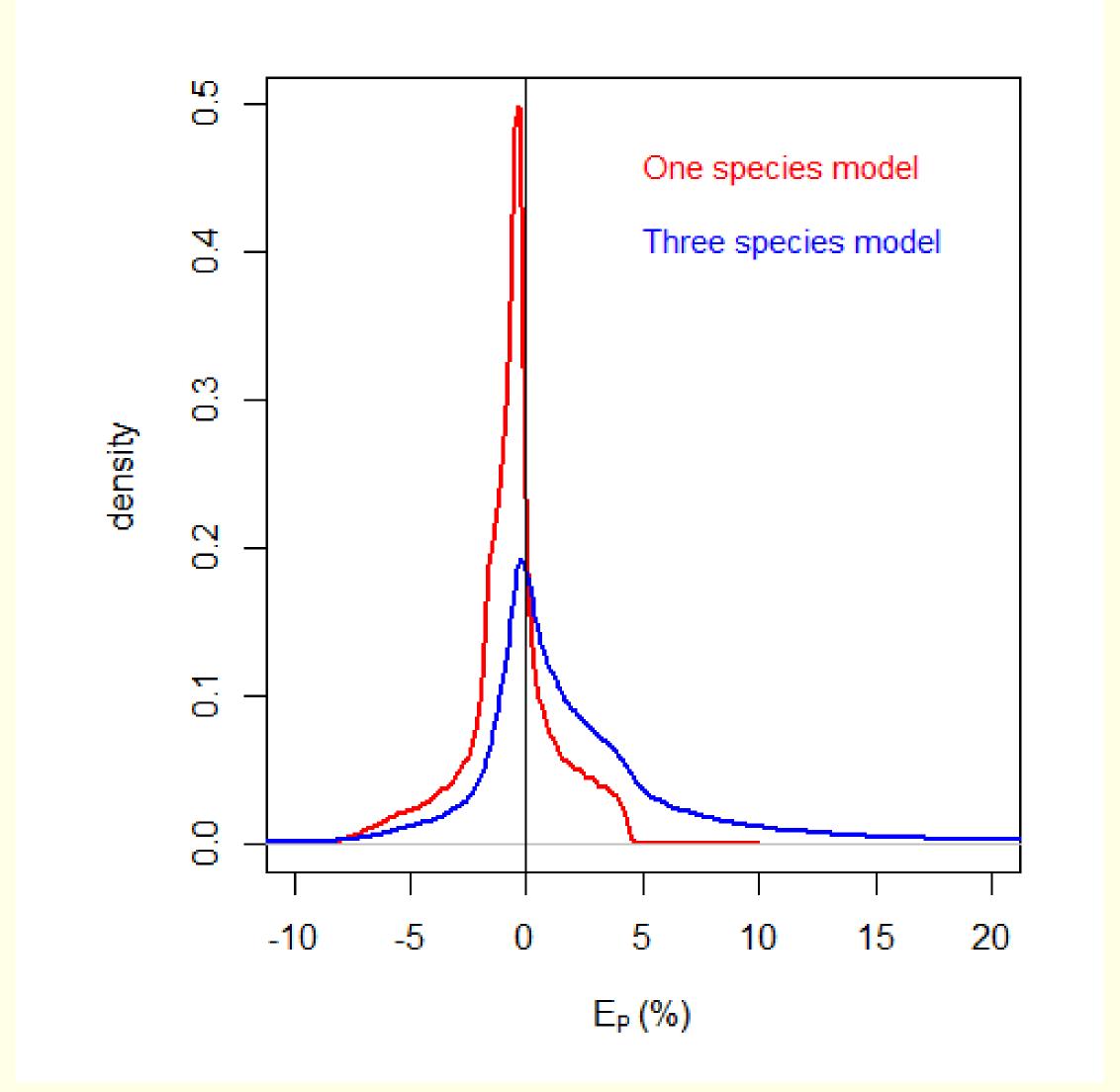


Fig. 3. Kernel density function of the relative error of prediction (E_P) for one species model and three species model.

Discussion and conclusion

The proposed analytical solutions constitute relatively efficient surrogates to the PDE models, even in the complex present in solution is only partial. In this situation, the mathematical treatment stresses the importance of the reaction layer thickness (μ) as a crucial parameter governing the dynamics of the complex in the soil-plant system. The reaction layer is the zone close to the root where the too slow kinetics of the complexation process limits the replenishment of the metal in solution, and thus its flux toward the sink. If, during the time needed for the complex does not dissociate, it behaves as inert. Thus, for the complex to contribute to the uptake of the metal, μ must be smaller than the diffusion layer thickness for a totally-inert complex (δ_{IN}).

References

van Leeuwen, H.P. Metal speciation dynamics and bioavailability: inert and labile complexes. *Environ. Sci. Technol.* **1999**, *33*, 3743–3748



